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Surname	Other names

Pearson Edexcel
International
Advanced Level

Centre Number ☐☐☐☐☐

Candidate Number ☐☐☐☐

Core Mathematics C34

Advanced

Monday 16 June 2014 – Morning
Time: 2 hours 30 minutes

Paper Reference
WMA02/01

You must have:
Mathematical Formulae and Statistical Tables (Blue)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 125.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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PEARSON

1. $$\mathrm{f}(x) = 2x^3 + x - 10$$

(a) Show that the equation $\mathrm{f}(x) = 0$ has a root α in the interval $[1.5, 2]$
(2)

The only real root of $\mathrm{f}(x) = 0$ is α

The iterative formula

$$x_{n+1} = \left(5 - \frac{1}{2}x_n\right)^{\frac{1}{3}}, \quad x_0 = 1.5$$

can be used to find an approximate value for α

(b) Calculate x_1, x_2 and x_3, giving your answers to 4 decimal places.
(3)

(c) By choosing a suitable interval, show that $\alpha = 1.6126$ correct to 4 decimal places.
(2)

Question Number	Scheme	Marks
1. (a)	$\mathrm{f}(1.5) = -1.75$, $\mathrm{f}(2) = 8$	M1
	Sign change (and $\mathrm{f}(x)$ is continuous) therefore there is a root α {lies in the interval $[1.5,\ 2]$}	A1
		[2]
(b)	$x_1 = \left(5 - \frac{1}{2}(1.5)\right)^{\frac{1}{3}}$	M1
	$x_1 = 1.6198$, $x_1 = 1.6198$ cao	A1cao
	$x_2 = 1.612159576...$, $x_3 = 1.612649754...$ $x_2 =$ awrt 1.6122 and $x_3 =$ awrt 1.6126	A1
		[3]
(c)	$\mathrm{f}(1.61255) = -0.001166022687...$, $\mathrm{f}(1.61265) = 0.0004942645692...$	
	Sign change (and as $\mathrm{f}(x)$ is continuous) therefore a root α lies in the interval $[1.61255,\ 1.61265] \Rightarrow \alpha = 1.6126$ (4 dp)	M1A1
		[2] **7**

Notes

(a) **M1:** Attempts to evaluate both $\mathrm{f}(1.5)$ and $\mathrm{f}(2)$ and finds at **least one** of $\mathrm{f}(1.5) =$ awrt -1.8 or truncated -1.7 **or** $\mathrm{f}(2) = 8$ **Must be using this interval or a sub interval e.g.[1.55, 1.95] not interval which goes outside the given interval such as [1.6, 2.1]**

A1: **both** $\mathrm{f}(1.5) =$ awrt -1.8 or truncated -1.7 **and** $\mathrm{f}(2) = 8$, **states sign change** { or f(1.5) < 0 < f(2) or f(1.5) f(2) < 0 } or f(1.5) <0 and f(2) >0; **and conclusion** e.g. therefore a root α [lies in the interval $[1.5,\ 2]$]or "so result shown" or qed or "tick" etc…

(b) **M1:** An attempt to substitute $x_0 = 1.5$ into the iterative formula

e.g. see $\left(5 - \frac{1}{2}(1.5)\right)^{\frac{1}{3}}$. Or can be implied by $x_1 =$ awrt 1.6

A1: $x_1 = 1.6198$ This **exact answer to 4 decimal** places is required for this mark

A1: $x_2 =$ awrt 1.6122 **and** $x_3 =$ awrt 1.6126 (so e.g. 1.61216 and 1.6126498 would be acceptable here)

(c) **M1:** Choose suitable interval for x, e.g. $[1.61255,\ 1.61265]$ and at least one attempt to evaluate $\mathrm{f}(x)$.
A minority of candidate may choose a tighter range which should include1.61262 (alpha to 5dp),
e.g. $[1.61259,\ 1.61263]$ This would be acceptable for both marks, provided the conditions for the A mark are met.

A1: needs (i) both evaluations correct to 1 sf, (either rounded or truncated) e.g. -0.001 and 0.0005 or 0.0004
(ii) sign change stated and
(iii)some form of conclusion which may be :
$\Rightarrow \alpha = 1.6126$ or "so result shown" or qed or tick or equivalent

N.B. f(1.61264)=0.0003 (to 1 sf)

2. A curve C has the equation

$$x^3 - 3xy - x + y^3 - 11 = 0$$

Find an equation of the tangent to C at the point (2, –1), giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(6)

Question Number	Scheme	Marks
2.	$\underline{3x^2} - \left(\underline{\underline{3y + 3x\frac{\mathrm{d}y}{\mathrm{d}x}}}\right) \underline{-1 + 3y^2\frac{\mathrm{d}y}{\mathrm{d}x}} = \underline{0}$	M1 $\underline{\text{A1}}$ $\underline{\underline{\text{M1}}}$
	$\left\{\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3x^2 - 3y - 1}{3x - 3y^2}\right\}$ *not necessarily required.*	
	At $(2, -1)$, $\mathrm{m}(\mathbf{T}) = \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3(2)^2 - 3(-1) - 1}{3(2) - 3(-1)^2}$ $\left\{= \frac{14}{3}\right\}$	M1
	T: $y - -1 = \frac{14}{3}(x - 2)$	dM1
	T: $14x - 3y - 31 = 0$ or equivalent	A1
		[6] **6**

Notes

1st M1: Differentiates implicitly to include either $\pm ky^2\frac{\mathrm{d}y}{\mathrm{d}x}$ **or** $\pm 3x\frac{\mathrm{d}y}{\mathrm{d}x}$.

(Ignore $\left(\frac{\mathrm{d}y}{\mathrm{d}x} = \right)$ at start and omission of = 0 at end.)

1st A1: $x^3 \rightarrow \underline{3x^2}$ **and** $-x + y^3 - 11 \rightarrow \underline{-1 + 3y^2\frac{\mathrm{d}y}{\mathrm{d}x}}$ (so the -11 should have gone) **and** = 0 needed **here or implied by further work**. Ignore $\left(\frac{\mathrm{d}y}{\mathrm{d}x} = \right)$ at start.

2nd M1: An attempt to apply the product rule: $-3xy \rightarrow -\left(3y + 3x\frac{\mathrm{d}y}{\mathrm{d}x}\right)$ or $\pm 3y \pm 3x\frac{\mathrm{d}y}{\mathrm{d}x}$ o.e.

3rd M1: Correct method to collect **two (not three)** dy/dx terms and to evaluate the gradient at $x = 2$ $y = -1$ (This stage may imply the earlier "=0")

4th dM1: This is dependent on all previous method marks

Uses line equation with their $\frac{14}{3}$. May use $y = \frac{14}{3}x + c$ and attempt to evaluate c by substituting $x = 2$ and $y = -1$.
(May be implied by correct answer)

2nd A1: Any positive or negative whole number multiple of $14x - 3y - 31 = 0$ is acceptable. Must have = 0.

N.B. If anyone attempts the question using $\frac{\mathrm{d}x}{\mathrm{d}y}$ instead of $\frac{\mathrm{d}y}{\mathrm{d}x}$, please send to review

Leave blank

3. Given that

$$y = \frac{\cos 2\theta}{1 + \sin 2\theta}, \qquad -\frac{\pi}{4} < \theta < \frac{3\pi}{4}$$

show that

$$\frac{\mathrm{d}y}{\mathrm{d}\theta} = \frac{a}{1 + \sin 2\theta}, \qquad -\frac{\pi}{4} < \theta < \frac{3\pi}{4}$$

where a is a constant to be determined.

(4)

Question Number	Scheme		Marks
	Apply quotient rule :	Or apply product rule to $y = \cos 2\theta(1+\sin 2\theta)^{-1}$	
3.	$\left\{\begin{matrix} u = \cos 2\theta & v = 1+\sin 2\theta \\ \frac{du}{d\theta} = -2\sin 2\theta & \frac{dv}{d\theta} = 2\cos 2\theta \end{matrix}\right\}$	$\left\{\begin{matrix} u = \cos 2\theta & v = (1+\sin 2\theta)^{-1} \\ \frac{du}{d\theta} = -2\sin 2\theta & \frac{dv}{d\theta} = -2\cos 2\theta(1+\sin 2\theta)^{-2} \end{matrix}\right\}$	
	$\frac{dy}{d\theta} = \frac{-2\sin 2\theta(1+\sin 2\theta) - 2\cos^2 2\theta}{(1+\sin 2\theta)^2}$	$-2(1+\sin 2\theta)^{-1}\sin 2\theta - 2\cos^2 2\theta(1+\sin 2\theta)^{-2}$	M1 A1
	$= \frac{-2\sin 2\theta - 2\sin^2 2\theta - 2\cos^2 2\theta}{(1+\sin 2\theta)^2}$	$= (1+\sin 2\theta)^{-2}\{-2\sin 2\theta - 2\sin^2 2\theta - 2\cos^2 2\theta$	
	$= \frac{-2\sin 2\theta - 2}{(1+\sin 2\theta)^2}$	$= (1+\sin 2\theta)^{-2}\{-2\sin 2\theta - 2\}$	M1
	$= \frac{-2(1+\sin 2\theta)}{(1+\sin 2\theta)^2} = \frac{-2}{1+\sin 2\theta}$		A1 cso
			[4] **4**

Notes

M1: **Applies the Quotient rule**, a form of which appears in the formula book, to $\frac{\cos 2\theta}{1+\sin 2\theta}$

If the formula is quoted it must be correct. There must have been some attempt to differentiate both terms. If the rule is not quoted nor implied by their working, meaning that terms are written out $u = \cos 2\theta,\ v = 1+\sin 2\theta,\ u' = ..,\ v' = ...$ followed by their $\frac{vu' - uv'}{v^2}$, then only accept answers of the form $\frac{(1+\sin 2\theta)A\sin 2\theta - \cos 2\theta \times (B\cos 2\theta)}{(1+\sin 2\theta)^2}$ where A and B are constant (could be 1) Condone "invisible" brackets for the M mark. If double angle formulae are used give marks for correct work.

Alternatively applies the product rule with $u = \cos 2\theta,\ v = (1+\sin 2\theta)^{-1}$

If the formula is quoted it must be correct. There must have been some attempt to differentiate both terms. If the rule is not quoted nor implied by their working, meaning that terms are written out $u = \cos 2\theta,\ v = (1+\sin 2\theta)^{-1},\ u' = ..,\ v' = ...$ followed by their $vu' + uv'$,

then only accept answers of the form $(1+\sin 2\theta)^{-1} \times A\sin 2\theta \pm \cos 2\theta \times (1+\sin 2\theta)^{-2} \times B\cos 2\theta$.

Condone "invisible brackets" for the M. If double angle formulae are used give marks for correct work.

A1: Any fully correct (unsimplified) form of $\frac{dy}{d\theta}$ If double angle formulae are used give marks for correct work.

Accept versions of $\frac{dy}{d\theta} = \frac{-2\sin 2\theta(1+\sin 2\theta) - 2\cos^2 2\theta}{(1+\sin 2\theta)^2}$ for use of the quotient rule or versions of

$\frac{dy}{d\theta} = (1+\sin 2\theta)^{-1} \times -2\sin 2\theta + \cos 2\theta \times (-1) \times (1+\sin 2\theta)^{-2} \times 2\cos 2\theta$ for use of the product rule.

M1: **Applies** $\sin^2 2\theta + \cos^2 2\theta \equiv 1$ or $-2\sin^2 2\theta - 2\cos^2 2\theta \rightarrow -2$ **correctly to eliminate squared trig. terms from the numerator** to obtain an expression of the form $k\sin 2\theta + \lambda$ where k and λ are constants (including 1) If double angle formulae have been used give marks only if correct work leads to answer in correct form. (If in doubt, send to review)

A1: Need to see **factorisation of numerator then** answer, which is cso

so $\frac{-2}{1+\sin 2\theta}$ or $\frac{a}{1+\sin 2\theta}$ and a = - 2, with no previous errors

4. Find

(a) $\int (2x+3)^{12}\,\mathrm{d}x$

(2)

(b) $\int \frac{5x}{4x^2+1}\,\mathrm{d}x$

(2)

Question Number	Scheme	Marks
4. (a)	$\left\{\int (2x+3)^{12}\,\mathrm{d}x\right\} = \dfrac{(2x+3)^{13}}{(13)(2)}\ \{+c\}$ $\pm\lambda(2x+3)^{13}$	M1
	$\dfrac{(2x+3)^{13}}{(13)(2)}\ \{+c\}$ (Ignore '+ c')	A1
		[2]
(b)	$\left\{\int \dfrac{5x}{4x^2+1}\,\mathrm{d}t\right\} = \dfrac{5}{8}\ln(4x^2+1)\ \{+c\}$ or $\dfrac{5}{8}\ln(x^2+\tfrac{1}{4})\ \{+k\}$	M1 A1
		[2] **4**

Notes

(a) **M1**: Gives $\pm\lambda(2x+3)^{13}$ where λ is a constant or $\pm\mu(x+\tfrac{3}{2})^{13}$

A1: Coefficient does not need to be simplified so is awarded for $\dfrac{(2x+3)^{13}}{(13)(2)}$ or for $\dfrac{2^{12}}{13}(x+\tfrac{3}{2})^{13}$ i.e. $\dfrac{4096}{13}(x+\tfrac{3}{2})^{13}$

Ignore subsequent errors and condone lack of constant c

N.B. If a binomial expansion is attempted, then it needs all thirteen terms to be correctly integrated for M1A1

(b) **M1**: Gives $\pm\mu\ln(4x^2+1)$ where μ is a constant or $\pm\mu\ln(x^2+\tfrac{1}{4})$ or indeed $\pm\mu\ln(k(4x^2+1))$

May also be awarded for $\dfrac{5}{8}\ln(4x+1)$ or $\dfrac{5}{8}\ln(x^2+1)$, where coefficient 5/8 is correct and there is a slip writing down the bracket.

It may also be given for $\pm\mu\ln(u)$ where u is clearly defined as $(4x^2+1)$ or equivalent substitutions such as $\pm\mu\ln(4u+1)$ where $u=x^2$

A1: $\dfrac{5}{8}\ln(4x^2+1)$ or $\dfrac{5}{8}\ln(x^2+\tfrac{1}{4})$ o.e. The modulus sign is not needed but allow $\dfrac{5}{8}\ln\left|4x^2+1\right|$

Also allow $0.625\ln(4x^2+1)$ and condone lack of constant c

N.B. $\dfrac{5}{8}\ln 4x^2+1$ with no bracket can be awarded M1A0

Leave blank

5. $$\mathrm{f}(x) = (8 + 27x^3)^{\frac{1}{3}}, \quad |x| < \frac{2}{3}$$

Find the first three non-zero terms of the binomial expansion of f(x) in ascending powers of x. Give each coefficient as a simplified fraction.

(5)

Question Number	Scheme	Marks
5.	$(8+27x^3)^{\frac{1}{3}} = \underline{(8)^{\frac{1}{3}}}\left(1+\frac{27x^3}{8}\right)^{\frac{1}{3}} = \underline{2}\left(1+\frac{27x^3}{8}\right)^{\frac{1}{3}}$ $\quad \underline{(8)^{\frac{1}{3}}}$ or $\underline{2}$ $=\{2\}\left[1+\left(\tfrac{1}{3}\right)\left(kx^3\right)+\frac{\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)}{2!}\left(kx^3\right)^2+\ldots\right]$ $=\{2\}\left[1+\left(\frac{1}{3}\right)\left(\frac{27x^3}{8}\right)+\frac{\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)}{2!}\left(\frac{27x^3}{8}\right)^2+\ldots\right]$ $=2\left[1+\frac{9}{8}x^3;-\frac{81}{64}x^6+\ldots\right]$ $=2+\frac{9}{4}x^3;-\frac{81}{32}x^6+\ldots$	<u>B1</u> M1 A1 A1; A1 **[5]**
Method 2	$\left\{(8+27x^3)^{\frac{1}{3}}\right\} = (8)^{\frac{1}{3}}+\left(\frac{1}{3}\right)(8)^{-\frac{2}{3}}(27x^3)+\frac{\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)}{2!}(8)^{-\frac{5}{3}}(27x^3)^2$ $(8)^{\frac{1}{3}}$ or 2 Any two of three (un-simplified or simplified) terms correct All three (un-simplified or simplified) terms correct. $=2+\frac{9}{4}x^3;-\frac{81}{32}x^6+\ldots$	 B1 M1 A1 A1; A1 **[5]** **5**

Notes

<u>Method 1:</u>

<u>B1</u>: $\underline{(8)^{\frac{1}{3}}}$ or $\underline{2}$ outside brackets then isw or $\underline{(8)^{\frac{1}{3}}}$ or $\underline{2}$ as candidate's constant term in their binomial expansion.

M1: Expands $\left(\ldots+kx^3\right)^{\frac{1}{3}}$ to give any 2 terms out of 3 terms correct for their *k* simplified or un-simplified

Eg: $1+\left(\tfrac{1}{3}\right)\left(kx^3\right)$ or $\left(\tfrac{1}{3}\right)\left(kx^3\right)+\frac{\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)}{2!}\left(kx^3\right)^2$ or $1+\ldots\ldots..+\frac{\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)}{2!}\left(kx^3\right)^2$ [Allow $\left(\tfrac{1}{3}-1\right)$ for $\left(-\tfrac{2}{3}\right)$]
where $k\neq 1$ are acceptable for M1. Allow omission of brackets. [*k* will usually be 27, 27/8 or 27/2…]

A1: A correct simplified or un-simplified $1+\left(\tfrac{1}{3}\right)\left(kx^3\right)+\frac{\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)}{2!}\left(kx^3\right)^2$ expansion with consistent $\left(kx^3\right)$ {or (*kx*) – for special case only}. **Note** that $k\neq 1$. The bracketing must be correct and now need all three terms correct for their *k*.

A1: $2+\frac{9}{4}x^3$ **- allow** $2+2.25x^3$ **or** $2+2\tfrac{1}{4}x^3$

A1: $-\frac{81}{32}x^6$ allow $-2.53125x^6$ or $-2\tfrac{17}{32}x^6$ (Ignore extra terms of higher power)

<u>Method 2:</u>

B1: $(8)^{\frac{1}{3}}$ or 2

M1: Any two of three (un-simplified or simplified) terms correct – condone missing brackets
A1: All three (un-simplified or simplified) terms correct. The bracketing must be correct but it is acceptable for them to recover this mark following "invisible" brackets.
A1A1: as above.

Special case (either method) uses *x* instead of x^3 **throughout to obtain** $=2+\tfrac{9}{4}x;-\tfrac{81}{32}x^2+\ldots$ **gets B1M1A1A0A0**

6. (a) Express $\dfrac{5-4x}{(2x-1)(x+1)}$ in partial fractions.

(3)

(b) (i) Find a general solution of the differential equation

$$(2x-1)(x+1)\frac{\mathrm{d}y}{\mathrm{d}x} = (5-4x)y, \quad x > \frac{1}{2}$$

Given that $y = 4$ when $x = 2$,

(ii) find the particular solution of this differential equation.
Give your answer in the form $y = \mathrm{f}(x)$.

(7)

Question Number	Scheme	Marks
6. (a)	$\frac{5-4x}{(2x-1)(x+1)} \equiv \frac{A}{(2x-1)} + \frac{B}{(x+1)}$ so $5-4x \equiv A(x+1)+B(2x-1)$	B1
	Let $x=-1,\ 9 = B(-3) \Rightarrow B=$ Let $x=\frac{1}{2},\ 3 = A\left(\frac{3}{2}\right) \Rightarrow A=$	M1
	A = 2 and B = -3 or $\left\{\frac{5-4x}{(2x-1)(x+1)} \equiv \frac{2}{(2x-1)} - \frac{3}{(x+1)}\right\}$	A1
		[3]
(b) (i), (ii)	$\int \frac{1}{y}\,dy = \int \frac{5-4x}{(2x-1)(x+1)}\,dx$	B1
	$= \int \frac{2}{(2x-1)} - \frac{3}{(x+1)}\,dx = C\ \ln(2x-1) + D\ln(x+1)$	M1
	$= \frac{"2"}{2}\ln(2x-1) - "3"\ln(x+1)$	A1ft
	$\ln y = \ln(2x-1) - 3\ln(x+1) + c$	A1
Method 1 for (ii)	$\ln 4 = \ln(2(2)-1) - 3\ln(2+1) + c \quad \Rightarrow c = \{\ln 36\}$	M1
	$\ln y = \ln(2x-1) - 3\ln(x+1) + \ln 36$ so $\ln y = \ln\left(\frac{36(2x-1)}{(x+1)^3}\right)$ So $y = \frac{36(2x-1)}{(x+1)^3}$	M1 A1 **[7]**
Method 2 for (ii)	Solution as Method 1 up to $\ln y = \ln(2x-1) - 3\ln(x+1) + c$ so first four marks as before	B1M1A1A1
	Writes $y = \frac{A(2x-1)}{(x+1)^3}$ as general solution which would earn the 3rd M1 mark.	M1
	Then may substitute to find their constant A, which would earn the 2nd M1 mark.	M1
	Then A1 for $y = \frac{36(2x-1)}{(x+1)^3}$ as before.	A1
		[7] **10**

Notes

(a) B1: Forming the linear identity (this may be implied).

Note: A & B are not assigned in this question – so other letters may be used

M1: A valid method to find the value of one of either their A or their B.

A1: $A = 2$ and $B = -3$ (This is sufficient without rewriting answer provided it is clear what A and B are)

Note: In part (a), $\left\{\frac{5-4x}{(2x-1)(x+1)} \equiv\right\} \frac{2}{(2x-1)} - \frac{3}{(x+1)}$, from no working, is B1M1A1 (cover-up rule).

(b) You can mark parts (b)(i) and (b)(ii) together.

(i) **B1**: Separates variables as shown. (Can be implied.) Need **both sides** correct, but condone missing integral signs.

M1: Uses partial fractions on RHS and obtains two log terms after integration. The coefficients may be wrong e.g. 2 ln $(2x - 1)$ or may follow their wrong partial fractions. Ignore LHS for this mark.

A1ft: RHS correct integration for **their** partial fractions – do not need LHS nor $+c$ for this mark

A1 : All three terms correct (LHS and RHS) including $+c$.

(ii) **M1**: Substitutes $y = 4$ and $x = 2$ into **their general solution** with a constant of integration to obtain c =.

M1: A fully correct method of removing the logs – must have a constant of integration which must be treated Correctly. Must have had ln y = ….earlier

A1: $y = \frac{36(2x-1)}{(x+1)^3}$ isw.

NB If Method 2 is used the third method mark is earned at the end of part (i), then the second method mark is earned when the values are substituted.

Special case1: A common error using method 2:

$y = \frac{(2x-1)}{(x+1)^3} + A$, then $4 = \frac{(3)}{(3)^3} + A$ so $A =$ would earn M1 (substitution); M0 (not fully correct removing logs); A0

Special case2: A possible error using method 1 or 2:

$y = (2x-1) - 3(x+1) + A$, then $4 = 3 - 9 + A$ so $A =$ would earn M0 (too bad an error); M0 (not fully correct removing logs); A0

i.e. M0M0A0

If there is no constant of integration they are likely to lose the last four marks.

Leave blank

7. The function f is defined by

$$\mathrm{f}: x \mapsto \frac{3x-5}{x+1}, \quad x \in \mathbb{R}, x \neq -1$$

(a) Find an expression for $\mathrm{f}^{-1}(x)$

(3)

(b) Show that

$$\mathrm{ff}(x) = \frac{x+a}{x-1}, \quad x \in \mathbb{R}, x \neq -1, x \neq 1$$

where a is an integer to be determined.

(4)

The function g is defined by

$$\mathrm{g}: x \mapsto x^2 - 3x, \quad x \in \mathbb{R}, 0 \leqslant x \leqslant 5$$

(c) Find the value of fg(2)

(2)

(d) Find the range of g

(3)

Question Number	Scheme		Marks
	Method 1	Method 2	
7. (a)	$y = \frac{3x-5}{x+1}$	$y = 3 - \frac{8}{x+1}$	
	$y(x+1) = 3x-5 \Rightarrow xy + y = 3x - 5$	$\frac{8}{x+1} = 3 - y$ so $x + 1 = \frac{8}{3-y}$	M1
	$y + 5 = 3x - xy \Rightarrow y + 5 = x(3-y)$ $\Rightarrow \frac{y+5}{3-y} = x$	$x = \frac{8}{3-y} - 1$	M1
	Hence $\left(\mathrm{f}^{-1}(x)\right) = \frac{x+5}{3-x}$ $(x \in \mathbb{R},\ x \neq 3)$	Hence $\left(\mathrm{f}^{-1}(x)\right) = \frac{8}{3-x} - 1$ $(x \in \mathbb{R},\ x \neq 3)$	A1 oe
			[3]
(b)	$\mathrm{ff}(x) = \frac{3\left(\frac{3x-5}{x+1}\right) - 5}{\left(\frac{3x-5}{x+1}\right) + 1}$	$\mathrm{ff}(x) = 3 - \frac{8}{3 - \frac{8}{x+1} + 1}$	M1 A1
	$= \frac{\frac{3(3x-5) - 5(x+1)}{x+1}}{\frac{(3x-5) + (x+1)}{x+1}}$	$\mathrm{ff}(x) = 3 - \frac{8(x+1)}{4x-4}$	M1
	$= \frac{9x - 15 - 5x - 5}{3x - 5 + x + 1} = \frac{4x - 20}{4x - 4}$		
	$= \frac{x-5}{x-1}$ (note that $a = -5$.)	$= \frac{x-5}{x-1}$	A1
			[4]
(c)	$\mathrm{fg}(2) = \mathrm{f}(4-6) = \mathrm{f}(-2) = \frac{3(-2) - "5"}{-2+1}$;$= 11$ or substitute 2 into $\mathrm{fg}(x) = \frac{3(x^2-3x) - 5}{x^2 - 3x + 1}$;$= 11$		M1; A1
			[2]
(d)	$\mathrm{g}(x) = x^2 - 3x = (x-1.5)^2 - 2.25$. Hence $\mathrm{g}_{\min} = -2.25$		M1
	Either $\mathrm{g}_{\min} = -2.25$ or $\mathrm{g}(x) \geqslant -2.25$ or $\mathrm{g}(5) = 25 - 15 = 10$		B1
	$\underline{-2.25 \leqslant \mathrm{g}(x) \leqslant 10}$ or $\underline{-2.25 \leqslant y \leqslant 10}$		A1
			[3] **12**

Notes

Method 2 is less likely and the notes apply to Method 1.

(a) **M1**: Brings $(x+1)$ to the LHS and multiplies out by y

or if x and y swapped first $(y+1)$ to the LHS and multiplies out by x

M1: A full method to make x (or swapped y) the subject by collecting terms and factorising.

A1: $\dfrac{x+5}{3-x}$ or equivalent e.g. $-\dfrac{x+5}{x-3}$ or $\dfrac{-x-5}{x-3}$ or $-1+\dfrac{8}{3-x}$ etc Ignore LHS.

Does not need to include domain i.e does not need statement that $x \in □$, $x \neq 3$ Should now be in x, not y, for this mark.

N.B. Use of quotient rule to differentiate and to find f' is M0M0A0. This is NOT a misread.

(b) **M1**: An attempt to substitute f into itself. e.g. $\mathrm{ff}(x) = \dfrac{3\mathrm{f}(x)-5}{\mathrm{f}(x)+1}$. Squaring f(x) is M0.

Allow $\mathrm{ff}(x) = \dfrac{3\mathrm{f}(x)-5}{x+1}$ or $\mathrm{ff}(x) = \dfrac{3x-5}{\mathrm{f}(x)+1}$ for M1A0

A1: Correct expression. This mark implies the previous method mark.

M1: An **attempt** to combine each of the numerator and the denominator into single rational fraction with same common denominator

A1: See $\dfrac{x-5}{x-1}$ Does not need to include domain or statement that $x \in □$, $x \neq -1$, $x \neq 1$

NB If they use a mixture of methods 1 and 2 then mark accordingly – attempt M1, correct A1, combined into single rational function M1 then answer is A1

so may see $= \dfrac{3\left(3-\dfrac{8}{x+1}\right)-5}{\left(3-\dfrac{8}{x+1}\right)+1}$ or $3-\dfrac{8}{\left(\dfrac{3x-5}{x+1}\right)+1}$

(c) **M1**: ***Full method*** of inserting $\mathrm{g}(2)$ (i.e. -2) into $\mathrm{f}(x)$. Or substitutes 2 into $\mathrm{fg}(x) = \dfrac{3(x^2-3x)-5}{x^2-3x+1}$

A1: cao

(d) **M1**: **Full method** to establish the minimum of g. (Or correct answer with no method)

e.g.: $(x \pm \alpha)^2 + \beta$ leading to $\mathrm{g}_{\min} = \beta$.

Or finding derivative, setting to zero, finding x (=1.5) and then finding $\mathrm{g}(1.5)$ in order to find the minimum.

Or obtaining roots of $x = 0, 3$ and using symmetry to obtain $\mathrm{g}_{\min} = g(1.5) = \beta$.

Or listing values leading to $\mathrm{g}_{\min} = g(1.5) = \beta$.

This mark may also be implied by -2.25.

B1: For finding **either** the correct minimum value of g (can be implied by $\mathrm{g}(x) \geqslant -2.25$ or $\mathrm{g}(x) > -2.25$) **or** for stating that $\mathrm{g}(5) = 10$ or finding the value 10 as a maximum

A1: $\underline{-2.25 \leqslant \mathrm{g}(x) \leqslant 10}$ or $\underline{-2.25 \leqslant y \leqslant 10}$ or $\underline{-2.25 \leqslant \mathrm{g} \leqslant 10}$.

Note that: $-2.25 \leqslant x \leqslant 10$ (wrong variable)is A0**;** $-2.25 < y < 10$ (wrong inequality) is A0;

$-2.25 \leqslant \mathrm{f} \leqslant 10$ (wrong function) is A0; Accept [-2.25, 10] (correct notation) for A1

but not (-2.25, 10) (strict inequality) which is A0

A correct answer with no working gains M1 B1 A1 i.e. 3/3

8. The volume V of a spherical balloon is increasing at a constant rate of 250 $\text{cm}^3\ \text{s}^{-1}$.
Find the rate of increase of the radius of the balloon, in cm s^{-1}, at the instant when the volume of the balloon is 12 000 cm^3.
Give your answer to 2 significant figures.

(5)

[*You may assume that the volume V of a sphere of radius r is given by the formula* $V = \frac{4}{3}\pi r^3$.]

Question Number	Scheme	Marks
8.	$\frac{\mathrm{d}V}{\mathrm{d}t}=250$	
	$\left\{V=\frac{4}{3}\pi r^3 \Rightarrow\right\} \frac{\mathrm{d}V}{\mathrm{d}r}=4\pi r^2$	B1
	$V=12\,000 \Rightarrow 12\,000=\frac{4}{3}\pi r^3 \Rightarrow r=\sqrt[3]{\frac{9000}{\pi}} \quad (=14.202480...)$	B1
	$\frac{\mathrm{d}r}{\mathrm{d}t}\left\{=\frac{\mathrm{d}r}{\mathrm{d}V}\times\frac{\mathrm{d}V}{\mathrm{d}t}\right\}=\frac{1}{4\pi r^2}\times 250$	M1
	When $r=\sqrt[3]{\frac{9000}{\pi}}$, $\frac{\mathrm{d}r}{\mathrm{d}t}=\frac{250}{4\pi\left(\sqrt[3]{\frac{9000}{\pi}}\right)^2}$	dM1
	So, $\frac{\mathrm{d}r}{\mathrm{d}t}=0.0986283...\left(\mathrm{cm\,s^{-1}}\right)$ awrt 0.099	A1
		[5] **5**

Notes

B1: $\frac{\mathrm{d}V}{\mathrm{d}r}=4\pi r^2$. This may be stated or used and need not be simplified

Applies $12\,000=\frac{4}{3}\pi r^3$ and rearranges to find r using division then cube root with accurate algebra

May state $r=\sqrt[3]{\frac{3V}{4\pi}}$ then substitute V = 12000 later which is equivalent. r does not need to be evaluated.

M1: **Uses** chain rule correctly so $\frac{1}{\left(\text{their } \frac{\mathrm{d}V}{\mathrm{d}r}\right)}\times 250$

dM1: Substitutes their r **correctly** into their equation for $\frac{\mathrm{d}r}{\mathrm{d}t}$ This depends on the previous method mark

A1: awrt 0.099 (Units may be ignored) If this answer is seen, then award A1 and isw.
Premature approximation usually results in all marks being earned prior to this one.

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9.

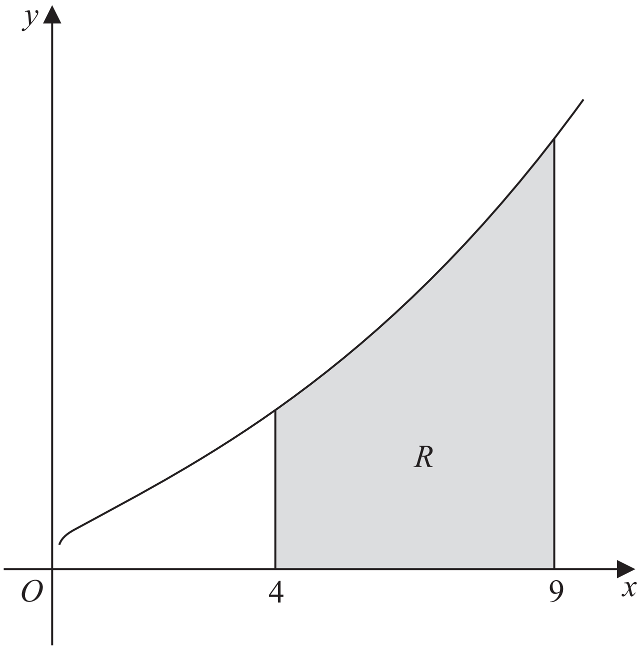

Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = e^{\sqrt{x}}, \; x > 0$

The finite region *R*, shown shaded in Figure 1, is bounded by the curve, the *x*-axis and the lines $x = 4$ and $x = 9$

(a) Use the trapezium rule, with 5 strips of equal width, to obtain an estimate for the area of *R*, giving your answer to 2 decimal places.
(4)

(b) Use the substitution $u = \sqrt{x}$ to find, by integrating, the exact value for the area of *R*.
(7)

Question Number	Scheme	Marks
9. (a)	$\begin{array}{r\|c\|c\|c\|c\|c\|c} x & 4 & 5 & 6 & 7 & 8 & 9 \\ \hline y & e^2 & e^{\sqrt{5}} & e^{\sqrt{6}} & e^{\sqrt{7}} & e^{\sqrt{8}} & e^3 \\ \hline & 7.389056... & 9.356469... & 11.582435... & 14.094030... & 16.918828... & 20.085536... \end{array}$	M1
	$\frac{1}{2}\times 1\times \underline{\{..........................\}}$	B1 oe
	$\frac{1}{2}\times 1\times \underline{\left\{e^2+e^3+2\left(e^{\sqrt{5}}+e^{\sqrt{6}}+e^{\sqrt{7}}+e^{\sqrt{8}}\right)\right\}}$ $\quad \left\{= \tfrac{1}{2}(27.47459302...+103.903526...)\right\}$	M1
	$= 65.6890595... = 65.69$ (2 dp)	A1
	Special case (s.c.) Uses h = 5/4 with 5 ordinates giving answer 65.76 – award M0B0M1A1(s.c.) *See note below*	**[4]**
(b)	$\left\{u=\sqrt{x}\Rightarrow\right\}\ \frac{\mathrm{d}u}{\mathrm{d}x}=\frac{1}{2}x^{-\frac{1}{2}}$ or $\frac{\mathrm{d}x}{\mathrm{d}u}=2u$	B1
	$\left\{\int e^{\sqrt{x}}\,\mathrm{d}x\right\} = \int e^u\,2u\,\mathrm{d}u$	M1 A1
	$= \{2\}\left(ue^u - \int e^u\,\mathrm{d}u\right)$	M1
	$= \{2\}\left(ue^u - e^u\right)$	A1
	$\left[2\left(ue^u - e^u\right)\right]_2^3 = 2\left(3e^3-e^3\right)-2\left(2e^2-e^2\right)$	ddM1
	$4e^3-2e^2$ or $2e^2(2e-1)$ etc.	A1
		[7] **11**

Notes

(a) **M1**: Finds y for x = 4, 5, 6, 7, 8 and 9. Need **six** y values for this mark. May leave as on middle row of table – give mark if correct unsimplified answers given, then isw if errors appear later. If given as decimals only, without prior expressions, need to be accurate to 2 **significant figures**.(Allow one slip) May not appear as table, but only in trapezium rule.

B1: Outside brackets $\frac{1}{2}\times 1$ or $\frac{1}{2}$ or h = 1 stated. This is independent of the method marks

M1: For structure of $\underline{\{\ldots\ldots\ldots\ldots\ldots\}}$ ft their ***y* values** and allow for 5 or 6 y values so may follow wrong h or table which has x from 5 to 9 or from 4 to 8 NB {4+9+2(5+6+7+8)} is M0

A1: 65.69 N.B. Wrong brackets e.g. $\frac{1}{2}\times 1\times(e^2+e^3)+2\left(e^{\sqrt{5}}+e^{\sqrt{6}}+e^{\sqrt{7}}+e^{\sqrt{8}}\right)$ is M0 **unless** followed by correct answer 65.69 which implies M1A1

Special case: uses five ordinates (i.e. four strips)

x	4	5.25	6.5	7.75	9
y	e^2	$e^{\sqrt{5.25}}$	$e^{\sqrt{6.5}}$	$e^{\sqrt{7.75}}$	e^3
	7.389056...	9.887663..	12.800826..	16.181719..	20.085536...

Then $\frac{1}{2}\times\frac{5}{4}\times\underline{\{\ldots\ldots\ldots\ldots\ldots\}}$

Giving $\frac{1}{2}\times\frac{5}{4}\times\underline{\left\{e^2+e^3+2\left(e^{\sqrt{5.25}}+e^{\sqrt{6.5}}+e^{\sqrt{7.75}}\right)\right\}}$ = 65.76

This complete method for special case earns M0 B0 M1 A1 i.e. 2/4

(b) **B1**: States or uses $\frac{du}{dx}=\frac{1}{2}x^{-\frac{1}{2}}$ or $\frac{dx}{du}=2u$

M1: Obtains $\pm\lambda\int ue^u\,du$ for a constant value λ **A1**: Obtains $2\int ue^u du$

M1: An attempt at integration by parts in the right direction **on** λue^u **.** This mark is implied by the correct answer. There is no need for limits. If the rule is quoted it must be correct. A version of the rule appears in the formula booklet. Accept for this mark expressions of the form $\int ue^u\,du = ue^u - \int e^u du$

A1: $\lambda ue^u \rightarrow \lambda ue^u - \lambda e^u$. (Candidates just quoting this answer earn M1A1)

ddM1: Substitutes limits of 3 and 2 in u (or 9 and 4 in x) in **their integrand** and subtracts the correct way round. (Allow one slip) This mark depends on both previous method marks having been earned

A1: Obtains $4e^3-2e^2$ or $2e^2(2e-1)$ with terms collected. If then given as a decimal isw.

Leave blank

10. (a) Use the identity for sin($A + B$) to prove that

$$\sin 2A \equiv 2\sin A \cos A$$

(2)

(b) Show that

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[\ln\left(\tan\left(\tfrac{1}{2}x\right)\right)\right] = \operatorname{cosec} x$$

(4)

A curve C has the equation

$$y = \ln\left(\tan\left(\tfrac{1}{2}x\right)\right) - 3\sin x, \qquad 0 < x < \pi$$

(c) Find the x coordinates of the points on C where $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$

Give your answers to 3 decimal places.
(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(6)

Question Number	Scheme	Marks
10. (a)	$A = B \Rightarrow \sin 2A = \underline{\underline{\sin(A+A)}} = \underline{\sin A\cos A + \cos A\sin A}$ *or* $\underline{\sin A\cos A + \sin A\cos A}$	M1
	Hence, $\underline{\underline{\sin 2A = 2\sin A\cos A}}$ (as required) *	A1 * **[2]**
(b)	Way 1A: $\left\{y = \ln\left[\tan\left(\tfrac{1}{2}x\right)\right] \Rightarrow\right\} \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\frac{1}{2}\sec^2\left(\frac{1}{2}x\right)}{\tan\left(\frac{1}{2}x\right)}$ Way 1B $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\frac{1}{2}\sec^2\left(\frac{1}{2}x\right)}{\tan\left(\frac{1}{2}x\right)}$	M1 A1
	$= \frac{1}{2\tan\left(\frac{1}{2}x\right)\cos^2\left(\frac{1}{2}x\right)} = \frac{1}{\frac{2\sin\left(\frac{1}{2}x\right)}{\cos\left(\frac{1}{2}x\right)}\cdot\frac{\cos^2\left(\frac{1}{2}x\right)}{1}}$ $= \frac{1+\tan^2\left(\frac{1}{2}x\right)}{2\tan\left(\frac{1}{2}x\right)} = \; = \frac{\cos^2\left(\frac{1}{2}x\right)+\sin^2\left(\frac{1}{2}x\right)}{2\sin\left(\frac{1}{2}x\right)\cos\left(\frac{1}{2}x\right)}$	dM1
	$= \frac{1}{2\sin\left(\frac{1}{2}x\right)\cos\left(\frac{1}{2}x\right)} = \frac{1}{\sin x} = \operatorname{cosec} x$ * $= \frac{1}{2\sin\left(\frac{1}{2}x\right)\cos\left(\frac{1}{2}x\right)} = \frac{1}{\sin x} = \operatorname{cosec} x$ *	A1 * **[4]**
	Way 2: $\left\{y = \ln\left[\sin\left(\tfrac{1}{2}x\right)\right] - \ln\left[\cos\left(\tfrac{1}{2}x\right)\right] \Rightarrow\right\} \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\frac{1}{2}\cos\left(\frac{1}{2}x\right)}{\sin\left(\frac{1}{2}x\right)} - \frac{-\frac{1}{2}\sin\left(\frac{1}{2}x\right)}{\cos\left(\frac{1}{2}x\right)}$	M1 A1
	$= \frac{\cos^2\left(\frac{1}{2}x\right)+\sin^2\left(\frac{1}{2}x\right)}{2\sin\left(\frac{1}{2}x\right)\cos\left(\frac{1}{2}x\right)}\;; = \frac{1}{\sin x} = \operatorname{cosec} x$	M1;A1 [4]
	Way3: quotes $\int \operatorname{cosec} x\,\mathrm{d}x = \ln\left(\tan\left(\tfrac{1}{2}x\right)\right)$	M1 A1
	(As differentiation is reverse of integration) $\frac{\mathrm{d}}{\mathrm{d}x}\left[\tan\left(\tfrac{1}{2}x\right)\right] = \operatorname{cosec} x$	M1 A1 [4]
(c)	$\left\{y = \ln\left[\tan\left(\tfrac{1}{2}x\right)\right] - 3\sin x \Rightarrow\right\} \frac{\mathrm{d}y}{\mathrm{d}x} = \operatorname{cosec} x - 3\cos x$	B1
	$\left\{\frac{\mathrm{d}y}{\mathrm{d}x} = 0 \Rightarrow\right\}$ $\operatorname{cosec} x - 3\cos x = 0 \Rightarrow \frac{1}{\sin x} - 3\cos x = 0$	M1
	$\Rightarrow 1 = 3\sin x\cos x \Rightarrow 1 = \tfrac{3}{2}(2\sin x\cos x)$ so $\sin 2x = k$, where $-1 < k < 1$ and $k \neq 0$	M1
	So $\sin 2x = \frac{2}{3}$	A1
	$\{2x = \{0.729727..., 2.411864...\}\}$ So $x = \{0.364863..., 1.205932...\}$	A1 A1 **[6]** **12**
Way2 10 (c)	Method (Squaring Method) $\left\{y = \ln\left[\tan\left(\tfrac{1}{2}x\right)\right] - 3\sin x \Rightarrow\right\} \frac{\mathrm{d}y}{\mathrm{d}x} = \operatorname{cosec} x - 3\cos x$	**B1**
	$\left\{\frac{\mathrm{d}y}{\mathrm{d}x} = 0 \Rightarrow\right\}$ $\operatorname{cosec} x - 3\cos x = 0 \Rightarrow \frac{1}{\sin x} - 3\cos x = 0$	M1
	$\Rightarrow \frac{1}{1-\cos^2 x} = 9\cos^2 x$ so $9\cos^4 x - 9\cos^2 x + 1 = 0$ or $9\sin^4 x - 9\sin^2 x + 1 = 0$	M1
	So $\cos^2 x = 0.873$ *or* 0.127 or $\sin^2 x = 0.873$ *or* 0.127	A1
	So $x = \{0.364863..., 1.205932...\}$	A1 A1 **[6]**

Way 3 10c)	"t" method $\left\{y = \ln\left[\tan\left(\tfrac{1}{2}x\right)\right] - 3\sin x \Rightarrow\right\}\ \frac{\mathrm{d}y}{\mathrm{d}x} = \operatorname{cosec} x - 3\cos x$	B1
	$\left\{\frac{\mathrm{d}y}{\mathrm{d}x} = 0 \Rightarrow\right\}\ \operatorname{cosec} x - 3\cos x = 0 \Rightarrow \frac{1}{\sin x} - 3\cos x = 0$	M1
	$\Rightarrow \frac{1+t^2}{2t} - 3\frac{1-t^2}{1+t^2} = 0$ so $t^4 + 6t^3 + 2t^2 - 6t + 1 = 0$	M1
	$t = 0.1845$ or 0.6885	A1
	So $x = \{0.364863..., 1.205932...\}$	A1 A1
		[6]

Notes

(a) M1: This mark is for the underlined equation in either form

$$\underline{\sin A\cos A + \cos A\sin A} \quad or \quad \underline{\sin A\cos A + \sin A\cos A}$$

A1: For this mark need to see :

sin2*A* at the start of the proof , or as part of a conclusion

sin(*A* + *A*) = at the start

$= \underline{\sin A\cos A + \cos A\sin A} \quad or \quad \underline{\sin A\cos A + \sin A\cos A}$

= 2sin*A*cos*A* at the end

(b)M1: For expression of the form $\frac{\pm k\sec^2\left(\frac{1}{2}x\right)}{\tan\left(\frac{1}{2}x\right)}$, where k is constant (could even be 1)

A1: Correct differentiation so $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\frac{1}{2}\sec^2\left(\frac{1}{2}x\right)}{\tan\left(\frac{1}{2}x\right)}$

Way 1A:

dM1: Use both $\tan\left(\frac{1}{2}x\right) = \frac{\sin\left(\frac{1}{2}x\right)}{\cos\left(\frac{1}{2}x\right)}$ and $\sec^2\left(\frac{1}{2}x\right) = \frac{1}{\cos^2\left(\frac{1}{2}x\right)}$ in their differentiated expression. This may be implied.

This depends on **the** previous Method mark.

A1*: Simplify the fraction, use double angle formula, see $\frac{1}{\sin x}$ and obtain correct answer with completely correct work and no errors seen (NB Answer is given)

Way 1B

dM1: Use both $\sec^2\left(\frac{1}{2}x\right) = 1 + \tan^2\left(\frac{1}{2}x\right)$ and $\tan\left(\frac{1}{2}x\right) = \frac{\sin\left(\frac{1}{2}x\right)}{\cos\left(\frac{1}{2}x\right)}$

A1*: Simplify the fraction, use double angle formula, see $\frac{1}{\sin x}$ and obtain correct answer with completely correct work and no errors seen (NB Answer is given)

Way 2:

M1:Split into $\left\{y = \ln\left[\sin\left(\frac{1}{2}x\right)\right] - \ln\left[\cos\left(\frac{1}{2}x\right)\right] \Rightarrow\right\}$ then differentiate to give $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{k\cos\left(\frac{1}{2}x\right)}{\sin\left(\frac{1}{2}x\right)} - \frac{c\sin\left(\frac{1}{2}x\right)}{\cos\left(\frac{1}{2}x\right)}$

A1: Correct answer $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\frac{1}{2}\cos\left(\frac{1}{2}x\right)}{\sin\left(\frac{1}{2}x\right)} - \frac{-\frac{1}{2}\sin\left(\frac{1}{2}x\right)}{\cos\left(\frac{1}{2}x\right)}$

M1: Obtain $= \frac{\cos^2\left(\frac{1}{2}x\right) + \sin^2\left(\frac{1}{2}x\right)}{2\sin\left(\frac{1}{2}x\right)\cos\left(\frac{1}{2}x\right)}$ A1*: As before

Way 3:

Alternative method: This is rare, but is acceptable. Must be completely correct.

Quotes $\int \operatorname{cosec} x\,\mathrm{d}x = \ln(\tan\left(\frac{1}{2}x\right))$ and follows this by $\frac{\mathrm{d}}{\mathrm{d}x}\left[\tan\left(\frac{1}{2}x\right)\right] = \operatorname{cosec} x$ gets 4/4

(c) B1: Correct differentiation – so see $\frac{\mathrm{d}y}{\mathrm{d}x} = \operatorname{cosec} x - 3\cos x$

M1: Sets their $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ and uses $\operatorname{cosec} x = \frac{1}{\sin x}$

Way 1:

M1: Rearranges and uses double angle formula to obtain $\sin 2x = k$, where $-1 < k < 1$ and $k \neq 0$

(This may be implied by $a + b\sin 2x = 0$ followed by correct answer)

A1: $\sin 2x = \frac{2}{3}$ (This may be implied by correct answer)

A1: **Either** awrt 0.365 **or** awrt 1.206 (answers in degrees lose both final marks)

A1: **Both** awrt 0.365 **and** awrt 1.206

Ignore y values. Ignore extra answers outside range. Lose the last A mark for extra answers in the range.

Way 2:

M1: Obtain quadratic in $\sin x$ or in $\cos x$. Condone $\cos ec^2 x - 9\cos^2 x = 0$ as part of the working

A1 A1 A1: See scheme

Way 3:

This method is unlikely and uses $t = \tan(\frac{x}{2})$. See scheme for detail

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11.

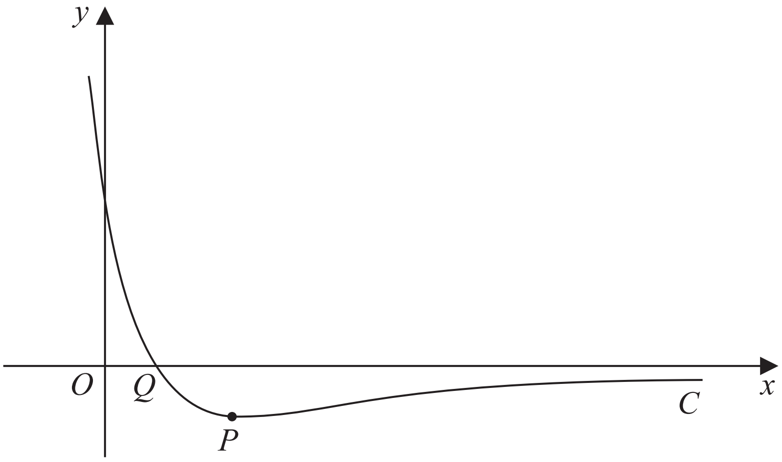


Figure 2

Figure 2 shows a sketch of part of the curve C with equation

$$y = e^{a-3x} - 3e^{-x}, \quad x \in \mathbb{R}$$

where a is a constant and $a > \ln 4$

The curve C has a turning point P and crosses the x-axis at the point Q as shown in Figure 2.

(a) Find, in terms of a, the coordinates of the point P.

(6)

(b) Find, in terms of a, the x coordinate of the point Q.

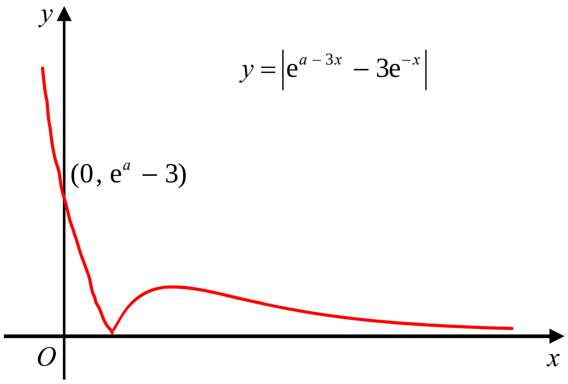
(3)

(c) Sketch the curve with equation

$$y = \left|e^{a-3x} - 3e^{-x}\right|, \quad x \in \mathbb{R}, \ a > \ln 4$$

Show on your sketch the exact coordinates, in terms of a, of the points at which the curve meets or cuts the coordinate axes.

(3)

Question Number	Scheme	Marks
11.		
(a)	$\frac{\mathrm{d}y}{\mathrm{d}x} = -3\mathrm{e}^{a-3x} + 3\mathrm{e}^{-x}$	M1 A1
	$-3\mathrm{e}^{a-3x} + 3\mathrm{e}^{-x} = 0 \Rightarrow \mathrm{e}^{-x} = \mathrm{e}^{a-3x} \Rightarrow -x = a - 3x \Rightarrow x =$	M1
	$x = \frac{1}{2}a$	A1
	So, $y_P = \mathrm{e}^{a-3\left(\frac{a}{2}\right)} - 3\mathrm{e}^{-\left(\frac{a}{2}\right)}; = -2\mathrm{e}^{-\frac{a}{2}}$	ddM1; A1
		[6]
	Mark parts (b) and (c) together.	
	Method 1 | Method 2 | Method 3	
(b)	Method 1: $0 = \mathrm{e}^{a-3x} - 3\mathrm{e}^{-x} \Rightarrow \mathrm{e}^{a-\text{"2"}x} = 3$ Method 2: $0 = \mathrm{e}^{a-3x} - 3\mathrm{e}^{-x} \Rightarrow \mathrm{e}^{\text{"2"}x} = \frac{\mathrm{e}^a}{3}$ Method 3: $0 = \mathrm{e}^{a-3x} - 3\mathrm{e}^{-x} \Rightarrow 3\mathrm{e}^{\text{"2"}x} = \mathrm{e}^a$	M1
	Method 1: $\Rightarrow a - \text{"2"}x = \ln 3$ Method 2: $\text{"2"}x = a - \ln 3$ Method 3: $\ln 3 + \text{"2"}x = a$	dM1
	$\Rightarrow x = \frac{a - \ln 3}{2}$ or equivalent e.g. $\frac{1}{2}\ln\left(\frac{e^a}{3}\right)$ or $-\ln\sqrt{\left(\frac{3}{e^a}\right)}$ etc	A1
		[3]
	Method 4 $0 = \mathrm{e}^{a-3x} - 3\mathrm{e}^{-x} \Rightarrow \mathrm{e}^{a-3x} = 3\mathrm{e}^{-x}$ and so $a - 3x = \ln 3 - x$	M1
	$\text{"2"}x = a - \ln 3$	dM1
	$\Rightarrow x = \frac{a - \ln 3}{2}$ o.e. e.g. $\frac{1}{2}\ln\left(\frac{e^a}{3}\right)$ or $-\ln\sqrt{\left(\frac{3}{e^a}\right)}$ etc	A1 **[3]**
(c)	 	
	Shape	B1
	Cusp and behaviour for large x	B1
	$(0, \mathrm{e}^a - 3)$.	B1
		[3] **12**

Notes

(a) **M1**: At least one term differentiated correctly
A1: Correct differentiation of both terms

M1: Sets $\frac{\mathrm{d}y}{\mathrm{d}x}$ to 0 and applies a correct method for eliminating the exponentials e^{x} to reach $x =$

(At this stage the RHS may include $\ln(\mathrm{e}^{a})$ term but should include no x terms)

A1: $x_P = \frac{1}{2}a$ **after correct work**

ddM1: (Needs both previous M marks) Substitutes their x-coordinate into y (not into $\frac{\mathrm{d}y}{\mathrm{d}x}$)

A1: $y_P = -2\mathrm{e}^{-\frac{a}{2}}$ given as one term

(b) Parts (b) and (c) may be marked together.
Methods 1, 2and 3:

M1: Put $y = 0$ and attempt to obtain $\mathrm{e}^{\mathrm{f}(x)} = k$ e.g. $\mathrm{e}^{a \pm \lambda x} = 3$ (Method 1) or $\mathrm{e}^{\lambda x} = \frac{\mathrm{e}^{a}}{3}$ (Method 2) or $3\mathrm{e}^{"2"x} = \mathrm{e}^{a}$ (Method 3) Must have all x terms on one side of the equation for any of these methods
dM1: This depends on previous M mark. Take logs correctly.
e.g. $a \pm \lambda x = \ln 3$ (Method 1) or $\lambda x = a - \ln 3$ (Method 2) or $\ln 3 + "2"x = a$ (Method 3)

A1: cao $x_Q = \frac{a - \ln 3}{2}$ (must be exact)
Method 4:
M1: Puts $\mathrm{e}^{a-3x} = 3\mathrm{e}^{-x}$ then takes lns correctly (see scheme) $a - 3x = \ln 3 - x$
dM1: Collects x terms on one side
A1: $x_Q = \frac{a - \ln 3}{2}$ cao (must be exact to answer requirements of (c))

(c) **B1**: **Correct overall shape**, so $y \geq 0$ for all x, curve crossing positive y axis and small portion seen to left of y axis, meets x axis once, one maximum turning point
B1: Cusp at $x = x_Q$ (not zero gradient) and no appearance of curve clearly increasing as x becomes large
B1: Either writes full coordinates $(0, \mathrm{e}^{a} - 3)$ in the text or $(0, \mathrm{e}^{a} - 3)$ or $\mathrm{e}^{a} - 3$ marked on the y-axis or even $(\mathrm{e}^{a} - 3, 0)$ if marked on the y axis (must be exact) – allow $\left|\mathrm{e}^{a} - 3\right|$ i.e. allow modulus sign, Can be earned without the graph.

No requirement for $x_Q = \frac{a - \ln 3}{2}$ to be repeated for this mark. It has been credited in part (b)

Leave blank

12.

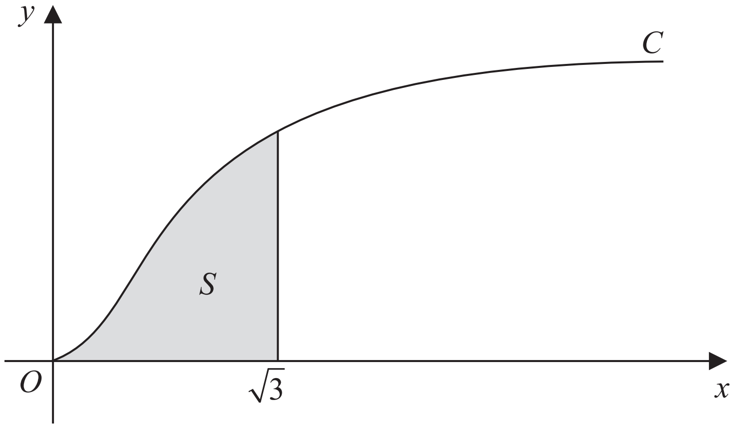


Figure 3

Figure 3 shows a sketch of part of the curve C with parametric equations

$$x = \tan t, \quad y = 2\sin^2 t, \quad 0 \leqslant t < \frac{\pi}{2}$$

The finite region S, shown shaded in Figure 3, is bounded by the curve C, the line $x = \sqrt{3}$ and the x-axis. This shaded region is rotated through 2π radians about the x-axis to form a solid of revolution.

(a) Show that the volume of the solid of revolution formed is given by

$$4\pi \int_0^{\frac{\pi}{3}} (\tan^2 t - \sin^2 t)\, \mathrm{d}t$$

(6)

(b) Hence use integration to find the exact value for this volume.

(6)

Question Number	Scheme	Marks
12 (a)	change limits: $x = 0 \to t = 0$ **and** $x = \sqrt{3} \to t = \frac{\pi}{3}$	B1
	Uses $V = (\pi)\int \underline{y^2}\mathrm{d}\underline{x}$ - in terms of the parameter t	M1
	$(\pi)\int y^2\mathrm{d}x = (\pi)\int y^2\frac{\mathrm{d}x}{\mathrm{d}t}\mathrm{d}t = (\pi)\int (2\sin^2 t)^2\sec^2 t\,\mathrm{d}t$	A1
	$= \{\pi\}\int 4\tan^2 t\sin^2 t\,\mathrm{d}t$ or $= \{\pi\}\int 4\sin^2 t\sin^2 t\frac{1}{\cos^2 t}\mathrm{d}t$	A1
	$= \{\pi\}\int 4\tan^2 t(1-\cos^2 t)\,\mathrm{d}t$ or $= \{\pi\}\int 4\sin^2 t(\sec^2 t - 1)\,\mathrm{d}t$	dM1
	$V = \pi\int_0^{\sqrt{3}} y^2\mathrm{d}x = 4\pi\int_0^{\frac{\pi}{3}}(\tan^2 t - \sin^2 t)\ \mathrm{d}t$ * Correct proof.	A1 * **[6]**
(b)	$\int(\tan^2 t - \sin^2 t)\ \mathrm{d}t = \int \sec^2 t - 1 - \left(\frac{1-\cos 2t}{2}\right)\mathrm{d}t$ Uses $1 + \tan^2 t = \sec^2 t$ (may be implied)	M1
	Uses $\cos 2t = 1 - 2\sin^2 t$ (may be implied)	M1
	$\left\{= \int \sec^2 t - 1 - \frac{1}{2} + \frac{1}{2}\cos 2t\ \mathrm{d}t\right\} = \tan t - t - \frac{1}{2}t + \frac{1}{4}\sin 2t$	M1 A1
	$= \left(\tan\left(\frac{\pi}{3}\right) - \frac{3}{2}\left(\frac{\pi}{3}\right) + \frac{1}{4}\sin\left(\frac{2\pi}{3}\right)\right) - (0)$ Applies limit of $\frac{\pi}{3}$	ddM1
	$= \sqrt{3} - \frac{\pi}{2} + \frac{\sqrt{3}}{8} = \frac{9\sqrt{3}}{8} - \frac{\pi}{2}$	
	$V = 4\pi\left(\frac{9\sqrt{3}}{8} - \frac{\pi}{2}\right)$ or $\pi\left(\frac{9\sqrt{3}}{2} - 2\pi\right)$ oe Two term exact answer	A1 **[6]**
	*See back page for methods using integration by parts	**12**

Notes

(a) **B1**: See **both** $x = 0 \to t = 0$ **and** $x = \sqrt{3} \to t = \frac{\pi}{3}$; Allow if just stated as in scheme- must be in part (a)

M1: attempt at $V = (\pi)\int \underline{y^2}\mathrm{d}\underline{x}$ **- ignore limits and** π **but** need to replace both y^2 and dx by expressions in terms of the parameter t. Methods using Cartesian approach are M0 unless parameters are reintroduced.

A1: $\int(2\sin^2 t)^2\sec^2 t\,\mathrm{d}t$ ignoring limits and π $\left[= \{\pi\}\int 4\sin^4 t\sec^2 t\,\mathrm{d}t = \{\pi\}\int\frac{4\sin^4 t}{\cos^2 t}\mathrm{d}t = \{\pi\}\int 4\tan^4 t\cos^2 t\,\mathrm{d}t\right]$

A1: Obtain $\int 4\tan^2 t\sin^2 t\,\mathrm{d}t$ at some point or $= \{\pi\}\int 4\sin^2 t\sin^2 t\frac{1}{\cos^2 t}\mathrm{d}t$

d**M1**: *Applies* $\sin^2 t = 1 - \cos^2 t$ or $\tan^2 t = \sec^2 t - 1$ after reaching $\int 4\tan^2 t\sin^2 t\,\mathrm{d}t$ or $\int 4\sin^2 t\sin^2 t\frac{1}{\cos^2 t}\mathrm{d}t$

A1*: Obtains given answer with no errors seen (**To obtain this mark** π **must have been included in** $V = \underline{\pi}\int \underline{y^2}\mathrm{d}x$)

This answer must include limits, but can follow B0 scored earlier. Any use of d*x* where d*t* should be used is M0

(b) **M1**: Uses $1 + \tan^2 t = \sec^2 t$

M1: Uses $\cos 2t = 1 - 2\sin^2 t$

M1: At least two terms of $\pm A\tan t \pm Bt \pm C\sin 2t$ **A1**: Correct integration of $\tan^2 t - \sin^2 t$ with all signs correct

ddM1: (depends upon the first two M1 marks being awarded in part (b)) Substitutes $\frac{\pi}{3}$ into their integrand (can be implied by answer or by 4.75) **A1**: Two term **exact** answer for V

13. (a) Express $2\sin\theta + \cos\theta$ in the form $R\sin(\theta + \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < 90°$. Give your value of α to 2 decimal places. **(3)**

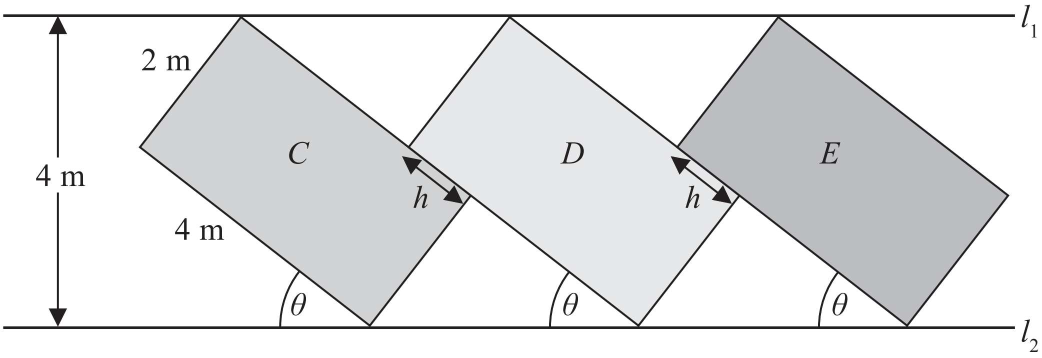


Figure 4

Figure 4 shows the design for a logo that is to be displayed on the side of a large building. The logo consists of three rectangles, *C*, *D* and *E*, each of which is in contact with two horizontal parallel lines l_1 and l_2. Rectangle *D* touches rectangles *C* and *E* as shown in Figure 4.

Rectangles *C*, *D* and *E* each have length 4 m and width 2 m. The acute angle θ between the line l_2 and the longer edge of each rectangle is shown in Figure 4.

Given that l_1 and l_2 are 4 m apart,

(b) show that

$$2\sin\theta + \cos\theta = 2$$

(2)

Given also that $0 < \theta < 45°$,

(c) solve the equation

$$2\sin\theta + \cos\theta = 2$$

giving the value of θ to 1 decimal place. **(3)**

Rectangles *C* and *D* and rectangles *D* and *E* touch for a distance h m as shown in Figure 4.

Using your answer to part (c), or otherwise,

(d) find the value of h, giving your answer to 2 significant figures. **(3)**

Question Number	Scheme	Marks
13. (a)	$R=\sqrt{5}=$ 2.23606... (must be given in part (a))	B1
	$\tan\alpha=\frac{1}{2}$ or $\sin\alpha=\frac{1}{\sqrt{5}}$ or $\cos\alpha=\frac{2}{\sqrt{5}}$ (see notes for other values which gain M1)	M1
	$\Rightarrow\ \alpha=26.56505...^{\circ}$ (must be given in part (a))	A1
		[3]
(b)	Way 1: Uses distance between two lines is 4 (or half distance is 2) with correct trigonometry **may** state $4\sin\theta+2\cos\theta=4$ **or** show sketch	M1
	Need sketch **and** $4\sin\theta+2\cos\theta=4$ **and** deduction that $2\sin\theta+\cos\theta=2$ or $\cos\theta+2\sin\theta=2$ *	A1 * **[2]**
	Way 2: **Alternative method:** Uses diagonal of rectangle as hypotenuse of right angle triangle and obtains $\sqrt{20}\sin(\theta+\alpha)=4$	M1
	So from (a) $2\sin\theta+\cos\theta=2$ or $\cos\theta+2\sin\theta=2$	A1 **[2]**
	Way 3: They may state and verify the result provided the work is correct and accurate See notes below. **Substitution of 36.9** (obtained in (c) is a circular argument and is **M0A0**)	
(c)	Way1: Uses $\sqrt{5}\sin(\theta+26.57)=2$ to obtain Way 2 $\cos^2\theta+4\cos\theta\sin\theta+4\sin^2\theta=4$ See notes for variations	
	$\sin(\theta+"26.57")=\frac{2}{"\sqrt{5}"}$ $(=0.8944...)$ $4\cos\theta\sin\theta-3\cos^2\theta=0$ $\cos\theta(4\sin\theta-3\cos\theta)=0$ so $\tan\theta=\frac{3}{4}$	M1
	$\theta=\arcsin\left(\frac{2}{\text{their }"\sqrt{5}"}\right)-"26.57"$ $\theta=\arctan\frac{3}{4}$ or equivalent	M1
	Hence, $\theta=36.8699...^{\circ}$	A1
		[3]
(d)	Way 1: $"x"=\frac{2}{\tan"36.9"}$ Way 2: $“y”=\frac{4}{\sin\theta}$	B1
	$\{h+x=4\Rightarrow\}\ h+\frac{2}{\tan"36.9"}=4$ $\{h+y=8\Rightarrow\}\ h+\frac{4}{\sin"36.9"}=8$	M1
	$h=4-\frac{2}{\tan 36.9}=1.336...$ or $\frac{4}{3}$ or <u>1.3</u> (2sf) $h=8-\frac{4}{\sin 36.9}=\frac{4}{3}$ or <u>1.3</u> (2sf)	<u>A1</u> **cao** **[3]** **11**

Notes

(a) **B1**: $R=\sqrt{5}$ or awrt 2.24 no working needed – must be in part (a)

M1: $\tan\alpha=\frac{1}{2}$ **or** $\tan\alpha=2$ or $\sin\alpha=\frac{1}{\sqrt{5}}$ or $\sin\alpha=\frac{2}{\sqrt{5}}$ or $\cos\alpha=\frac{2}{\sqrt{5}}$ or $\cos\alpha=\frac{1}{\sqrt{5}}$ and attempt to find alpha. Method mark may be implied by correct alpha.

A1: accept $\alpha=$ awrt 26.57; also accept $\sqrt{5}\sin\left(\theta+26.57\right)$ - must be in part (a)

Answers in radians (0.46) are A0

(b) **Way 1:**

M1: Uses distance between two lines is 4 (or half distance is 2) states $4\sin\theta+2\cos\theta=4$ **or** shows sketch (may be on Figure 4 on question paper) with some trigonometry

A1*: **Shows sketch** with implication of two right angled triangles (may be on Figure 4 on question paper) **and follows** $4\sin\theta+2\cos\theta=4$ by stating printed answer or equivalent (given in the mark scheme) and no errors seen.

Way 2:

on scheme (not a common method)

Way 3:

They may state and verify the result provided the work is correct and accurate.

M1: Verification with correct accurate work e.g. $2\times\frac{x}{4}+\frac{4-x}{2}=2$, with x shown on figure

A1: Needs conclusion that $2\sin\theta+\cos\theta=2$

Substitution of 36.9 (obtained in (c) is a circular argument and is **M0A0**)

(c) Way 1:

M1: $\sin\left(\theta+\text{their }\alpha\right)=\frac{2}{\text{their }R}$ (Uses part (a) to solve equation)

M1: $\theta=\arcsin\left(\frac{2}{\text{their }R}\right)-\text{their }\alpha$ (operations undone in the correct order with subtraction)

A1: awrt 36.9 (answer in radians is 0.644 and is A0)

Way 2:

M1: Squares both sides, uses appropriate trig identities and reaches $\tan\theta=\frac{3}{4}$ or $\sin\theta=\frac{3}{5}$ or $\cos\theta=\frac{4}{5}$ or $\sin 2\theta=\frac{24}{25}$

{One example is shown in the scheme. Another popular one is $2\sin\theta=2-\cos\theta\rightarrow 4(1-\cos^2\theta)=4-4\cos\theta+\cos^2\theta\rightarrow 5\cos^2\theta-4\cos\theta=0$ and so $\cos\theta=\frac{4}{5}$ for M1}

M1: $\theta=\arctan\frac{3}{4}$ or other correct inverse trig value e.g. $arc\sin\theta\ (\frac{3}{5})$ or $arc\cos\theta\ (\frac{4}{5})$

A1: awrt 36.9 (answer in radians is 0.644 and is A0)

(d) Way 1: (Most popular)

B1 : States $x=\frac{2}{\tan\theta}$, where x (not defined in the question) is the non-overlapping length of rectangle

M1: Writes equation $h+\frac{2}{\tan\theta}=4$ - must be this expression or equivalent e.g. $\tan\theta=\frac{2}{4-h}$ gets B1 M1

A1: accept decimal which round to 1.3 or the exact answer i.e. $\frac{4}{3}$ (may follow slight inaccuracies in earlier angle being rounded wrongly)

N.B. There is a variation which states $\sin\theta=\frac{2\cos\theta}{4-h}$ or $\frac{\sin\theta}{2}=\frac{\sin(90-\theta)}{4-h}$ for B1 M1 then A1 as before

Way 2: (Less common)

B1 : States $y = \dfrac{4}{si\text{n}\,\theta}$, where y (not defined in question) is the non-overlapping length of two rectangles

M1: Writes equation $h + \dfrac{4}{si\text{n}\,\theta} = 8$ - must be this expression or equivalent e.g. $\sin\theta = \dfrac{4}{8-h}$ gets B1 M1

A1: as in Way 1

There are other longer trig methods – possibly using Pythagoras for showing that *h* = 1.3 to 2sf. If the method is clear award B1M1A1 – otherwise send to review.

14. Relative to a fixed origin O, the line l has vector equation

$$\mathbf{r} = \begin{pmatrix} -1 \\ -4 \\ 6 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$$

where λ is a scalar parameter.

Points A and B lie on the line l, where A has coordinates $(1, a, 5)$ and B has coordinates $(b, -1, 3)$.

(a) Find the value of the constant a and the value of the constant b.

(3)

(b) Find the vector $\overrightarrow{AB}$.

(2)

The point C has coordinates $(4, -3, 2)$

(c) Show that the size of the angle CAB is 30°

(3)

(d) Find the exact area of the triangle CAB, giving your answer in the form $k\sqrt{3}$, where k is a constant to be determined.

(2)

The point D lies on the line l so that the area of the triangle CAD is twice the area of the triangle CAB.

(e) Find the coordinates of the two possible positions of D.

(4)

Question Number	Scheme	Marks
14.	$A(1, a, 5)$, $B(b, -1, 3)$, $l: \mathbf{r} = -\mathbf{i} - 4\mathbf{j} + 6\mathbf{k} + \lambda(2\mathbf{i} + \mathbf{j} - \mathbf{k})$	
(a)	**Either** at point $A: \lambda = 1$ **or** at point $B: \lambda = 3$	M1
	leading to either $a = -3$ **or** $b = 5$	A1
	leading to both $a = -3$ **and** $b = 5$	A1
		[3]
(b)	Attempts $\pm\left[('5\mathbf{i}' - \mathbf{j} + 3\mathbf{k}) - (\mathbf{i}\ '-3\mathbf{j}' + 5\mathbf{k})\right]$ subtraction either way round	M1
	$\overrightarrow{AB} = 4\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$ o.e. subtraction correct way round	A1
		[2]
(c)	Way 1 $(\overrightarrow{AC}) = \begin{pmatrix} 3 \\ "0" \\ -3 \end{pmatrix}$ or $(\overrightarrow{CA}) = \begin{pmatrix} -3 \\ "0" \\ 3 \end{pmatrix}$ Way 2 $AB = 2\sqrt{6},\ AC = 3\sqrt{2},\ BC = \sqrt{6}$	M1
	Way 1 $\cos C\hat{A}B = \dfrac{\begin{pmatrix} 4 \\ 2 \\ -2 \end{pmatrix} \bullet \begin{pmatrix} 3 \\ 0 \\ -3 \end{pmatrix}}{\sqrt{(4)^2 + (2)^2 + (-2)^2} . \sqrt{(3)^2 + (0)^2 + (-3)^2}}$ Way 2 $\cos C\hat{A}B = \dfrac{24 + 18 - 6}{2\sqrt{24}\sqrt{18}}$ Or right angled triangle and $\cos C\hat{A}B = \dfrac{\sqrt{3}}{2}$ o.e.	dM1
	Way 1 $\cos C\hat{A}B = \dfrac{12 + 0 + 6}{\sqrt{24} . \sqrt{18}} = \dfrac{\sqrt{3}}{2}$ (o.e.) $\Rightarrow C\hat{A}B = 30^\circ$ * Way 2 so $C\hat{A}B = 30^\circ$	A1 * cso
		[3]
(d)	Area $CAB = \dfrac{1}{2}\sqrt{24}\sqrt{18}\sin 30^\circ$	M1
	$= 3\sqrt{3}$ (or $k = 3$)	A1
		[2]
(e)	$\left(\overrightarrow{OD_1}\right) = \begin{pmatrix} -1 \\ -4 \\ 6 \end{pmatrix} + 5\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$ **or** $= \begin{pmatrix} 1 \\ a \\ 5 \end{pmatrix} + 2\begin{pmatrix} 4 \\ 2 \\ -2 \end{pmatrix}$ or $= \begin{pmatrix} b \\ -1 \\ 3 \end{pmatrix} + \begin{pmatrix} 4 \\ 2 \\ -2 \end{pmatrix}$; = or…	M1; oe
	$\begin{pmatrix} 9 \\ 1 \\ 1 \end{pmatrix}$	A1
	$\overrightarrow{OD_2} = \begin{pmatrix} -1 \\ -4 \\ 6 \end{pmatrix} - 3\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$ **or** $= \begin{pmatrix} 1 \\ a \\ 5 \end{pmatrix} - 2\begin{pmatrix} 4 \\ 2 \\ -2 \end{pmatrix}$ or $= \begin{pmatrix} b \\ -1 \\ 3 \end{pmatrix} - 3\begin{pmatrix} 4 \\ 2 \\ -2 \end{pmatrix}$;	M1; oe
	$= \begin{pmatrix} -7 \\ -7 \\ 9 \end{pmatrix}$	A1
		[4]
	See notes for a common approach to part (e) using the length of *AD*	14

Notes

Throughout – allow vectors to be written as a row, with commas, as this is another convention.

(a) **M1**: Finds, or implies, correct value of λ for at least one of the two given points

A1: **At least one** of *a* **or** *b* correct

A1: **Both** *a* **and** *b* correct

(b) **M1**: Subtracts the position vector of *A* from that of *B* or the position vector of *B* from that of *A*. Allow any notation. Even allow coordinates to be subtracted. Follow through their *a* and *b* for this method mark.

A1: Need correct answer : so $\overrightarrow{AB} = 4\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$ or $\overrightarrow{AB} = \begin{pmatrix} 4 \\ 2 \\ -2 \end{pmatrix}$ or (4, 2, -2) This is not ft.

(c) Way 1:

M1: Subtracts the position vector of *A* from that of *C* or the position vector of *C* from that of *A*. Allow any notation. Even allow coordinates to be subtracted. Follow through their *a* for this method mark.

dM1: Applies dot product formula between their $\left(\overrightarrow{AB} \text{ or } \overrightarrow{BA}\right)$ and their $\left(\overrightarrow{AC} \text{ or } \overrightarrow{CA}\right)$.

A1*: Correctly proves that $C\hat{A}B = 30^\circ$. This is a printed answer. Must have used $\left(\overrightarrow{AB} \text{ with } \overrightarrow{AC}\right)$ or $\left(\overrightarrow{BA} \text{ with } \overrightarrow{CA}\right)$ for this mark and must not have changed a negative to a positive to falsely give the answer, that would result in M1M1A0

Do not need to see $\frac{\sqrt{3}}{2}$ but should see equivalent value. Allow $\frac{\pi}{6}$ as final answer.

Way 2:

M1: Finds lengths of AB, AC and BC

dM1: Uses cosine rule or trig of right angled triangle, either sin, cos or tan

A1: Correct proof that angle = 30 degrees

(d) **M1**: Applies $\frac{1}{2}\left|\overrightarrow{AB}\right|\left|\overrightarrow{AC}\right|\sin 30^\circ$ **-** must try to use their vectors (b – a) and (c – a) or state formula and try to use it. Could use vector product. Must not be using $\frac{1}{2}\left|\overrightarrow{OB}\right|\left|\overrightarrow{OC}\right|\sin 30^\circ$

A1: $3\sqrt{3}$ cao – must be exact and in this form (see question)

(e) **M1**: Realises that *AD* is twice the length of *AB* and uses **complete method** to find one of the points. Then uses one of the three possible starting points on the line (*A*, *B*, or the point with position vector $-\mathbf{i} - 4\mathbf{j} + 6\mathbf{k}$) to reach *D*. See one of the equations in the mark scheme and ft their *a* or *b*.

So accept $\left(\overrightarrow{OD_1}\right) = \begin{pmatrix} -1 \\ -4 \\ 6 \end{pmatrix} + 5\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$ or $= \begin{pmatrix} 1 \\ a \\ 5 \end{pmatrix} + 4\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$ or $= \begin{pmatrix} b \\ -1 \\ 3 \end{pmatrix} + 2\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$

A1: Accept (9, 1, 1) or 9**i** + **j** + **k** or $\begin{pmatrix} 9 \\ 1 \\ 1 \end{pmatrix}$ cao

M1: Realises that *AD* is twice the length of *AB* but is now in the opposite direction so uses one of the three possible starting points to reach *D*. See one of the equations in the mark scheme and ft their *a* or *b*.

<table>
<tr><td colspan="4">So accept $\left(\overrightarrow{OD_2}\right) = \begin{pmatrix}-1\\-4\\6\end{pmatrix} - 3\begin{pmatrix}2\\1\\-1\end{pmatrix}$ or $= \begin{pmatrix}1\\a\\5\end{pmatrix} - 2\begin{pmatrix}4\\2\\-2\end{pmatrix}$ or $= \begin{pmatrix}b\\-1\\3\end{pmatrix} - 3\begin{pmatrix}4\\2\\-2\end{pmatrix}$
A1: Accept (-7, -7, 9) or -7i -7j + 9k or $\begin{pmatrix}-7\\-7\\9\end{pmatrix}$ cao
NB Many long methods still contain unknown variables x, y and z or λ. These are not complete methods so usually earn M0A0M0A0 on part (e) PTO.</td></tr>
<tr><td>(e)</td><td>Mark scheme for a common approach to part (e) using the length of <i>AD</i> is given below:
$(2\lambda-2)^2+(\lambda-1)^2+("1"-\lambda)^2="96"$ then obtain $\lambda^2-2\lambda-15=0$ so $\lambda=$, then substitute value of λ to find coordinates. May make a slip in algebra expanding brackets or collecting terms (even if results in two term quadratic)
This may be simplified to $\sqrt{6}(\lambda-1)=4\sqrt{6}$ or to $\sqrt{6}(1-\lambda)=4\sqrt{6}$
NB $6(1-\lambda)^2=4\sqrt{6}$ is M0 as one side has dimension (length)^2 and the other is length
$\begin{pmatrix}9\\1\\1\end{pmatrix}$ (from λ=5)
Substitute other value of λ. May make a slip in algebra
$=\begin{pmatrix}-7\\-7\\9\end{pmatrix}$(from λ=-3)
Special case – uses AD is half AB instead of double AB
$(2\lambda-2)^2+(\lambda-1)^2+("1"-\lambda)^2="6"$ then obtain $\lambda^2-2\lambda=0$ so $\lambda=$, then substitute value of λ to find coordinates
$\begin{pmatrix}-1\\-4\\6\end{pmatrix}$ (from λ=0)
Substitute other value of λ
$=\begin{pmatrix}3\\-2\\4\end{pmatrix}$(from λ=2)
For this solution score M1A0M1A0 i.e. 2/4</td><td>M1
A1
M1
A1</td><td>[4]</td></tr>
</table>

Qu 12(b) using integration by parts

Qu 12 (b) Some return to $V=\{\pi\}\int 4\tan^2 t\sin^2 t\,\mathrm{d}t$ .There **are two ways to proceed and both use integration by parts**

(b)	Way 1: $\int(\tan^2 t\ \ \sin^2 t)\ \mathrm{d}t = \int(\sec^2 t-1)\sin^2 t\ \mathrm{d}t$ Uses $1+\tan^2 t=\sec^2 t$	M1
	Uses $\cos 2t = 1-2\sin^2 t$	M1
	$\left\{= \sin^2 t\tan t-\int 2\sin t\cos t\tan t\,\mathrm{d}t-\int\frac{1-\cos 2t}{2}\mathrm{d}t\right\} = \sin^2 t\tan t-\frac{3}{2}t+\frac{3}{4}\sin 2t$	M1 A1
	$= -\left(\frac{3}{4}\tan\left(\frac{\pi}{3}\right)-\left(\frac{\pi}{2}\right)+\frac{3}{4}\sin\left(\frac{2\pi}{3}\right)\right)-(0)$ Applies limit of $\frac{\pi}{3}$	ddM1
	$V = 4\pi\left(\frac{9\sqrt{3}}{8}-\frac{\pi}{2}\right)$ or $\pi\left(\frac{9\sqrt{3}}{2}-2\pi\right)$ oe Two term exact answer	A1
		[6]
	Way 2: Try to use parts on $\int(\sec^2 t-1)\sin^2 t\ \mathrm{d}t$ using $u=\sin^2 t$ and $v=\tan t-t$	
	Award first two M marks as before Uses $1+\tan^2 t=\sec^2 t$ and Uses $\cos 2t = 1-2\sin^2 t$	M1 M1
	This needs parts twice and to get down to $=\sin^2 t(\tan t-t)-t+\frac{1}{2}\sin 2t-\frac{t}{2}\cos 2t+\frac{1}{4}\sin 2t$	M1A1
	Then limits as before to give $V = 4\pi\left(\frac{9\sqrt{3}}{8}-\frac{\pi}{2}\right)$ or $\pi\left(\frac{9\sqrt{3}}{2}-2\pi\right)$ oe	ddM1A1