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Surname		Other names	
Pearson Edexcel nternational Advanced Level	Centre Number		Candidate Number
CORE IVIAT Advanced			C34 Paper Reference
COFE IVIAL Advanced Tuesday 19 January 2016 – Time: 2 hours 30 minutes	Morning		C34 Paper Reference WMA02/01

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 125.
- The marks for **each** question are shown in brackets – use this as a guide as to how much time to spend on each question.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.





Turn over 🕨



WMA02

This resource was created and owned by Pearson Edexcel Past Paper Leave blank $f(x) = (3 - 2x)^{-4}, \quad |x| < \frac{3}{2}$ 1. Find the binomial expansion of f(x), in ascending powers of x, up to and including the term in x^2 , giving each coefficient as a simplified fraction. (4) 2 P 4 6 9 5 8 A 0 2 4 4

Question Number	Scheme	Marks
1	$(3-2x)^{-4} = 3^{-4} \left(1 - \frac{2}{3}x\right)^{-4} \qquad 3^{-4} \operatorname{or} \frac{1}{81}$	B1
	$=\frac{1}{81}\times\left(1+(-4)\left(-\frac{2}{3}x\right)+\frac{(-4)(-5)}{2}\left(-\frac{2}{3}x\right)^{2}+\right)$	<u>M1A1</u>
	$= \frac{1}{81} + \frac{8}{243}x + \frac{40}{729}x^2 + \dots$	A1 (4 marks)
	Alternative: $(3-2x)^{-4} = 3^{-4} + (-4)(3)^{-5}(-2x) + \frac{(-4)(-5)}{2}(3)^{-6}(-2x)^2 + \dots$	B1 M1 A1
	$= \frac{1}{81} + \frac{8}{243}x + \frac{40}{729}x^2 + \dots$	A1
		(4 marks)

- B1 For taking out a factor of 3^{-4} Evidence would be seeing either 3^{-4} or $\frac{1}{81}$ before the bracket.
- M1 For the form of the binomial expansion with n = -4 and a term of (kx)To score M1 it is sufficient to see just the second and third term with the correct coefficient multiplied by the correct power of x. Condone sign slips. Look for $\dots + (-4)(kx) + \frac{(-4)(-5)}{2!}(kx)^2 \dots$
- A1 Any (unsimplified) form of the binomial expansion. Ignore the factor before the bracket.

The bracketing must be correct but it is acceptable for them to recover from "missing" brackets for full marks.

Look for
$$1 + (-4)\left(-\frac{2}{3}x\right) + \frac{(-4)(-5)}{2}\left(-\frac{2}{3}x\right)^2 + \text{ or } 1 + \frac{8}{3}x + \frac{40}{9}x^2 + A1$$

A1 $cao = \frac{1}{81} + \frac{8}{243}x + \frac{40}{729}x^2 + \dots$ Ignore any further terms.

Alternative

B1 For seeing either 3^{-4} or $\frac{1}{81}$ as the first term

M1 It is sufficient to see the second and third term (unsimplified or simplified) condoning missing brackets.

ie. Look for ... + $(-4)(3)^{-5}(kx) + \frac{(-4)(-5)}{2}(3)^{-6}(kx)^2$

- A1 Any (un simplified) form of the binomial expansion. $\dots + (-4)(3)^{-5}(-2x) + \frac{(-4)(-5)}{2}(3)^{-6}(-2x)^2$
- A1 Must now be simplified cao

WMA02

www.mystudybro.com This resource was created and owned by Pearson Edexcel Past Paper Leave blank 2. (a) Show that $\cot^2 x - \csc x - 11 = 0$ may be expressed in the form $\csc^2 x - \csc x + k = 0$, where k is a constant. (1) (b) Hence solve for $0 \le x < 360^{\circ}$ $\cot^2 x - \csc x - 11 = 0$ Give each solution in degrees to one decimal place. (Solutions based entirely on graphical or numerical methods are not acceptable.) (5) 4

Que Nur	estion mber	Scheme	Marks
2	(a)	$\csc^2 x - \csc x - 12 = 0$ or $k = -12$	B1
(b)	$\csc^2 x - \csc x - 12 = 0$ so $(\csc x - 4)(\csc x + 3) = 0 \Rightarrow \csc x =$	M1
		$\sin x = \frac{1}{4} \text{ or } -\frac{1}{3}$	dM1
		$\Rightarrow x = 14.5^{\circ} \text{ or } 165.5^{\circ} \text{ or } 340.5^{\circ} \text{ or } 199.5^{\circ}$	dM1, A1 A1
			[5] (6 marks)
a) 31: b)	Accep If they	t $\csc^2 x - \csc x - 12 = 0$ or $k = -12$. No working is required. write $\csc^2 x - \csc x - 12 = 0$ followed by $k = 12$ allow isw	
a) 31: b) ⁄11	Accep If they Solves compl	t $\csc^2 x - \csc x - 12 = 0$ or $k = -12$. No working is required. write $\csc^2 x - \csc x - 12 = 0$ followed by $k = 12$ allow isw s quadratic in $\csc x$ by any method – factorising, formula (accept answers to etion of square. Correct answers (for $\csc x$ of 4 and –3) imply this M mark.	o 1 dp),
a) 31: b) /11	Accep If they Solves compl Quadr Uses	t $\csc^2 x - \csc x - 12 = 0$ or $k = -12$. No working is required. write $\csc^2 x - \csc x - 12 = 0$ followed by $k = 12$ allow isw s quadratic in $\csc x$ by any method – factorising, formula (accept answers to etion of square. Correct answers (for $\csc x$ of 4 and –3) imply this M mark. ratic equations that have 'imaginary' roots please put into review. $\sin x = \frac{1}{\csc x}$ by taking the reciprocal of at least one of their previous answers	o 1 dp),
a) 31: b) /11	Accep If they Solves compl Quadr Uses This is For us Implie	t $\csc^2 x - \csc x - 12 = 0$ or $k = -12$. No working is required. write $\csc^2 x - \csc x - 12 = 0$ followed by $k = 12$ allow isw s quadratic in $\csc x$ by any method – factorising, formula (accept answers to etion of square. Correct answers (for $\csc x$ of 4 and –3) imply this M mark. atic equations that have 'imaginary' roots please put into review. $\sin x = \frac{1}{\csc x}$ by taking the reciprocal of at least one of their previous answers s dependent upon having scored the first M1 ing arcsin to produce one answer inside the range 0 to 360 from their values. ed by any of 14.5° or 165.5° or 340.5° or 199.5° following ($\csc x -4$) ($\csc x = 3$)	(x+3) = 0
a) 31: b) /11 M1 M1	Accep If they Solves compl Quadr Uses This is For us Implie Two c	t $\csc^2 x - \csc x - 12 = 0$ or $k = -12$. No working is required. write $\csc^2 x - \csc x - 12 = 0$ followed by $k = 12$ allow isw s quadratic in $\csc x$ by any method – factorising, formula (accept answers to etion of square. Correct answers (for $\csc x$ of 4 and –3) imply this M mark. atic equations that have 'imaginary' roots please put into review. $\sin x = \frac{1}{\csc x}$ by taking the reciprocal of at least one of their previous answers s dependent upon having scored the first M1 ing arcsin to produce one answer inside the range 0 to 360 from their values. ed by any of 14.5° or 165.5° or 340.5° or 199.5° following ($\csc x -4$) ($\csc x -4$) correct answers inside the range 0 to 360	o 1 dp), x+3) =0
a) 31: b) M1 IM1 IM1 A1 A1	Accep If they Solves compl Quadr Uses This is For us Implie Two c All for	t $\csc^2 x - \csc x - 12 = 0$ or $k = -12$. No working is required. <i>x</i> write $\csc^2 x - \csc x - 12 = 0$ followed by $k = 12$ allow isw s quadratic in $\csc x$ by any method – factorising, formula (accept answers to etion of square. Correct answers (for $\csc x$ of 4 and –3) imply this M mark. atic equations that have 'imaginary' roots please put into review. $\sin x = \frac{1}{\csc x}$ by taking the reciprocal of at least one of their previous answers s dependent upon having scored the first M1 ing arcsin to produce one answer inside the range 0 to 360 from their values. ed by any of 14.5° or 165.5° or 340.5° or 199.5° following ($\csc x -4$) ($\csc x$ - to correct answers inside the range 0 to 360 ur answers in the range, $x = awrt14.5° 165.5° 340.5° 199.5°$	o 1 dp), x+3) =0

Radian solutions will be unlikely, but could be worth dM1 for one solution and dM1A1 A0 for all four solutions (maximum penalty is 1 mark) but accuracy marks are awarded for solutions to 3dp FYI: Solutions awrt are 0.253, 2.889, 3.481, 5.943

The first two M marks may be achieved 'the other way around' if a candidate uses $\csc x = \frac{1}{\sin x}$ in line 1 and

produces a quadratic in $\sin x$.

Award M1 for using $\operatorname{cosec} x = \frac{1}{\sin x}$ (twice) and producing a quadratic in $\sin x$ and dM1 for solving as above.

Mathematics C34

WMA02

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	$3^x + 6y = \frac{3}{2}xy^2$	
Find the exact value	ue of $\frac{dy}{dx}$ at the point on <i>C</i> with coordinates (2, 2)	3). Give your answer in
the form $\frac{a + \ln b}{8}$, where a and b are integers.	-
0		(7)

W	MA	02
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Question Number	Scheme	Marks
3	Differentiates wrt x $3^x \ln 3 + 6 \frac{dy}{dx} = \frac{3}{2}y^2 + 3xy \frac{dy}{dx}$	$B1 \underline{B1}, \underline{M1}, \underline{A1}$
	Substitutes (2, 3) AND rearranges to get $\frac{dy}{dx}$ $\Rightarrow 9 \ln 3 + 6 \frac{dy}{dx} = \frac{27}{2} + 18 \frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{9 \ln 3 - \frac{27}{2}}{12} = \frac{6 \ln 3 - 9}{8}, = \frac{-9 + \ln 729}{8}$	M1 A1, A1
		(7)
		(7 marks)

- B1 Differentiates $3^x \rightarrow 3^x \ln 3$ or $e^{x \ln 3} \rightarrow e^{x \ln 3} \ln 3$
- B1 Differentiates $6y \rightarrow 6\frac{dy}{dr}$
- M1 Uses the product rule to differentiate $\frac{3}{2}xy^2$. Evidence could be sight of $\frac{3}{2}y^2 + kxy\frac{dy}{dx}$ If the rule is quoted it must be correct. It could be implied by u=.., u'=.., v'=.. followed by their vu'+uv'. For this M to be scored y^2 must differentiate to $ky\frac{dy}{dx}$, it cannot differentiate to 2y.
- A1 A completely correct differential of $\frac{3}{2}xy^2$. It need not be simplified.
- M1 Substitutes x = 2, y = 3 into their expression containing a derivative to find a 'numerical' value for $\frac{dy}{dx}$ The candidate may well have attempted to change the subject. Do not penalise accuracy errors on this method mark
- A1 Any correct numerical answer in the form $\frac{p \ln q r}{s}$ where p, q, r and s are constants e.g. $\frac{9 \ln 3 \frac{27}{2}}{12}$
- A1 Exact answer. Accept either $\frac{-9 + \ln 729}{8}$ or $\frac{\ln 729 9}{8}$
- Note: There may be candidates who multiply by 2 first and start with $2 \times 3^{x} + 12y = 3xy^{2}$

This is perfectly acceptable and the mark scheme can be applied in a similar way.

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Past Paper

4.

Leave blank







The curve *C* with equation $y = \frac{2}{(4+3x)}$, $x > -\frac{4}{3}$ is shown in Figure 1

The region bounded by the curve, the x-axis and the lines x = -1 and $x = \frac{2}{3}$, is shown shaded in Figure 1

This region is rotated through 360 degrees about the x-axis.

(a) Use calculus to find the exact value of the volume of the solid generated.

(5)



Figure 2

Figure 2 shows a candle with axis of symmetry AB where AB = 15 cm. A is a point at the centre of the top surface of the candle and B is a point at the centre of the base of the candle. The candle is geometrically similar to the solid generated in part (a).

(b) Find the volume of this candle.

(2)



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Question Number	Scheme	Marl	٢S
4(a)	$(V) = \pi \int_{-1}^{\frac{2}{3}} \frac{4}{(4+3x)^2} dx$	B1	
	$(\pi)\int \frac{4}{(4+3x)^2} \mathrm{d}x = (\pi)\left(-\frac{4}{3}(4+3x)^{-1}\right)$	M1A1	
	$= (\pi) \left[-\frac{4}{3} (4+3x)^{-1} \right]_{-1}^{\frac{2}{3}} = (\pi) \left[-\frac{4}{3} (4+2)^{-1}\frac{4}{3} (4-3)^{-1} \right]$	M1	
	$=\frac{10}{9}\pi$	A1	[5]
(b)	Length scale factor is 9 so volume scale factor is 9^3		
	So volume = $9^3 \times \frac{10}{9} \pi = 810\pi$ or $2545 (\text{cm}^3)$	M1 A1	
		(7 marks)	[2]

(a)

Need a correct statement including π and correct limits and dx . Allow $(V) = \pi \int_{-\infty}^{\frac{1}{3}} \left(\frac{2}{4+3x}\right)^2 dx$ B1

Allow if candidate initially writes down $V = \pi \int \left(\frac{2}{(4+3x)}\right)^2 dx$ attempts to integrate and later uses the

correct limits either way around. Also allow if the π is later multiplied by their $\int_{-\infty}^{\frac{\pi}{3}} \left(\frac{2}{4+3x}\right)^2 dx$

Uses substitution or reverse chain rule to do integral achieving $(k(4+3x)^{-1})$ M1

For $-\frac{4}{3}(4+3x)^{-1}$ They do not need π or the limits A1

Substitutes the correct limits in a changed/integrated function and subtracts (either way around) M1

This answer or equivalent fraction. Accept answer with recurring decimals ie 1.1π A1

(b)

Attempts to multiply their answer in (a) by 729. May be implied by $\frac{10}{9}\pi \rightarrow 810$ (missing the π) M1

This may be implied by $(a) \times \left(\frac{15}{1+\frac{2}{2}}\right)^3$

Any correct equivalent awrt 2540 or 2550 or 810π A1

Leave

5.

$$f(x) = -x^3 + 4x^2 - 6$$

- (a) Show that the equation f(x) = 0 has a root between x = 1 and x = 2
- (b) Show that the equation f(x) = 0 can be rewritten as

$$x = \sqrt{\left(\frac{6}{4-x}\right)}$$

(2)

(2)

- (c) Starting with $x_1 = 1.5$ use the iteration $x_{n+1} = \sqrt{\left(\frac{6}{4 x_n}\right)}$ to calculate the values of x_2 , x_3 and x_4 giving all your answers to 4 decimal places.
- (d) Using a suitable interval, show that 1.572 is a root of f(x) = 0 correct to 3 decimal places.

(2)

(3)



Winter 2016 Past Paper (Mark Scheme)

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Question Number	Scheme	Mark	S
5. (a)	f(1) = -3, $f(2) = 2$	M1	
	Sign change (and as $f(x)$ is continuous) therefore a root α lies in the		
	interval [1, 2]	A1	[2]
(b)	$f(x) = -x^3 + 4x^2 - 6 = 0 \Longrightarrow x^2(4 - x) = 6$	M1	[2]
	$\Rightarrow x^2 = \left(\frac{6}{4-x}\right)$ and so $x = \sqrt{\left(\frac{6}{4-x}\right)} *$	A1*	[2]
(c)	$x_2 = \sqrt{\left(\frac{6}{4 - 1.5}\right)}$	M1	
	$x_2 = awrt 1.5492$,	A1	
	$x_3 = awrt 1.5647$, and $x_4 = awrt 1.5696 / 1.5697$	A1	
			[3]
(d)	f(1.5715) = -0.00254665, f(1.5725) = 0.0026157969		
	Sign change (and as $f(x)$ is continuous) therefore a root α lies in the interval $[1.5715, 1.5725] \Rightarrow \alpha = 1.572$ (3 dp)	M1A1	
		(9 mar	[2] ks)

(a)

M1 Attempts to evaluate **both** f(1) and f(2) and achieves at least one of f(1) = -3 or f(2) = 2If a smaller interval is chosen, eg 1.57 and 1.58, the candidate must refer back to the region 1 to 2 A1 Requires (i) both f(1) = -3 and f(2) = 2 correct,

(ii) sign change stated or equivalent Eg $f(1) \times f(2) < 0$ and (iii) some form of conclusion which may be : or "so result shown" or qed or tick or equivalent

- **(b)**
- M1 Must either state f(x) = 0 or set $-x^3 + 4x^2 6 = 0$ before writing down at least the line equivalent to $\pm x^2(x-4) = \pm 6$

A1* Completely correct with all signs correct. There is no requirement to show $\frac{-6}{4-x} \rightarrow \frac{6}{x-4}$

Expect to see a minimum of the equivalent to $x^2 = \left(\frac{-6}{4-x}\right)$ and $x = \sqrt{\left(\frac{6}{x-4}\right)}$

Alternative working backwards

M1 Starts with answer and squares, multiplies across and expands

$$x = \sqrt{\left(\frac{6}{4-x}\right)} \Longrightarrow x^2 = \frac{6}{4-x} \Longrightarrow x^2(4-x) = 6 \Longrightarrow 4x^2 - x^3 = 6$$

A1 Completely correct $-x^3 + 4x^2 - 6 = 0$ and states "therefore f(x) = 0" or similar

(c) Ignore any reference to labelling. Mark as the first, second and third values given.

- M1 An attempt to substitute $x_0 = 1.5$ into the iterative formula. Eg. Sight of $\sqrt{\left(\frac{6}{4-1.5}\right)}$ or $x_2 = a \text{ wrt } 1.55$
- A1 $x_2 = awrt 1.5492$
- A1 **Both** $x_3 = a w rt 1.5647$ **and** $x_4 = a w rt 1.5696$ or 1.5697
- **(d)**
- M1 Choose suitable interval for *x*, e.g. [1.5715, 1.5725] and at least one attempt to evaluate f(x) not the iterative formula. A minority of candidate may choose a tighter range which should include 1.57199 (alpha to 5dp). This would be acceptable for both marks, provided the conditions for the A mark are met. Continued iteration is M0

A1 Needs

- (i) both evaluations correct to 1 sf, (either rounded or truncated)
- Eg f(1.5715) = -0.003 rounded f(1.5715) = -0.002 truncated
- (ii) sign change stated or equivalent Eg $f(a) \times f(b) < 0$ and

(iii)some form of conclusion which may be : or "so result shown" or qed or tick or equivalent

x	f(x)
1	-3
1.1	-2.491
1.2	-1.968
1.3	-1.437
1.4	-0.904
1.5	-0.375
1.6	0.144
1.7	0.647
1.8	1.128
1.9	1.581
2	2

x	f(x)
1.5715	-0.002546651
1.5716	-0.002030342
1.5717	-0.001514047
1.5718	-0.000997766
1.5719	-0.0004815
1.572	3.4752E-05
1.5721	0.00055099
1.5722	0.001067213
1.5723	0.001583422
1.5724	0.002099617
1.5725	0.002615797



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Ques	stion nber	Scheme	Mar	٢S			
6(a	a)	320 (°C)	B1	[1]			
(b))	$T = 180 \Rightarrow 300e^{-0.04t} = 160, \Rightarrow e^{-0.04t} = \frac{160}{300} (awrt 0.53)$	M1, A1	[1]			
((•)	$t = \frac{1}{-0.04} \ln\left(\frac{160}{300}\right) or \frac{1}{0.04} \ln\left(\frac{300}{160}\right)$ $15.7 \text{ (minutes) cao}$ $\frac{\mathrm{d}T}{\mathrm{d}T} = (-0.04) \times 300 \mathrm{e}^{-0.04t} = (-0.04) \times (T - 20)$	dM1 A1cso M1 A1	[4]			
	.)	$\frac{dt}{dt} = (-0.04) \times 300e^{-t} = (-0.04) \times (1 - 20)$ $= \frac{20 - T}{25} *$					
Alt	(b)	Puts $T = 180$ so $180 = 300e^{-0.04t} + 20$ and $300e^{-0.04t} = 160$	M1				
		$\ln 300 - 0.04t = \ln 160 \Longrightarrow t =, \qquad \frac{\ln 300 - \ln 160}{0.04}$	dM1, A1				
		15.7 (minutes) cao	Alcso	[4]			
(a) B1 (b) M1 A1	320 Sub Cor For Ad	e cao - do not need ° C estitutes $T= 180$ and proceeds to a form $Ae^{-0.04t} = B$ or $Ce^{0.04t} = D$ adone slips on the power for this mark. For example condone $Ae^{-0.4t} = B$ er $e^{-0.04t} = \frac{160}{300}$ or $e^{0.04t} = \frac{300}{160}$ or exact equivalent such as $e^{-0.04t} = \frac{8}{15}$ except decimals here $e^{-0.04t} = 0.53$, or $e^{0.04t} = 1.875$					
dM1	Dep	bendent upon having scored the first M1, it is for moving from $e^{kt} = c, c > 0 \Rightarrow$	$rac{\ln c}{k}$				
A1 (c) M1	15.7 Diff This	correct answer and correct solution only. Do not accept awrt ferentiates to give $\frac{dT}{dt} = ke^{-0.04t}$. Condone $\frac{dT}{dt} = ke^{-0.4t}$ following $T = 300e^{-0.4t}$ is can be achieved from $T=300e^{-0.04t} + 20 \Rightarrow t = \frac{1}{-0.04} \ln\left(\frac{T-20}{300}\right) \Rightarrow \frac{dt}{dT} = \frac{k}{(T-20)}$	κ + 20				
A1	Corr	Correct derivative and correctly eliminates t to achieve $\frac{dT}{dt} = (-0.04) \times (T - 20)$ oe					
	If ca	and idate changes the subject it is for $\frac{dt}{dT} = \frac{-25}{(T-20)}$ oe					
	Alt	ernatively obtains the correct derivative, substitutes T in $\frac{dT}{dt} = \frac{20 - T}{25} \rightarrow \frac{dT}{dt}$	$= -12e^{-0.04t}$	and			
A1*	con Obta	npares. To score the A1* under this method there must be a statement. ains printed answer correctly – no errors					

Mathematics C34



(b) Using your answer to (a), find the exact coordinates of the stationary point on the curve C for which x > 0. Write each coordinate in its simplest form.

The finite region *R*, shown shaded in Figure 3, is bounded by the curve *C*, the *x*-axis and the line x = 3

(c) Complete the table below with the value of *y* corresponding to x = 1

x	0	1	2	3
у	0		$\frac{3}{5}\ln 5$	$\frac{3}{10}\ln 10$

(d) Use the trapezium rule with all the y values in the completed table to find an approximate value for the area of R, giving your answer to 4 significant figures.

(3)

(1)

(5)



Question Number	Scheme	Marks
7.(a)	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{(x^2+1) \times \frac{6x}{x^2+1} - 6x \times \ln(x^2+1)}{(x^2+1)^2} \text{Or} \frac{6x(1-\ln(x^2+1))}{(x^2+1)^2}$	M1A1 [2]
(b)	$\frac{\mathrm{d}y}{\mathrm{d}x} = 0 \Rightarrow \underbrace{(x^2 + 1)}_{(x^2 + 1)} = 3\ln(x^2 + 1)(2x) = 0$	M1
	$\ln(x^2+1) = 1$ so $x = \sqrt{e-1}$	M1A1
(c)	$y = \frac{3}{e}$ $\frac{3}{2} \ln 2$ or 1.0397	ddM1A1 [5] B1 [1]
(d)	$\frac{1}{2} \times 1 \times \underbrace{\{\dots,\dots,\}}$	B1 oe
	$\frac{1}{2} \times 1 \times \left\{ 0 + \frac{3}{10} \ln 10 + 2 \left(\frac{3}{2} \ln 2 + \frac{3}{5} \ln 5 \right) \right\}$ $\left\{ = \frac{1}{2} \left(0.6907755279 + 4.010767 \right) \right\}$	M1
	= 2.351 (awrt 4 sf)	A1 [3]
		(11 marks)

(a)

M1 Applies the Quotient rule, a form of which appears in the formula book, to $\frac{3\ln(x^2+1)}{(x^2+1)}$

If the formula is quoted it must be correct. There must have been some attempt to differentiate both terms. If the rule is not quoted nor implied by their working, meaning that terms are written out

 $u = 3\ln(x^2 + 1), v = x^2 + 1, u' = ..., v' = ...$ followed by their $\frac{vu' - uv'}{v^2}$, then only accept answers of the form

$$\frac{dy}{dx} = \frac{(x^2+1)A\frac{x}{x^2+1} - Bx\ln(x^2+1)}{(x^2+1)^2} or \frac{Ax - Bx\ln(x^2+1)}{(x^2+1)^2}.$$
 Condone invisible brackets for the M.

Alternatively applies the product rule with $u = 3\ln(x^2 + 1), v = (x^2 + 1)^{-1}$

If the formula is quoted it must be correct. There must have been some attempt to differentiate both terms. If the rule is not quoted nor implied by their working, meaning that terms are written out

 $u = 3\ln(x^{2} + 1), v = (x^{2} + 1)^{-1}, u' = ..., v' = ... \text{ followed by their } vu' + uv', \text{ then only accept answers of the form}$ $(x^{2} + 1)^{-1} \times A \frac{x}{x^{2} + 1} + (x^{2} + 1)^{-2} \times Bx \ln(x^{2} + 1).$

Condone invisible brackets for the M.

A1 Any fully correct (unsimplified) form of f'(x). Remember to isw.

Using quotient rule look for variations of
$$\frac{dy}{dx} = \frac{(x^2+1) \times \frac{6x}{x^2+1} - 6x \times \ln(x^2+1)}{(x^2+1)^2}$$

Using the product rule look for $\frac{dy}{dx} = (x^2+1)^{-1} \times \frac{6x}{x^2+1} - (x^2+1)^{-2} \times 2x \times 3 \ln(x^2+1)$
(b)
M1 Setting their numerator (with more than one term) of their $f'(x) = 0$ and proceeds to a form that does not include fractional terms.
If the product rule has been applied in (a) they also need an equation without fractions to score this.
Allow all marks in part (b) if **denominator** was incorrect in (a), for example *v* rather than v^2 in their quotient rule.
M1 Proceeds using correct work to $\ln(x^2+1) = A \Rightarrow x = ..$
A1 $x = \sqrt{e-1}$ achieved from $a \pm$ correct numerator. Ie condone it arising from $\pm (yu'-uy')$

Dependent upon both M's having been scored. It is for substituting in their value of xdM1 (which may be decimal) and finding a value of y from the correct function

A1 Correct solution only
$$y = \frac{3}{e}$$
 and no other solution for $x > 0$. Ignore solutions $x \le 0$

A1

(b)

B1
$$\frac{3}{2}\ln 2$$
 or 1.0397 or exact equivalent such as $\frac{1}{2}\ln 8$

B1 for
$$h = 1$$
. This is implied by $\frac{1}{2} \times 1$ or $\frac{1}{2}$ outside the (main) bracket

For inside the brackets: $0 + \frac{3}{10} \ln 10 + 2 \left(\frac{3}{2} \ln 2 + \frac{3}{5} \ln 5 \right)$ You can follow through on their $\frac{3}{2} \ln 2$ M1 The decimal equivalent is 0 + 0.691 + 2(1.040 + 0.966)

Allow if you have an invisible bracket. That is you see $\frac{1}{2} \times 0 + \frac{3}{10} \ln 10 + 2 \left(\frac{3}{2} \ln 2 + \frac{3}{5} \ln 5 \right)$

```
A1
       awrt 2.351
```

Vinter 2016 ast Paper	This resource was created and owned by Pearson Edexcel	nematics	5 C3 /MAC
	,]	Leave
8.	$f(\theta) = 9\cos^2\theta + \sin^2\theta$		
(a)	Show that $f(\theta) = a + b \cos 2\theta$, where <i>a</i> and <i>b</i> are integers which should be found	l. (3)	
(b)	Using your answer to part (a) and integration by parts, find the exact value of		
	$\int^{\frac{\pi}{2}} \theta^2 f(\theta) d\theta$		
		(6)	
22		I	

Winter 2016

Past Paper (Mark Scheme)

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WMA02

Question Number	Scheme	Marks
8 (a)	Either $f(\theta) = 9\cos^2 \theta + \sin^2 \theta = 9\cos^2 \theta + 1 - \cos^2 \theta$	M1
	$= 8\cos^2\theta + 1 = 8\frac{(\cos 2\theta + 1)}{2} + 1$	M1
	$=5+4\cos 2\theta$	A1 [3]
	Or $f(\theta) = 9 \frac{(\cos 2\theta + 1)}{2} + 1 \frac{(1 - \cos 2\theta)}{2}$	M1 M1
	$=5+4\cos 2\theta$	A1 [3]
(b)	Either :Way1 splits as $\int_{0}^{\frac{\pi}{2}} a\theta^2 d\theta + \int_{0}^{\frac{\pi}{2}} b\theta^2 \cos 2\theta d\theta$	[0]
	$\int_{0}^{\frac{\pi}{2}} b\theta^2 \cos 2\theta \mathrm{d}\theta = \dots \theta^2 \sin 2\theta \pm \int \dots \theta \sin 2\theta \mathrm{d}\theta$	M1
	$=\theta^{2} \sin 2\theta \pm\theta \cos 2\theta \pm \int\cos 2\theta d\theta$	dM1
	Integral = $\left[2\theta^2 \sin 2\theta + 2\theta \cos 2\theta - \sin 2\theta\right] + \frac{5}{3}\theta^3$	Al B1ft
жажаюнанала	Use limits to give $\left[\frac{5\left(\frac{\pi}{2}\right)^3}{3} - \pi\right] - [0] = \left[\frac{5\pi^3}{24} - \pi\right]$	ddM1 A1 [6] (9 marks)
1st 4 marks	Or: Way 2 $\int_{0}^{\frac{\pi}{2}} \theta^2 f(\theta) d\theta = \int_{0}^{\frac{\pi}{2}} \theta^2 (a + b\cos 2\theta) d\theta =$	
	$=\theta^{2}(\theta\pm\sin 2\theta)-\int\theta(\theta\pm\sin 2\theta)d\theta$	M1
	$=\theta^{2}(\theta\pm\sin 2\theta)\theta(\theta^{2}\pm\cos 2\theta)\pm\int.(\theta^{2}\pm\cos 2\theta)d\theta$	dM1
	$= \theta^{2} (5\theta + 2\sin 2\theta) - 2\theta \left(\frac{5\theta^{2}}{2} - \cos 2\theta\right) + \left(\frac{5\theta^{3}}{3} - \sin 2\theta\right)$	A1 B1ft
1 st 4 marks	Or: Way 3 Way 2 that goes back to Way One	
	$\int_{0}^{\frac{\pi}{2}} \theta^{2} (a+b\cos 2\theta) d\theta = \theta^{2} (\theta \pm\sin 2\theta) - \int\theta (\theta \pm\sin 2\theta) d\theta$	M1
	$= \theta^2 (\theta \pm\sin 2\theta) - (\int\theta^2 d\theta) \pm \int\theta \sin 2\theta d\theta$	
	$=\theta^{2}(\theta\pm\sin 2\theta)-\left(\int\theta^{2}d\theta\right)\pm\theta\cos 2\theta\pm\int\cos 2\theta d\theta$	dM1
	$=\boldsymbol{\theta}^{2}(5\boldsymbol{\theta}+2\sin 2\theta)-\frac{10}{3}\boldsymbol{\theta}^{3}+2\theta\cos 2\theta-\sin 2\theta$	A1 B1ft

- (a)
- M1 Uses $\sin^2 \theta = 1 \cos^2 \theta$ or $\cos^2 \theta = 1 \sin^2 \theta$ to reach an expression in either $\sin^2 \theta$ or $\cos^2 \theta$
- M1 Attempts to use the double angle formula $\cos 2\theta = \pm 1 \pm 2 \sin^2 \theta$ or $\cos 2\theta = \pm 2 \cos^2 \theta \pm 1$ to convert their expression in $\sin^2 \theta$ or $\cos^2 \theta$ to form an expression $a + b \cos 2\theta$
- A1 cao = $5 + 4\cos 2\theta$

.....

Alternative

- M1 One attempted application of double angle formula on either $\sin^2 \theta$ or $\cos^2 \theta$. See above for rules
- M1 A second attempted applications of double angle formula to form an expression $a + b \cos 2\theta$
- A1 $cao = 5 + 4\cos 2\theta$
- (b) Note: On e pen this is marked up M1 M1 A1 M1 M1 A1. We are scoring it M1 M1 A1 B1 M1 A1
- M1 An attempt at using integration by parts the correct way around. IF THE CANDIDATE DOES NOT STATE OR IMPLY AN INCORRECT FORMULA ACCEPT

In Way One look for $\int b\theta^2 \cos 2\theta \, d\theta \rightarrow \pm ..\theta^2 \sin 2\theta \pm \int ..\theta \sin 2\theta \, d\theta$

In Way Two look for
$$\int \theta^2 (a + b\cos 2\theta) d\theta \rightarrow \left[\theta^2 (..\theta \pm ..\sin 2\theta)\right] - \int ..\theta (..\theta \pm ..\sin 2\theta) d\theta$$

dM1 Dependent upon M1 having been scored, it is for an attempted use of integration by parts the correct way around for a second time.
 In Way One look for

In Way One look for

$$\Rightarrow \pm ..\theta^2 \sin 2\theta \pm ..\theta \cos 2\theta \pm \int ..\cos 2\theta d\theta$$

In Way Two look for

$$\rightarrow \left[\theta^2(..\theta\pm..\sin 2\theta)\right] - ..\theta(..\theta^2\pm..\cos 2\theta) \pm \int (..\theta^2\pm..\cos 2\theta) d\theta$$

Way 3 : You may see a candidate multiplying out their second integral and reverting to a type one integral.

$$\rightarrow \left[\theta^2 (..\theta \pm ..\sin 2\theta)\right] \pm ..\theta^3 \pm ...\theta \cos 2\theta \pm \int ..\cos 2\theta \, \mathrm{d}\theta$$

A1 cao $\left[2\theta^2 \sin 2\theta + 2\theta \cos 2\theta - \sin 2\theta\right]$ Accept in any unsimplified form

- B1ft $\int a\theta^2 d\theta \rightarrow a \frac{\theta^3}{3}$ It is scored for the term independent of the trigonometrical terms.
- ddM1 Dependent upon both previous M's. For using **both** limits although you may not see the 0. A decimal answer of 3.318 following correct working implies this mark
- A1 cso. Note that a correct answer does not necessarily imply a correct solution

Mathematics C34

WMA02

DO NOT WRITE IN THIS AR

Question Number	Scheme	Marks
9(a)	$\frac{3x^2 - 4}{x^2(3x - 2)} \equiv \frac{A}{x} + \frac{B}{x^2} + \frac{C}{3x - 2}$	
	$\frac{2}{x^2}, \frac{-6}{3x-2}$ (B = 2, C = -6)	B1, B1,
	$3x^2 - 4 \equiv Ax(3x - 2) + B(3x - 2) + Cx^2 \Longrightarrow A =$	M1
	$\frac{3}{r}$ (A = 3) is one of the fractions	A1
		[4]
(b)	$\int \frac{1}{y} \mathrm{d}y = \int \frac{3x^2 - 4}{x^2 (3x - 2)} \mathrm{d}x$	B1
	$\ln y = \int \frac{A}{x} + \frac{B}{x^2} + \frac{C}{3x - 2} dx$	M1
	$= A \ln x - \frac{B}{x} + \frac{C}{3} \ln(3x - 2) \qquad (+k)$	M1A1ft
	$y = e^{A \ln x - \frac{B}{x} + \frac{C}{3} \ln(3x-2) + D}$ or $y = De^{A \ln x - \frac{B}{x} + \frac{C}{3} \ln(3x-2)}$	M1
	$y = Kx^{3}(3x-2)^{-2}e^{-\frac{2}{x}}$ or $\frac{Kx^{3}e^{-\frac{2}{x}}}{(3x-2)^{2}}$ or $\frac{e^{k}x^{3}e^{-\frac{2}{x}}}{(3x-2)^{2}}$ oe	Alcso
		[6] (10 marks)

(a)
B1 For either
$$+\frac{2}{x^2}$$
 or $\frac{-6}{3x-2}$ being one of the "partial" fractions
B1 For two of the partial fractions being $+\frac{2}{x^2}$ and $\frac{-6}{3x-2}$
M1 Need three terms in pfs and correct method either compares coefficients or substitutes a value to obtain A
Look for $3x^2 - 4 = Ax(3x-2) + B(3x-2) + Cx^2 \Rightarrow 4 = ..$
A1 $\frac{3}{x}$
(b)
B1: Separates variables correctly. No need for integral signs
M1 Integrates left hand side to give lny and uses their partial fractions from part (a) (may only have two pf's)
M2 Obtains two In terms and one reciprocal term on rhs (need not have constant of integration for this
mark) (must have 3 pf's here). Condone a missing bracket on the ln(3x-2)
A1ft Correct (unsimplified) answer for rhs for their A , B and C (do not need constant of integration at this stage)
M3 For undoing the logs correctly to get $y = ...$ now need constant of integration.
Accept $y = e^{Ainx - \frac{B}{x} + \frac{C}{3in(3x-2)+d}}$ OR $y = De^{Ainx - \frac{B}{x} + \frac{C}{3in(3x-2)}}$ BUT NOT $y = e^{Ainx - \frac{B}{x} + \frac{C}{3in(3x-2)}} + D$
A1 cso One of the forms of the answer given in the scheme o.e.
Special case: For students who use two partial fractions
Very common incorrect solutions using two partial fractions are
 $\frac{3x^2 - 4}{x^2(3x-2)} = \frac{A}{x^2} + \frac{B}{3x-2} = \frac{2}{x^2} + \frac{-6}{3x-2}$ using substitution and comparing terms in x^2

Both of these will scoring B1B1M0A0 in SC in (a) In part (b) this could score B1, M1 M0 A0 M1 A0 for a total of 5 out of 10.

For the final M1 they must have the correct form	$y = e^{-\frac{\dots}{x} + \dots \ln(3x-2) + D}$	or $y = D e^{-\frac{1}{x} + \dots \ln(3x-2)}$	or equivalent

WMA02

Leave blank 10. (a) Express $3\sin 2x + 5\cos 2x$ in the form $R\sin(2x + \alpha)$, where R > 0 and $0 < \alpha < \frac{\pi}{2}$ Give the exact value of R and give the value of α to 3 significant figures. (3) (b) Solve, for $0 < x < \pi$, $3\sin 2x + 5\cos 2x = 4$ (Solutions based entirely on graphical or numerical methods are not acceptable.) (5) $g(x) = 4(3\sin 2x + 5\cos 2x)^2 + 3$ (c) Using your answer to part (a) and showing your working, (i) find the greatest value of g(x), (ii) find the least value of g(x). (4)



Question Number	Scheme	Marks
10. (a)	$R = \sqrt{34}$	B1
	$\tan \alpha = \frac{5}{3}$	M1
	$\Rightarrow \alpha = 1.03$ awrt 1.03	A1
(b)	$3 \sin 2x + 5 \cos 2x = 4 \implies \sqrt{34} \sin (2x + 1.03) = 4$	[3]
	$\sin(2x + "1.03") = \frac{4}{"\sqrt{34}"} (= 0.68599)$	M1
	One solution in range Eg. $2x + "1.03" = 2\pi + \arcsin\left(\frac{4}{"\sqrt{34}"}\right) \Rightarrow x =$	M1
	Either $x = awrt 3.0$ or $awrt 0.68$	A1
	Second solution in range Eg $2x + "1.03" = \pi - \arcsin\left(\frac{4}{"\sqrt{34}"}\right) \Rightarrow x =$	M1
	Both $x = awrt 2sf 3.0$ and 0.68	A1 [5]
(c)	Greatest value is $4(\sqrt{34})^2 + 3 = 139$	M1 A1
	Least value is $4(0) + 3 = 3$	M1 A1
		[4]
		(12 marks)

- (a)
- B1 $R = \sqrt{34}$ Condone $\pm \sqrt{34}$
- M1 For $\tan \alpha = \pm \frac{5}{3}$ or $\tan \alpha = \pm \frac{3}{5}$ This may be implied by awrt 1.0 rads or awrt 59 degrees If R is used to find α only accept $\cos \alpha = \pm \frac{3}{their R}$ or $\sin \alpha = \pm \frac{5}{their R}$
- A1 accept $\alpha = \text{awrt 1.03}$; also accept $\sqrt{34} \sin(2x + 1.03)$. If the question is done in degrees only the first accuracy mark is withheld. The answer in degrees (59.04) is A0
- (b) On epen this is marked up M1M1M1A1A1. We are scoring it M1M1A1M1A1
- M1 For reaching $\sin(2x \pm \text{their } \alpha) = \frac{4}{\text{their } R}$ (Uses part (a) to solve equation) It may be implied by $(2x \pm \text{their } \alpha) = \arcsin\left(\frac{4}{\text{their } R}\right) = 0.75$ rads
- M1 For an attempt at one solution in the range. It is acceptable to find the negative solution, -0.14 and add π Look for $2x \pm \text{their } \alpha = 2\pi + \arcsin\left(\frac{4}{\text{their } R}\right) \Rightarrow x = ..$ (correct order of operations) Alternatively $2x \pm \text{their } \alpha = \pi - \arcsin\left(\frac{4}{\text{their } R}\right) \Rightarrow x = ..$
- A1 Awrt 3.0 or awrt 0.68. Condone 3 for 3.0. In degrees accept awrt 38.8 or 172.1
- M1 For an attempt at a second solution in the range. This can be scored from their " $\arcsin\left(\frac{4}{\text{their }R}\right)$ "

Look for
$$2x \pm \text{their } \alpha = \pi - \text{their } \arcsin\left(\frac{4}{\text{their } R}\right) \Rightarrow x = .. \text{ (correct order of operations)}$$

Or $2x \pm \text{their } \alpha = 2\pi + \text{their } \arcsin\left(\frac{4}{\text{their } R}\right) \Rightarrow x = ..$

- A1 Awrt 3.0 AND awrt 0.68 in radians or awrt 38.8 and awrt 172.1 in degrees. Condone 3 for 3.0
- (c) (i)
- M1 Attempts to find $4(R)^2 + 3$
- A1 139 cao
- (c)(ii)
- M1 Uses 0 for minimum value. Accept $4(0)^2 + 3$
- A1 3

Leave blank





Figure 4 shows a sketch of part of the curve with equation y = f(x), $x \in \mathbb{R}$

The curve meets the coordinate axes at the points A(0, -3) and $B(-\frac{1}{3}\ln 4, 0)$ and the curve has an asymptote with equation y = -4

In separate diagrams, sketch the graph with equation

(a) $y = |\mathbf{f}(x)|$ (4)

(b)
$$y = 2f(x) + 6$$

On each sketch, give the exact coordinates of the points where the curve crosses or meets the coordinate axes and the equation of any asymptote.

Given that

$$f(x) = e^{-3x} - 4, \qquad x \in \mathbb{R}$$
$$g(x) = \ln\left(\frac{1}{x+2}\right), \qquad x > -2$$

(c) state the range of f,

(1)

(3)

(3)

(3)

- (d) find $f^{-1}(x)$,
- (e) express fg(x) as a polynomial in *x*.



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Question Marks Scheme Number **B**1 Shape --11(a) --Asymptote y = 4y = 4**B**1 (0,3) **B**1 y intercept (0,3) $\left[-\frac{1}{3}\ln 4,0\right]$ Touches x axis at $\left(-\frac{1}{3}\ln 4, 0\right)$ B1 [4] Shape **B**1 Asymptote y = -2**B**1 **(b) B**1 Passes though origin ----- y = -2[3] **B**1 f(x) > -4(c) [1] $y = e^{-3x} - 4 \qquad \Rightarrow e^{-3x} = y + 4$ (**d**) M1 $\Rightarrow -3x = \ln(y+4)$ and x =dM1 $f^{-1}(x) = -\frac{1}{3}\ln(x+4)$ or $\ln\frac{1}{(x+4)^{\frac{1}{3}}}$, (x > -4) cao A1 [3] $fg(x) = e^{-3\ln\left(\frac{1}{x+2}\right)} - 4$ = $(x+2)^3 - 4$, = $x^3 + 6x^2 + 12x + 4$ M1 **(e)** dM1, A1 [3] (14 marks)

Winter 2016

(a)	
B1	For a correct shape. The curve must lie completely in first and second quadrants with a cusp on $-$ ve x axis and approaching an asymptote as x becomes large and positive. The curvature must be correct on both sections. The lh section does not need to extend above the asymptote. On the rh section do
	not allow if the curve drops below the <i>y</i> intercept.
	See practice items for clarification
B1	The asymptote of the curve is given as $y = 4$. Do not award if a second asymptote is given or the curve does not appear to have an asymptote at $y = 4$
B1	y intercept is (0, 3). Allow if given in the body of their answer say as $A=$.
	3 sufficient if given on the y-axis. Condone (3,0) being marked on the correct axis.
	Do not award if there are two intercepts
B1	x intercept is $(-1/3 \ln 4, 0)$. Allow if given in the body of their answer say as $B=$.
	$-1/3 \ln 4$ is sufficient if given on the x axis. Do not award if there are two intercepts
(b)	
B1	shape – similar to original graph (do not try to judge the stretch)
B1	Equation of the asymptote given as $y = -2$ Do not award if a second asymptote is given or the curve
	does not appear to have an asymptote here.
B1	Curve passes through <i>O</i> only.
(c)	
B1	cao f(x) > -4 . Accept alternatives such as $(-4,\infty)$
	Note that $f(x) \ge -4$ is B0. Accept range or y for $f(x)$
(d)	
M1	For an attempt to make x (or a switched y) the subject of the formula. For this to be scored they must make e^{-3x} or e^{3x} as subject. Allow numerical slips.
dM1	This is dependent upon the first M being scored. It is for undoing the exp correctly by using ln's.
	Condone imaginary brackets for this mark. Accept <i>x</i> being given as a function of <i>y</i> and involving ln's.
	If the rhs is written $ln(4x)$ this implies taking the ln of each term which is dM0
A1	This is cao. Accept any correct equivalent. The domain is not required for this mark but the bracket is.
	Accept $y =$
(e)	
N/1	Correct order of operations Allow for $e^{\pm 3\ln \frac{1}{(x+2)}} - 4$
AM1	Context order of operations. Allow for $c = -4$
ulvi i	Dependent internou wark. Simplifies this expression using firstly the power law and then the fact that $\frac{1}{2}$
	$e^{-x} = \dots$ to reach $Ig(x)$ as a function of x.

 $e^{\ln x} = ...$ to reach Ig(x) as a function of x. You may condone sign errors here so tolerate $e^{-3\ln(\frac{1}{x+2})} - 4 \rightarrow \frac{1}{(x+2)^3} - 4$

A1 Correct expansion to give this answer.

Leave blank

12. With respect to a fixed origin O, the lines l_1 and l_2 are given by the equations

$$l_1: \mathbf{r} = \begin{pmatrix} 12\\ -4\\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 5\\ -4\\ 2 \end{pmatrix}, \qquad l_2: \mathbf{r} = \begin{pmatrix} 2\\ 2\\ 0 \end{pmatrix} + \mu \begin{pmatrix} 0\\ 6\\ 3 \end{pmatrix}$$

where λ and μ are scalar parameters.

- (a) Show that l_1 and l_2 meet, and find the position vector of their point of intersection A.
- (b) Find, to the nearest 0.1°, the acute angle between l_1 and l_2

The point *B* has position vector $\begin{pmatrix} 7 \\ 0 \\ 2 \end{pmatrix}$.

- (c) Show that *B* lies on l_1
- (d) Find the shortest distance from B to the line l_2 , giving your answer to 3 significant figures.

(4)

(1)

(6)

(3)

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WMA02

Question Number	Scheme	Marks	
12 (a)	$\begin{pmatrix} 12\\-4\\5 \end{pmatrix} + \lambda \begin{pmatrix} 5\\-4\\2 \end{pmatrix} = \begin{pmatrix} 2\\2\\0 \end{pmatrix} + \mu \begin{pmatrix} 0\\6\\3 \end{pmatrix} \Rightarrow \begin{array}{c} 12+5\lambda = 2\\-4-4\lambda = 2+6\mu \text{any two of these}\\5+2\lambda = 3\mu \end{array}$	M1	
	Full method to find either λ or μ	M1	
	(1) $\Rightarrow \lambda = -2$ Sub $\lambda = -2$ into (2) to give $\mu = \frac{1}{2}$ (need both)	A1	
	Check values in 3^{rd} equation $5+2(-2) = 3(\frac{1}{2})$ and make statement eq. True	B1	
	Position vector of intersection is $\begin{pmatrix} 12\\-4\\5 \end{pmatrix} - 2\begin{pmatrix} 5\\-4\\2 \end{pmatrix} \mathbf{OR} \begin{pmatrix} 2\\2\\0 \end{pmatrix} + \frac{1}{3}\begin{pmatrix} 0\\6\\3 \end{pmatrix} = \begin{pmatrix} 2\\4\\1 \end{pmatrix}$	M1,A1	[7]
	$\begin{pmatrix} 5 \end{pmatrix} \begin{pmatrix} 0 \end{pmatrix}$		[0]
(b)	$\cos\theta = \frac{\begin{pmatrix} -4\\2 \end{pmatrix} \cdot \begin{pmatrix} 6\\3 \end{pmatrix}}{\sqrt{5^2 + (-4)^2 + 2^2}\sqrt{6^2 + 3^2}} = \frac{-18}{45} = -0.4$	M1 A1	
	So acute angle is 66.4 degrees	A1	[0]
(c)	When $\lambda = -1$ this gives $\begin{pmatrix} 7\\0\\3 \end{pmatrix}$ so <i>B</i> lies on l_1	B1	[3]
	ez ez		
(d)	Way 1: $AB = \sqrt{45}$ $h = \sqrt{45} \times \sin 66.4$	M1 A1 M1	
	h = 6.15	A1	
	Way 2 : \overrightarrow{XB} : $=\pm \begin{pmatrix} 5\\ -2-6\mu\\ 3-3\mu \end{pmatrix}$	M1A1	[4]
	Find $\overrightarrow{XB} \cdot \begin{pmatrix} 0\\6\\3 \end{pmatrix} = 0 \Rightarrow \mu = \left(-\frac{1}{15}\right)$ followed by calculation of $\left \overrightarrow{XB}\right $ by Pythag	M1	
	h = 6.15	A1	[4]
		(14 mark	s)

Mathematics C34

WMA02

www.mystudybro.com This resource was created and owned by Pearson Edexcel Past Paper Leave blank 13. A curve C has parametric equations $x = 6\cos 2t$, $y = 2\sin t$, $-\frac{\pi}{2} < t < \frac{\pi}{2}$ (a) Show that $\frac{dy}{dx} = \lambda \operatorname{cosec} t$, giving the exact value of the constant λ . (4) (b) Find an equation of the normal to C at the point where $t = \frac{\pi}{3}$ Give your answer in the form y = mx + c, where m and c are simplified surds. (6) The cartesian equation for the curve C can be written in the form $x = f(y), \quad -k < y < k$ where f(y) is a polynomial in y and k is a constant. (c) Find f(y). (3) (d) State the value of *k*. (1)



Question Number	Scheme	Marks	
13 (a)	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\frac{\mathrm{d}y}{\mathrm{d}t}}{\frac{\mathrm{d}x}{\mathrm{d}t}} = \frac{2\cos t}{-12\sin 2t}$	M1	
	$=\frac{2\cos t}{-24\sin t\cos t}$	dM1	
	$=\frac{2\cos t}{12\cos t} = -\frac{1}{12}\cos t$	M1 A1	F 43
	$-24\sin t \cos t$		[4]
(b)	When $t = \frac{\pi}{3}$, $\frac{dy}{dx} = -\frac{1}{12 \times \sqrt{3}/2} = \left(-\frac{\sqrt{3}}{18}\right)$	M1 A1	
	So Normal has gradient $-\frac{1}{m} = 6\sqrt{3}$	M1	
	When $t = \frac{\pi}{3}$, $x = -3$ and $y = \sqrt{3}$	B1	
	Equation of normal is $y - \sqrt{3} = 6\sqrt{3}(x+3)$ so $y = 6\sqrt{3}x + 19\sqrt{3}$	M1 A1	[6]
(c)	$x=6(1-2\sin^2 t) \Longrightarrow x=f(y)$	M1	[0]
	So $x = 6 - 3y^2$ or $f(y) = 6 - 3y^2$	dM1 A1	[3]
		B 1	[9]
(d)	-2 < y < 2 or $k = 2$		[1]
Alt (a)	Via cartesian must start with $x = A \pm B y^2$ or $y = \sqrt{C \pm D x}$	(14 marks))
	$\frac{dx}{dy} = ky$ or $\frac{dy}{dx} = b\left(2 - \frac{x}{3}\right)^{-\frac{1}{2}}$ then as before	M1	
	followed by correct (double angle) substitution	dM1	
Alt (b)	Must start with $x = A \pm By^2$ or $y = \sqrt{C \pm D x}$		
	$\frac{dx}{dy} = -6y$ or $\frac{dy}{dx} = -\frac{1}{6}\left(2-\frac{x}{3}\right)^{-\frac{1}{2}}$		
	For substituting their $y = \sqrt{3}$ into a $\frac{dx}{dy}$ of the form Py	1st M1	
	Or alternatively substituting their $x = -3$ into a $\frac{dy}{dx}$ of the form $P(Q \pm Rx)^{-\frac{1}{2}}$		
	For using the 'correct numerical' grad of the normal either $-\frac{dx}{dy}$ or $-\frac{1}{\frac{dy}{dx}}$	2nd M1	

- (a)
- M1 Differentiates both x and y wrt t and establishes $\frac{dy}{dt}_{dt} = \frac{\pm A \cos t}{\pm B \sin 2t}$

They may use any double angle formula for cos first. Condone sign slips in this formula

Eg cos 2t = ±2 cos² t ±1 to get
$$\frac{dy}{dt}_{dt} = \frac{\pm A \cos t}{\pm B \sin t \cos t}$$

dM1 Correct double angle formula used $\sin 2t = 2 \sin t \cos t$ In the alternative method the correct double angle formula must have been used

M1 Cancels cost and replaces 1/sint by cosect correctly achieving a form $\frac{dy}{dx} = \lambda cosect$

A1 cao
$$\frac{dy}{dx} = -\frac{1}{12} \csc t$$

(b)

- M1 Substitute $t = \frac{\pi}{3}$ into their $\frac{dy}{dx} = \lambda \operatorname{cosec} t$
- A1 $\frac{dy}{dx} = -\frac{1}{12 \times \sqrt{3}/2}$ or exact equivalent. It may be implied by normal gradient of $6\sqrt{3}$

Accept decimals here. $\frac{dy}{dx} = -0.096$ or implied by normal gradient of 10.4

- M1 Use of negative reciprocal in finding the gradient of the normal.
- B1 for x = -3, $y = \sqrt{3}$
- M1 Correct method for line equation using their **normal** gradient and their (-3, $\sqrt{3}$) allowing a sign slip on one of their coordinates.

Look for
$$y - y_1 = -\frac{dx}{dy}\Big|_{t=\frac{\pi}{3}} (x - x_1)$$
 or $x - x_1 = -\frac{dy}{dx}\Big|_{t=\frac{\pi}{3}} (y - y_1)$

If the candidate uses y = mx + c they must proceed as far as c = ... for this mark

- A1 cao $y = 6\sqrt{3}x + 19\sqrt{3}$
- (c)
- M1 Attempts to use the double angle formula $\cos 2t = \pm 1 \pm 2 \sin^2 t$ leading to an equation linking x and y If $\cos 2t = \cos^2 t - \sin^2 t$ is initially used there must be an attempt to replace the $\cos^2 t$ by $1 - \sin^2 t$

dM1 Uses correct $\cos 2t = 1 - 2\sin^2 t$ and attempts to replace $\sin t$ by $\frac{y}{2}$ and $\cos 2t$ by $\frac{x}{6}$

Condone poor bracketing in cases such as $\cos 2t = 1 - 2\sin^2 t \Rightarrow \frac{x}{6} = 1 - 2\frac{y^2}{2}$

- A1 Correct equation. Accept $x = 6 3y^2$ or $f(y) = 6 3y^2$
- (d)
- B1 States k = 2 or writes the range of y as -2 < y < 2