Write your name here

Surname	Other names

Pearson Edexcel
International Advanced Level

Centre Number

Candidate Number

Core Mathematics C34

Advanced

Monday 27 January 2014 – Morning
Time: 2 hours 30 minutes

Paper Reference
WMA02/01

You must have:
Mathematical Formulae and Statistical Tables (Blue)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 125.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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1. $$f(x) = \frac{2x}{x^2 + 3}, \qquad x \in \mathbb{R}$$

Find the set of values of x for which $f'(x) > 0$

You must show your working.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(6)

Question Number	Scheme	Marks
1.	$\mathrm{f}(x)=\dfrac{2x}{x^2+3} \quad \Rightarrow \mathrm{f}'(x)=\dfrac{(x^2+3)2-2x\times 2x}{(x^2+3)^2}=\left(\dfrac{6-2x^2}{(x^2+3)^2}\right)$	M1A1
	$\mathrm{f}'(x)>0 \Rightarrow \dfrac{6-2x^2}{(x^2+3)^2}>0$	
	Critical values $\quad 6-2x^2=0 \Rightarrow x=\pm\sqrt{3}$	M1A1
	Inside region chosen $\quad -\sqrt{3}<x<\sqrt{3}$	dM1A1
		(6 marks)

Notes

M1 Applies the Quotient rule, a form of which appears in the formula book, to $\dfrac{2x}{x^2+3}$

If the formula is quoted it must be correct. There must have been some attempt to differentiate both terms.
If the rule is not quoted nor implied by their working, meaning that terms are written out
$u=2x,\ v=x^2+3,\ u'=..,\ v'=...$ followed by their $\dfrac{vu'-uv'}{v^2}$, then only accept answers of the form

$\dfrac{(x^2+3)A-2x\times Bx}{(x^2+3)^2}$ $\quad A,B>0$. Condone invisible brackets for the M.

Alternatively applies the product rule with $u=2x,\ v=\left(x^2+3\right)^{-1}$

If the formula is quoted it must be correct. There must have been some attempt to differentiate both terms.
If the rule is not quoted nor implied by their working, meaning that terms are written out
$u=2x,\ v=\left(x^2+3\right)^{-1},\ u'=..,\ v'=...$ followed by their $vu'+uv'$, then only accept answers of the form
$\left(x^2+3\right)^{-1}\times A\pm 2x\times\left(x^2+3\right)^{-2}\times Bx$.
Condone invisible brackets for the M.

A1 Any fully correct (unsimplified) form of $\mathrm{f}'(x)$

Accept versions of $\mathrm{f}'(x)=\dfrac{(x^2+3)2-2x\times 2x}{(x^2+3)^2}$ for the quotient rule or

Versions of $\mathrm{f}'(x)=\left(x^2+3\right)^{-1}\times 2-2x\times\left(x^2+3\right)^{-2}\times 2x$ for use of the product rule.

M1 Setting their numerator of $\mathrm{f}'(x)=0$ or >0, and proceeding to find two critical values.

A1 Both critical values $\pm\sqrt{3}$ are found. Accept for this mark expressions like $x>\pm\sqrt{3}$ and ± 1.73

dM1 For choosing the inside region of their critical values.
The inequality (if seen) must have been of the correct form. Either $Ax^2....-B<0$, $C......-Dx^2>0$ or $x^2<C$ .It is dependent upon having set the numerator >0 or =0.

A1 Correct solution only. $-\sqrt{3}<x<\sqrt{3}$. Accept $\left(-\sqrt{3},\sqrt{3}\right)$ $\ x<\sqrt{3}$ and $x>-\sqrt{3}$

Do not accept $x<\sqrt{3}$ or $x>-\sqrt{3}$ or $-1.73<x<1.73$.
Do not accept a correct answer coming from an incorrect inequality. This would be dM0A0.
Condone a solution $x^2<3 \Rightarrow x<\pm\sqrt{3} \Rightarrow -\sqrt{3}<x<\sqrt{3}$
Do not accept a solution without seeing a correct $\mathrm{f}'(x)$ first. Note that this is a demand of the question.

Leave blank

2. Solve, for $0 \leqslant x \leqslant 270°$, the equation

$$\frac{\tan 2x + \tan 50°}{1 - \tan 2x \tan 50°} = 2$$

Give your answers in degrees to 2 decimal places.

(6)

Question Number	Scheme	Marks
2	$\dfrac{\tan 2x + \tan 50°}{1 - \tan 2x \tan 50°} = 2 \quad \Rightarrow \tan(2x + 50°) = 2$	M1A1
	$\Rightarrow 2x + 50° = 63.43°, (243.43°, 423.43°)$	
	$\Rightarrow x = \text{awrt } 6.72° \text{ or } 96.72° \text{ or } 186.72°$	dM1, A1
	$\Rightarrow 2x + 50° = 243.43° \ (423.43°) \Rightarrow x = ..$	dM1
	$x = \text{awrt } 6.72°, 96.72°, 186.72°$	A1
		(6 marks)

Notes

M1 Uses the compound angle identity $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$ to write the equation in the form $\tan(2x \pm 50°) = 2$. Accept a sign error in bracket.

A1 $\tan(2x + 50°) = 2$

dM1 Uses the correct order of operations to find one solution in the range.
Moves from $\tan(2x \pm 50°) = 2 \Rightarrow 2x \pm 50° = \arctan 2 \Rightarrow x = ...$
This is dependent upon having scored the first M1

A1 One correct answer, usually awrt $6.72°$, but accept any of $6.72°, 96.72°, 186.72°$

dM1 Uses the correct order of operations to find a second solution in the range.
This can be scored by $2x \pm 50° = 180 + \text{their } 63 \ or \ 360 + \text{their } 63 \Rightarrow x = ..$
It may be implied by $90 + \text{their } 6.7, \ or \ 180 + \text{their } 6.7$ as long as no incorrect working is seen.
This is dependent upon having scored the first M1

A1 All three answers in the range, $x = \text{awrt } 6.72°, 96.72°, 186.72°$
Any extra solutions in the range withhold the last A mark.
Ignore any solutions outside the range $0 \leqslant x \leqslant 270°$
Radian solutions will be unlikely, but could be worth marks only if $50° \rightarrow 0.873$ radians.
$\tan(2x + 50)° = 2 \Rightarrow 2x + 50° = 1.107..$ will score M1A1dM0 and nothing else.

Question Number	Scheme	Marks
2(alt 1)	$\dfrac{\tan 2x + \tan 50°}{1 - \tan 2x \tan 50°} = 2 \quad \Rightarrow \tan 2x + \tan 50° = 2\left(1 - \tan 2x \tan 50°\right)$	
	$\tan 2x = \dfrac{2 - \tan 50°}{1 + 2\tan 50°} \qquad = (0.239...)$	M1A1
	$2x = 13.435° \Rightarrow x = \text{awrt } 6.72°$	dM1 A1
	$\Rightarrow 2x° = 193.435°\ (373.435°) \Rightarrow x = ..$	dM1
	$x = \text{awrt } 6.72°, 96.72°, 186.72°$	A1
		(6 marks)

Notes

M1 Cross mulitiplies, collects terms in $\tan 2x$ and makes $\tan 2x$ the subject.
Allow for $\tan 2x = ...$

A1 Accept $\tan 2x = \dfrac{2 - \tan 50°}{1 + 2\tan 50°}$ or the decimal equivalent $\tan 2x = \text{awrt } 0.239$

dM1 Correct order of operations to find one solution $\tan 2x = ... \Rightarrow 2x = \arctan .. \Rightarrow x = ..$
This is dependent upon having scored the first M1

A1 One correct solution usually awrt $6.72°$

dM1 Uses the correct order of operations to find another solution in the range.
This can be scored by Either $2x = 180 + \text{their } 13.4$ *or* $360 + \text{their } 13.4 \Rightarrow x = ..$
Or $90 + \text{their } 6.7$, *or* $180 + \text{their } 6.7$
This is dependent upon having scored the first M1

A1 All three answers in the range, $x = \text{awrt } 6.72°, 96.72°, 186.72°$
Any extra solutions in the range withhold the last A mark.
Ignore any solutions outside the range $0 \leqslant x \leqslant 270°$

Question Number	Scheme	Marks
2(alt 2)	Similar to alt 1 but additionally uses $\tan 2x = \dfrac{2\tan x}{1 - \tan^2 x}$	
	$(2 - \tan 50)\tan^2 x + (2 + 4\tan 50)\tan x + (\tan 50 - 2) = 0$	
	$\tan x = \text{awrt } 0.118, -8.49$	M1A1
	$x = 6.72, 96.72, 186.72$	dM1 A1dM1A1

Leave blank

3. Given that

$$4x^3 + 2x^2 + 17x + 8 \equiv (Ax + B)(x^2 + 4) + Cx + D$$

(a) find the values of the constants A, B, C and D.

(4)

(b) Hence find

$$\int_1^4 \frac{4x^3 + 2x^2 + 17x + 8}{x^2 + 4}\,\mathrm{d}x$$

giving your answer in the form $p + \ln q$, where p and q are integers.

(6)

Question Number	Scheme	Marks
3(a)	$4x^3+2x^2+17x+8 \equiv (Ax+B)(x^2+4)+Cx+D$	
	Compare x^3 terms: $A=4$	B1
	Compare x^2 terms: $B=2$	B1
	Compare either x term or constant term: $4A+C=17$ or $4B+D=8$	
	$\Rightarrow C=..\text{or } D=..$	M1
	$\Rightarrow C=1,\ D=0$	A1
		(4)
(b)	$\int_1^4 \frac{4x^3+2x^2+17x+8}{x^2+4}\,\mathrm{d}x = \int_1^4 4x+2+\frac{x}{x^2+4}\,\mathrm{d}x$	M1
	$=\left[2x^2+2x,+\frac{1}{2}\ln(x^2+4)\right]_1^4$	M1, M1A1
	$=\left[2\times16+2\times4+\frac{1}{2}\ln(20)\right]-\left[2\times1+2\times1+\frac{1}{2}\ln(5)\right]$	
	$=36+\frac{1}{2}\ln\left(\frac{20}{5}\right)$	dM1
	$=36+\ln(2)$	A1
		(6)
		(10 marks)

Notes for Question 3

(a)

B1 States that A=4. It may be implied by writing out the rhs as $(4x+B)(x^2+4)+Cx+D$

B1 States that B=2. It may be implied by writing out the rhs as $(Ax+2)(x^2+4)+Cx+D$

M1 Compares either the x or constant terms and proceeding to find a numerical value of either C or D. This mark may be implied by a correct value of either C or D.

A1 Both values correct $C=1, D=0$

Alternatively can be scored via 'division'.

$$\begin{array}{r}
4x+2 \\
x^2+0x+4\overline{)4x^3+2x^2+17x+8} \\
\underline{4x^3+0x^2+16x} \\
.\ +2x^2+1x+8 \\
\underline{2x^2+0x+8} \\
1x+0
\end{array}$$

B1, B1 for sight of the 4 and the 2 in quotient $4x+2$

M1 for proceeding to get a linear remainder

A1 The correct linear remainder. Accept x. If their division is unclear accept the answers in the correct place in part b

(b)

M1 For using their answers to part (a) to rewrite the integral in the form

$$\int \frac{4x^3+2x^2+17x+8}{x^2+4}\,dx = \int 'A'x+'B'+\frac{'C'x+'D'}{x^2+4}\,dx$$ with numerical values for $A,B,C,$ and D.

We will condone a restart to this question in (b) but it would not score marks in (a)

M1 For the correct method of integrating the $'A'x+'B'$ part. Follow through on their 'A' and 'B'

Accept $\frac{A}{2}x^2+Bx$ or $\frac{(Ax+B)^2}{2}$. It cannot be scored for an attempt to integrate $('A'x+'B')(x^2+4)$

M1 For the correct method of integrating $\frac{'C'x}{x^2+4}$. Accept constant$\times\ln(x^2+4)$ or constant$\times x\frac{\ln(x^2+4)}{x}$

This can be scored for values of $D\neq 0$ as long as $\int \frac{'C'x+'D'}{x^2+4}\,dx = \int \frac{'C'x}{x^2+4}\,dx + \cancel{\int \frac{'D'}{x^2+4}\,dx}$.

A1 Correct integral $2x^2+2x+\frac{1}{2}\ln(x^2+4)+(c)$. There is no requirement for $+c$.

dM1 For putting in limits, subtracting **and** correctly collecting terms in ln's using subtraction law.

It is dependent upon having scored the previous M. Allow, for example, $\frac{1}{2}\ln 20-\frac{1}{2}\ln 5=\ln 2$

A1 CSO and CAO $36+\ln(2)$

Leave blank

4.

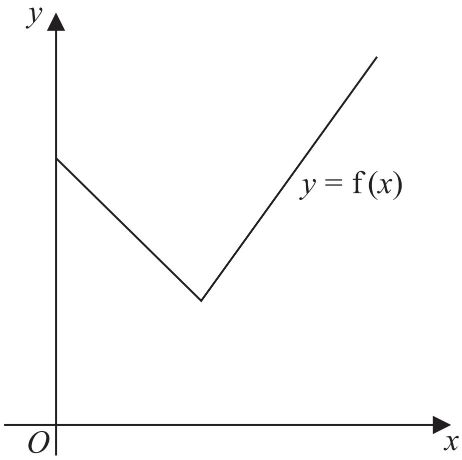

Figure 1

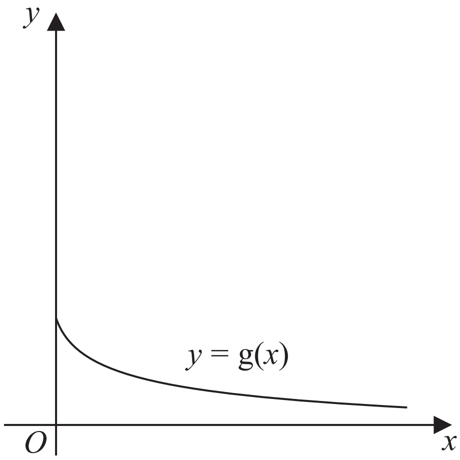

Figure 2

Figure 1 shows a sketch of part of the graph $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = 2|3 - x| + 5, \quad x \geqslant 0$$

Figure 2 shows a sketch of part of the graph $y = \mathrm{g}(x)$, where

$$\mathrm{g}(x) = \frac{x+9}{2x+3}, \quad x \geqslant 0$$

(a) Find the value of fg(1)

(2)

(b) State the range of g

(2)

(c) Find $\mathrm{g}^{-1}(x)$ and state its domain.

(4)

Given that the equation $\mathrm{f}(x) = k$, where k is a constant, has exactly two roots,

(d) state the range of possible values of k.

(3)

Question Number	Scheme	Marks
4(a)	$\mathrm{fg}(1) = \mathrm{f}(2) = 7$	M1A1
		(2)
(b)	Either g(0)=3 or $g(x \to \infty) \to 0.5$	M1
	$0.5 < \mathrm{g}(x) \leqslant 3$	A1
		(2)
(c)	Attempt change of subject of $y = \dfrac{x+9}{2x+3} \Rightarrow y(2x+3) = x+9$	
	$\Rightarrow 2xy - x = 9 - 3y$	M1
	$\Rightarrow x(2y-1) = 9 - 3y \Rightarrow x = \dfrac{9-3y}{2y-1}$	dM1
	$\mathrm{g}^{-1}(x) = \dfrac{9-3x}{2x-1}, \quad 0.5 < x \leqslant 3$	A1, B1 ft
		(4)
(d)	Attempts f(0) = 2×3+5=11 $\Rightarrow k \leqslant 11$ Or f(3) = 2×0+5=5 $\Rightarrow k > 5$	M1A1
	$5 < k \leqslant 11$	A1
		(3)
		(11 marks)

Notes for Question 4

(a)

M1 Full method for their answer to g(1) being subbed into f. The order must be correct.

Can be scored for seeing 1 being substituted into $2\left|3-\dfrac{x+9}{2x+3}\right|+5$

A1 cso 7. Do not accept multiple answers.
Just '7' would score both marks as long as no incorrect working is seen

(b)

M1 Calculates the value of g at either 'end. Sight of 3 or 0.5 is sufficient.

A1 $0.5<\text{g}(x)\leqslant 3$. Accept $0.5<y\leqslant 3$

Also accept variations such as $(0.5,3]$, All values (of y) bigger than 0.5 but less than or equal to 3.

Do not accept this in terms of x.

(c)

M1 For an attempt to make x (or a switched y) the subject of the formula. For this to be scored they must cross multiply and get both x (or switched y) terms on the same side of the equation. Allow slips.

dM1 This is dependent upon the first M being scored. In addition to collecting like terms they must factorise and divide. Condone just **one numerical/sign** slip. Accept x being given as a function of y.

A1 $\text{g}^{-1}(x)=\dfrac{9-3x}{2x-1}$ or $\text{g}^{-1}(x)=\dfrac{3x-9}{1-2x}$. Accept the form $y=$, $\text{g}^{-1}=$ instead of $\text{g}^{-1}(x)$ but the function must be in terms of x.

B1ft Accept either $0.5<x\leqslant 3$, $(0.5,3]$ or follow through on the candidates range to part (b).

The domain cannot be expressed in terms of y.
Do not follow through on $y\in\mathbb{R}\Rightarrow x\in\mathbb{R}$

(d)

M1 Attempts to find either f(0) or f(3). Evidence could be seeing $2|3-0|+5$ or $2|3-3|+5$
or the sight of 5 or 11.

A1 For a range with both ends, for example $5<k<11$, $5\leqslant k\leqslant 11$, $5\leqslant k<11$ or alternatively getting one end completely correct $k\leqslant 11$, or $k>5$. Accept $y\leqslant 11$ etc

A1 cao $5<k\leqslant 11$. Accept $(5,11]$, $k\leqslant 11$ **and** $k>5$

Do not accept $k\leqslant 11$ **or** $k>5$ **or** $5<y\leqslant 11$ for the final mark

Leave blank

5. (a) Prove, by using logarithms, that

$$\frac{\mathrm{d}}{\mathrm{d}x}(2^x) = 2^x \ln 2$$

(3)

The curve C has the equation

$$2x + 3y^2 + 3x^2y + 12 = 4 \times 2^x$$

The point P, with coordinates (2, 0), lies on C.

(b) Find an equation of the tangent to C at P.

(6)

Question Number	Scheme	Marks
5 (a)	Sets $y=2^x$ and takes ln of both sides to get $\ln y = x\ln 2$	M1
	Differentiates wrt x to get $\frac{1}{y}\frac{\mathrm{d}y}{\mathrm{d}x}=\ln 2 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=..$	dM1
	Rearranges to achieve $\frac{\mathrm{d}y}{\mathrm{d}x}=2^x\ln 2$ cao	A1*
		(3)
(b)	Differentiates wrt x $\quad \underline{\underline{2+6y\frac{\mathrm{d}y}{\mathrm{d}x}}}+\underline{6xy+3x^2\frac{\mathrm{d}y}{\mathrm{d}x}}=4\times 2^x\ln 2$ oe	M1, B1, A1
	Substitutes (2, 0) **AND** rearranges to get $\frac{\mathrm{d}y}{\mathrm{d}x}$	
	$\Rightarrow 2+12\frac{\mathrm{d}y}{\mathrm{d}x}=16\ln 2 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{16\ln 2-2}{12}$ $\quad (=0.758)$	M1
	Find equation of tangent using (2, 0) and their numerical $\frac{\mathrm{d}y}{\mathrm{d}x}$	dM1
	$y=\frac{(16\ln 2-2)(x-2)}{12}$ oe	A1
	Accept $y=0.76x-1.52$	**(6)**
		(9 marks)
Alt 1 5 (a)	Writes $2^x = \mathrm{e}^{x\ln 2}$	M1
	Differentiates wrt x to get $\frac{\mathrm{d}}{\mathrm{d}x}\left(\mathrm{e}^{x\ln 2}\right)=\mathrm{e}^{x\ln 2}\ln 2 = 2^x\ln 2$ cao	dM1 A1*
		(3)
Alt 2 5 (a)	Sets $y=2^x$ and takes $\ln_2$ of both sides to get	
	$\ln_2 y = x \Rightarrow \frac{\ln y}{\ln 2}=x \Rightarrow \ln y = x\ln 2$	M1
		(3)

Notes for Question 5

(a)

M1 Sets $y = 2^x$, takes ln of both sides, then uses index law to get $\ln y = x\ln 2$

dM1 Differentiates wrt x to get $\frac{1}{y}\frac{\mathrm{d}y}{\mathrm{d}x} = \ln 2$ and then proceeds to $\frac{\mathrm{d}y}{\mathrm{d}x} = ..$

Alternatively differentiates wrt y to get $\frac{1}{y} = \ln 2\frac{\mathrm{d}x}{\mathrm{d}y}$

A1* $\frac{\mathrm{d}y}{\mathrm{d}x} = 2^x \ln 2$. This is a given answer. All aspects of the proof must be present.

(**Alt a**)

M1 Writes $2^x = \mathrm{e}^{x\ln 2}$

dM1 Differentiates wrt x to get $\frac{\mathrm{d}}{\mathrm{d}x}\left(\mathrm{e}^{x\ln 2}\right) = A\mathrm{e}^{x\ln 2}$

A1* $\frac{\mathrm{d}}{\mathrm{d}x}\left(\mathrm{e}^{x\ln 2}\right) = \mathrm{e}^{x\ln 2}\ln 2 = 2^x \ln 2$. This is a given answer. All aspects of the proof must be present

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(b)

M1 Uses the product rule to differentiate $3x^2y$. Evidence could be sight of $Axy + 3x^2\frac{\mathrm{d}y}{\mathrm{d}x}$

If the rule is quoted it must be correct. It could be implied by u=.., u'=.., v=.., v'=.. followed by their vu'+uv'.

For this M to be scored y must differentiate to $\frac{\mathrm{d}y}{\mathrm{d}x}$, it cannot differentiate to 1.

B1 Differentiates $2x + 3y^2 \to 2 + 6y\frac{\mathrm{d}y}{\mathrm{d}x}$

Watch for people who divide by 4 first. Then $\frac{1}{2}x + \frac{3}{4}y^2 \to \frac{1}{2} + \frac{3}{2}y\frac{\mathrm{d}y}{\mathrm{d}x}$

A1 A completely correct differential. It need not be simplified.

Accept the form $2\mathrm{d}x + 6y\mathrm{d}y + 6xy\mathrm{d}x + 3x^2\mathrm{d}y = 4\times 2^x \ln 2\mathrm{d}x$ for the first three marks

Note that $\frac{\mathrm{d}y}{\mathrm{d}x} = 2 + 6y\frac{\mathrm{d}y}{\mathrm{d}x} + 6xy + 3x^2\frac{\mathrm{d}y}{\mathrm{d}x} = 4\times 2^x \ln 2$ is A0 but they can recover if their intention is clear.

M1 Substitutes $x = 2,\ y = 0$ into their expression, and rearranges to find a 'numerical' value for $\frac{\mathrm{d}y}{\mathrm{d}x}$

dM1 Uses their numerical value to $\frac{\mathrm{d}y}{\mathrm{d}x}$ and (2, 0) to find an equation of a tangent.

Accept $\frac{y-0}{x-2} = \text{Numerical } \frac{\mathrm{d}y}{\mathrm{d}x}$

If $y = mx + c$ is used then a full method must be seen to find 'c' using both (2, 0) and a numerical m.

A1 $y = \frac{(16\ln 2 - 2)(x-2)}{12}$.

Accept alternatives such as $\frac{y-0}{x-2} = \frac{(16\ln 2 - 2)}{12}$, $\frac{y-0}{x-2} = \frac{(2\ln 16 - 1)}{6}$

As the form of the answer is not required accept awrt 2dp $y = 0.76x - 1.52$, $\frac{y-0}{x-2} = 0.76$

Leave blank

6. Given that the binomial expansion, in ascending powers of x, of

$$\frac{6}{\sqrt{(9 + Ax^2)}}, \qquad |x| < \frac{3}{\sqrt{|A|}}$$

is $$B - \frac{2}{3}x^2 + Cx^4 + \ldots$$

(a) find the values of the constants A, B and C.

(7)

(b) Hence find the coefficient of x^6

(2)

Question Number	Scheme	Marks
6 (a)	$\dfrac{6}{\sqrt{(9+Ax^2)}} = 6(9+Ax^2)^{-\frac{1}{2}} = 6\times 9^{-\frac{1}{2}}\left(1+\dfrac{A}{9}x^2\right)^{-\frac{1}{2}}$ $\qquad 9^{-\frac{1}{2}}$ or $\dfrac{1}{3}$	B1
	$= 2\times\underline{\left(1+\left(-\dfrac{1}{2}\right)\left(\dfrac{A}{9}x^2\right)+\dfrac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2}\left(\dfrac{A}{9}x^2\right)^2+\ldots\right)}$	<u>M1A1</u>
	$= 2\times\left(1-\dfrac{A}{18}x^2+\dfrac{A^2}{216}x^4+\ldots\right)$	
	$= 2-\dfrac{A}{9}x^2+\dfrac{A^2}{108}x^4+\ldots$	
	Compare to $B-\dfrac{2}{3}x^2+Cx^4 \Rightarrow$ $\quad B=2$	B1
	$A=6$	B1
	$C = '\dfrac{1}{108}'\times 'A^2' = \dfrac{1}{3}$	dM1A1
		(7)
(b)	Coefficient of $x^6 = 2\times\dfrac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)}{3!}\times\left(\dfrac{A}{9}\right)^3 = -\dfrac{5}{27}$	M1A1
		(2)
		(9 marks)

Notes for Question 6

For this question (a) and (b) can be treated as one whole question. Marks for (a) can be gained in (b)
(a)

B1 For taking out a factor of $9^{-\frac{1}{2}}$

Evidence would seeing either $6\times 9^{-\frac{1}{2}}$ or $6\times\frac{1}{3}$ or 2 before the bracket.

M1 For the form of the binomial expansion with $n=-\frac{1}{2}$ and a term of $\left(\frac{A}{9}x^2\right)$

To score M1 it is sufficient to see just either the first two terms. ie. $1+\left(-\frac{1}{2}\right)\left(\frac{A}{9}x^2\right)+\ldots$

or the first term and a later term if an error was made on term two. Condone poor bracketing
If the 9 has been removed 'incorrectly' accept for this M mark

$$(1+kAx^2)^{-\frac{1}{2}}=1+\left(-\frac{1}{2}\right)\left(kAx^2\right)+\frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2}\left(kAx^2\right)^2$$

A1 Any (unsimplified) form of the binomial expansion. Ignore the factor preceeding the bracket.

$1+\left(-\frac{1}{2}\right)\left(\frac{A}{9}x^2\right)+\frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2}\left(\frac{A}{9}x^2\right)^2+\ldots$ is acceptable.

The bracketing must be correct but it is OK for them to recover.

B1 For writing down $B=2$
Note that this could be found by substituting $x=0$ into both sides of expression

B1 For writing down $A=6$

dM1 For substituting their numerical value of A into their coefficient of x^4 (involving A or A^2) to find C.
The coefficient does not need to be correct but the previous M1 must have been scored.

A1 For $C=\frac{1}{3}$. Accept equivalents like $C=\frac{36}{108}=0.\dot{3}$

(b)

M1 For a correct unsimplified term in x^6 of the binomial exp with $n=-\frac{1}{2}$ and term = $\left(\frac{their\,'A'}{9}x^2\right)$.

Sight of $\cancel{2\times}\frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)}{3!}\times\left(\frac{A}{9}\right)^3$ with or without the factor of 2 with their numerical A

Accept with or without correct bracketing but must involve A^3.

You may have to check $-\frac{5A^3}{11664}$ or $2\times -\frac{5A^3}{11664}=-\frac{5A^3}{5832}$ if there is little working

Allow this mark if a candidate has a correct unsimplified fraction in part a, but then uses an incorrect simplification in calculating the term in x^6. This incorrect simplification must include $\ldots A^3$

A1 $-\frac{5}{27}$ or other exact correct equivalents such as $-\frac{15}{81}$, $-\frac{3240}{17496}$. Accept with the x^6

Leave blank

7.

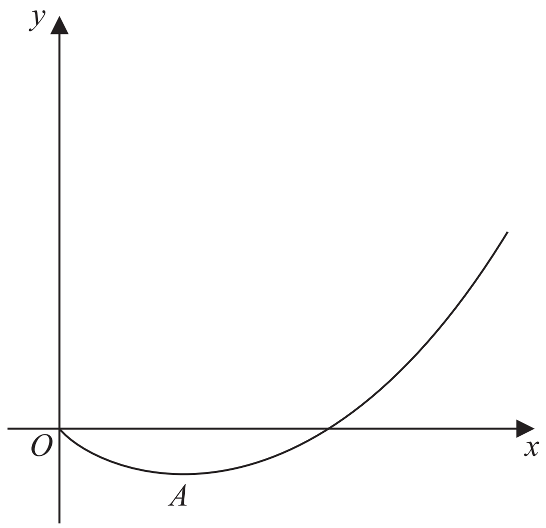


Figure 3

Figure 3 shows a sketch of part of the curve with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = 2x(1 + x)\ln x, \quad x > 0$$

The curve has a minimum turning point at A.

(a) Find $\mathrm{f}'(x)$

(3)

(b) Hence show that the x coordinate of A is the solution of the equation

$$x = \mathrm{e}^{-\frac{1+x}{1+2x}}$$

(3)

(c) Use the iteration formula

$$x_{n+1} = \mathrm{e}^{-\frac{1+x_n}{1+2x_n}}, \qquad x_0 = 0.46$$

to find the values of x_1, x_2 and x_3 to 4 decimal places.

(3)

(d) Use your answer to part (c) to estimate the coordinates of A to 2 decimal places.

(2)

Question Number	Scheme	Marks
7 (a)	Applies $vu'+uv'$ with $u=2x+2x^2$ and $v=\ln x$ or vice versa	
	$$f'(x)=\ln(x)(2+4x)+(2x+2x^2)\times\frac{1}{x}$$	M1A1A1
		(3)
(b)	Sets $\ln(x)(2+4x)+(2x+2x^2)\times\frac{1}{x}=0$ and makes $\ln x$ the subject	M1
	$$\ln(x)=-\frac{1+x}{1+2x}\Rightarrow x=e^{-\frac{1+x}{1+2x}}$$	dM1A1*
		(3)
(c)	Subs $x_0=0.46$ into $x=e^{-\frac{1+x}{1+2x}}$	M1
	x_1= awrt 0.4675, x_2= awrt 0.4684 x_3= awrt 0.4685	A1,A1
		(3)
(d)	A= (0.47, -1.04)	M1A1
		(2)
		(11 marks)
Alt 7 (a)	Writes $f(x)=2x\ln x+2x^2\ln x$ and applies $vu'+uv'$	
	$f'(x)=2\ln(x)+2x\times\frac{1}{x}+2x^2\times\frac{1}{x}+4x\ln x$	M1A1A1
		(3)

Notes for Question 7

(a)

M1 Fully applies the product rule to $2x(1+x)\ln x$. This can be achieved by
Either setting $u=2x+2x^2$ and $v=\ln x$.
If the rule is quoted it must be correct and $\ln x \to \frac{1}{x}$ when differentiated. If the rule is not quoted, it can be implied by $u=2x+2x^2,\ v=\ln x,\ u'=A+Bx,\ v'=\frac{1}{x}$ followed by their $vu'+uv'$ only accept answers of the form $f'(x)=\ln(x)\left(A+Bx\right)+\left(2x+2x^2\right)\times\frac{1}{x}$
Or writing $f(x)=2x\ln x+2x^2\ln x$ and attempting to apply the product rule to both parts. It must be seen to be correctly applied to at least one of the products. See above for the rules.
Again insist on $\ln x \to \frac{1}{x}$ when differentiating.
Or applying the product rule to a triple product.
Look for expressions like $\frac{d}{dx}(uvw)=u'vw+uv'w+uvw'$

A1 Two out of the four separate terms correct (unsimplified).

A1 All four terms correct (unsimplified) $f'(x)=\ln(x)\left(2+4x\right)+\left(2x+2x^2\right)\times\frac{1}{x}$
or if two applications $f'(x)=2\ln(x)+4x\ln(x)+2x\times\frac{1}{x}+2x^2\times\frac{1}{x}$

(b)

M1 Sets or implies that their $f'(x)=0$ and proceeds to make the $\ln x$ term the subject of the formula,
To score this $f'(x)$ does not need to be correct but it must be of equal difficulty. Look for more than one term in lnx and two other 'unlike terms'

dM1 Dependent upon the last M1. For moving from $\ln x=...\Rightarrow x=e^{...}$

A1* CSO $x=e^{-\frac{1+x}{1+2x}}$.
All aspects of the proof must be correct including the position of the minus sign and the bracketing.

(b) Alt working backwards

M1 By taking ln's proceeds from $x=e^{-\frac{1+x}{1+2x}}$ to $\pm\ln x\times(1+2x)=\pm(1+x)$

dM1 Dependent upon the last M1. Moves to $\pm\ln x\times(2+4x)\pm(2+2x)=0$

A1 For a statement that completes the proof. Accept 'hence $f'(x)=0\Rightarrow$ solution is the x coordinate of A. All aspects must be correct including their $f'(x)$.

(c)

M1 For an attempt to find x_1 from the value of x_0 by substituting 0.46 into $e^{-\frac{1+x}{1+2x}}$
Possible ways this can be scored could be sight of $e^{-\frac{1+0.46}{1+2\times0.46}}$ or awrt 0.47

A1 x_1 =awrt 0.4675.

A1 x_2 = awrt 0.4684 x_3 = awrt 0.4685

(d)

M1 For **either** $x=0.47$, **or** $y=-1.04$ as a result of using $x_3=$ 0.46 truncated or $x_3=$ 0.47 rounded from part c. Alternatively for substituting their answer for x_3 in part (c) either to 4dp or rounded to 2dp into $f(x)$ to find the y coordinate of A. Accept sight of $2\times x_3(1+x_3)\ln x_3$

A1 A= (0.47, -1.04).

Leave blank

8. (a) Prove that

$$2\operatorname{cosec} 2A - \cot A \equiv \tan A, \qquad A \neq \frac{n\pi}{2}, n \in \mathbb{Z}$$

(4)

(b) Hence solve, for $0 \leqslant \theta \leqslant \frac{\pi}{2}$

(i) $2\operatorname{cosec} 4\theta - \cot 2\theta = \sqrt{3}$

(ii) $\tan\theta + \cot\theta = 5$

Give your answers to 3 significant figures.

(6)

Question Number	Scheme	Marks
8 (a)	$2\text{cosec}2A - \cot A = \dfrac{2}{\sin 2A} - \dfrac{1}{\tan A}$ $\quad 2\cos\text{ec}2A = \dfrac{2}{\sin 2A}$	B1
	$= \dfrac{2}{2\sin A\cos A} - \dfrac{\cos A}{\sin A}$	M1
	$= \dfrac{2-2\cos^2 A}{2\sin A\cos A}$	M1
	$\dfrac{2(1-\cos^2 A)}{2\sin A\cos A} = \dfrac{2\sin^2 A}{2\sin A\cos A} = \dfrac{\sin A}{\cos A} = \tan A$	A1*
		(4)
(b)(i)	$2\text{cosec}4\theta - \cot 2\theta = \sqrt{3} \Rightarrow \tan 2\theta = \sqrt{3}$	M1
	$\Rightarrow \theta = \dfrac{\arctan\sqrt{3}}{2} = \dfrac{\pi}{6}$ Accept awrt 0.524	A1
(ii)	$\tan\theta + \cot\theta = 5 \Rightarrow \text{cosec}2\theta = \dfrac{5}{2}$	M1
	$\Rightarrow \theta = \dfrac{1}{2}\arcsin\left(\dfrac{2}{5}\right) = \text{awrt } 0.206, 1.37$	dM1A1A1
		(6)
		(10 marks)
Alt 8 (a)	$2\text{cosec}2A - \cot A = \tan A \Rightarrow \dfrac{2}{\sin 2A} - \dfrac{1}{\tan A} = \tan A$	B1
	$\Rightarrow \dfrac{2}{2\sin A\cos A} - \dfrac{\cos A}{\sin A} = \dfrac{\sin A}{\cos A}$	M1
	$\times 2\sin A\cos A \Rightarrow 2 - 2\cos^2 A = 2\sin^2 A$	M1
	$\Rightarrow 2(1-\cos^2 A) = 2\sin^2 A$	
	$\Rightarrow 2\sin^2 A = 2\sin^2 A$ *QED* (minimal statement must be seen)	A1* **(4)**
Alt 8b(ii)	$\tan\theta + \cot\theta = 5 \Rightarrow \dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta} = 5 \Rightarrow \dfrac{1}{\frac{1}{2}\sin 2\theta} = 5 \Rightarrow \sin 2\theta = \dfrac{2}{5}$ This can now score all of the marks as it is effectively using part (a)	M1
SC 8b(ii)	$\tan\theta + \cot\theta = 5 \Rightarrow \tan\theta + \dfrac{1}{\tan\theta} = 5 \Rightarrow \tan^2\theta - 5\tan\theta + 1 = 0$ $\Rightarrow \theta = \text{awrt } 0.206, 1.37$ This is not using part (a) and is a special case with one mark per correct answer. One answer =1000 Two answers=1100	1,1,0,0

Notes for Question 8

(a)

B1 Writes $2\operatorname{cosec}2A$ as $\dfrac{2}{\sin 2A}$

M1 Uses the double angle formula for $\sin 2A$ (see below) and $\cot A = \dfrac{\cos A}{\sin A}$ to write the given expression in terms of just $\sin A$ and $\cos A$

For the double angle formula accept sight of $\dfrac{2}{2\sin A\cos A}$ or $\dfrac{2}{\sin A\cos A + \cos A\sin A}$

Condone $\sin 2A = 2\sin\cos$ for this method within their solution.

M1 For writing the given expression as a single fraction in terms of just $\sin A$ and $\cos A$
The denominator must be correct for their fraction and at least one numerator must have been modified.

Accept $\dfrac{2}{2\sin A\cos A} - \dfrac{\cos A}{\sin A} \rightarrow \dfrac{2\sin A - 2\sin A\cos^2 A}{2\sin^2 A\cos A}$ - not the lowest common denominator.

Accept $\dfrac{1}{2\sin A\cos A} - \dfrac{\cos A}{\sin A} \rightarrow \dfrac{\sin A - 2\cos^2 A}{2\sin A\cos A}$ Incorrect 'fraction' but denominator correct

A1* A completely correct proof. This is a given solution and there must not be any errors.
For this mark do not condone expressions like $\sin 2A = 2\sin\cos$
The 2's must be cancelled at some point. It is OK for them to 'just' disappear

The $1-\cos^2 A$ term must be replaced with $\sin^2 A$. $\dfrac{2\sin^2 A}{2\sin A\cos A}$

The expression $\dfrac{\sin A}{\cos A}$ must be clearly seen before being replaced by $\tan A$ but $\dfrac{2\sin^{2}\cancel{A}}{2\sin \cancel{A}\cancel{\cos}A}$ is OK

(b (i))

M1 Uses part (a) to write given equation in form $\tan 2\theta = \sqrt{3}$ and proceeding to $\theta = ..$
This may be implied by $A = 2\theta$ as long as they proceed to $\theta = ..$
Accept a restart as long as they get to the line $\tan 2\theta = \sqrt{3}$

A1 $\theta = \dfrac{\pi}{6}$. Accept awrt 0.524 (radians) but not $30°$. Ignore extra solutions outside the range.
Withhold this mark if extra solutions are given inside the range.
The answer without working **does not** score any marks. The demand of the question is clearly stated – Hence solve…meaning that you should see an equivalent statement to $\tan 2\theta = \sqrt{3}$

(b (ii))

M1 Uses part (a) to write given equation in form $\operatorname{cosec} 2\theta = C$, where C is a constant.
It may be implied by $\sin 2\theta = ..$

dM1 Dependent upon the first method and is scored for the correct 'order' of operations.
The mark is scored for $\operatorname{cosec} 2\theta = C \Rightarrow \sin 2\theta = \dfrac{1}{C} \Rightarrow 2\theta = \arcsin\left(\dfrac{1}{C}\right) \Rightarrow \theta = ..$
It can be implied by a correct answer only if the line $\operatorname{cosec} 2\theta = C$ is present.

A1 One correct solution awrt 0.206 or 1.37. Accept awrt $0.0656\pi,\ 0.436\pi$. Remember to isw here.
Accept awrt $11.8°$ or $78.2°$ if the mark in (bi) had been lost for $30°$

A1 Both solutions correct and no extras inside the range. See above for alternatives.
The correct answers without working **does not** score any marks.
Special case where they produce answers from a quadratic in $\tan\theta$ can score 1100 if they get both answers and no others inside the range.

Leave blank

9. (a) Use the substitution $u = 4 - \sqrt{x}$ to find

$$\int \frac{\mathrm{d}x}{4 - \sqrt{x}}$$

(6)

A team of scientists is studying a species of slow growing tree.

The rate of change in height of a tree in this species is modelled by the differential equation

$$\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{4 - \sqrt{h}}{20}$$

where h is the height in metres and t is the time measured in years after the tree is planted.

(b) Find the range in values of h for which the height of a tree in this species is increasing.
(2)

(c) Given that one of these trees is 1 metre high when it is planted, calculate the time it would take to reach a height of 10 metres. Write your answer to 3 significant figures.
(7)

Question Number	Scheme	Marks
9(a)	$u = 4 - \sqrt{x} \Rightarrow x = (4-u)^2 \Rightarrow \dfrac{dx}{du} = -2(4-u)$	M1A1
	$\displaystyle\int \frac{dx}{4-\sqrt{x}} = \int \frac{-2(4-u)du}{u} = \int -\frac{8}{u} + 2du$	M1A1
	$= -8\ln u + 2u \quad (+c)$	dM1
	$= -8\ln\left\|4-\sqrt{x}\right\| + 2\left(4-\sqrt{x}\right)(+c)$ oe	A1
		(6)
(b)	Height increases when $\dfrac{dh}{dt} = \dfrac{4-\sqrt{h}}{20} > 0 \Rightarrow (0<)\ h < 16$	M1A1
		(2)
(c)	$\dfrac{dh}{dt} = \dfrac{4-\sqrt{h}}{20} \Rightarrow \displaystyle\int \frac{dh}{4-\sqrt{h}} = \int \frac{dt}{20}$	B1
	$\Rightarrow -8\ln\left(4-\sqrt{h}\right) + 2\left(4-\sqrt{h}\right) = \dfrac{t}{20} + c$	M1A1
	Substitute t=0, h=1 $\quad -8\ln 3 + 6 = c$	dM1
	$\Rightarrow -8\ln\left(4-\sqrt{h}\right) + 2\left(4-\sqrt{h}\right) = \dfrac{t}{20} - 8\ln 3 + 6$ oe	A1
	Substitute h=10 into $\Rightarrow -8\ln\left(4-\sqrt{10}\right) + 2\left(4-\sqrt{10}\right) = \dfrac{t}{20} - 8\ln 3 + 6$	ddM1
	$\Rightarrow t = \text{awrt } 118 \text{(years)}$	A1
		(7)
		(15 marks)
Alt (c)	$\dfrac{dh}{dt} = \dfrac{4-\sqrt{h}}{20} \Rightarrow \displaystyle\int \frac{dh}{4-\sqrt{h}} = \int \frac{dt}{20}$	B1
	$\Rightarrow -8\ln\left(4-\sqrt{h}\right) + 2\left(4-\sqrt{h}\right) = \dfrac{t}{20}$	M1
	$\Rightarrow \left[-8\ln\left(4-\sqrt{h}\right) + 2\left(4-\sqrt{h}\right)\right]_{h=1}^{h=10} = \left[\dfrac{t}{20}\right]_{t=0}^{T}$	A1
	$\Rightarrow \left(-8\ln\left(4-\sqrt{10}\right) + 2\left(4-\sqrt{10}\right)\right) - \left(-8\ln\left(4-\sqrt{1}\right) + 2\left(4-\sqrt{1}\right)\right) = \dfrac{T}{20}$	dM1,ddM1, A1
	$\Rightarrow T = \text{awrt } 118 \text{ (years)}$	A1

Notes for Question 9

(a)

M1 Scored for an attempt to write x in terms of u **and** differentiating to get either dx in terms of du or $\frac{\mathrm{d}x}{\mathrm{d}u}$ in terms of u. Accept for the M incorrect expressions like $x=16-u^2 \Rightarrow \frac{\mathrm{d}x}{\mathrm{d}u}=-2u$
The minimum expectation is that the expression in u is quadratic and the derivative in u is linear.
Alternatively uses $u=4-\sqrt{x}$ to find $\frac{\mathrm{d}u}{\mathrm{d}x}=Cx^{-0.5}$ and attempts to get $\frac{\mathrm{d}x}{\mathrm{d}u}=\mathrm{f}(u)$ or similar.

A1 Either $\frac{\mathrm{d}x}{\mathrm{d}u}=-2(4-u)$ or $\mathrm{d}x=-2(4-u)\mathrm{d}u$ or $x'=-2(4-u)$ or equivalents. Condone $\mathrm{d}x=-8+2u$
Accept these within the integral for both marks as long as no incorrect working is seen.

M1 An attempt to divide their dx by u to get an integral of the form $\int \frac{A}{u}+B\,(\mathrm{d}u)$.

A1 A fully correct integral in terms of u. Condone the omission of du if the intention is clear.
Accept forms such as $\int -\frac{8}{u}+2\mathrm{d}u$ $\int -8u^{-1}+2\mathrm{d}u$ $-2\left(\int \frac{4}{u}-1\right)$

dM1 For integrating $\frac{1}{u} \to \ln u$ and increasing the power of any other term(s). It is dependent upon the previous method mark. There is no need to set the answer in terms of x. There is no need to have $+c$

A1 For a correct answer in terms of x with or without $+c$. There is no requirement for modulus signs.
Accept either $=-8\ln\left|4-\sqrt{x}\right|+2\left(4-\sqrt{x}\right)$ $(+c)$ or $=-8\ln\left(4-\sqrt{x}\right)-2\sqrt{x}$ $(+c)$ oe

(b)

M1 Setting $\frac{\mathrm{d}h}{\mathrm{d}t}=0 \Rightarrow h=...$ or $\frac{\mathrm{d}h}{\mathrm{d}t}>0 \Rightarrow h<...$. Accept h=16 for this mark.

A1 Stating either $h<16$ or $0<h<16$ or $0 \le h<16$ or all values up to 16.
A correct answer can score both marks as long as no incorrect working is seen.

(c)

B1 Writing $\int \frac{\mathrm{d}h}{4-\sqrt{h}}=\int \frac{\mathrm{d}t}{20}$ or equivalent. It must include dh and dt but $\int$ could be implied

M1 For an attempt to integrate **both** sides, no need for c
Follow through on **their** answer to part (a) for x or u with 'h' on the lhs with At on the rhs.

A1 A fully correct answer with $+c$ $-8\ln\left(4-\sqrt{h}\right)+2\left(4-\sqrt{h}\right)=\frac{t}{20}+c$

dM1 Substitute t = 0, h = 1 in an attempt to find c. Minimal evidence is required. Accept t = 0, h = 1 $\Rightarrow c=..$ The previous M must have been awarded.

A1 A correct equation $\Rightarrow -8\ln\left(4-\sqrt{h}\right)+2\left(4-\sqrt{h}\right)=\frac{t}{20}-8\ln 3+6$ oe.

Accept $\Rightarrow -8\ln\left(4-\sqrt{h}\right)+2\left(4-\sqrt{h}\right)=\frac{t}{20}+awrt\ 2.79$

ddM1 Substitute h=10 into their equation involving h, t and their value of c in an attempt to find t.
It is dependent upon both M's being scored in this part of the question. Again accept minimal evidence

A1 Awrt 118 (years). The answer without any correct working scores 0 marks.
Condone x and h being interchanged in this part of the question.

(c Alt)

A1 Setting the limits is equivalent to understanding that there is a constant.

dM1 Using limits of 1 and 10 and subtracting ddM1 Using limits of 't' and 0 and subtracting

A1 A fully correct expression involving just 't' A1 Awrt 118 (years). No working = 0 marks

Leave blank

10. With respect to a fixed origin O, the lines l_1 and l_2 are given by the equations

$$l_1 : \mathbf{r} = (\mathbf{i} + 5\mathbf{j} + 5\mathbf{k}) + \lambda(2\mathbf{i} + \mathbf{j} - \mathbf{k})$$

$$l_2 : \mathbf{r} = (2\mathbf{j} + 12\mathbf{k}) + \mu(3\mathbf{i} - \mathbf{j} + 5\mathbf{k})$$

where λ and μ are scalar parameters.

(a) Show that l_1 and l_2 meet and find the position vector of their point of intersection. **(6)**

(b) Show that l_1 and l_2 are perpendicular to each other. **(2)**

The point A, with position vector $5\mathbf{i} + 7\mathbf{j} + 3\mathbf{k}$, lies on l_1

The point B is the image of A after reflection in the line l_2

(c) Find the position vector of B. **(3)**

Question Number	Scheme	Marks
10 (a)	$\begin{pmatrix}1\\5\\5\end{pmatrix}+\lambda\begin{pmatrix}2\\1\\-1\end{pmatrix}=\begin{pmatrix}0\\2\\12\end{pmatrix}+\mu\begin{pmatrix}3\\-1\\5\end{pmatrix}\Rightarrow$ $\begin{matrix}1+2\lambda=3\mu\\5+\lambda=2-\mu\\5-\lambda=12+5\mu\end{matrix}$ any two of	M1
	Full method to find both λ and μ	dM1
	(2) +(3) $\Rightarrow 10=14+4\mu\Rightarrow\mu=-1$	
	Sub $\mu=-1$ into (2) $\Rightarrow 5+1\lambda=2-(-1)\Rightarrow\lambda=-2$	A1
	Check values in 3rd equation $1+2(-2)=3(-1)$.	B1
	Position vector of intersection is $\begin{pmatrix}1\\5\\5\end{pmatrix}+-2\begin{pmatrix}2\\1\\-1\end{pmatrix}$ OR $\begin{pmatrix}0\\2\\12\end{pmatrix}+(-1)\begin{pmatrix}3\\-1\\5\end{pmatrix}=\begin{pmatrix}-3\\3\\7\end{pmatrix}$	ddM1,A1
		(6)
(b)	$\begin{pmatrix}2\\1\\-1\end{pmatrix}.\begin{pmatrix}3\\-1\\5\end{pmatrix}=2\times3+1\times-1+-1\times5=0$	M1
	Scalar product =0, lines are perpendicular	A1
		(2)
(c)	Let X be the point of intersection $\overrightarrow{OX}=\begin{pmatrix}-3\\3\\7\end{pmatrix}$	
	$\overrightarrow{AX}=\overrightarrow{OX}-\overrightarrow{OA}=\begin{pmatrix}-8\\-4\\4\end{pmatrix}$	M1
	$\overrightarrow{OB}=\overrightarrow{OX}+\overrightarrow{AX}=\begin{pmatrix}-3\\3\\7\end{pmatrix}+\begin{pmatrix}-8\\-4\\4\end{pmatrix}=\begin{pmatrix}-11\\-1\\11\end{pmatrix}$	M1A1
		(3)
		(11 marks)

Notes for Question 10

(a)

M1 For writing down any two equations that give the coordinates of the point of intersection.
Accept two of $1+2\lambda=3\mu,\quad 5+\lambda=2-\mu,\quad 5-\lambda=12+5\mu$
There must be an attempt to set the coordinates equal but condone slips.

dM1 A full method to find **both** λ **and** μ.
Don't be overly concerned with the mechanics of the method but it must end with values for both. It is dependent upon the previous method

A1 Both values correct $\mu=-1\ \lambda=-2$

B1 The correct values of λ and μ must be substituted into **both** sides of the third equation with some calculation (or statement) showing both sides are equal. This can also be scored via the substitution of $\mu=-1\ \lambda=-2$ into **both** equations of the lines resulting in the same coordinate.

ddM1 Substitutes their value of λ into l_1 to find the coordinates or position vector of the point of intersection. It is dependent upon having scored both methods so far.
Alternatively substitutes their value of μ into l_2 to find the coordinates or position vector of the point of intersection. It may be implied by 2 out of 3 correct coordinates.

A1 Correct answer only. Accept as a vector or a coordinate. Accept (-3, 3, 7)

(b)

M1 A clear attempt to find the scalar product of the gradient vectors $\begin{pmatrix}2\\1\\-1\end{pmatrix}.\begin{pmatrix}3\\-1\\5\end{pmatrix}$

You must see an attempt to multiply and add. Eg $2\times3+1\times-1+-1\times5$ or $6-1-5$. Allow for slips.

A1 The above method must be followed by a reason and a conclusion. The scalar product must be zero.
Accept '=0, hence perpendicular'. Accept '=0, therefore proven'

©

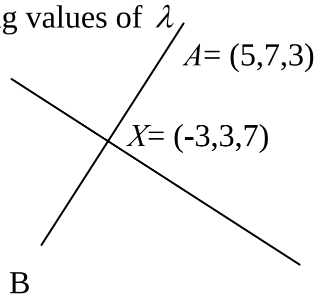
M1 An attempt to find the vector $\overrightarrow{AX}$ where X is their point of intersection using $\overrightarrow{AX}=\overrightarrow{OX}-\overrightarrow{OA}$
This is scored if the 'difference' between the vectors or coordinates are attempted

M1 Attempts to find the coordinate or vector of B using $\overrightarrow{OB}=\overrightarrow{OX}+\overrightarrow{AX}$ or $\overrightarrow{OB}=\overrightarrow{OA}+2\times\overrightarrow{AX}$
Allow a misunderstanding on the direction of $\overrightarrow{AX}$

A1 Correct answer only. $\overrightarrow{OB}=\begin{pmatrix}-11\\-1\\11\end{pmatrix}$ *or* $-11i-1j+11k$

Do NOT accept the coordinate for this mark. Correct answer with no working scores all 3 marks.
The correct coordinate would score 2 out of 3.

(c Alt) Using values of λ



M1 Attempts to find λ on l_1 at X $(\lambda=-2)$ and A $(\lambda=2)$

M1 Uses the difference between λ values to find λ (-6) at B

A1 cao

Leave blank

11. The curve C has parametric equations

$$x = 10\cos 2t, \quad y = 6\sin t, \quad -\frac{\pi}{2} \leqslant t \leqslant \frac{\pi}{2}$$

The point A with coordinates $(5, 3)$ lies on C.

(a) Find the value of t at the point A.

(1)

(b) Show that an equation of the normal to C at A is

$$3y = 10x - 41$$

(6)

The normal to C at A cuts C again at the point B.

(c) Find the exact coordinates of B.

(8)

Question Number	Scheme	Marks
11(a)	$6\sin t = 3 \Rightarrow \sin t = 0.5 \Rightarrow t = \frac{\pi}{6}$	B1
		(1)
(b)	$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{6\cos t}{-20\sin 2t}$	M1A1
	Sub $t = \frac{\pi}{6}$ into $\frac{dy}{dx} = \frac{6\cos t}{-20\sin 2t} = \frac{6\cos\left(\frac{\pi}{6}\right)}{-20\sin\left(\frac{\pi}{3}\right)} = -\frac{3}{10}$	M1A1
	Uses normal gradient with (5, 3)$\Rightarrow \frac{y-3}{x-5} = \frac{10}{3}$	M1
	$\Rightarrow 3y = 10x - 41$	A1*
		(6)
©	Sub $x = 10\cos 2t,\quad y = 6\sin t,$ into $3y = 10x - 41$	
	$\Rightarrow 18\sin t = 100\cos 2t - 41$	M1
	$\Rightarrow 18\sin t = 100(1 - 2\sin^2 t) - 41$	
	$\Rightarrow 200\sin^2 t + 18\sin t - 59 = 0$	M1, A1
	$\Rightarrow (2\sin t - 1)(100\sin t + 59) = 0$	
	$\Rightarrow \sin t = -\frac{59}{100}$ $(\Rightarrow t = -0.63106...)$	M1A1
	Using either their t or $\sin t$ to find either coord of B	M1
	Hence B has co-ordinates (3.038, - 3.54). These are exact values	A1,A1
	The equivalent fractional answers are $\left(\frac{1519}{500}, -\frac{177}{50}\right)$	**(8)**
		(15 marks)

Notes for Question 11

(a)

B1 $t=\frac{\pi}{6}$ Accept awrt 0.5236 (4dp). Answers in degrees $30°$ is B0

(b)

M1 Uses $\frac{dy}{dx}=\frac{dy/dt}{dx/dt}$ and differentiates to obtain a gradient function of the form $\frac{A\cos t}{B\sin 2t}$

A1 $\frac{dy}{dx}=\frac{6\cos t}{-20\sin 2t}=\left(-\frac{3\cos t}{10\sin 2t}\right)$. There is no requirement to simplify this.

M1 Substitutes their value of t (from part a) into their $\frac{dy}{dx}$ to get a numerical value for the gradient.

Accept also their t being substituted into $\frac{dx}{dy}$ or $-\frac{dx}{dy}$

A1 Achieves a correct numerical answer for $\frac{dy}{dx}=-\frac{3}{10}$ $\left(\frac{dx}{dy}=-\frac{10}{3}\ or\ -\frac{dx}{dy}=\frac{10}{3}\right)$.

It needs to be attributed to the correct derivative. Do not accept $\frac{dx}{dy}=-\frac{3}{10}$

This may be implied by the correct value in the gradient of the normal.

M1 Award for a correct method to find the equation of the normal.

They must use (5, 3) and their numerical value of $"-\frac{dx}{dy}"$. Eg $\frac{y-5}{x-3}=-\frac{dx}{dy}$

If $y=mx+c$ is used then it must be a full method using (5, 3) and with $m="-\frac{dx}{dy}"$ as far as c =..

A1* cso. This is a proof and you must be convinced of all aspects including the sight of an intermediate line between $\frac{y-3}{x-5}=\frac{10}{3}$ and $3y=10x-41$

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M1 An attempt to substitute both $x=10\cos 2t$ and $y=6\sin t$ into $3y=10x-41$ forming a trigonometrical equation in just the variable t.

M1 Uses the identity $\cos 2t=1-2\sin^2 t$ and rearranges to produce a quadratic equation in $\sin t$.
If the identity $\cos 2t=\cos^2 t-\sin^2 t$ **or** $\cos 2t=2\cos^2 t-1$ is used instead, one further step, using the identity $\cos^2 t=1-\sin^2 t$, must be seen before the mark can be awarded.

A1 A correct 3TQ=0 in $\sin t$. Look for $200\sin^2 t+18\sin t-59=0$ or equivalent.

M1 For a correct attempt at solving the 3TQ=0 (usual rules) in $\sin t$. Accept a correct answer (from a graphical calculator) as justification.

A1 Award for either $\sin t=-\frac{59}{100}oe$ or $t=-0.63..$

M1 Using either their t or $\sin t$ to find either the x or y coordinate of B.
Accept as evidence sight of $10\cos 2\times 't'$ or $6\sin 't'$ or one correct answer (awrt 2dp).

A1 One coordinate both correct and exact. These are exact answers (3.038, - 3.54).

A1 Both coordinates correct and exact. Cso and cao (3.038, - 3.54).

Alternative solution to parts b and c using the Cartesian equation of C

Question Number	Scheme	Marks
Alt11(b)	$x = 10(1-2\sin^2 t) \Rightarrow x = 10 - \frac{5}{9}y^2$	M1A1
	$\left.\frac{\mathrm{d}x}{\mathrm{d}y}\right\vert_{y=3} = -\frac{10y}{9} = -\frac{10\times 3}{9} = -\frac{10}{3}$	M1A1
	$\frac{y-3}{x-5} = \frac{10}{3} \Rightarrow 3y = 10x - 41$	M1A1
		(6)
Alt(c)	Sub $x = 10 - \frac{5}{9}y^2$ into $3y = 10x - 41 \Rightarrow 3y = 10\left(10 - \frac{5}{9}y^2\right) - 41$	M1
	$\Rightarrow 50y^2 + 27y - 531 = 0$	M1A1
	$\Rightarrow (y-3)(50y+177) = 0$	M1A1
	$\Rightarrow y = ...$	
	Substitutes $y = ...$ into $x = 10 - \frac{5}{9}y^2$	M1
	$\Rightarrow x = 3.038,\ y = -3.54$	A1A1
		(8)
Alt(c)	Sub $y = 6\sqrt{\frac{10-x}{20}}$ into $3y = 10x - 41 \Rightarrow 36\sqrt{\frac{10-x}{20}} = 10x - 41$	M1
	$\Rightarrow 36^2\left(\frac{10-x}{20}\right) = (10x-41)^2$	
	$\Rightarrow 500x^2 - 4019x + 7595 = 0$	M1A1
	$\Rightarrow (500x - 1519)(x-5) = 0$	M1A1
	Substitutes $x = ...$ into $y = 6\sqrt{\frac{10-x}{20}}$	M1
	$\Rightarrow x = 3.038,\ y = -3.54$	A1A1
		(8)

Notes for Question 11

(b)

M1 Uses the double angle formula $\cos 2t = 1 - 2\sin^2 t$ to get the equation of C in the form $x = \mathrm{f}(y^2)$.

A1 The correct equation is obtained. That is $x = 10 - \frac{5}{9}y^2$ or equivalent $x = 10\left(1 - 2\left(\frac{y}{6}\right)^2\right)$

M1 Differentiates wrt y (usual rules) and subs y=3 to get a numerical value to $\frac{\mathrm{d}x}{\mathrm{d}y}$

A1 $\frac{\mathrm{d}x}{\mathrm{d}y} = -\frac{10}{3}$ This may be implied by the correct value in the gradient of the normal.

M1 Award for a correct method to find the equation of the normal.
They must use (5, 3) and their numerical value of $-\frac{\mathrm{d}x}{\mathrm{d}y}$. Eg $\frac{y-5}{x-3} = -\frac{\mathrm{d}x}{\mathrm{d}y}$
If $y = mx + c$ is used then it must be a full method with (5, 3) and with $m = "-\frac{\mathrm{d}x}{\mathrm{d}y}"$ as far as c =..

A1* cso. This is a proof and you must be convinced of all aspects including the last line of

$$\frac{y-3}{x-5} = \frac{10}{3} \Rightarrow 3y = 10x - 41$$

(c)

M1 Sub **their** $x = 10 - \frac{5}{9}y^2$ into $3y = 10x - 41$ to produce an equation in y
Alternatively subs **their** $y = 6\sqrt{\frac{10-x}{20}}$ into $3y = 10x - 41$ to produce an equation in x

M1 Forms a quadratic equation in y (or x)

A1 For achieving $50y^2 + 27y - 531 = 0$ / $500x^2 - 4019x + 7595 = 0$ or equivalent

M1 For a correct attempt at solving the 3TQ=0 in y (or x).
If you see the answers you can award this. We are accepting answers from a calculator.

A1 Correct factors. If the correct y (or x) is given then this mark is automatically awarded.

M1 Substitutes their $y =$ into their $x = \mathrm{f}(y^2) \Rightarrow x = ..$ or vice versa

A1 One correct, either $x = 3.038$ or y =-3.54 The values must be exact

A1 Both correct. $x = 3.038$ and y =-3.54
Accept $x = \frac{1519}{500}, y = -\frac{177}{50}$,

Leave blank

12.

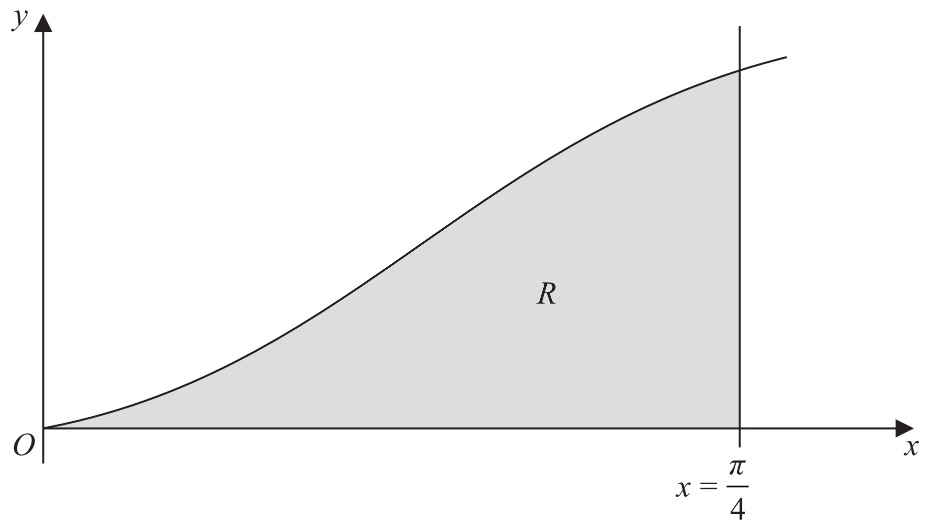

Figure 4

Figure 4 shows a sketch of part of the curve with equation

$$y = x(\sin x + \cos x), \qquad 0 \leqslant x \leqslant \frac{\pi}{4}$$

The finite region R, shown shaded in Figure 4, is bounded by the curve, the x-axis and the line $x = \frac{\pi}{4}$. This shaded region is rotated through 2π radians about the x-axis to form a solid of revolution, with volume V.

(a) Assuming the formula for volume of revolution show that $V = \int_0^{\frac{\pi}{4}} \pi x^2 (1 + \sin 2x)\,\mathrm{d}x$

(3)

(b) Hence using calculus find the exact value of V.

You must show your working.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(9)

Question Number	Scheme	Marks
12(a)	$y^2 = \left(x(\sin x + \cos x)\right)^2 = x^2(\sin x + \cos x)^2$	
	$= x^2(\sin^2 x + \cos^2 x + 2\sin x \cos x)$	M1
	$= x^2\left(1 + \sin 2x\right)$	A1
	$V = \int_0^{\frac{\pi}{4}} \pi y^2 dx = \int_0^{\frac{\pi}{4}} \pi x^2 (1 + \sin 2x) dx$	A1*
		(3)
(b)	$V = \int_0^{\frac{\pi}{4}} \pi x^2 (1 + \sin 2x) dx = \int_0^{\frac{\pi}{4}} (\pi x^2 + \pi x^2 \sin 2x) dx$	
	$= \int_0^{\frac{\pi}{4}} \pi x^2 \mathrm{d}x + \int_0^{\frac{\pi}{4}} \pi x^2 \sin 2x \mathrm{d}x$	
	$\int_0^{\frac{\pi}{4}} \pi x^2 \mathrm{d}x = \left[\pi \frac{x^3}{3}\right]_0^{\frac{\pi}{4}} = \pi \frac{\left(\frac{\pi}{4}\right)^3}{3}$ OR $\int_0^{\frac{\pi}{4}} x^2 \mathrm{d}x = \left[\frac{x^3}{3}\right]_0^{\frac{\pi}{4}} = \frac{\left(\frac{\pi}{4}\right)^3}{3}$	M1A1
	$\cancel{\pi} \int x^2 \sin 2x \, \mathrm{d}x = \cancel{\pi}\left(\pm Bx^2 \cos 2x \pm C \int x \cos 2x \, \mathrm{d}x\right)$	M1
	$= \cancel{\pi}\left(-x^2 \frac{\cos 2x}{2} + \int x \cos 2x \, \mathrm{d}x\right)$	A1
	$= \cancel{\pi}\left(\pm Bx^2 \cos 2x \pm Cx \sin 2x \pm \int D \sin 2x \mathrm{d}x\right)$	dM1
	$= \cancel{\pi}\left(-x^2 \frac{\cos 2x}{2} + x \frac{\sin 2x}{2} + \frac{\cos 2x}{4}\right)$	A1
	$\cancel{\pi} \int_{x=0}^{x=\frac{\pi}{4}} x^2 \sin 2x \mathrm{d}x = \cancel{\pi}\left[\pm Bx^2 \cos 2x \pm Cx \sin 2x \pm D \cos 2x\right]_0^{\frac{\pi}{4}} = \cancel{\pi}\left(\frac{\pi}{8} - \frac{1}{4}\right)$	ddM1
	$V = \int_0^{\frac{\pi}{4}} \pi x^2 (1 + \sin 2x) dx = \int_0^{\frac{\pi}{4}} \pi x^2 \mathrm{d}x + \int_0^{\frac{\pi}{4}} \pi x^2 \sin 2x \mathrm{d}x$	
	$= \left(\frac{\pi^4}{192}\right) + \left(\frac{\pi^2}{8} - \frac{\pi}{4}\right)$ oe	A1,A1
		(9)
		(12 marks)

Notes for Question 12

(a)

M1 For squaring y AND attempting to multiply out the bracket. The minimum requirement is that $y^2 = x^2\left(\sin^2 x + \cos^2 x + ...\right)$. There is no need to include $'\pi'$ for this mark.

A1 Using $\sin^2 x + \cos^2 x = 1$ **and** $2\sin x\cos x = \sin 2x$ to achieve $y^2 = x^2\left(1 + \sin 2x\right)$

There is no need to include $'\pi'$ for this mark.

You may accept $\sin^2 x + 2\sin x\cos x + \cos^2 x = 1 + \sin 2x$

A1* It must be stated or implied that $V = \int_0^{\frac{\pi}{4}} \pi y^2 \mathrm{d}x$.

It may be implied by replacing y^2 by $\left(x(\sin x + \cos x)\right)^2$

A correct proof must follow involving all that is required for the previous M1A1

The limits could just appear in the final line without any explanation. Note that this is a given answer

(b)

M1 For splitting the given integral into a sum **and** integrating x^2 or πx^2 to Ax^3.

There is no need for limits at this stage

A1 $\int_0^{\frac{\pi}{4}} x^2 \mathrm{d}x = \left[\frac{x^3}{3}\right]_0^{\frac{\pi}{4}} = \frac{\left(\frac{\pi}{4}\right)^3}{3}$. There is no need to simplify this. Accept $\int_0^{\frac{\pi}{4}} \pi x^2 \mathrm{d}x = \pi\left[\frac{x^3}{3}\right]_0^{\frac{\pi}{4}} = \pi\frac{\left(\frac{\pi}{4}\right)^3}{3}$

M1 For integrating $\int \pi x^2 \sin 2x \,\mathrm{d}x$ or $\int x^2 \sin 2x \,\mathrm{d}x$ by parts. The integration must be the correct way around. There is no need for limits. If the rule is quoted it must be correct, a version of which appears in the formula booklet.

Accept for this mark expressions of the form $\int x^2 \sin 2x \,\mathrm{d}x = \pm Bx^2 \cos 2x \pm \int Cx\cos 2x \,\mathrm{d}x$

A1 $\int x^2 \sin 2x \,\mathrm{d}x = -x^2 \frac{\cos 2x}{2} + \int x\cos 2x \,\mathrm{d}x$ OR $\int \pi x^2 \sin 2x \,\mathrm{d}x = -\pi x^2 \frac{\cos 2x}{2} + \int \pi x \cos 2x \,\mathrm{d}x$

dM1 A second application by parts, the correct way around. No need for limits. See the previous M1 for how to award. It is dependent upon this having been awarded.

Look for $\int x^2 \sin 2x \,\mathrm{d}x = \pm Bx^2 \cos 2x \pm Cx\sin 2x \pm \int D\sin 2x \,\mathrm{d}x$

A1 A fully correct answer to the integral of $\int x^2 \sin 2x \,\mathrm{d}x = -x^2\frac{\cos 2x}{2} + x\frac{\sin 2x}{2} + \frac{\cos 2x}{4}$

ddM1 For substituting in both limits and subtracting. The two M's for int by parts must have been scored.

A1 Either of $\left(\frac{\pi^4}{192}\right)$ linked to first M or $\left(\frac{\pi^2}{8} - \frac{\pi}{4}\right)$ linked to ddM. Accept in the form $\pi\left(\frac{\pi^3}{192} + ...\right)$

A1 Correct answer and correct solution only. Accept exact equivalents $V = \pi\left(\frac{\pi^3}{192} + \frac{\pi}{8} - \frac{1}{4}\right)$

Alt way (2)- Where candidate does not split up first.

(b)	$V=\int_0^{\frac{\pi}{4}}\cancel{\pi}x^2(1+\sin 2x)dx=\cancel{\pi}x^2\left(x\pm A\cos 2x\right)-\int_0^{\frac{\pi}{4}}B\cancel{\pi}x\left(x\pm A\cos 2x\right)dx$	2nd M1
	$=\cancel{\pi}x^2\left(x-\frac{\cos 2x}{2}\right)-\int_0^{\frac{\pi}{4}}2\cancel{\pi}x\left(x-\frac{\cos 2x}{2}\right)dx$	A1
	$=\cancel{\pi}\left(x^2\left(x\pm A\cos 2x\right)-Bx\left(Cx^2\pm D\sin 2x\right)\pm\int_0^{\frac{\pi}{4}}Ex^2\pm F\sin 2x\,dx\right)$	3rd dM1
	$=\cancel{\pi}\left(x^2\left(x-\frac{\cos 2x}{2}\right)-2x\left(\frac{x^2}{2}-\frac{\sin 2x}{4}\right)+\int_0^{\frac{\pi}{4}}2\left(\frac{x^2}{2}-\frac{\sin 2x}{4}\right)dx\right)$	A1
	$=\cancel{\pi}\left(x^2\left(x-\frac{\cos 2x}{2}\right)-2x\left(\frac{x^2}{2}-\frac{\sin 2x}{4}\right)+\mathbf{2}\left(\frac{\boldsymbol{x}^{\mathbf{3}}}{\mathbf{6}}+\frac{\cos 2x}{8}\right)\right)$	**1st M1A1**
	$V=\cancel{\pi}\left[x^2\left(x-\frac{\cos 2x}{2}\right)-2x\left(\frac{x^2}{2}-\frac{\sin 2x}{4}\right)+2\left(\frac{x^3}{6}+\frac{\cos 2x}{8}\right)\right]_0^{\frac{\pi}{4}}$	ddM1
	$=\frac{\pi^4}{192},+\frac{\pi^2}{8}-\frac{\pi}{4}$	A1, A1
		(9)

1ST M1,A1 Seen after two (not necessarily) correct applications of integration by parts, it is for integrating the x^2 term

2nd M1A1 It is for the first attempt at an application of integration by parts on $\int x^2(1+\sin 2x)dx$

Look for $x^2\left(x\pm A\cos 2x\right)-\int_0^{\frac{\pi}{4}}Bx\left(x\pm A\cos 2x\right)dx$ for the method

3rd dM1A1 It is for a further attempt at an application of integration by parts the correct way around. It is dependent upon the first method having been awarded.

Look for $=x^2\left(x\pm A\cos 2x\right)-Bx\left(Cx^2\pm D\sin 2x\right)\pm\int_0^{\frac{\pi}{4}}Ex^2\pm F\sin 2x\,dx$