Write your name here

Surname	Other names

Pearson Edexcel International Advanced Level

Centre Number	Candidate Number

Core Mathematics C34

Advanced

Tuesday 19 January 2016 – Morning
Time: 2 hours 30 minutes

Paper Reference
WMA02/01

You must have:
Mathematical Formulae and Statistical Tables (Blue)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 125.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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1. $$f(x) = (3 - 2x)^{-4}, \qquad |x| < \frac{3}{2}$$

Find the binomial expansion of f(x), in ascending powers of x, up to and including the term in x^2, giving each coefficient as a simplified fraction.

(4)

Question Number	Scheme	Marks
1	$(3-2x)^{-4} = 3^{-4}\left(1-\frac{2}{3}x\right)^{-4}$ $\quad 3^{-4}$ or $\frac{1}{81}$	B1
	$=\frac{1}{81}\times\left(1+(-4)\left(-\frac{2}{3}x\right)+\frac{(-4)(-5)}{2}\left(-\frac{2}{3}x\right)^2+\ldots\right)$	M1A1
	$=\frac{1}{81}+\frac{8}{243}x+\frac{40}{729}x^2+\ldots$	A1
		(4 marks)
	Alternative: $(3-2x)^{-4} = 3^{-4}+(-4)(3)^{-5}(-2x)+\frac{(-4)(-5)}{2}(3)^{-6}(-2x)^2+\ldots$	B1 M1 A1
	$=\frac{1}{81}+\frac{8}{243}x+\frac{40}{729}x^2+\ldots$	A1
		(4 marks)

B1 For taking out a factor of 3^{-4}

Evidence would be seeing either 3^{-4} or $\frac{1}{81}$ before the bracket.

M1 For the form of the binomial expansion with $n=-4$ and a term of (kx)

To score M1 it is sufficient to see just the second and third term with the correct coefficient multiplied by the correct power of x. Condone sign slips. Look for $\ldots+(-4)(kx)+\frac{(-4)(-5)}{2!}(kx)^2\ldots$

A1 Any (unsimplified) form of the binomial expansion. Ignore the factor before the bracket.

The bracketing must be correct but it is acceptable for them to recover from "missing" brackets for full marks.

Look for $1+(-4)\left(-\frac{2}{3}x\right)+\frac{(-4)(-5)}{2}\left(-\frac{2}{3}x\right)^2+$ or $1+\frac{8}{3}x+\frac{40}{9}x^2+$

A1 cao $=\frac{1}{81}+\frac{8}{243}x+\frac{40}{729}x^2+\ldots$. Ignore any further terms.

Alternative

B1 For seeing either 3^{-4} or $\frac{1}{81}$ as the first term

M1 It is sufficient to see the second and third term (unsimplified or simplified) condoning missing brackets.

ie. Look for $\ldots+(-4)(3)^{-5}(kx)+\frac{(-4)(-5)}{2}(3)^{-6}(kx)^2$

A1 Any (un simplified) form of the binomial expansion. $\ldots+(-4)(3)^{-5}(-2x)+\frac{(-4)(-5)}{2}(3)^{-6}(-2x)^2$

A1 Must now be simplified cao

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2. (a) Show that

$$\cot^2 x - \operatorname{cosec} x - 11 = 0$$

may be expressed in the form $\operatorname{cosec}^2 x - \operatorname{cosec} x + k = 0$, where k is a constant.

(1)

(b) Hence solve for $0 \leqslant x < 360°$

$$\cot^2 x - \operatorname{cosec} x - 11 = 0$$

Give each solution in degrees to one decimal place.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(5)

Question Number	Scheme	Marks
2 (a)	$\text{cosec}^2 x - \text{cosec}x - 12 = 0$ or $k = -12$	B1 [1]
(b)	$\text{cosec}^2 x - \text{cosec}\,x - 12 = 0$ so $(\text{cosec}x - 4)(\text{cosec}x + 3) = 0 \Rightarrow \text{cosec}x = ...$	M1
	$\sin x = \frac{1}{4}$ or $-\frac{1}{3}$	dM1
	$\Rightarrow x = 14.5°$ or $165.5°$ or $340.5°$ or $199.5°$	dM1, A1 A1 [5]
		(6 marks)

(a)

B1: Accept $\text{cosec}^2 x - \text{cosec}x - 12 = 0$ or $k = -12$. No working is required.
If they write $\text{cosec}^2 x - \text{cosec}x - 12 = 0$ followed by $k = 12$ allow isw

(b)

M1 Solves quadratic in cosec x by any method – factorising, formula (accept answers to 1 dp), completion of square. Correct answers (for cosec x of 4 and –3) imply this M mark.
Quadratic equations that have 'imaginary' roots please put into review.

dM1 Uses $\sin x = \frac{1}{\text{cosec}x}$ by taking the reciprocal of at least one of their previous answers
This is dependent upon having scored the first M1

dM1 For using arcsin to produce one answer inside the range 0 to 360 from their values.
Implied by any of $14.5°$ or $165.5°$ or $340.5°$ or $199.5°$ following (cosec x -4) (cosec x+3) =0

A1 Two correct answers inside the range 0 to 360

A1 All four answers in the range, $x = \text{awrt}\,14.5°\ 165.5°\ 340.5°\ 199.5°$
Any extra solutions in the range withhold the last A mark.
Ignore any solutions outside the range $0 \leqslant x \leqslant 360°$

....

Radian solutions will be unlikely, but could be worth dM1 for one solution and dM1A1 A0 for all four solutions (maximum penalty is 1 mark) but accuracy marks are awarded for solutions to 3dp
FYI: Solutions awrt are 0.253, 2.889, 3.481, 5.943

....

The first two M marks may be achieved 'the other way around' if a candidate uses $\text{cosec}x = \frac{1}{\sin x}$ in line 1 and produces a quadratic in $\sin x$.

Award M1 for using $\text{cosec}x = \frac{1}{\sin x}$ (twice) and producing a quadratic in $\sin x$ and dM1 for solving as above.

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3. A curve C has equation

$$3^x + 6y = \frac{3}{2}xy^2$$

Find the exact value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ at the point on C with coordinates (2, 3). Give your answer in the form $\frac{a + \ln b}{8}$, where a and b are integers.

(7)

Question Number	Scheme	Marks
3	Differentiates wrt x $\quad 3^x \ln 3 + 6\dfrac{dy}{dx} = \frac{3}{2}y^2 + 3xy\dfrac{dy}{dx}$	B1 B1,M1, A1
	Substitutes (2, 3) **AND** rearranges to get $\dfrac{dy}{dx}$ $\Rightarrow 9\ln 3 + 6\dfrac{dy}{dx} = \dfrac{27}{2} + 18\dfrac{dy}{dx} \Rightarrow \dfrac{dy}{dx} = \dfrac{9\ln 3 - \frac{27}{2}}{12} = \dfrac{6\ln 3 - 9}{8}, = \dfrac{-9 + \ln 729}{8}$	M1 A1, A1
		(7)
		(7 marks)

B1 Differentiates $3^x \to 3^x \ln 3$ or $e^{x\ln 3} \to e^{x\ln 3}\ln 3$

B1 Differentiates $6y \to 6\dfrac{dy}{dx}$

M1 Uses the product rule to differentiate $\dfrac{3}{2}xy^2$. Evidence could be sight of $\frac{3}{2}y^2 + kxy\dfrac{dy}{dx}$

If the rule is quoted it must be correct. It could be implied by u=.., u'=.., v =.., v'=.. followed by their vu'+uv'. For this M to be scored y^2 must differentiate to $ky\dfrac{dy}{dx}$, it cannot differentiate to $2y$.

A1 A completely correct differential of $\dfrac{3}{2}xy^2$. It need not be simplified.

M1 Substitutes $x = 2,\ y = 3$ into their expression containing a derivative to find a 'numerical' value for $\dfrac{dy}{dx}$

The candidate may well have attempted to change the subject. Do not penalise accuracy errors on this method mark

A1 Any correct numerical answer in the form $\dfrac{p\ln q - r}{s}$ where p, q, r and s are constants e.g. $\dfrac{9\ln 3 - \frac{27}{2}}{12}$

A1 Exact answer. Accept either $\dfrac{-9 + \ln 729}{8}$ or $\dfrac{\ln 729 - 9}{8}$

....

Note: There may be candidates who multiply by 2 first and start with $2\times 3^x + 12y = 3xy^2$

This is perfectly acceptable and the mark scheme can be applied in a similar way.

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4.

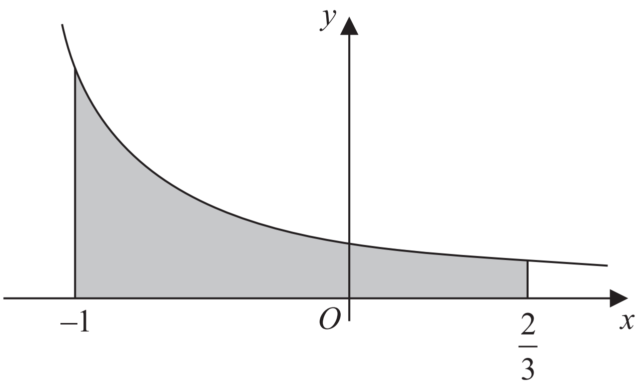

Figure 1

The curve C with equation $y = \frac{2}{(4 + 3x)}$, $x > -\frac{4}{3}$ is shown in Figure 1

The region bounded by the curve, the x-axis and the lines $x = -1$ and $x = \frac{2}{3}$, is shown shaded in Figure 1

This region is rotated through 360 degrees about the x-axis.

(a) Use calculus to find the exact value of the volume of the solid generated.

(5)

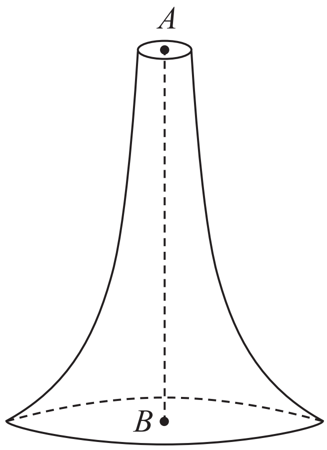

Figure 2

Figure 2 shows a candle with axis of symmetry AB where $AB = 15$ cm. A is a point at the centre of the top surface of the candle and B is a point at the centre of the base of the candle. The candle is geometrically similar to the solid generated in part (a).

(b) Find the volume of this candle.

(2)

Question Number	Scheme	Marks
4(a)	$(V)=\pi\int_{-1}^{\frac{2}{3}}\frac{4}{(4+3x)^2}\,dx$	B1
	$(\pi)\int\frac{4}{(4+3x)^2}\,dx=(\pi)\left(-\frac{4}{3}(4+3x)^{-1}\right)$	M1A1
	$=(\pi)\left[-\frac{4}{3}(4+3x)^{-1}\right]_{-1}^{\frac{2}{3}}=(\pi)\left[-\frac{4}{3}(4+2)^{-1}--\frac{4}{3}(4-3)^{-1}\right]$	M1
	$=\frac{10}{9}\pi$	A1 [5]
(b)	Length scale factor is 9 so volume scale factor is 9^3 So volume $=9^3\times\frac{10}{9}\pi=810\pi$ or $2545\,(\text{cm}^3)$	M1 A1 [2]
		(7 marks)

(a)

B1 Need a correct statement including π and correct limits and dx . Allow $(V)=\pi\int_{-1}^{\frac{2}{3}}\left(\frac{2}{4+3x}\right)^2 dx$

Allow if candidate initially writes down $V=\pi\int\left(\frac{2}{(4+3x)}\right)^2 dx$ attempts to integrate and later uses the correct limits either way around. Also allow if the π is later multiplied by their $\int_{-1}^{\frac{2}{3}}\left(\frac{2}{4+3x}\right)^2 dx$

M1 Uses substitution or reverse chain rule to do integral achieving $\left(k(4+3x)^{-1}\right)$

A1 For $-\frac{4}{3}(4+3x)^{-1}$ They do not need π or the limits

M1 Substitutes the correct limits in a changed/integrated function and subtracts (either way around)

A1 This answer or equivalent fraction. Accept answer with recurring decimals ie $1.\dot{1}\pi$

(b)

M1 Attempts to multiply their answer in (a) by 729. May be implied by $\frac{10}{9}\pi\rightarrow 810$ (missing the π)

This may be implied by $(a)\times\left(\frac{15}{1+\frac{2}{3}}\right)^3$

A1 Any correct equivalent awrt 2540 or 2550 or 810π

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5.

$$f(x) = -x^3 + 4x^2 - 6$$

(a) Show that the equation $f(x) = 0$ has a root between $x = 1$ and $x = 2$

(2)

(b) Show that the equation $f(x) = 0$ can be rewritten as

$$x = \sqrt{\left(\frac{6}{4 - x}\right)}$$

(2)

(c) Starting with $x_1 = 1.5$ use the iteration $x_{n+1} = \sqrt{\left(\frac{6}{4 - x_n}\right)}$ to calculate the values of x_2, x_3 and x_4 giving all your answers to 4 decimal places.

(3)

(d) Using a suitable interval, show that 1.572 is a root of $f(x) = 0$ correct to 3 decimal places.

(2)

Question Number	Scheme	Marks
5. (a)	$f(1) = -3\ ,\quad f(2) = 2$	M1
	Sign change (and as $f(x)$ is continuous) therefore a root α lies in the interval $[1,\ 2]$	A1
		[2]
(b)	$f(x) = -x^3 + 4x^2 - 6 = 0 \Rightarrow x^2(4-x) = 6$	M1
	$\Rightarrow x^2 = \left(\dfrac{6}{4-x}\right)$ and so $x = \sqrt{\left(\dfrac{6}{4-x}\right)}$ *	A1*
		[2]
(c)	$x_2 = \sqrt{\left(\dfrac{6}{4-1.5}\right)}$	M1
	$x_2 = \text{awrt } 1.5492$,	A1
	$x_3 = \text{awrt } 1.5647$, **and** $x_4 = \text{awrt } 1.5696 / 1.5697$	A1
		[3]
(d)	$f(1.5715) = -0.00254665...,\quad f(1.5725) = 0.0026157969$	
	Sign change (and as $f(x)$ is continuous) therefore a root α lies in the interval $[1.5715,\ 1.5725] \Rightarrow \alpha = 1.572$ (3 dp)	M1A1
		[2]
		(9 marks)

(a)

M1 Attempts to evaluate **both** $f(1)$ and $f(2)$ and achieves at least one of $f(1) = -3$ **or** $f(2) = 2$

If a smaller interval is chosen, eg 1.57 and 1.58, the candidate must refer back to the region 1 to 2

A1 Requires (i) both $f(1) = -3$ **and** $f(2) = 2$ correct,

(ii) sign change stated or equivalent Eg $f(1) \times f(2) < 0$ and

(iii) some form of conclusion which may be : or "so result shown" or qed or tick or equivalent

(b)

M1 Must either state f(x) = 0 or set $-x^3 + 4x^2 - 6 = 0$ before writing down at least the line equivalent to $\pm x^2(x-4) = \pm 6$

A1* Completely correct with all signs correct. There is no requirement to show $\dfrac{-6}{4-x} \rightarrow \dfrac{6}{x-4}$

Expect to see a minimum of the equivalent to $x^2 = \left(\dfrac{-6}{4-x}\right)$ and $x = \sqrt{\left(\dfrac{6}{x-4}\right)}$

Alternative working backwards

M1 Starts with answer and squares, multiplies across and expands

$$x = \sqrt{\left(\frac{6}{4-x}\right)} \Rightarrow x^2 = \frac{6}{4-x} \Rightarrow x^2(4-x) = 6 \Rightarrow 4x^2 - x^3 = 6$$

A1 Completely correct $-x^3 + 4x^2 - 6 = 0$ **and** states "therefore f(x) = 0" or similar

(c) Ignore any reference to labelling. Mark as the first, second and third values given.

M1 An attempt to substitute $x_0 = 1.5$ into the iterative formula. Eg. Sight of $\sqrt{\left(\frac{6}{4-1.5}\right)}$ or $x_2 = \text{awrt} 1.55$

A1 $x_2 = \text{awrt } 1.5492$

A1 **Both** $x_3 = \text{awrt } 1.5647$ **and** $x_4 = \text{awrt } 1.5696$ ***or*** 1.5697

(d)

M1 Choose suitable interval for x, e.g. $[1.5715, 1.5725]$ and at least one attempt to evaluate f(x) not the iterative formula. A minority of candidate may choose a tighter range which should include1.57199 (alpha to 5dp). This would be acceptable for both marks, provided the conditions for the A mark are met. Continued iteration is M0

A1 Needs
(i) both evaluations correct to 1 sf, (either rounded or truncated)
Eg $f(1.5715) = -0.003$ rounded $f(1.5715) = -0.002$ truncated
(ii) sign change stated or equivalent Eg $f(a) \times f(b) < 0$ and
(iii)some form of conclusion which may be : or "so result shown" or qed or tick or equivalent

x	f(x)
1	-3
1.1	-2.491
1.2	-1.968
1.3	-1.437
1.4	-0.904
1.5	-0.375
1.6	0.144
1.7	0.647
1.8	1.128
1.9	1.581
2	2

x	f(x)
1.5715	-0.002546651
1.5716	-0.002030342
1.5717	-0.001514047
1.5718	-0.000997766
1.5719	-0.0004815
1.572	3.4752E-05
1.5721	0.00055099
1.5722	0.001067213
1.5723	0.001583422
1.5724	0.002099617
1.5725	0.002615797

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6. A hot piece of metal is dropped into a cool liquid. As the metal cools, its temperature T degrees Celsius, t minutes after it enters the liquid, is modelled by

$$T = 300e^{-0.04t} + 20, \quad t \geqslant 0$$

(a) Find the temperature of the piece of metal as it enters the liquid.
(1)

(b) Find the value of t for which $T = 180$, giving your answer to 3 significant figures.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)
(4)

(c) Show, by differentiation, that the rate, in degrees Celsius per minute, at which the temperature of the metal is changing, is given by the expression

$$\frac{20 - T}{25}$$

(3)

Question Number	Scheme	Marks
6(a)	$320\ (°C)$	B1 [1]
(b)	$\text{T}=180 \Rightarrow 300\text{e}^{-0.04t}=160, \Rightarrow \text{e}^{-0.04t}=\dfrac{160}{300}(\text{awrt } 0.53)$	M1, A1
	$t=\dfrac{1}{-0.04}\ln\left(\dfrac{160}{300}\right)$ *or* $\dfrac{1}{0.04}\ln\left(\dfrac{300}{160}\right)$	dM1
	15.7 (minutes) cao	A1cso [4]
(c)	$\dfrac{\text{d}T}{\text{d}t}=(-0.04)\times 300\text{e}^{-0.04t}=(-0.04)\times(T-20)$	M1 A1
	$=\dfrac{20-T}{25}$ *	A1* [3]
		(8 marks)
Alt **(b)**	Puts $T = 180$ so $180 = 300\text{e}^{-0.04t} + 20$ and $300\text{e}^{-0.04t}=160$	M1
	$\ln 300 - 0.04t = \ln 160 \Rightarrow t = ..,\ \dfrac{\ln 300 - \ln 160}{0.04}$	dM1, A1
	15.7 (minutes) cao	A1cso [4]

(a)

B1 320 cao - do not need ° C

(b)

M1 Substitutes T= 180 and proceeds to a form $Ae^{-0.04t}=B$ or $Ce^{0.04t}=D$
Condone slips on the power for this mark. For example condone $Ae^{-0.4t}=B$

A1 For $e^{-0.04t}=\dfrac{160}{300}$ or $e^{0.04t}=\dfrac{300}{160}$ or exact equivalent such as $e^{-0.04t}=\dfrac{8}{15}$
Accept decimals here $e^{-0.04t}=0.53..$ or $e^{0.04t}=1.875$

dM1 Dependent upon having scored the first M1, it is for moving from $\text{e}^{kt}=c, c>0 \Rightarrow t=\dfrac{\ln c}{k}$

A1 15.7 correct answer and correct solution only. Do not accept awrt

(c)

M1 Differentiates to give $\dfrac{\text{d}T}{\text{d}t}=k\text{e}^{-0.04t}$. Condone $\dfrac{\text{d}T}{\text{d}t}=k\text{e}^{-0.4t}$ following $\text{T}=300\text{e}^{-0.4t}+20$
This can be achieved from $\text{T}=300\text{e}^{-0.04t}+20 \Rightarrow t=\dfrac{1}{-0.04}\ln\left(\dfrac{T-20}{300}\right) \Rightarrow \dfrac{\text{d}t}{\text{d}T}=\dfrac{k}{(T-20)}$ for M1

A1 Correct derivative and correctly eliminates t to achieve $\dfrac{\text{d}T}{\text{d}t}=(-0.04)\times(T-20)$ oe
If candidate changes the subject it is for $\dfrac{\text{d}t}{\text{d}T}=\dfrac{-25}{(T-20)}$ oe
Alternatively obtains the correct derivative, substitutes T in $\dfrac{\text{d}T}{\text{d}t}=\dfrac{20-T}{25} \rightarrow \dfrac{\text{d}T}{\text{d}t}=-12\text{e}^{-0.04t}$ and compares. To score the A1* under this method there must be a statement.

A1* Obtains printed answer correctly – no errors

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7.

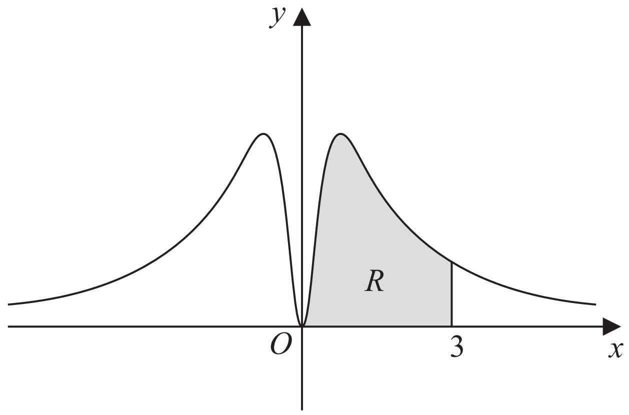

Figure 3

Figure 3 shows part of the curve C with equation

$$y = \frac{3\ln(x^2 + 1)}{(x^2 + 1)}, \quad x \in \mathbb{R}$$

(a) Find $\frac{dy}{dx}$ **(2)**

(b) Using your answer to (a), find the exact coordinates of the stationary point on the curve C for which $x > 0$. Write each coordinate in its simplest form. **(5)**

The finite region R, shown shaded in Figure 3, is bounded by the curve C, the x-axis and the line $x = 3$

(c) Complete the table below with the value of y corresponding to $x = 1$

x	0	1	2	3
y	0		$\frac{3}{5}\ln 5$	$\frac{3}{10}\ln 10$

(1)

(d) Use the trapezium rule with all the y values in the completed table to find an approximate value for the area of R, giving your answer to 4 significant figures. **(3)**

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Question Number	Scheme	Marks
7.(a)	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{(x^2+1)\times\frac{6x}{x^2+1}-6x\times\ln(x^2+1)}{(x^2+1)^2}$ Or $\frac{6x\left(1-\ln(x^2+1)\right)}{(x^2+1)^2}$	M1A1 [2]
(b)	$\frac{\mathrm{d}y}{\mathrm{d}x}=0 \Rightarrow (x^2+1)3\frac{2x}{(x^2+1)}-3\ln(x^2+1)(2x)=0$	M1
	$\ln(x^2+1)=1$ *so* $x=\sqrt{e-1}$	M1A1
	$y=\frac{3}{e}$	ddM1A1 [5]
(c)	$\frac{3}{2}\ln 2$ or 1.0397	B1 [1]
(d)	$\frac{1}{2}\times1\times\{.........................\}$	B1 oe
	$\frac{1}{2}\times1\times\left\{0+\frac{3}{10}\ln10+2\left(\frac{3}{2}\ln2+\frac{3}{5}\ln5\right)\right\}$	M1
	$\{=\tfrac{1}{2}(0.6907755279..+4.010767..)\}$	
	$=2.351$ (awrt 4 sf)	A1 [3]
		(11 marks)

(a)

M1 Applies the Quotient rule, a form of which appears in the formula book, to $\frac{3\ln(x^2+1)}{(x^2+1)}$

If the formula is quoted it must be correct. There must have been some attempt to differentiate both terms. If the rule is not quoted nor implied by their working, meaning that terms are written out $u=3\ln\left(x^2+1\right)$, $v=x^2+1$, $u'=..$, $v'=...$ followed by their $\frac{vu'-uv'}{v^2}$, then only accept answers of the form

$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{(x^2+1)A\frac{x}{x^2+1}-Bx\ln(x^2+1)}{(x^2+1)^2}$ *or* $\frac{Ax-Bx\ln(x^2+1)}{(x^2+1)^2}$. Condone invisible brackets for the M.

Alternatively applies the product rule with $u=3\ln(x^2+1)$, $v=\left(x^2+1\right)^{-1}$

If the formula is quoted it must be correct. There must have been some attempt to differentiate both terms. If the rule is not quoted nor implied by their working, meaning that terms are written out $u=3\ln(x^2+1)$, $v=\left(x^2+1\right)^{-1}$, $u'=..$, $v'=...$ followed by their $vu'+uv'$, then only accept answers of the form

$\left(x^2+1\right)^{-1}\times A\frac{x}{x^2+1}+\left(x^2+1\right)^{-2}\times Bx\ln(x^2+1)$.

Condone invisible brackets for the M.

A1 Any fully correct (unsimplified) form of $f'(x)$. Remember to isw.

Using quotient rule look for variations of $\dfrac{dy}{dx} = \dfrac{(x^2+1)\times\dfrac{6x}{x^2+1} - 6x\times\ln(x^2+1)}{(x^2+1)^2}$

Using the product rule look for $\dfrac{dy}{dx} = (x^2+1)^{-1}\times\dfrac{6x}{x^2+1} - (x^2+1)^{-2}\times 2x\times 3\ln(x^2+1)$

(b)

M1 Setting their numerator (with more than one term) of their $f'(x)=0$ and proceeds to a form that does not include fractional terms.
If the product rule has been applied in (a) they also need an equation without fractions to score this.
Allow all marks in part (b) if **denominato**r was incorrect in (a), for example v rather than v^2 in their quotient rule.

M1 Proceeds using correct work to $\ln(x^2+1)=A \Rightarrow x=..$

A1 $x=\sqrt{e-1}$ achieved from a $\pm$ correct numerator. Ie condone it arising from $\pm(vu'-uv')$

dM1 Dependent upon both M's having been scored. It is for substituting in their value of x (which may be decimal) and finding a value of y from the correct function

A1 Correct solution only $y=\dfrac{3}{e}$ and no other solution for $x>0$. Ignore solutions $x\leqslant 0$

(c)

B1 $\dfrac{3}{2}\ln 2$ or 1.0397 or exact equivalent such as $\dfrac{1}{2}\ln 8$

(d)

B1 for $h = 1$. This is implied by $\dfrac{1}{2}\times 1$ or $\dfrac{1}{2}$ outside the (main) bracket

M1 For inside the brackets: $0+\dfrac{3}{10}\ln 10 + 2\left(\dfrac{3}{2}\ln 2 + \dfrac{3}{5}\ln 5\right)$ You can follow through on their $\dfrac{3}{2}\ln 2$
The decimal equivalent is $0+0.691+2(1.040+0.966)$
Allow if you have an invisible bracket. That is you see $\dfrac{1}{2}\times 0 + \dfrac{3}{10}\ln 10 + 2\left(\dfrac{3}{2}\ln 2 + \dfrac{3}{5}\ln 5\right)$

A1 awrt 2.351

Leave blank

8. $$f(\theta) = 9\cos^2\theta + \sin^2\theta$$

(a) Show that $f(\theta) = a + b\cos 2\theta$, where a and b are integers which should be found.

(3)

(b) Using your answer to part (a) and integration by parts, find the exact value of

$$\int_0^{\frac{\pi}{2}} \theta^2 f(\theta)\,d\theta$$

(6)

Question Number	Scheme	Marks
8(a)	**Either** $f(\theta)=9\cos^2\theta+\sin^2\theta=9\cos^2\theta+1-\cos^2\theta$	M1
	$=8\cos^2\theta+1=8\dfrac{(\cos 2\theta+1)}{2}+1$	M1
	$=5+4\cos 2\theta$	A1 [3]
	Or $f(\theta)=9\dfrac{(\cos 2\theta+1)}{2}+1\dfrac{(1-\cos 2\theta)}{2}$	M1 M1
	$=5+4\cos 2\theta$	A1 [3]
(b)	**Either :Way1** splits as $\int_0^{\frac{\pi}{2}} a\theta^2\,d\theta+\int_0^{\frac{\pi}{2}} b\theta^2\cos 2\theta\,d\theta$	
	$\int_0^{\frac{\pi}{2}} b\theta^2\cos 2\theta\,d\theta=\ ...\theta^2\sin 2\theta\pm\int ...\theta\sin 2\theta d\theta$	M1
	$=\ =..\theta^2\sin 2\theta\pm..\theta\cos 2\theta\pm\int ..\cos 2\theta d\theta$	dM1
	Integral $=\underline{\underline{\left[2\theta^2\sin 2\theta+2\theta\cos 2\theta-\sin 2\theta\right]}}+\frac{5}{3}\theta^3$	A1 B1ft
	Use limits to give $\left[\dfrac{5\left(\frac{\pi}{2}\right)^3}{3}-\pi\right]-[0]\quad=\quad\left[\dfrac{5\pi^3}{24}-\pi\right]$	ddM1 A1 [6]
		(9 marks)
1st 4 marks	**Or: Way 2** $\int_0^{\frac{\pi}{2}}\theta^2 f(\theta)d\theta=\int_0^{\frac{\pi}{2}}\theta^2(a+b\cos 2\theta)d\theta=$	
	$=\theta^2(..\theta\pm..\sin 2\theta)-\int ..\theta(..\theta\pm..\sin 2\theta)d\theta$	M1
	$=\theta^2(...\theta\pm..\sin 2\theta)-..\theta\left(..\theta^2\pm...\cos 2\theta\right)\pm\int .\left(..\theta^2\pm...\cos 2\theta\right)d\theta$	dM1
	$=\theta^2(5\theta+2\sin 2\theta)-2\theta\left(\dfrac{5\theta^2}{2}-\cos 2\theta\right)+\left(\dfrac{5\theta^3}{3}-\sin 2\theta\right)$	A1 **B1ft**
1st 4 marks	**Or: Way 3 Way 2 that goes back to Way One**	
	$\int_0^{\frac{\pi}{2}}\theta^2(a+b\cos 2\theta)d\theta=\theta^2(..\theta\pm..\sin 2\theta)-\int ..\theta(..\theta\pm..\sin 2\theta)d\theta$	M1
	$=\theta^2(..\theta\pm..\sin 2\theta)-\left(\int ...\theta^2 d\theta\right)\pm\int ..\theta\sin 2\theta d\theta$	
	$=\theta^2(..\theta\pm..\sin 2\theta)-\left(\int ...\theta^2 d\theta\right)\pm..\theta\cos 2\theta\pm\int ..\cos 2\theta d\theta$	dM1
	$=\theta^2(5\theta+2\sin 2\theta)-\dfrac{10}{3}\theta^3+2\theta\cos 2\theta-\sin 2\theta$	A1 **B1ft**

(a)

M1 Uses $\sin^2\theta = 1-\cos^2\theta$ or $\cos^2\theta = 1-\sin^2\theta$ to reach an expression in either $\sin^2\theta$ or $\cos^2\theta$

M1 Attempts to use the double angle formula $\cos 2\theta = \pm 1 \pm 2\sin^2\theta$ or $\cos 2\theta = \pm 2\cos^2\theta \pm 1$ to convert their expression in $\sin^2\theta$ or $\cos^2\theta$ to form an expression $a+b\cos 2\theta$

A1 cao $=5+4\cos 2\theta$

Alternative

M1 One attempted application of double angle formula on either $\sin^2\theta$ or $\cos^2\theta$. See above for rules

M1 A second attempted applications of double angle formula to form an expression $a+b\cos 2\theta$

A1 cao$=5+4\cos 2\theta$

(b) Note: On e pen this is marked up M1 M1 A1 M1 M1 A1. We are scoring it M1 M1 A1 B1 M1 A1

M1 An attempt at using integration by parts the correct way around.
IF THE CANDIDATE DOES NOT STATE OR IMPLY AN INCORRECT FORMULA ACCEPT

In Way One look for $\int b\theta^2 \cos 2\theta \, d\theta \to \pm..\theta^2 \sin 2\theta \pm \int ..\theta \sin 2\theta d\theta$

In Way Two look for $\int \theta^2 (a+b\cos 2\theta) d\theta \to \left[\theta^2(..\theta \pm ..\sin 2\theta)\right] - \int ..\theta(..\theta \pm ..\sin 2\theta) d\theta$

dM1 Dependent upon M1 having been scored, it is for an attempted use of integration by parts the correct way around for a second time.

In Way One look for

$$\to \pm..\theta^2 \sin 2\theta \pm ..\theta \cos 2\theta \pm \int ..\cos 2\theta d\theta$$

In Way Two look for

$$\to \left[\theta^2(..\theta \pm ..\sin 2\theta)\right] - ..\theta(..\theta^2 \pm ..\cos 2\theta) \pm \int (..\theta^2 \pm ..\cos 2\theta) d\theta$$

Way 3 : You may see a candidate multiplying out their second integral and reverting to a type one integral.

$$\to \left[\theta^2(..\theta \pm ..\sin 2\theta)\right] \pm ..\theta^3 \pm ...\theta \cos 2\theta \pm \int ..\cos 2\theta \, d\theta$$

A1 cao $\left[2\theta^2 \sin 2\theta + 2\theta \cos 2\theta - \sin 2\theta\right]$ Accept in any unsimplified form

B1ft $\int a\theta^2 \, d\theta \to a\frac{\theta^3}{3}$ It is scored for the term independent of the trigonometrical terms.

ddM1 Dependent upon both previous M's. For using **both** limits although you may not see the 0.
A decimal answer of 3.318 following correct working implies this mark

A1 cso. Note that a correct answer does not necessarily imply a correct solution

Leave blank

9. (a) Express $\dfrac{3x^2 - 4}{x^2(3x - 2)}$ in partial fractions.

(4)

(b) Given that $x > \dfrac{2}{3}$, find the general solution of the differential equation

$$x^2(3x - 2)\,\frac{dy}{dx} = y(3x^2 - 4)$$

Give your answer in the form $y = f(x)$.

(6)

P 4 6 9 5 8 A 0 2 6 4 4

Question Number	Scheme	Marks
9(a)	$\dfrac{3x^2-4}{x^2(3x-2)} \equiv \dfrac{A}{x}+\dfrac{B}{x^2}+\dfrac{C}{3x-2}$	
	$\dfrac{2}{x^2}, \dfrac{-6}{3x-2}$ $(B=2,\ C=-6)$	B1, B1,
	$3x^2-4 \equiv Ax(3x-2)+B(3x-2)+Cx^2 \Rightarrow A=..$	M1
	$\dfrac{3}{x}$ $(A=3)$ is one of the fractions	A1
		[4]
(b)	$\displaystyle\int \frac{1}{y}\,\mathrm{d}y = \int \frac{3x^2-4}{x^2(3x-2)}\,\mathrm{d}x$	B1
	$\ln y = \displaystyle\int \frac{A}{x}+\frac{B}{x^2}+\frac{C}{3x-2}\,\mathrm{d}x$	M1
	$= A\ln x - \dfrac{B}{x}+\dfrac{C}{3}\ln(3x-2)$ $(+k)$	M1A1ft
	$y = \mathrm{e}^{A\ln x - \frac{B}{x}+\frac{C}{3}\ln(3x-2)+D}$ or $y = D\mathrm{e}^{A\ln x-\frac{B}{x}+\frac{C}{3}\ln(3x-2)}$	M1
	$y = Kx^3(3x-2)^{-2}e^{-\frac{2}{x}}$ or $\dfrac{Kx^3e^{-\frac{2}{x}}}{(3x-2)^2}$ or $\dfrac{\mathrm{e}^k x^3\mathrm{e}^{-\frac{2}{x}}}{(3x-2)^2}$ oe	A1cso
		[6] **(10 marks)**

(a)

B1 For either $+\frac{2}{x^2}$ or $\frac{-6}{3x-2}$ being one of the "partial" fractions

B1 For two of the partial fractions being $+\frac{2}{x^2}$ **and** $\frac{-6}{3x-2}$

M1 Need three terms in pfs and correct method either compares coefficients or substitutes a value to obtain A
Look for $3x^2 - 4 \equiv Ax(3x-2) + B(3x-2) + Cx^2 \Rightarrow A = ..$

A1 $\frac{3}{x}$

(b)

B1: Separates variables correctly. No need for integral signs

M1 Integrates left hand side to give ln$y$ and uses their partial fractions from part (a) (may only have two pf 's)

M1 Obtains two ln terms and one reciprocal term on rhs (need not have constant of integration for this mark) (must have 3 pf 's here). Condone a missing bracket on the ln(3x-2)

A1ft Correct (unsimplified) answer for rhs for their A, B and C (do not need constant of integration at this stage)

M1 For undoing the logs correctly to get y = …. now need constant of integration.
Accept $y = \mathrm{e}^{A\ln x - \frac{B}{x} + \frac{C}{3}\ln(3x-2)+d}$ OR $y = D\mathrm{e}^{A\ln x - \frac{B}{x} + \frac{C}{3}\ln(3x-2)}$ BUT NOT $y = \mathrm{e}^{A\ln x - \frac{B}{x} + \frac{C}{3}\ln(3x-2)} + D$

A1 cso One of the forms of the answer given in the scheme o.e.

....

Special case: For students who use two partial fractions

Very common incorrect solutions using two partial fractions are

$$\frac{3x^2-4}{x^2(3x-2)} \equiv \frac{A}{x^2} + \frac{B}{3x-2} = \frac{\mathbf{2}}{\mathbf{x^2}} + \frac{\mathbf{3}}{\mathbf{3x-2}}$$ using substitution and comparing terms in x^2

or $$\frac{3x^2-4}{x^2(3x-2)} \equiv \frac{A}{x^2} + \frac{B}{3x-2} = \frac{\mathbf{2}}{\mathbf{x^2}} + \frac{\mathbf{-6}}{\mathbf{3x-2}}$$ using substitution

Both of these will scoring B1B1M0A0 in SC in (a)

In part (b) this could score B1, M1 M0 A0 M1 A0 for a total of 5 out of 10.

For the final M1 they must have the correct form $y = \mathrm{e}^{-\frac{...}{x}+...\ln(3x-2)+D}$ or $y = D\mathrm{e}^{-\frac{...}{x}+...\ln(3x-2)}$ or equivalent

....

Leave blank

10. (a) Express $3\sin 2x + 5\cos 2x$ in the form $R\sin(2x + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the exact value of R and give the value of α to 3 significant figures.

(3)

(b) Solve, for $0 < x < \pi$,

$$3\sin 2x + 5\cos 2x = 4$$

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(5)

$$g(x) = 4(3\sin 2x + 5\cos 2x)^2 + 3$$

(c) Using your answer to part (a) and showing your working,

(i) find the greatest value of $g(x)$,

(ii) find the least value of $g(x)$.

(4)

Question Number	Scheme	Marks
10. (a)	$R = \sqrt{34}$	B1
	$\tan\alpha = \frac{5}{3}$	M1
	$\Rightarrow \alpha = 1.03$ awrt 1.03	A1
		[3]
(b)	$3\sin 2x + 5\cos 2x = 4 \Rightarrow \sqrt{34}\sin(2x+1.03) = 4$	
	$\sin(2x+"1.03") = \frac{4}{"\sqrt{34}"}$ $(= 0.68599...)$	M1
	One solution in range Eg. $2x+"1.03"=2\pi+\arcsin\left(\frac{4}{"\sqrt{34}"}\right)\Rightarrow x=...$	M1
	Either $x =$ awrt 3.0 **or** awrt 0.68	A1
	Second solution in range Eg $2x+"1.03"=\pi-\arcsin\left(\frac{4}{"\sqrt{34}"}\right)\Rightarrow x=...$	M1
	Both $x =$ awrt 2sf 3.0 **and** 0.68	A1
		[5]
(c)	Greatest value is $4\left(\sqrt{34}\right)^2+3$ $= 139$	M1 A1
	Least value is $4(0)+3=3$.	M1 A1
		[4]
		(12 marks)

(a)

B1 $R = \sqrt{34}$ Condone $\pm\sqrt{34}$

M1 For $\tan\alpha = \pm\frac{5}{3}$ or $\tan\alpha = \pm\frac{3}{5}$ This may be implied by awrt 1.0 rads or awrt 59 degrees

If R is used to find α only accept $\cos\alpha = \pm\frac{3}{their\ R}$ or $\sin\alpha = \pm\frac{5}{their\ R}$

A1 accept $\alpha =$ awrt 1.03; also accept $\sqrt{34}\sin(2x+1.03)$.

If the question is done in degrees only the first accuracy mark is withheld. The answer in degrees (59.04) is A0

(b) On epen this is marked up M1M1M1A1A1. We are scoring it M1M1A1M1A1

M1 For reaching $\sin(2x \pm \text{their } \alpha) = \frac{4}{\text{their } R}$ (Uses part (a) to solve equation)

It may be implied by $(2x \pm \text{their } \alpha) = \arcsin\left(\frac{4}{\text{their } R}\right)$ = 0.75 rads

M1 For an attempt at one solution in the range. It is acceptable to find the negative solution, -0.14 and add π

Look for $2x \pm \text{their } \alpha = 2\pi + \arcsin\left(\frac{4}{\text{their } R}\right) \Rightarrow x = ..$ (correct order of operations)

Alternatively $2x \pm \text{their } \alpha = \pi - \arcsin\left(\frac{4}{\text{their } R}\right) \Rightarrow x = ..$

A1 Awrt 3.0 **or** awrt 0.68. Condone 3 for 3.0. In degrees accept awrt 38.8 or 172.1

M1 For an attempt at a second solution in the range. This can be scored from their "$\arcsin\left(\frac{4}{\text{their } R}\right)$"

Look for $2x \pm \text{their } \alpha = \pi - \text{their } \arcsin\left(\frac{4}{\text{their } R}\right) \Rightarrow x = ..$ (correct order of operations)

Or $2x \pm \text{their } \alpha = 2\pi + \text{their } \arcsin\left(\frac{4}{\text{their } R}\right) \Rightarrow x = ..$

A1 Awrt 3.0 **AND** awrt 0.68 in radians or awrt 38.8 and awrt 172.1 in degrees. Condone 3 for 3.0

(c) (i)

M1 Attempts to find $4(R)^2 + 3$

A1 139 cao

(c)(ii)

M1 Uses 0 for minimum value. Accept $4(0)^2 + 3$

A1 3

Leave blank

11.

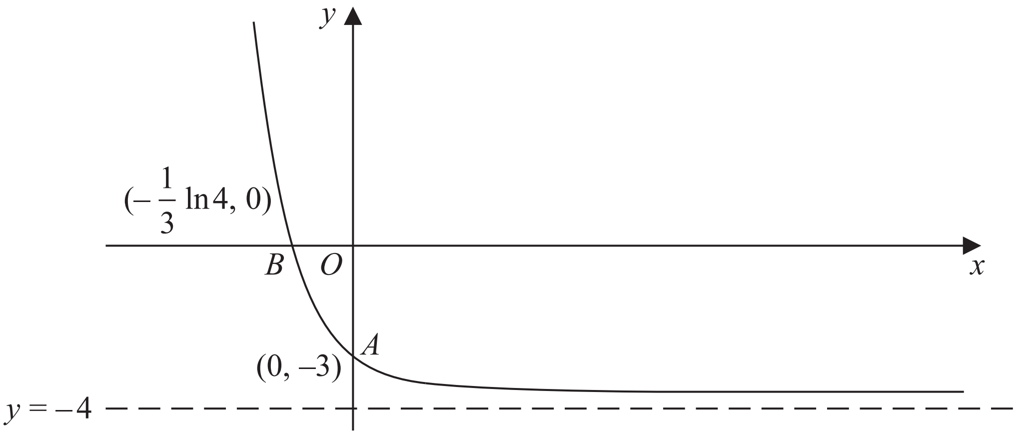


Figure 4

Figure 4 shows a sketch of part of the curve with equation $y = f(x)$, $x \in \mathbb{R}$

The curve meets the coordinate axes at the points $A(0, -3)$ and $B(-\frac{1}{3}\ln 4, 0)$ and the curve has an asymptote with equation $y = -4$

In separate diagrams, sketch the graph with equation

(a) $y = |f(x)|$

(4)

(b) $y = 2f(x) + 6$

(3)

On each sketch, give the exact coordinates of the points where the curve crosses or meets the coordinate axes and the equation of any asymptote.

Given that

$$f(x) = e^{-3x} - 4, \qquad x \in \mathbb{R}$$

$$g(x) = \ln\left(\frac{1}{x+2}\right), \qquad x > -2$$

(c) state the range of f,

(1)

(d) find $f^{-1}(x)$,

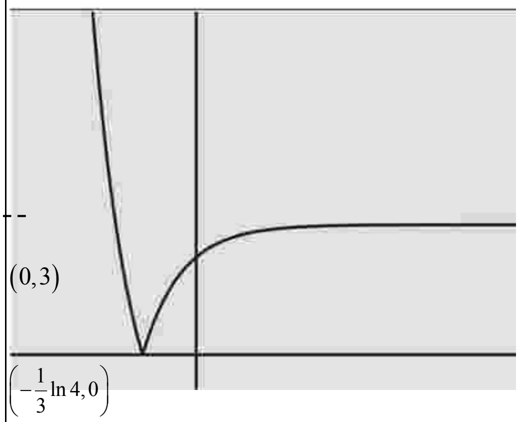
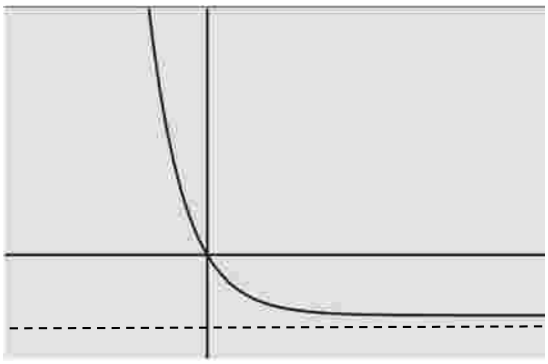
(3)

(e) express $fg(x)$ as a polynomial in x.

(3)

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

P 4 6 9 5 8 A 0 3 4 4 4

Question Number	Scheme	Marks
11(a)	 $y = 4$ Shape	B1
	Asymptote $y = 4$	B1
	y intercept $(0, 3)$	B1
	Touches x axis at $\left(-\frac{1}{3}\ln 4, 0\right)$	B1
		[4]
(b)	 $y = -2$ Shape	B1
	Asymptote $y = -2$	B1
	Passes though origin	B1
		[3]
(c)	$f(x) > -4$	B1
		[1]
(d)	$y = e^{-3x} - 4 \quad \Rightarrow e^{-3x} = y + 4$	M1
	$\Rightarrow -3x = \ln(y+4)$ and $x =$	dM1
	$f^{-1}(x) = -\frac{1}{3}\ln(x+4)$ or $\ln\frac{1}{(x+4)^{\frac{1}{3}}}$, $(x > -4)$ cao	A1
		[3]
(e)	$fg(x) = e^{-3\ln\left(\frac{1}{x+2}\right)} - 4$	M1
	$= (x+2)^3 - 4$, $\quad = x^3 + 6x^2 + 12x + 4$	dM1, A1
		[3]
		(14 marks)

(a)

B1 For a correct shape. The curve must lie completely in first and second quadrants with a cusp on – ve x axis and approaching an asymptote as x becomes large and positive. The curvature must be correct on both sections. The lh section does not need to extend above the asymptote. On the rh section do not allow if the curve drops below the y intercept.
See practice items for clarification

B1 The asymptote of the curve is given as $y = 4$. Do not award if a second asymptote is given or the curve does not appear to have an asymptote at $y = 4$

B1 y intercept is (0, 3). Allow if given in the body of their answer say as A= .
3 sufficient if given on the y-axis. Condone (3,0) being marked on the correct axis.
Do not award if there are two intercepts

B1 x intercept is $(-1/3 \ln 4, 0)$. Allow if given in the body of their answer say as B= .
$-1/3 \ln 4$ is sufficient if given on the x axis. Do not award if there are two intercepts

(b)

B1 shape – similar to original graph (do not try to judge the stretch)

B1 Equation of the asymptote given as $y = -2$ Do not award if a second asymptote is given or the curve does not appear to have an asymptote here.

B1 Curve passes through O only.

(c)

B1 cao $f(x) > -4$. Accept alternatives such as $(-4,\infty)$
Note that $f(x) \geq -4$ is B0. Accept range or y for $f(x)$

(d)

M1 For an attempt to make x (or a switched y) the subject of the formula. For this to be scored they must make e^{-3x} or e^{3x} as subject. Allow numerical slips.

dM1 This is dependent upon the first M being scored. It is for undoing the exp correctly by using ln's. Condone imaginary brackets for this mark. Accept x being given as a function of y and involving ln's. If the rhs is written ln(4x) this implies taking the ln of each term which is dM0

A1 This is cao. Accept any correct equivalent. The domain is not required for this mark but the bracket is. Accept y =..

(e)

M1 Correct order of operations. Allow for $e^{\pm 3\ln\frac{1}{(x+2)}} - 4$

dM1 Dependent Method Mark. Simplifies this expression using firstly the power law and then the fact that $e^{\ln ..} = ..$ to reach fg(x) as a function of x.

You may condone sign errors here so tolerate $e^{-3\ln\left(\frac{1}{x+2}\right)} - 4 \rightarrow \frac{1}{(x+2)^3} - 4$

A1 Correct expansion to give this answer.

Leave blank

12. With respect to a fixed origin O, the lines l_1 and l_2 are given by the equations

$$l_1 : \mathbf{r} = \begin{pmatrix} 12 \\ -4 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 5 \\ -4 \\ 2 \end{pmatrix}, \qquad l_2 : \mathbf{r} = \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 0 \\ 6 \\ 3 \end{pmatrix}$$

where λ and μ are scalar parameters.

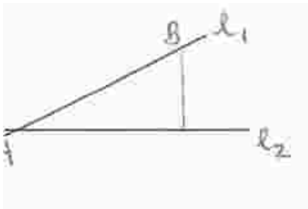
(a) Show that l_1 and l_2 meet, and find the position vector of their point of intersection A.
(6)

(b) Find, to the nearest 0.1°, the acute angle between l_1 and l_2
(3)

The point B has position vector $\begin{pmatrix} 7 \\ 0 \\ 3 \end{pmatrix}$.

(c) Show that B lies on l_1
(1)

(d) Find the shortest distance from B to the line l_2, giving your answer to 3 significant figures.
(4)

Question Number	Scheme	Marks
12 (a)	$\begin{pmatrix}12\\-4\\5\end{pmatrix}+\lambda\begin{pmatrix}5\\-4\\2\end{pmatrix}=\begin{pmatrix}2\\2\\0\end{pmatrix}+\mu\begin{pmatrix}0\\6\\3\end{pmatrix}\Rightarrow$ $12+5\lambda=2$, $-4-4\lambda=2+6\mu$, $5+2\lambda=3\mu$ **any two** of these	M1
	Full method to find either λ **or** μ	M1
	$(1)\Rightarrow\lambda=-2$	
	Sub $\lambda=-2$ into (2) to give $\mu=\frac{1}{3}$ (need both)	A1
	Check values in 3rd equation $5+2(-2)=3(\frac{1}{3})$ and make statement eg. True	B1
	Position vector of intersection is $\begin{pmatrix}12\\-4\\5\end{pmatrix}-2\begin{pmatrix}5\\-4\\2\end{pmatrix}$ **OR** $\begin{pmatrix}2\\2\\0\end{pmatrix}+\frac{1}{3}\begin{pmatrix}0\\6\\3\end{pmatrix}=\begin{pmatrix}2\\4\\1\end{pmatrix}$	M1,A1
		[6]
(b)	$\cos\theta=\dfrac{\begin{pmatrix}5\\-4\\2\end{pmatrix}.\begin{pmatrix}0\\6\\3\end{pmatrix}}{\sqrt{5^2+(-4)^2+2^2}\sqrt{6^2+3^2}}==\dfrac{-18}{45}=-0.4$	M1 A1
	So acute angle is 66.4 degrees	A1
		[3]
(c)	When $\lambda=-1$ this gives $\begin{pmatrix}7\\0\\3\end{pmatrix}$ so B lies on l_1	B1
		[1]
		
(d)	**Way** 1: $AB=\sqrt{45}$	M1 A1
	$h=\sqrt{45}\times\sin 66.4$	M1
	$h=6.15$	A1
		[4]
	Way 2: $\overrightarrow{XB}:\ =\pm\begin{pmatrix}5\\-2-6\mu\\3-3\mu\end{pmatrix}$	M1A1
	Find $\overrightarrow{XB}.\begin{pmatrix}0\\6\\3\end{pmatrix}=0\Rightarrow\mu=\left(-\dfrac{1}{15}\right)$ followed by calculation of $\left\lvert\overrightarrow{XB}\right\rvert$ by Pythag	M1
	$h=6.15$	A1
		[4]
		(14 marks)

(a)

M1 For writing down **any two equations** that give the coordinates of the point of intersection. Accept two of $12+5\lambda = 2$, $-4-4\lambda = 2+6\mu$, $5+2\lambda = 3\mu$ condoning slips.

M1 A full method to find **either** λ **or** μ .

A1 **Both** values correct $\mu=\frac{1}{3}$ and $\lambda=-2$

B1 The correct values must be substituted into **both** sides of the third equation. There must be some minimal statement (a tick will suffice) that the values are the same. This can also be scored via the substitution of $\mu=\frac{1}{3}$ $\lambda=-2$ into **both** of the equations of the lines but there must be the same minimal statement.

M1 Substitutes their value of λ into l_1 to find the coordinates or position vector of the point of intersection. It is dependent upon having scored second method mark. Alternatively substitutes their value of μ into l_2 to find the coordinates or position vector of the point of intersection.

A1 Correct answer only. Accept as a vector or a coordinate. Accept (2, 4, 1) (A correct answer here implies previous M mark). Note that the correct answer can be achieved by solving just the first equation.

(b)

M1 A clear attempt to use the correct formula for $a.b=|a||b|\cos\theta$ (where a and b are the gradient vectors)

Expect to see $5\times0+-4\times6+2\times3=\sqrt{5^2+(-4)^2+2^2}\times\sqrt{6^2+3^2}\cos\theta$ allowing for slips.

A1 For $\cos\theta=\pm0.4$. This may be implied by 66.4 or 113.6.Also accept $\cos\theta=\frac{0-24+6}{\sqrt{45}\sqrt{45}}$ oe

A1 cao for awrt 66.4

(c)

B1 States or uses $\lambda=-1$ and checks all 3 coordinates

(d) **Way 1:**

M1 Finds distance AB using a correct method

Using Pythagoras look for $\sqrt{(7-"2")^2+(0-"4")^2+(3-"1")^2}$ or 'one' gradient $\sqrt{5^2+(-4)^2+2^2}$

A1 Correct answer $(AB)=\sqrt{45}$ or $(AB)=3\sqrt{5}$ or (AB) = awrt 6.71

M1 Reaches $h=\sqrt{45}\times\sin 66.4$ with their values for AB and $\sin\theta$ to find h

A1 awrt 6.15 (allow also if it follows 113.6). The exact answer of $\frac{3}{5}\sqrt{105}$ is fine

Way 2: Setting up a point X on l_2

M1 Finds distance XB^2 or vector $\overrightarrow{XB}$ using a correct method. For this to be scored $X=(2,2+6\mu,3\mu)$ $B=(7,0,3)$ and there must be an attempt at differences

A1 Correct answer $h^2=\left|\overrightarrow{XB}\right|^2=45\mu^2+6\mu+38$ or $\overrightarrow{XB}=\pm(-5,2+6\mu,3\mu-3)$

M1 Find minimum value of h by completion of square or differentiation giving h =

Alternatively by the vector method uses $\overrightarrow{XB}.\begin{pmatrix}0\\6\\3\end{pmatrix}=0\Rightarrow\mu=..$ followed by substitution of this $\mu\left(=-\frac{1}{15}\right)$

into l_2 to find length of $BX=\left|\pm\begin{pmatrix}-5\\1.6\\-3.2\end{pmatrix}\right|$ using pythagoras' theorem.

A1 awrt 6.15

Leave blank

13. A curve C has parametric equations

$$x = 6\cos 2t, \qquad y = 2\sin t, \qquad -\frac{\pi}{2} < t < \frac{\pi}{2}$$

(a) Show that $\frac{dy}{dx} = \lambda \operatorname{cosec} t$, giving the exact value of the constant λ.

(4)

(b) Find an equation of the normal to C at the point where $t = \frac{\pi}{3}$

Give your answer in the form $y = mx + c$, where m and c are simplified surds.

(6)

The cartesian equation for the curve C can be written in the form

$$x = f(y), \qquad -k < y < k$$

where $f(y)$ is a polynomial in y and k is a constant.

(c) Find $f(y)$.

(3)

(d) State the value of k.

(1)

Question Number	Scheme	Marks
13 (a)	$\frac{dy}{dx}=\frac{dy/dt}{dx/dt}=\frac{2\cos t}{-12\sin 2t}$	M1
	$=\frac{2\cos t}{-24\sin t\cos t}$	dM1
	$=\frac{2\cancel{\cos t}}{-24\sin t\cancel{\cos t}}= -\frac{1}{12}\ \text{cosec}\ t$	M1 A1 [4]
(b)	When $t=\frac{\pi}{3}$, $\frac{dy}{dx}=-\frac{1}{12\times\sqrt{3}/2}=\left(-\frac{\sqrt{3}}{18}\right)$	M1 A1
	So Normal has gradient $-\frac{1}{m}=6\sqrt{3}$	M1
	When $t=\frac{\pi}{3}$, $x=-3$ and $y=\sqrt{3}$	B1
	Equation of normal is $y-\sqrt{3}=6\sqrt{3}(x+3)$ so $y=6\sqrt{3}x+19\sqrt{3}$	M1 A1 [6]
(c)	$x=6(1-2\sin^2 t)\Rightarrow x=f(y)$	M1
	So $x=6-3y^2$ or $f(y)=6-3y^2$	dM1 A1 [3]
(d)	$-2<y<2$ or $k=2$	B1 [1]
		(14 marks)
Alt (a)	Via cartesian must start with $x=A\pm By^2$ or $y=\sqrt{C\pm Dx}$ $\frac{dx}{dy}=ky$ or $\frac{dy}{dx}=b\left(2-\frac{x}{3}\right)^{-\frac{1}{2}}$ then as before followed by correct (double angle) substitution	M1 dM1
Alt (b)	Must start with $x=A\pm By^2$ or $y=\sqrt{C\pm Dx}$ $\frac{dx}{dy}=-6y$ or $\frac{dy}{dx}=-\frac{1}{6}\left(2-\frac{x}{3}\right)^{-\frac{1}{2}}$ For substituting their $y=\sqrt{3}$ into a $\frac{dx}{dy}$ of the form Py Or alternatively substituting their $x=-3$ into a $\frac{dy}{dx}$ of the form $P(Q\pm Rx)^{-\frac{1}{2}}$ For using the 'correct numerical' grad of the normal either $-\frac{dx}{dy}$ or $-\frac{1}{dy/dx}$	 1st M1 2nd M1

(a)

M1 Differentiates both x and y wrt t and establishes $\dfrac{\mathrm{d}y/\mathrm{d}t}{\mathrm{d}x/\mathrm{d}t} = \dfrac{\pm A\cos t}{\pm B\sin 2t}$

They may use any double angle formula for cos first. Condone sign slips in this formula

Eg $\cos 2t = \pm 2\cos^2 t \pm 1$ to get $\dfrac{\mathrm{d}y/\mathrm{d}t}{\mathrm{d}x/\mathrm{d}t} = \dfrac{\pm A\cos t}{\pm B\sin t\cos t}$

dM1 Correct double angle formula used $\sin 2t = 2\sin t\cos t$
In the alternative method the correct double angle formula must have been used

M1 Cancels cos$t$ and replaces 1/sin$t$ by cosec$t$ correctly achieving a form $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \lambda\mathrm{cosec}t$

A1 cao $\dfrac{\mathrm{d}y}{\mathrm{d}x} = -\dfrac{1}{12}\mathrm{cosec}t$

(b)

M1 Substitute $t = \dfrac{\pi}{3}$ into their $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \lambda\mathrm{cosec}t$

A1 $\dfrac{\mathrm{d}y}{\mathrm{d}x} = -\dfrac{1}{12\times \sqrt{3}/2}$ or exact equivalent. It may be implied by normal gradient of $6\sqrt{3}$

Accept decimals here. $\dfrac{\mathrm{d}y}{\mathrm{d}x} = -0.096$ or implied by normal gradient of 10.4

M1 Use of negative reciprocal in finding the gradient of the normal.

B1 for $x = -3$, $y = \sqrt{3}$

M1 Correct method for line equation using their **normal** gradient and their (-3, $\sqrt{3}$) allowing a sign slip on one of their coordinates.

Look for $y - y_1 = -\left.\dfrac{\mathrm{d}x}{\mathrm{d}y}\right|_{t=\frac{\pi}{3}}(x - x_1)$ or $x - x_1 = -\left.\dfrac{\mathrm{d}y}{\mathrm{d}x}\right|_{t=\frac{\pi}{3}}(y - y_1)$

If the candidate uses $y = mx + c$ they must proceed as far as c =.. for this mark

A1 cao $y = 6\sqrt{3}x + 19\sqrt{3}$

(c)

M1 Attempts to use the double angle formula $\cos 2t = \pm 1 \pm 2\sin^2 t$ leading to an equation linking x and y
If $\cos 2t = \cos^2 t - \sin^2 t$ is initially used there must be an attempt to replace the $\cos^2 t$ by $1 - \sin^2 t$

dM1 Uses correct $\cos 2t = 1 - 2\sin^2 t$ and attempts to replace sin$t$ by $\dfrac{y}{2}$ and $\cos 2t$ by $\dfrac{x}{6}$

Condone poor bracketing in cases such as $\cos 2t = 1 - 2\sin^2 t \Rightarrow \dfrac{x}{6} = 1 - 2\dfrac{y}{2}^2$

A1 Correct equation. Accept $x = 6 - 3y^2$ or $\mathrm{f}(y) = 6 - 3y^2$

(d)

B1 States $k = 2$ or writes the range of y as $-2 < y < 2$