Write your name here

Surname	Other names

Pearson Edexcel
International
Advanced Level

Centre Number	Candidate Number

Core Mathematics C34

Advanced

Monday 26 January 2015 – Afternoon
Time: 2 hours 30 minutes

Paper Reference
WMA02/01

You must have:
Mathematical Formulae and Statistical Tables (Blue)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 125.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

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1. The curve C has equation

$$y = \frac{3x-2}{(x-2)^2}, \quad x \neq 2$$

The point P on C has x coordinate 3

Find an equation of the normal to C at the point P in the form $ax + by + c = 0$, where a, b and c are integers.

(6)

Question Number	Scheme	Marks
1	$y = 7$ at point P	B1
	$y = \dfrac{3x-2}{(x-2)^2} \Rightarrow \dfrac{dy}{dx} = \dfrac{(x-2)^2 \times 3 - (3x-2)\times 2(x-2)}{(x-2)^4}$	M1A1
	Sub $x = 3$ into $\dfrac{dy}{dx} = (-11)$	M1
	$\dfrac{1}{11} = \dfrac{y-7}{x-3} \Rightarrow x - 11y + 74 = 0$ cso	M1A1 cso
		(6 marks)

B1 For seeing $y = 7$ when $x = 3$. This may be awarded if embedded within an equation.

M1 Application of Quotient rule. If the rule is quoted it must be correct.

It may be implied by their $u = 3x-2, u' = .., v = (x-2)^2, v' = ..$ followed by their $\dfrac{vu' - uv'}{v^2}$

If the rule is neither stated nor implied only accept expressions of the form $\dfrac{(x-2)^2 \times A - (3x-2)\times B(x-2)}{\left((x-2)^2\right)^2}$ $A, B > 0$ condoning missing brackets

Alternatively applies the Product rule to $(3x-2)(x-2)^{-2}$ If the rule is quoted it must be correct.

It may be implied by their u or $v = 3x-2, u', v$ or $u = (x-2)^{-2}, v'$ followed by their $vu' + uv'$

If the rule is neither stated nor implied only accept expressions of the form $A(x-2)^{-2} \pm B(3x-2)(x-2)^{-3}$

If they use partial fractions expect to see $y = \dfrac{3x-2}{(x-2)^2} \Rightarrow y = \dfrac{P}{(x-2)} + \dfrac{Q}{(x-2)^2} (P = 3, Q = 4) \Rightarrow \dfrac{dy}{dx} = \pm\dfrac{R}{(x-2)^2} \pm \dfrac{S}{(x-2)^3}$

You may also see implicit differentiation etc where the scheme is easily applied.

A1 A correct (unsimplified) form of the derivative.

Accept from the quotient rule versions equivalent to $\dfrac{dy}{dx} = \dfrac{(x-2)^2 \times 3 - (3x-2)\times 2(x-2)}{\left((x-2)^2\right)^2}$

Accept from the product rule versions equivalent to $\dfrac{dy}{dx} = 3(x-2)^{-2} - 2(3x-2)(x-2)^{-3}$

Accept from partial fractions $\dfrac{dy}{dx} = -3(x-2)^{-2} - 8(x-2)^{-3}$

or $(x-2)^2 \dfrac{dy}{dx} + y \times 2(x-2) = 3$ from implicit differentiation

FYI: Correct simplified expressions are $\dfrac{dy}{dx} = \dfrac{-3x^2 + 4x + 4}{(x-2)^4}$ or $\dfrac{-3x-2}{(x-2)^3}$

M1 Sub $x = 3$ into what they believe is their derivative to find a numerical value of $\dfrac{dy}{dx}$.

M1 Uses $x = 3$ and their numerical value of y with their numerical $-\dfrac{dx}{dy}$ at $x = 3$ to form an equation of a normal. If the form $y = mx + c$ is used then it must be a full method reaching a value for c.

A1 Correct solution only Accept $\pm A(x - 11y + 74) = 0$ where $A \in \mathbb{N}$. from correct working.

Watch for correct answers coming from incorrect versions of $\dfrac{dy}{dx}$ with eg. $(x-2)^2$ on the denominator

Leave blank

2. Solve, for $0 \leqslant \theta < 2\pi$,

$$2\cos 2\theta = 5 - 13\sin\theta$$

Give your answers in radians to 3 decimal places.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(5)

Question Number	Scheme	Marks
2	$2\cos 2\theta = 5 - 13\sin\theta \Rightarrow 4\sin^2\theta - 13\sin\theta + 3 = 0$	M1A1
	$\Rightarrow (4\sin\theta - 1)(\sin\theta - 3) = 0$	
	$\sin\theta = \frac{1}{4}$	M1
	$\theta = \text{awrt } 0.253, \quad 2.889 \text{ (3dp)}$	A1,A1 cso
		(5 marks)

M1 Uses $\cos 2\theta = 1 - 2\sin^2\theta$ to get a quadratic equation in just $\sin\theta$.

If candidate uses $\cos 2\theta = \cos^2\theta - \sin^2\theta$ *or* $2\cos^2\theta - 1$ they must use $\cos^2\theta = 1 - \sin^2\theta$ to form a quadratic equation in just $\sin\theta$ before scoring the M.

A1 $\pm(4\sin^2\theta - 13\sin\theta + 3) = 0$. The = 0 may be implied by subsequent working

M1 Solves their 3TQ in $\sin\theta$ with usual rules by factorisation, formula or completing the square. They must proceed as far as $\sin\theta = ..$ Accept an answer from a calculator. You may have to pick up a calculator to check their values.

A1 Either of $\theta = \text{awrt } 0.25, 2.89 \text{ (2dp)}$ in radians or either of $\theta = \text{awrt } 14.5, 165.5 \text{ (1dp)}$ in degrees
Accept either of awrt $0.08\pi, 0.92\pi$

A1 Correct solution with only two solutions $\theta = \text{awrt } 0.253, 2.889 \text{ (3dp)}$ within the given range.
Accept equivalents such as awrt $0.0804\pi, 0.9196\pi$
Ignore any extra answers outside the range.
Note that incorrect factorisation $(4\sin\theta - 1)(\sin\theta + 3) = 0$ would lead to correct answers. As this mark is cso, it would be withheld in such circumstances.

Leave blank

3. The function g is defined by

$$\mathrm{g} : x \mapsto |8 - 2x|, \qquad x \in \mathbb{R}, \qquad x \geqslant 0$$

(a) Sketch the graph with equation $y = \mathrm{g}(x)$, showing the coordinates of the points where the graph cuts or meets the axes.

(3)

(b) Solve the equation

$$|8 - 2x| = x + 5$$

(3)

The function f is defined by

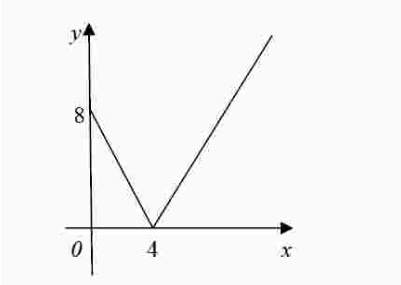
$$\mathrm{f} : x \mapsto x^2 - 3x + 1, \qquad x \in \mathbb{R}, \qquad 0 \leqslant x \leqslant 4$$

(c) Find fg(5).

(2)

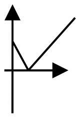
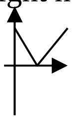
(d) Find the range of f. You must make your method clear.

(4)

Question Number	Scheme	Marks
3(a)	 V shape just in Quad 1and correct position	B1
	Meets/cuts y axis at (0,8)	B1
	Meets x axis at (4,0)	B1
		(3)
(b)	$x = 1$	B1
	$x + 5 = -(8 - 2x) \Rightarrow x = 13$	M1A1
		(3)
(c)	$\text{fg}(5) = \text{f}(2) = -1$	M1A1
		(2)
(d)	$\text{f}'(x) = 2x - 3 \Rightarrow \text{min at } x = \frac{3}{2} \Rightarrow \text{min} = -\frac{5}{4}$	M1A1
	Maximum value = 5	B1
	$-\frac{5}{4} \leqslant \text{f}(x) \leqslant 5$	A1
		(4)
		(12 marks)

(a)

B1 Accept a V shape **just in quadrant one** with the left hand end meeting the y - axis, the minimum point on the x - axis and the right hand section being at least as high as the left hand section.

Look for either  or  shape just in quadrant one. Don't accept a curved base.

B1 The graph meets **or** cuts the y axis at (0, 8) only. Allow just 8 and condone (8,0) written on the correct axis. There needs to be a graph for this to be awarded

B1 The graph meets the x axis at (4, 0) only. Allow 4 and condone (0,4) written on the correct axis There needs to be a graph for this to be awarded.

(b)

B1 For stating that $x = 1$

M1 For an attempt at the 'second' solution.
Accept $x + 5 = -(8 - 2x) \Rightarrow x = \ldots$ or $-(x+5) = 8 - 2x \Rightarrow x = \ldots$ or equivalent
Do NOT condone invisible brackets in this case
Accept $(x+5)^2 = (8-2x)^2 \Rightarrow x = ..$

A1 $x = 13$ and no other solutions (apart from $x = 1$). Accept this for both marks **as long as no incorrect working is seen.** Eg $x + 5 = -8 - 2x \Rightarrow x = 13$ is M0 A0

(c)

M1 Scored for a full method to find $\text{fg}(5)$.
Accept $x = 5$ being substituted into $|8 - 2x|$ and the result being substituted into $x^2 - 3x + 1$
Accept an attempt to substitute $x = 5$ into $(|8-2x|)^2 - 3|8-2x| + 1$
Accept for an attempt at $\text{f}(2)$ but not $\text{f}(-2)$

A1 -1 only. Accept this for both marks as long as no incorrect working is seen.

(d)

M1 An acceptable method of finding a turning point. A full method using calculus or a full method by completion of the square is acceptable. The y value must be attempted.
Using calculus look for $\text{f}'(x) = ax + b = 0 \Rightarrow x = ..$ followed by an attempt to find y.
Using completing the square look for $\left(x \pm \frac{3}{2}\right)^2 \pm \left(\frac{3}{2}\right)^2 + 1$ followed by a statement that $y = \pm\left(\frac{3}{2}\right)^2 + 1$

A1 For achieving the minimum value of $y = -\frac{5}{4}$. Award for $y > -1.25$ following the M mark

B1 For achieving the maximum value of $y = 5$.
This may be scored from an inequality. Accept $.. \leq \text{f} \leq 5$ and even $.. \leq \text{f} < 5$

A1 CSO Allow $[-1.25, 5]$ and $y \geq -1.25$ **and** $y \leq 5$
Do not allow $y \geq -1.25$ **or** $y \leq 5$ or $[-1.25, 5)$ or $-\frac{5}{4} \leq \text{f}(x) < 5$

Special case: Allow the answer from a graphical calculator as long as it is given with the evidence of a correct sketch. Allow from a table as long as the value at 1.5 is calculated. Score 4/4
Just the (correct) answer, no working, special case award 1,1,0,0

Leave blank

4. Use the substitution $x = 2\sin\theta$ to find the exact value of

$$\int_0^{\sqrt{3}} \frac{1}{(4-x^2)^{\frac{3}{2}}}\,dx$$

(7)

Question Number	Scheme	Marks
4	$x = 2\sin\theta \Rightarrow \dfrac{dx}{d\theta} = 2\cos\theta$	B1
	$\displaystyle\int \frac{1}{(4-x^2)^{3/2}}dx = \int \frac{1}{(4-4\sin^2\theta)^{3/2}} 2\cos\theta(d\theta)$	M1
	$\displaystyle = \int \frac{1}{4}\sec^2\theta(d\theta)\ OR \int \frac{1}{4}\times\frac{1}{\cos^2\theta}(d\theta)$	M1
	$= \dfrac{1}{4}\tan\theta$	dM1A1
	Uses limits 0 and $\dfrac{\pi}{3}$ in their integrated expression	
	$= \left[\dfrac{1}{4}\tan\theta\right]_0^{\frac{\pi}{3}} = \dfrac{\sqrt{3}}{4}$	M1A1
		(7 marks)

B1 States either $\dfrac{dx}{d\theta} = 2\cos\theta$ or $dx = 2\cos\theta\, d\theta$. Condone $x' = 2\cos\theta$

M1 Attempt to produce integral in just θ by substituting $x = 2\sin\theta$ and using $dx = \pm A\cos\theta\,(d\theta)$
You may condone a missing $d\theta$

M1 Uses $1-\sin^2\theta = \cos^2\theta$ and simplifies integral to $\displaystyle\int C\sec^2\theta(\,d\theta)$ or $\displaystyle\int \frac{C}{\cos^2\theta}(\,d\theta)$

Again you may condone a missing $d\theta$

dM1 Dependent upon previous M1 for $\displaystyle\int \sec^2\theta \to \tan\theta$

A1 $\dfrac{1}{4}\tan\theta(+c)$. No requirement for the $+c$

M1 Changes limits in x to limits in θ of 0 and $\dfrac{\pi}{3}$, then subtracts their integrated expression either way around. The subtraction of 0 can be implied if f (0) =0. If the candidate changes the limits to 0 and 60 (degrees) it scores M0, A0. Alternatively they could attempt to change their integrated expression in θ back to a function in x and use the original limits. Such a method would require seeing either $\cos\theta = \sqrt{1-\dfrac{x^2}{4}}$ or $\tan\theta = \dfrac{\frac{x}{2}}{\sqrt{1-\frac{x^2}{4}}}$

A1 $\dfrac{\sqrt{3}}{4}$.

Leave blank

5. (a) Use the binomial expansion, in ascending powers of x, of $\dfrac{1}{\sqrt{(1-2x)}}$ to show that

$$\frac{2+3x}{\sqrt{(1-2x)}} \approx 2 + 5x + 6x^2, \qquad |x| < 0.5$$

(4)

(b) Substitute $x = \dfrac{1}{20}$ into

$$\frac{2+3x}{\sqrt{(1-2x)}} = 2 + 5x + 6x^2$$

to obtain an approximation to $\sqrt{10}$

Give your answer as a fraction in its simplest form.

(3)

Question Number	Scheme	Marks
5 (a)	$\frac{1}{\sqrt{1-2x}} = (1-2x)^{-\frac{1}{2}} = 1 + \left(-\frac{1}{2}\right)(-2x) + \frac{\left(-\frac{1}{2}\right)\left(-\frac{1}{2}-1\right)}{2!}(-2x)^2$	M1A1
	$= 1 + x + \frac{3}{2}x^2 + \ldots$	
	$\frac{2+3x}{\sqrt{1-2x}} = (2+3x)(1 + x + \frac{3}{2}x^2 + \ldots$	M1
	$= 2 + 5x + 6x^2$	A1*
		(4)
(b)	Sub $x = \frac{1}{20}$ into both sides of $\frac{2+3x}{\sqrt{1-2x}} = 2 + 5x + 6x^2$	M1
	$\frac{43\sqrt{10}}{60} = \frac{453}{200}$ oe $\frac{43}{20} = \frac{453}{200} \times \frac{3}{\sqrt{10}}$	dM1
	$\sqrt{10} = \frac{1359}{430}$ Accept $\frac{4300}{1359}$	A1
		(3)
		(7 marks)

(a)

M1 Uses the correct form of the binomial expansion with $n=\pm\frac{1}{2}$ and $'x'=\pm 2x$ to achieve

$1+\left(\pm\frac{1}{2}\right)(\pm 2x)+\frac{\left(\pm\frac{1}{2}\right)\left(\pm\frac{1}{2}-1\right)}{2}(\pm 2x)^2\ldots$ You may condone missing/invisible brackets.

C**andidates cannot just write down the answer** $1+x+\frac{3}{2}x^2+.$

There must be an intermediate line showing some working for at least the x^2 term.

A1 Correct (unsimplified expression) $1+\left(-\frac{1}{2}\right)(-2x)+\frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2}(-2x)^2$

Condone poor notation such as $-2x^2$ for $(-2x)^2$ if it is subsequently corrected.

Evidence of this could be $1+-0.5\times -2x+\frac{-0.5\times -1.5}{2}\times -2x^2=1+x+\frac{3}{2}x^2+\ldots$

M1 An attempt to multiply their 'quadratic' binomial expansion by $(2+3x)$.

Look for at least 4 terms. If they have simplified their binomial expansion to $1+x+\frac{3}{2}x^2+.$

then it is possible to write out the final answer of $2+5x+6x^2$ from $(2+3x)(1+x+\frac{3}{2}x^2)$

This is acceptable for the final M1A1only if the quadratic expansion $1+x+\frac{3}{2}x^2$ has been simplified from an intermediate line.

A1* $2+5x+6x^2$ Correct solution only. This is a given answer and all aspects must be correct including bracketing.

(b)

M1 Sub $x=\frac{1}{20}$ into **both sides** of the given expression. Condone missing brackets.

Accept for this any equivalent to $\frac{2+3\times 0.05}{\sqrt{1-2\times 0.05}}=2.265$

dM1 For an attempt to simplify **both** sides of the expression resulting in an expression involving $\sqrt{10}$

Look for an equation of the from $\frac{a\sqrt{10}}{b}=\frac{c}{d}$ or equivalent where *a*, *b*, *c* and *d* are integers

Sight of $\frac{43\sqrt{10}}{60}$, on the left hand side and $\frac{453}{200}$ on the right hand side an example of correct work.

An alternative would be $\frac{43}{20}=\frac{453}{200}\times\frac{3}{\sqrt{10}}$ Accept mixed numbers for fractions such as $\frac{453}{200}$

A1 Accept $\sqrt{10}=\frac{1359}{430}=3\frac{69}{430}$ or by using the rationalised form $\sqrt{10}=\frac{4300}{1359}=3\frac{223}{1359}$

Leave blank

6. (i) Given $x = \tan^2 4y$, $0 < y < \frac{\pi}{8}$, find $\frac{\mathrm{d}y}{\mathrm{d}x}$ as a function of x.

Write your answer in the form $\frac{1}{A(x^p + x^q)}$, where A, p and q are constants to be found.

(5)

(ii) The volume V of a cube is increasing at a constant rate of 2 $\text{cm}^3\ \text{s}^{-1}$. Find the rate at which the length of the edge of the cube is increasing when the volume of the cube is 64 cm^3.

(5)

Question Number	Scheme	Marks
6(i)	$x=\tan^2 4y \Rightarrow \dfrac{dx}{dy}=8\tan 4y\sec^2 4y$ oe	M1A1
	$\dfrac{dy}{dx}=\dfrac{1}{8\tan 4y\sec^2 4y},=\dfrac{1}{8\tan 4y(1+\tan^2 4y)}=\dfrac{1}{8\sqrt{x}(1+x)}=\dfrac{1}{8\left(x^{0.5}+x^{1.5}\right)}$	M1,M1A1
		(5)
(ii)	$\dfrac{dV}{dt}=2,\quad V=x^3\Rightarrow\dfrac{dV}{dx}=3x^2$	B1,B1
	Uses $\dfrac{dV}{dt}=\dfrac{dV}{dx}\times\dfrac{dx}{dt}$	M1
	$\left.\dfrac{dx}{dt}\right\rvert_{x=4}=\dfrac{2}{3x^2}=\dfrac{1}{24}\left(\text{cm s}^{-1}\right)$	M1A1
		(5)
		(10 marks)

(i)

M1 Differentiates $\tan^2 4y$ to get an expression equivalent to the form $C\tan 4y\sec^2 4y$
You may see $\tan 4y\times A\sec^2 4y+\tan 4y\times B\sec^2 4y$ from the product rule or versions appearing from $\sqrt{x}=\tan 4y\Rightarrow Ax^{-0.5}\times...=B\sec^2 4y$ or
$Ax^{-0.5}=B\sec^2 4y\times...$ $x=\dfrac{\sin^2 4y}{\cos^2 4y}\Rightarrow\dfrac{dx}{dy}=\dfrac{\cos^2 4y\times A\sin 4y\cos 4y-\sin^2 4y\times B\cos 4y\sin 4y}{\left(\cos^2 4y\right)^2}$
from the quotient rule

A1 Any fully correct answer, or equivalent, including the left hand side. $\dfrac{dx}{dy}=2\tan 4y\times 4\sec^2 4y$
Also accept the equivalent by implicit differentiation $1=8\tan 4y\sec^2 4y\dfrac{dy}{dx}$

M1 Uses $\dfrac{dy}{dx}=1\Big/\dfrac{dx}{dy}$ Follow through on their $\dfrac{dx}{dy}$.
Condone issues with reciprocating the '8' but not the trigonometrical terms.
If implicit differentiation is used it is scored for writing $\dfrac{dy}{dx}$ as the subject.

M1 Uses $\sec^2 4y=1+\tan^2 4y$ where $x=\tan^2 4y$ to get their expression for $\dfrac{dy}{dx}$ or $\dfrac{dx}{dy}$ in terms of just x.
If they use other functions it is for using $\sin^2 4y=\dfrac{x}{1+x}$ and $\cos^2 4y=\dfrac{1}{1+x}$ where $x=\tan^2 4y$ to get their expression for $\dfrac{dy}{dx}$ or $\dfrac{dx}{dy}$ in terms of just x.

A1 Correct answer and solution. Accept $\dfrac{1}{8\left(x^{0.5}+x^{1.5}\right)},\dfrac{1}{8\left(x^{\frac{1}{2}}+x^{\frac{3}{2}}\right)}$ or A=8, p =0.5and q =1.5
Candidates do not have to explicitly state the values of A, p and q. Remember to isw after the sight of an acceptable answer.

Alt (i) using $y=\frac{1}{4}\arctan(\sqrt{x}) \Rightarrow \frac{dy}{dx}=\frac{1}{4}\left(\frac{1}{1+(\sqrt{x})^2}\right)\times\frac{1}{2}x^{-\frac{1}{2}}$

M1 Changes the subject of the formula to get $y=D\arctan(\sqrt{x})$ and proceeds to $\frac{dy}{dx}=\left(\frac{1}{1+(\sqrt{x})^2}\right)\times...$

A1 Achieves $y=\frac{1}{4}\arctan(\sqrt{x})$ and proceeds to $\frac{dy}{dx}=\left(\frac{1}{1+(\sqrt{x})^2}\right)\times...$

M1 Correctly proceeds to $\frac{dy}{dx}=E\left(\frac{1}{1+(\sqrt{x})^2}\right)\times x^{-\frac{1}{2}}$

M1 Writes $x^{-\frac{1}{2}}=\frac{1}{\sqrt{x}}$ and multiplies out bracket to get $\frac{dy}{dx}=E\left(\frac{1}{x^{\frac{1}{2}}+x^{\frac{3}{2}}}\right)$

A1 Correct answer and solution. Accept $\frac{1}{8(x^{0.5}+x^{1.5})},\frac{1}{8\left(x^{\frac{1}{2}}+x^{\frac{3}{2}}\right)}$

(ii)

B1 States or uses $\frac{dV}{dt}=2$. It may be awarded if embedded within the chain rule and assigned to $\frac{dV}{dt}$

B1 States or uses $\frac{dV}{dx}=3x^2$. It may be awarded if embedded within the chain rule and assigned to $\frac{dV}{dx}$ You may also see $x=V^{\frac{1}{3}} \Rightarrow \frac{dx}{dV}=\frac{1}{3}V^{-\frac{2}{3}}$

Accept any variable, for example *s*, *l*, *a* in place of *x*.

M1 Uses a correct chain rule, eg. $\frac{dV}{dt}=\frac{dV}{dx}\times\frac{dx}{dt}$ with $\frac{dV}{dt}=2$ and their value of $\frac{dV}{dx}$ OR $\frac{dx}{dV}$

You may see different versions of this. Eg $\frac{dV}{dx}=\frac{dV}{dt}\div\frac{dx}{dt}$

M1 Substitutes x=4 into their chain rule to find a numerical value for $\frac{dx}{dt}=..$.

Accept a substitution of $V=64 \Rightarrow \frac{dx}{dV}=\frac{1}{3}V^{-\frac{2}{3}}$ to find a numerical value for $\frac{dx}{dt}=..$

Condone poor notation for $\frac{dx}{dt}$ and the appearance of an answer from the substitution of x = 4 into an incorrect chain rule expression will be sufficient to award this mark.

A1 cso $\left.\frac{dx}{dt}\right|_{x=4}=\frac{1}{24}(\text{cm s}^{-1})$. Accept awrt 0.0417

Leave blank

7. (a) Given that

$$2\cos(x+30)^\circ = \sin(x-30)^\circ$$

without using a calculator, show that

$$\tan x^\circ = 3\sqrt{3} - 4$$

(5)

(b) Hence or otherwise solve, for $0 \leqslant \theta < 180$,

$$2\cos(2\theta+40)^\circ = \sin(2\theta-20)^\circ$$

Give your answers to one decimal place.

(4)

Question Number	Scheme	Marks
7(a)	$2\cos(x+30)° = \sin(x-30)°$	
	$2(\cos x°\cos 30° - \sin x°\sin 30°) = \sin x°\cos 30° - \cos x°\sin 30°$	M1A1
	$2\cos 30° - 2\tan x°\sin 30° = \tan x°\cos 30° - \sin 30°$	
	$\sin 30° = \frac{1}{2}, \quad \cos 30° = \frac{\sqrt{3}}{2}$	B1
	$\tan x° = \frac{2\sqrt{3}+1}{\sqrt{3}+2} \Rightarrow \times\frac{\sqrt{3}-2}{\sqrt{3}-2} \Rightarrow \tan x° = 3\sqrt{3}-4$	dM1A1*
		(5)
(b)	$\tan(2\theta+10)° = 3\sqrt{3}-4$	M1
	$2\theta+10 = 50.1, (230.1) \Rightarrow \theta = ..$	dM1
	$\theta = 20.1, \quad 110.1$	A1,A1
		(4)
		(9 marks)

(a)

M1 Uses identities for $\cos(A+B)$ and $\sin(A-B)$ with $A = x, B = 30$.
Condone missing bracket and incorrect signs but the terms must be correct.

A1 Fully correct equation in sin$x$ and cos$x$

B1 Replaces $\sin 30$ by $\frac{1}{2}$ and $\cos 30$ by $\frac{\sqrt{3}}{2}$ throughout their expanded equation.

If candidate divides by cos30 it will be for $\tan 30 = \frac{\sqrt{3}}{3}$ or equivalent

dM1 Either for collecting terms in sin$x$ and cos$x$ to reach $(...)\sin x = (...)\cos x$, before then dividing by cos$x$ to reach $\frac{\sin x}{\cos x} = \frac{(...)}{(...)}$ or $\tan x = \frac{(...)}{(...)}$.
Alternatively, by dividing by cos$x$ first, producing an equation in tan$x$, then collecting terms reaching $\tan x = \frac{(...)}{(...)}$
An intermediate line must be seen. $(\sqrt{3}+2)\tan x° = 2\sqrt{3}+1 \Rightarrow \tan x° = 3\sqrt{3}-4$ is dM0
Similarly $(\sqrt{3}+2)\sin x° = (2\sqrt{3}+1)\cos x \Rightarrow \tan x° = 3\sqrt{3}-4$ is dM0

A1* Reaches final answer by showing rationalisation with no errors.
Accept as a minimium $\tan x° = \frac{2\sqrt{3}+1}{\sqrt{3}+2} \times \frac{\sqrt{3}-2}{\sqrt{3}-2} \Rightarrow \tan x° = 3\sqrt{3}-4$

(b)

M1 For using part (a) to produce (or imply) an equation $\tan(2\theta \pm \alpha)° = 3\sqrt{3}-4$
Condone $\alpha = 0$ and θ being replaced by x.

dM1 Dependent upon the previous M. Score for an attempt at the correct method to find one value of θ
Look for $\tan(2\theta \pm \alpha)° = 3\sqrt{3}-4 \Rightarrow \theta = \frac{\text{invtan}\left(3\sqrt{3}-4\right) \pm \alpha}{2}$

A1 One correct answer awrt 1dp $\theta = 20.1$ or 110.1

A1 Both $\theta = 20.1$ and 110.1 awrt 1dp and no other solutions within the given range. Ignore extra solutions outside the given range.

An otherwise case for students starting again in part (b).

M1 Expands both sides (see part a) using correct identities, divides by $\cos 2\theta$ and proceeds to an equation of the form $\tan 2\theta = ...$
Note that the correct answer is $\tan 2\theta = \frac{2\cos 40 + \sin 20}{2\sin 40 + \cos 20} (= 0.84219)$

dM1 Uses correct order of operations from $\tan 2\theta = ... \Rightarrow \theta = \frac{1}{2}\arctan...$ to find at least one solution

A1A1 Follows

Correct answers without working scores B1, B1

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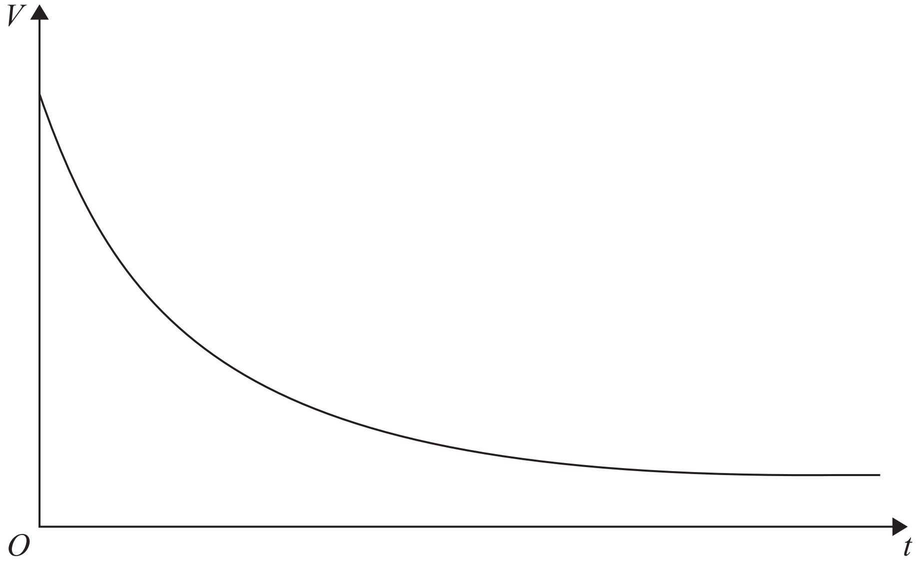

Figure 1

The value of Lin's car is modelled by the formula

$$V = 18\,000e^{-0.2t} + 4000e^{-0.1t} + 1000, \quad t \geqslant 0$$

where the value of the car is V pounds when the age of the car is t years.

A sketch of t against V is shown in Figure 1.

(a) State the range of V.

(2)

According to this model,

(b) find the rate at which the value of the car is decreasing when $t = 10$
Give your answer in pounds per year.

(3)

(c) Calculate the exact value of t when $V = 15000$

(4)

Question Number	Scheme	Marks
8 (a)	$1000 < V \le 23000$	B1,B1
		(2)
(b)	$\frac{dV}{dt} = 18000 \times -0.2e^{-0.2t} + 4000 \times -0.1e^{-0.1t}$	M1
	$\left.\frac{dV}{dt}\right\|_{t=10} = 18000 \times -0.2e^{-2} + 4000 \times -0.1e^{-1} = \text{awrt}(-)634$	M1A1
		(3)
(c)	$15000 = 18000e^{-0.2t} + 4000e^{-0.1t} + 1000$	
	$0 = 9e^{-0.2t} + 2e^{-0.1t} - 7$	
	$0 = (9e^{-0.1t} - 7)(e^{-0.1t} + 1)$	M1A1
	$9e^{-0.1t} = 7 \Rightarrow t = 10\ln\left(\frac{9}{7}\right)$ oe	dM1A1
		(4)
		(9 marks)

(a)

B1 Accept either boundary: $V < 23000$ or $V \le 23000$ or $V_{\max}\,23000$ for the upper boundary and $V > 1000$ or $V \ge 1000$ or $V_{\min}\,1000$ for the lower boundary. Answers like $V \ge 23000$ are B0

B1 Completely correct solution.
Accept $1000 < V \le 23000$, $1000 < Range$ or $y \le 23000$, (1000,23000], $V > 1000$ *and* $V \le 23000$

(b)

M1 Score for a $\frac{dV}{dt} = Ae^{-0.2t} + Be^{-0.1t}$, where $A \ne 18000,\ B \ne 4000$

M1 Sub $t = 10$ into a $\frac{dV}{dt}$ of the form $Ae^{-0.2t} + Be^{-0.1t}$ where $A \ne 18000,\ B \ne 4000$

Condone substitution of $t = 10$ into a $\frac{dV}{dt}$ of the form $Ae^{-0.2t} + Be^{-0.1t} + 1000$ $A \ne 18000,\ B \ne 4000$

A1 Correct solution and answer only. Accept ± 634 following correct $\frac{dV}{dt} = -3600e^{-0.2t} - 400e^{-0.1t}$

Watch for students who sub $t = 10$ into their V first and then differentiate. This is 0,0,0.

Watch for students who achieve +634 following $\frac{dV}{dt} = 3600e^{-0.2t} + 400e^{-0.1t}$. This is 1,1,0

A correct answer with no working can score all marks.

(c)

M1 Setting up 3TQ in $e^{\pm 0.1t}$ AND correct attempt to factorise or solve by the formula.

For this to be scored the $e^{\pm 0.2t}$ term must be the x^2 term.

A1 Correct factors $(9e^{-0.1t} - 7)(e^{-0.1t} + 1)$ or $(7e^{0.1t} - 9)(e^{0.1t} + 1)$ or a root $e^{-0.1t} = \frac{7}{9}$

dM1 Dependent upon the previous M1.
This is scored for setting the $ae^{\pm 0.1t} - b = 0$ and proceeding using correct ln work to $t = ...$

A1 $t = 10\ln\left(\frac{9}{7}\right)$. Accept alternatives such as $t = \frac{1}{0.1}\ln\left(\frac{9}{7}\right), \frac{1}{-0.1}\ln\left(\frac{7}{9}\right), -10\ln\left(\frac{7}{9}\right)$

If any extra solutions are given withhold this mark.

9.

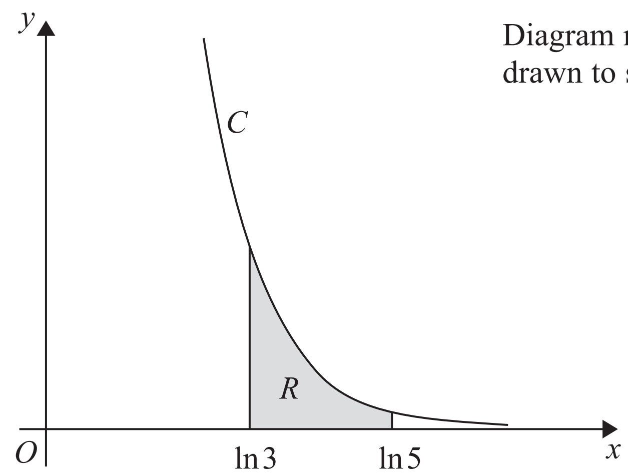

Figure 2

The curve C has parametric equations

$$x = \ln(t + 2), \quad y = \frac{4}{t^2} \qquad t > 0$$

The finite region R, shown shaded in Figure 2, is bounded by the curve C, the x-axis and the lines with equations $x = \ln 3$ and $x = \ln 5$

(a) Show that the area of R is given by the integral

$$\int_1^3 \frac{4}{t^2(t+2)}\,\mathrm{d}t$$

(3)

(b) Hence find an exact value for the area of R.

Write your answer in the form $(a + \ln b)$, where a and b are rational numbers.

(7)

(c) Find a cartesian equation of the curve C in the form $y = \mathrm{f}(x)$.

(2)

Question Number	Scheme	Marks
9 (a)	$\frac{dx}{dt}=\frac{1}{t+2}$, Area of R = $\int y\,dx=\int \frac{4}{t^2}\times\frac{1}{(t+2)}(dt)$	B1, M1
	Correct proof with limits and no errors Area = $\int_1^3 \frac{4}{t^2(t+2)}dt$	A1*
		(3)
(b)	$\frac{4}{t^2(t+2)}=\frac{A}{t}+\frac{B}{t^2}+\frac{C}{(t+2)}$ or $\frac{4}{t^2(t+2)}=\frac{A}{t^2}+\frac{B}{(t+2)}$	B1
	$4=At(t+2)+B(t+2)+Ct^2$	
	Sub $t=0\Rightarrow B=2$	
	Sub $t=-2\Rightarrow C=1$	
	Compare t^2 $A+C=0\Rightarrow A=-1$	M1A1
	$\int_1^3 \frac{4}{t^2(t+2)}dt=\int_1^3 \frac{-1}{t}+\frac{2}{t^2}+\frac{1}{(t+2)}dt=\left[-\ln t-\frac{2}{t}+\ln(t+2)\right]_1^3$	M1A1
	$=\left(-\ln 3-\frac{2}{3}+\ln 5\right)-\left(-\ln 1-\frac{2}{1}+\ln 3\right)$	
	$=\ln\left(\frac{5}{9}\right)+\frac{4}{3}$	dM1A1
		(7)
(c)	Sub $t=e^x-2$ into $y=\frac{4}{t^2}\Rightarrow y=\frac{4}{\left(e^x-2\right)^2}$, $(x>\ln 2)$	M1A1
		(2)
		(12 marks)

(a)

B1 States or implies $\frac{dx}{dt}=\frac{1}{t+2}$. Accept $dx=\frac{1}{t+2}dt$

You may award this if embedded within an integral **before the final answer** is given

For example accept Area = $\int y\,dx=\int_1^3 \frac{4}{t^2}\times\frac{1}{t+2}dt$

M1 States and uses Area = $\int y\,\mathrm{d}x$ with the y, the dx and the $\int$ sign and replaces both y and dx by functions of t.

Alternatively states and uses Area = $\int y\frac{\mathrm{d}x}{\mathrm{d}t}(\mathrm{d}t)$ with the y, the $\frac{\mathrm{d}x}{\mathrm{d}t}$ and the $\int$ sign and replaces both y and $\frac{\mathrm{d}x}{\mathrm{d}t}$ by functions of t. There is no need for limits and you can award even if there is a lack of a dt

A1* Correct proof with no errors or omissions **on any line for the integrand** and there must be a dt in all integrals in t. The limits need only be correct on the final line and they may have just been written in. The two separate fractions must be combined into a single fraction

(b)

B1 Scored for use of partial fractions. Accept the correct form $\left(\frac{4}{t^2(t+2)}\right)=\frac{A}{t}+\frac{B}{t^2}+\frac{C}{(t+2)}$ but also award for the form $\left(\frac{4}{t^2(t+2)}\right)=\frac{A}{t^2}+\frac{B}{(t+2)}$

M1 Substitute values of t and/or use inspection to determine A, B and C from a form equivalent to $4=At(t+2)+B(t+2)+Ct^2$. **The partial fraction must be of the correct form**

A1 For $\frac{4}{t^2(t+2)}=\frac{-1}{t}+\frac{2}{t^2}+\frac{1}{(t+2)}$.

M1 $\int \frac{A}{t^2}+\frac{C}{(t+2)}\mathrm{d}t=\frac{..}{t}+..\ln(t+2)$

Note this can be scored from an incorrect assumption that $\frac{4}{t^2(t+2)}=\frac{A}{t^2}+\frac{B}{(t+2)}$

A1 $\int \frac{4}{t^2(t+2)}\mathrm{d}t=\left[-\ln t-\frac{2}{t}+\ln(t+2)+(c)\right]$ There is no need to consider limits.

dM1 Dependent upon previous M. Sub in limits, subtracts either way around **and** uses a correct log law at least once to get expression of the form $a+\ln b$.

A1 Correct solution only $=\ln\left(\frac{5}{9}\right)+\frac{4}{3}$

(c)

M1 Rearranges $x=\ln(t+2)$ to reach $t=\mathrm{e}^x\pm 2$ and sub in $y=\frac{4}{t^2}$ to get y in terms of x

Alternatively substitutes $t=\sqrt{\frac{4}{y}}$ or equivalent into $x=\ln(t+2)$ and attempts to rearrange to $y=..$

A1 $y=\frac{4}{\left(e^x-2\right)^2}$. Remember to isw.

You can ignore any reference to the domain, $x>\ln 2$, for this mark.

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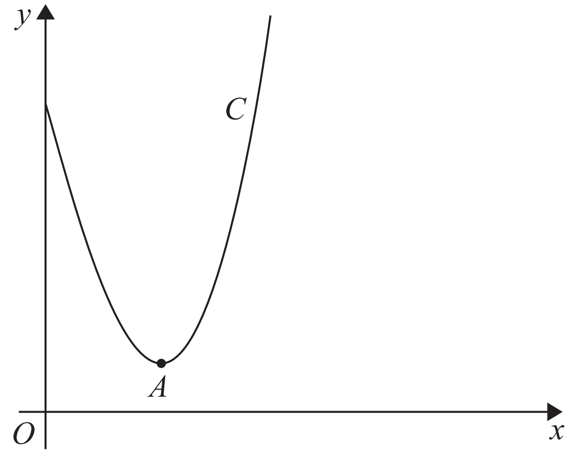


Figure 3

Figure 3 shows a sketch of part of the curve C with equation

$$y = \frac{x^2 \ln x}{3} - 2x + 4, \quad x > 0$$

Point A is the minimum turning point on the curve.

(a) Show, by using calculus, that the x coordinate of point A is a solution of

$$x = \frac{6}{1 + \ln(x^2)}$$

(5)

(b) Starting with $x_0 = 2.27$, use the iteration

$$x_{n+1} = \frac{6}{1 + \ln(x_n^{\ 2})}$$

to calculate the values of x_1, x_2 and x_3, giving your answers to 3 decimal places.

(3)

(c) Use your answer to part (b) to deduce the coordinates of point A to one decimal place.

(2)

P 4 5 0 5 8 A 0 3 2 4 8

Question Number	Scheme	Marks
10(a)	$y=\dfrac{x^2\ln x}{3}-2x+4\Rightarrow\dfrac{dy}{dx}=\underbrace{\dfrac{2x\ln x}{3}+\dfrac{x^2}{3x}},\underbrace{-2}$	M1A1, B1
	$\dfrac{2x\ln x}{3}+\dfrac{x^2}{3x}-2=0\Rightarrow x(2\ln x+1)=6\Rightarrow x=..$	dM1
	$\Rightarrow x=\dfrac{6}{1+\ln x^2}$	A1*
		(5)
(b)	$x_1=\dfrac{6}{1+\ln(2.27^2)}=$ awrt 2.273	M1A1
	x_2 = awrt 2.271 and x_3 =awrt 2.273	A1
		(3)
(c)	A= (2.3, 0.9)	M1 A1
		(2)
		(10 marks)

(a)

M1 Applying the product rule to $x^2 \ln x$ or multiples of it such as $\frac{x^2 \ln x}{3}$ and even $\frac{x^2}{3} \times \frac{\ln x}{3}$

If the rule is quoted it must be correct. It may be implied by, for example,

$u = \frac{x^2}{3}, v = \ln x, u' = .., v' = ..$ followed by their $vu' + uv'$

If it is not quoted nor implied only accept expressions of the form $Ax \ln x + Bx^2 \times \frac{1}{x}$

A1 A correct (unsimplified) derivative for $\frac{x^2 \ln x}{3} \rightarrow \frac{2x \ln x}{3} + \frac{x^2}{3x}$

B1 The derivative of the $-2x+4$ term is seen or implied to be -2

dM1 Dependent upon the previous M being scored. It is for setting their $\frac{dy}{dx} = 0$, taking out a common factor of x and proceeding to $x = ..$ Alternatively they could state that $\frac{dy}{dx} = 0$ and write out a line from their derivative equivalent to $\frac{2x \ln x}{3} + \frac{x^2}{3x} = 2$

A1* Correct solution only $x = \frac{6}{1 + \ln x^2}$. Note that this is a given answer.

All aspects need to be correct. $2 \ln x + 1 \Rightarrow \ln x^2 + 1$ may just be stated

Note: If the candidate multiplies by 3 to get $3y = x^2 \ln x - 6x + 12 \Rightarrow 3\frac{dy}{dx} = 2x \ln x + x - 6$ before setting $\frac{dy}{dx} = 0$ they can score all marks if they proceed to the given answer.

If they multiply by 3 and leave the subject as y (or perhaps ignore the lhs) they can score a special case 1 0 1 1 0 for $2x \ln x + x - 6 = 0 \Rightarrow x(2 \ln x + 1) = 6 \Rightarrow x = \frac{6}{(2 \ln x + 1)} \Rightarrow x = \frac{6}{(\ln x^2 + 1)}$

(b)

M1 Attempts $\frac{6}{1 + \ln 2.27^2}$. Awrt 2.273 implies this method

A1 x_1 = awrt 2.273. The subscript is not important. Mark as the first value given.

A1 x_2 =awrt 2.271(3dp) **and** x_3 = awrt 2.273 (3dp)

(c)

M1 Deduces the x coordinate of A is 2.3. The sight of 2.3 is sufficient to award this as long as their values in (b) round to this.

Alternatively uses their (rounded) answer from part (b) and substitutes it into equation for y to find the y coordinate of A. In a similar way to that of the x coordinate, the sight of 0.9 would be sufficient evidence for this award.

A1 (2.3, 0.9). Accept x =2.3, y =0.9

Leave blank

11. With respect to a fixed origin O the lines l_1 and l_2 are given by the equations

$$l_1\text{: } \mathbf{r} = \begin{pmatrix} 14 \\ -6 \\ -13 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix} \qquad l_2\text{: } \mathbf{r} = \begin{pmatrix} p \\ -7 \\ 4 \end{pmatrix} + \mu \begin{pmatrix} q \\ 2 \\ 1 \end{pmatrix}$$

where λ and μ are scalar parameters and p and q are constants.

Given that l_1 and l_2 are perpendicular,

(a) show that $q = 3$

(2)

Given further that l_1 and l_2 intersect at point X,

find

(b) the value of p,

(5)

(c) the coordinates of X.

(2)

The point A lies on l_1 and has position vector $\begin{pmatrix} 6 \\ -2 \\ 3 \end{pmatrix}$

Given that point B also lies on l_1 and that $AB = 2AX$

(d) find the two possible position vectors of B.

(3)

Question Number	Scheme	Marks
11 (a)	$\begin{pmatrix}-2\\1\\4\end{pmatrix}.\begin{pmatrix}q\\2\\1\end{pmatrix}=-2\times q+1\times 2+4\times 1=0\Rightarrow q=3$	M1A1*
		(2)
(b)	Equate the y and z coordinates	
	$\begin{pmatrix}14\\-6\\-13\end{pmatrix}+\lambda\begin{pmatrix}-2\\1\\4\end{pmatrix}=\begin{pmatrix}p\\-7\\4\end{pmatrix}+\mu\begin{pmatrix}3\\2\\1\end{pmatrix}\Rightarrow \begin{matrix}-6+1\lambda=-7+2\mu\\ -13+4\lambda=4+1\mu\end{matrix}$	M1
	Full method to find either λ or μ	dM1
	(2) $\Rightarrow \lambda=-1+2\mu$ subbed into (3) $-13+8\mu-4=4+\mu\Rightarrow \mu=3$	
	Sub $\mu=3$ into (2) $\Rightarrow -6+1\lambda=-7+2\times 3\Rightarrow \lambda=5$	A1either
	Sub values back into x coordinates $14-2\times 5=p+3\times 3\Rightarrow p=-5$	ddM1 A1
		(5)
(c)	Point of intersection is $\begin{pmatrix}14\\-6\\-13\end{pmatrix}+5\begin{pmatrix}-2\\1\\4\end{pmatrix}$ OR $\begin{pmatrix}'-5'\\-7\\4\end{pmatrix}+3\times\begin{pmatrix}3\\2\\1\end{pmatrix}=(4,-1,7)$	M1,A1
		(2)
(d)	$\overrightarrow{AX}=\begin{pmatrix}-2\\1\\4\end{pmatrix}$ $\quad\overrightarrow{OB}=\overrightarrow{OA}\pm 2\overrightarrow{AX}=\begin{pmatrix}6\\-2\\3\end{pmatrix}\pm 2\times\begin{pmatrix}-2\\1\\4\end{pmatrix}$	M1 either
	$\overrightarrow{OB}=\begin{pmatrix}2\\0\\11\end{pmatrix}$ $\quad\overrightarrow{OB}=\begin{pmatrix}10\\-4\\-5\end{pmatrix}$	A1A1
		(3) **(12 marks)**

(a)

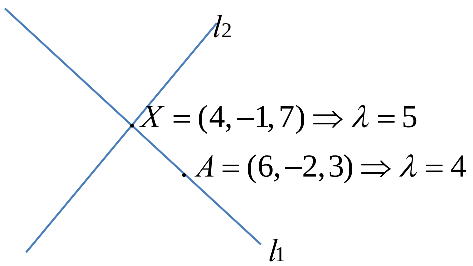
M1 Attempts to find a solution for q by setting the scalar product of the direction vectors = 0
Condone one sign error in $-2\times q+1\times 2+4\times 1=0$ leading to $q=..$
Alternatively set $q=3$ and attempt the scalar product

A1* $q=3$. This is a given answer.
In the alternative, there must be a statement (=0) and a conclusion, (hence true/hence perpendicular)

(b)

M1 Equate y and z coordinates. Condone sign errors
dM1 Dependent upon the previous M. Scored for a full method to find either λ or μ
A1 Either $\lambda = 5$ or $\mu = 3$
ddM1 Dependent upon both previous M's.
Either uses both of their values for λ and μ in the equation for the x coordinates in order to find a numerical value for p. Condone sign slips. Look for $14 - 2\lambda = p + 3\mu$ with their λ and μ leading to a solution of p.
Alternatively uses the value of one variable, expresses the other variable in terms of this and substitutes both in the equation for the x coordinates in order to find a numerical value for p.
A1 $p = -5$
(c)
M1 Uses their value of λ in l_1, or their values of μ, p and q in l_2 to find the coordinate of X
A1 Coordinates of $X = (4, -1, 7)$. Accept in vector form $\overrightarrow{OX} = \begin{pmatrix} 4 \\ -1 \\ 7 \end{pmatrix}$

(d)
M1 Uses either $\overrightarrow{OB} = \overrightarrow{OA} \pm 2\overrightarrow{AX}$ $\overrightarrow{AX} = \pm\begin{pmatrix} 6-'4' \\ -2-'-1' \\ 3-'7' \end{pmatrix}$ with an attempt to find one possible value of vector B
Or uses mid points. Sight of (a, b, c) appearing from solving $\frac{6+a}{2} = 4, \frac{-2+b}{2} = -1, \frac{3+c}{2} = 7$
Or attempts to find the value of one value of λ and uses it correctly to find one position of B
You may see $|\lambda_B - \lambda_A| = 2 \times |\lambda_A - \lambda_X|$ or a version of
$\sqrt{(14-2\lambda-6)^2 + (-6+\lambda+2)^2 + (-13+4\lambda-3)^2} = 2 \times \sqrt{(6-'4')^2 + (-2-'-1)^2 + (3-7)^2} \Rightarrow \lambda = ..$
followed by λ being substituted into line l_1
A more simple version of this could be using a diagram and deducing that $\lambda = 2$ or 6 followed by λ being substituted into line l_1



A1 Gives one possible vector $\overrightarrow{OB} = \begin{pmatrix} 2 \\ 0 \\ 11 \end{pmatrix} = (2i + 11k)$ or $\overrightarrow{OB} = \begin{pmatrix} 10 \\ -4 \\ -5 \end{pmatrix} = 10i - 4j - 5k$
A1 Gives both possible position vectors or coordinates

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12.

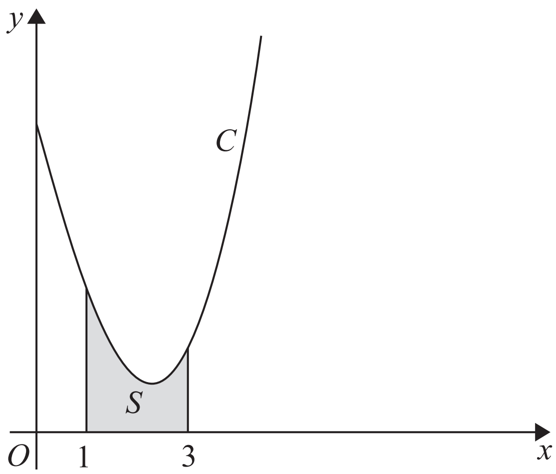


Figure 4

Figure 4 shows a sketch of part of the curve C with equation

$$y = \frac{x^2 \ln x}{3} - 2x + 4, \quad x > 0$$

The finite region S, shown shaded in Figure 4, is bounded by the curve C, the x-axis and the lines with equations $x = 1$ and $x = 3$

(a) Complete the table below with the value of y corresponding to $x = 2$. Give your answer to 4 decimal places.

x	1	1.5	2	2.5	3
y	2	1.3041		0.9089	1.2958

(1)

(b) Use the trapezium rule, with all the values of y in the completed table, to obtain an estimate for the area of S, giving your answer to 3 decimal places.

(3)

(c) Use calculus to find the exact area of S.

Give your answer in the form $\frac{a}{b} + \ln c$, where a, b and c are integers.

(6)

(d) Hence calculate the percentage error in using your answer to part (b) to estimate the area of S. Give your answer to one decimal place.

(2)

(e) Explain how the trapezium rule could be used to obtain a more accurate estimate for the area of S.

(1)

Question Number	Scheme	Marks
12(a)	0.9242 exactly	B1
		(1)
(b)	Strip width =0.5	B1
	Area $\approx \frac{0.5}{2}\left(\left(2+1.2958+2\times(1.3041+'0.9242'+0.9089\right)\right)$	M1
	=2.393	A1
		(3)
(c)	$\int \frac{x^2 \ln x}{3} - 2x + 4\,dx$	
	$= \frac{x^3}{9}\ln x - \int \frac{x^3}{9}\times\frac{1}{x}dx, \quad -x^2+4x$	M1A1, B1
	$= \frac{x^3}{9}\ln x - \frac{x^3}{27} \quad \left(-x^2+4x\right)$	A1
	Area = $\left[\frac{x^3}{9}\ln x - \frac{x^3}{27} - x^2 + 4x\right]_1^3 = \left(3\ln 3 - 1 - 9 + 12\right) - \left(-\frac{1}{27} - 1 + 4\right)$	dM1
	$= \ln 27 - \frac{26}{27}$	A1
		(6)
(d)	% error = $\pm\frac{\lvert real - approx\rvert}{real}\times 100$ =Accept awrt ±2.6%	M1A1
		(2)
(e)	Increase the number of 'strips'	B1
		(1)
		(13 marks)

(a)
B1 0.9242 exactly either in the table or within the trapezium rule in part (b)

(b)
B1 Uses a strip width of 0.5 or equivalent.
M1 Uses the correct form of the trapezium rule, a form of which appears in the formula booklet.
Look for $\frac{....}{2}\left(\left(2+1.2958+2\times(1.3041+\text{their } 0.9242+0.9089\right)\right)$
Accept for this the sum of four trapezia
A1 Awrt 2.393 (3dp)

(c)

M1 Uses integration by parts, the correct way around.

Accept integration on either $x^2 \ln x$ or multiples like $\frac{x^2 \ln x}{3}$ or even $\frac{x^2}{3} \times \frac{\ln x}{3}$

Accept as evidence an expression of the form $px^3 \ln x - \int \frac{qx^3}{x} \,(\mathrm{d}x)$

A1 Using integration by parts to arrive at an intermediate form $\frac{x^3}{k} \ln x - \int \frac{x^3}{k} \times \frac{1}{x} (\mathrm{d}x)$ where k most likely will be a multiple of 3.

B1 Integrates the $-2x+4$ to $-x^2+4x+(c)$. Ignore any constants.

Watch for candidates who take out a common factor $\frac{1}{3}(..-6x+12)$ to $\frac{1}{3}\left(...-3x^2+12x+(c)\right)$

A1 The correct integral for $\frac{x^2 \ln x}{3}$ Accept equivalent expressions to $\frac{x^3}{9} \ln x - \frac{x^3}{27} + (c)$

This is independent of the integral for the $-2x+4$ term.

dM1 Dependent upon the M mark – it is for substituting in both x=3 and x=1 and subtracting (either way around).

A1 Correct solution only $= \ln 27 - \frac{26}{27}$. The answer must be in this form and $3\ln 3 \rightarrow \ln 27$

(d)

M1 Uses their answer obtained by integration in part c and their answer obtained by the trapezium rule in part b and calculates $\pm \frac{|c-b|}{c}$

A1 Accept awrt ±2.6%.

(e)

B1 Makes a reference to increasing the number of strips. Accept decrease the width of the strips, use more trapezia Be generous with statements like more values or strips as the intention is clear. Also accept more x's, more y's but don't accept use more decimal places.

Leave blank

13. (a) Express $10\cos\theta - 3\sin\theta$ in the form $R\cos(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < 90°$

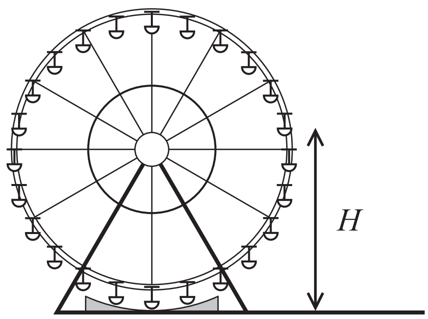
Give the exact value of R and give the value of α to 2 decimal places.

(3)

Alana models the height above the ground of a passenger on a Ferris wheel by the equation

$$H = 12 - 10\cos(30t)° + 3\sin(30t)°$$

where the height of the passenger above the ground is H metres at time t minutes after the wheel starts turning.



(b) Calculate

(i) the maximum value of H predicted by this model,

(ii) the value of t when this maximum first occurs.

Give each answer to 2 decimal places.

(4)

(c) Calculate the value of t when the passenger is 18 m above the ground for the first time.

Give your answer to 2 decimal places.

(4)

(d) Determine the time taken for the Ferris wheel to complete two revolutions.

(2)

Question Number	Scheme	Marks
13(a)	$R=\sqrt{109}$	B1
	$\tan\alpha=\frac{3}{10}\Rightarrow\alpha=\text{awrt}16.70°$	M1A1
		(3)
(b)(i)	Max height = 12+ ‘$\sqrt{109}$ ’= 22.44 m	M1A1
(ii)	Occurs when $30t+'16.70'=180\Rightarrow t=5.44$	M1A1
		(4)
(c)	$18=12-\sqrt{109}\cos(30t+16.70)\Rightarrow\cos(30t+16.70)=-\frac{6}{\sqrt{109}}\quad(-0.57..)$	M1A1
	$\Rightarrow 30t+16.70=\arccos\left(-\frac{6}{\sqrt{109}}\right)\Rightarrow t=..$	dM1
	$t=\text{awrt }3.61\,(\text{2dp})$	A1
		(4)
(d)	Attempting $30t=360\Rightarrow t=..$ or $30t=720\Rightarrow t=..$	M1
	2 revolutions in 24 **minutes**	A1
		(2)
		(13 marks)

(a)

B1 Accept $R = \pm\sqrt{109}$. Remember to isw after a correct answer. Eg $R = \sqrt{109} = 10.....$

M1 For $\tan\alpha = \pm\frac{3}{10}$ or $\tan\alpha = \pm\frac{10}{3}$. If R is used to find α , only accept $\cos\alpha = \pm\frac{10}{'R'}$ or $\sin\alpha = \pm\frac{3}{'R'}$

A1 $\alpha = \text{awrt}\,16.70°$ (2dp). Condone $\alpha = 16.7°$

Note that the answer of $\alpha = \text{awrt}\,0.29$ radians scores A0.

(b)(i)

M1 For 12 +their R

A1 Awrt 22.44 m. Accept 12+ $\sqrt{109}$

(b) (ii)

M1 For arriving at a solution for t from $30t \pm '16.70' = 180 \Rightarrow t = ..$

If radians were used in part a then accept $30t \pm '0.29' = \pi \Rightarrow t = ..$

A1 $t = \text{awrt}\,5.44$ only.

If multiple solutions are found, 5.44 must be referred to as their 'chosen' solution

....

Answers from calculus will be rare. They can be scored as follows.

From the original function:

For (b)(ii) M1 $\frac{dH}{dt} = 0 \to 30t = 180 - '\arctan\left(\frac{3}{10}\right)' \Rightarrow t = ..$ A1 $t = \text{awrt}\,5.44$ only

(b)(i) M1 Sub their $t = \text{awrt}\,5.44$ obtained from $\frac{dH}{dt} = 0$ A1 Awrt 22.44 m

From the adapted function:

For (b)(ii) M1 $\frac{dH}{dt} = 0 \to (30t + '16.70') = 180 \Rightarrow t = ..$ A1 $t = \text{awrt}\,5.44$ only

(b)(i) M1 Sub their $t = \text{awrt}\,5.44$ obtained from $\frac{dH}{dt} = 0$ A1 Awrt 22.44 m

....

(c)

M1 Attempts to substitute $H = 18$ into $H = 12 - 10\cos 30t + 3\sin 30t$ and use their answer to part (a) to proceed to $\cos(30t \pm their\,'16.70') = ...$

A1 $\cos(30t + their\,'16.70') = -\frac{6}{\sqrt{109}}$ or awrt -0.57 . It may be implied by $30t + '16.70' = awrt125°$

dM1 Dependent upon previous M. Score for $\cos(30t \pm '16.70') = -... \Rightarrow t = ..$

The cos(..) must be negative, the order of operations must be seen to be correct with the 'invcos' being attempted first and the second quadrant must be chosen for their calculation.

A1 $t = 3.61$.

The answer with no incorrect working scores all 4 marks.

If multiple solutions are found, 3.61 must be referred to as their 'chosen' solution

(d)

M1 Attempting $30t = 360 \Rightarrow t = ..$ or $30t = 720 \Rightarrow t = ..$

A1 24 minutes. **Both** 24 **and** minutes are required.