Centre No.						Paper Reference							Surname	Initial(s)
Candidate No.						6	6	6	5	/	0	1	Signature	

Paper Reference(s)

6665/01

Edexcel GCE

Core Mathematics C3

Advanced Level

Monday 23 January 2006 – Afternoon

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
1	
2	
3	
4	
5	
6	
7	
8	
Total	

Materials required for examination
Mathematical Formulae (Green)

Items included with question papers
Nil

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initial(s) and signature.
Check that you have the correct question paper.
You must write your answer for each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.
There are 20 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the examiner. Answers without working may gain no credit.

Printer's Log. No.
N23495A
W850/R6665/57570 7/7/3/3/3/3/3

N 2 3 4 9 5 A 0 1 2 0

Turn over

edexcel

Leave blank

1.

Figure 1

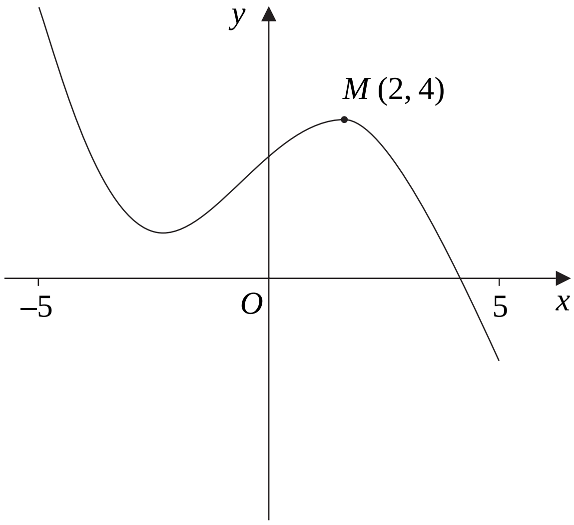


Figure 1 shows the graph of $y = \mathrm{f}(x)$, $-5 \leqslant x \leqslant 5$.
The point M (2, 4) is the maximum turning point of the graph.

Sketch, on separate diagrams, the graphs of

(a) $y = \mathrm{f}(x) + 3$,

(2)

(b) $y = |\mathrm{f}(x)|$,

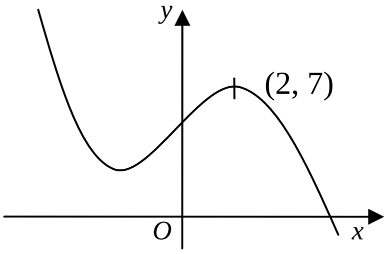
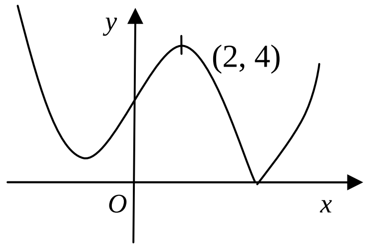
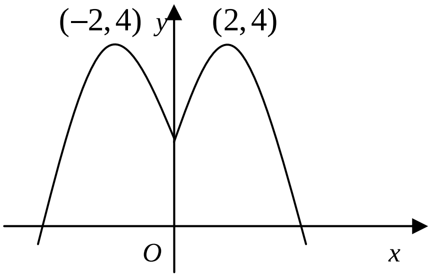
(2)

(c) $y = \mathrm{f}(|x|)$.

(3)

Show on each graph the coordinates of any maximum turning points.

N 2 3 4 9 5 A 0 2 2 0

Question Number	Scheme		Marks	
1.	(a) 	Shape unchanged Point	B1 B1	 **(2)**
	(b) 	Shape Point	B1 B1	 **(2)**
	(c) 	Shape (2, 4) (−2, 4)	B1 B1 B1	 **(3)** **[7]**

Leave blank

2. Express

$$\frac{2x^2+3x}{(2x+3)(x-2)}-\frac{6}{x^2-x-2}$$

as a single fraction in its simplest form.

(7)

Question Number	Scheme	Marks
2.	$x^2 - x - 2 = (x-2)(x+1)$ At any stage	B1
	$\dfrac{2x^2+3x}{(2x+3)(x-2)} = \dfrac{x(2x+3)}{(2x+3)(x-2)} = \dfrac{x}{x-2}$	B1
	$\dfrac{2x^2+3x}{(2x+3)(x-2)} - \dfrac{6}{x^2-x-2} = \dfrac{x(x+1)-6}{(x-2)(x+1)}$	M1
	$= \dfrac{x^2+x-6}{(x-2)(x+1)}$	A1
	$= \dfrac{(x+3)(x-2)}{(x-2)(x+1)}$	M1 A1
	$= \dfrac{x+3}{x+1}$	A1 **(7)**
		[7]
	Alternative method	
	$x^2 - x - 2 = (x-2)(x+1)$ At any stage	B1
	$(2x+3)$ appearing as a factor of the numerator at any stage	B1
	$\dfrac{2x^2+3x}{(2x+3)(x-2)} - \dfrac{6}{(x-2)(x+1)} = \dfrac{(2x^2+3x)(x+1)-6(2x+3)}{(2x+3)(x-2)(x+1)}$	M1
	$= \dfrac{2x^3+5x^2-9x-18}{(2x+3)(x-2)(x+1)}$ can be implied	A1
	$= \dfrac{(x-2)(2x^2+9x+9)}{(2x+3)(x-2)(x+1)}$ or $\dfrac{(2x+3)(x^2+x-6)}{(2x+3)(x-2)(x+1)}$ or $\dfrac{(x+3)(2x^2-x-6)}{(2x+3)(x-2)(x+1)}$ Any one linear factor × quadratic	M1
	$= \dfrac{(2x+3)(x-2)(x+3)}{(2x+3)(x-2)(x+1)}$ Complete factors	A1
	$= \dfrac{x+3}{x+1}$	A1 **(7)**

Leave blank

3. The point P lies on the curve with equation $y = \ln\left(\frac{1}{3}x\right)$. The x-coordinate of P is 3.

Find an equation of the normal to the curve at the point P in the form $y = ax + b$, where a and b are constants.

(5)

Question Number	Scheme	Marks
3.	$\frac{dy}{dx} = \frac{1}{x}$ accept $\frac{3}{3x}$	M1 A1
	At $x = 3$, $\frac{dy}{dx} = \frac{1}{3} \Rightarrow m' = -3$ Use of $mm' = -1$	M1
	$y - \ln 1 = -3(x-3)$	M1
	$y = -3x + 9$ Accept $y = 9 - 3x$	A1 **(5)**
		[5]
	$\frac{dy}{dx} = \frac{1}{3x}$ leading to $y = -9x + 27$ is a maximum of M1 A0 M1 M1 A0 =3/5	
4.	(a) (i) $\frac{d}{dx}\left(e^{3x+2}\right) = 3e^{3x+2}$ $\left(\text{or } 3e^2e^{3x}\right)$ At any stage	B1
	$\frac{dy}{dx} = 3x^2 e^{3x+2} + 2x e^{3x+2}$ Or equivalent	M1 A1+A1
		(4)
	(ii) $\frac{d}{dx}\left(\cos\left(2x^3\right)\right) = -6x^2 \sin\left(2x^3\right)$ At any stage	M1 A1
	$\frac{dy}{dx} = \frac{-18x^3 \sin\left(2x^3\right) - 3\cos\left(2x^3\right)}{9x^2}$	M1 A1 **(4)**
	Alternatively using the product rule for second M1 A1	
	$y = (3x)^{-1} \cos\left(2x^3\right)$	
	$\frac{dy}{dx} = -3(3x)^{-2} \cos\left(2x^3\right) - 6x^2 (3x)^{-1} \sin\left(2x^3\right)$	
	Accept equivalent unsimplified forms	
	(b) $1 = 8\cos(2y+6)\frac{dy}{dx}$ or $\frac{dx}{dy} = 8\cos(2y+6)$	M1
	$\frac{dy}{dx} = \frac{1}{8\cos(2y+6)}$	M1 A1
	$\frac{dy}{dx} = \frac{1}{8\cos\left(\arcsin\left(\frac{x}{4}\right)\right)}$ $\left(= (\pm)\frac{1}{2\sqrt{\left(16 - x^2\right)}}\right)$	M1 A1 **(5)**
		[13]

Leave blank

4. (a) Differentiate with respect to x

(i) x^2e^{3x+2},

(4)

(ii) $\dfrac{\cos(2x^3)}{3x}$.

(4)

(b) Given that $x = 4\sin(2y + 6)$, find $\dfrac{dy}{dx}$ in terms of x.

(5)

<table>
<tr><th>Question Number</th><th>Scheme</th><th>Marks</th></tr>
<tr><td>3.</td><td>$\frac{dy}{dx} = \frac{1}{x}$ accept $\frac{3}{3x}$
At $x = 3$, $\frac{dy}{dx} = \frac{1}{3} \Rightarrow m' = -3$ Use of $mm' = -1$
$y - \ln 1 = -3(x-3)$
$y = -3x + 9$ Accept $y = 9 - 3x$

$\frac{dy}{dx} = \frac{1}{3x}$ leading to $y = -9x + 27$ is a maximum of M1 A0 M1 M1 A0 =3/5</td><td>M1 A1
M1
M1
A1 (5)
[5]</td></tr>
<tr><td>4.</td><td>(a) (i) $\frac{d}{dx}\left(e^{3x+2}\right) = 3e^{3x+2}$ $\left(\text{or } 3e^2e^{3x}\right)$ At any stage
$\frac{dy}{dx} = 3x^2 e^{3x+2} + 2x e^{3x+2}$ Or equivalent

(ii) $\frac{d}{dx}\left(\cos\left(2x^3\right)\right) = -6x^2 \sin\left(2x^3\right)$ At any stage
$\frac{dy}{dx} = \frac{-18x^3 \sin\left(2x^3\right) - 3\cos\left(2x^3\right)}{9x^2}$
Alternatively using the product rule for second M1 A1
$y = (3x)^{-1}\cos\left(2x^3\right)$
$\frac{dy}{dx} = -3(3x)^{-2}\cos\left(2x^3\right) - 6x^2(3x)^{-1}\sin\left(2x^3\right)$
Accept equivalent unsimplified forms

(b) $1 = 8\cos(2y+6)\frac{dy}{dx}$ or $\frac{dx}{dy} = 8\cos(2y+6)$
$\frac{dy}{dx} = \frac{1}{8\cos(2y+6)}$
$\frac{dy}{dx} = \frac{1}{8\cos\left(\arcsin\left(\frac{x}{4}\right)\right)}$ $\left(= (\pm)\frac{1}{2\sqrt{\left(16 - x^2\right)}}\right)$</td><td>B1
M1 A1+A1
(4)
M1 A1
M1 A1 (4)

M1
M1 A1
M1 A1 (5)
[13]</td></tr>
</table>

Leave blank

5. $$\mathrm{f}(x) = 2x^3 - x - 4.$$

(a) Show that the equation $\mathrm{f}(x) = 0$ can be written as

$$x = \sqrt{\left(\frac{2}{x} + \frac{1}{2}\right)}.$$

(3)

The equation $2x^3 - x - 4 = 0$ has a root between 1.35 and 1.4.

(b) Use the iteration formula

$$x_{n+1} = \sqrt{\left(\frac{2}{x_n} + \frac{1}{2}\right)},$$

with $x_0 = 1.35$, to find, to 2 decimal places, the values of x_1, x_2 and x_3.

(3)

The only real root of $\mathrm{f}(x) = 0$ is α.

(c) By choosing a suitable interval, prove that $\alpha = 1.392$, to 3 decimal places.

(3)

Question Number	Scheme	Marks
5.	(a) $2x^2 - 1 - \frac{4}{x} = 0$ Dividing equation by x	M1
	$x^2 = \frac{1}{2} + \frac{4}{2x}$ Obtaining $x^2 = \ldots$	M1
	$x = \sqrt{\left(\frac{2}{x} + \frac{1}{2}\right)}$ ∗ cso	A1 **(3)**
	(b) $x_1 = 1.41, x_2 = 1.39, x_3 = 1.39$	B1, B1, B1
	If answers given to more than 2 dp, penalise first time then accept awrt above.	**(3)**
	(c) Choosing $(1.3915, 1.3925)$ or a tighter interval	M1
	$\text{f}(1.3915) \approx -3\times10^{-3}, \text{f}(1.3925) \approx 7\times10^{-3}$ Both, awrt	A1
	Change of sign (and continuity) $\Rightarrow \alpha \in (1.3915, 1.3925)$	
	$\Rightarrow \alpha = 1.392$ to 3 decimal places ∗ cso	A1 **(3)** **[9]**
6.	(a) $R\cos\alpha = 12, \quad R\sin\alpha = 4$	
	$R = \sqrt{(12^2 + 4^2)} = \sqrt{160}$ Accept if just written down, awrt 12.6	M1 A1
	$\tan\alpha = \frac{4}{12}, \Rightarrow \alpha \approx 18.43°$ awrt 18.4°	M1, A1**(4)**
	(b) $\cos(x + \text{ their } \alpha) = \frac{7}{\text{their } R} \quad (\approx 0.5534)$	M1
	$x + \text{their } \alpha = 56.4°$ awrt 56°	A1
	$= \ldots, 303.6°$ 360° – their principal value	M1
	$x = 38.0°, 285.2°$ Ignore solutions out of range	A1, A1 **(5)**
	If answers given to more than 1 dp, penalise first time then accept awrt above.	
	(c)(i) minimum value is $-\sqrt{160}$ ft their R	B1ft
	(ii) $\cos(x + \text{ their } \alpha) = -1$	M1
	$x \approx 161.57°$ cao	A1 **(3)** **[12]**

Leave blank

6. $$f(x) = 12\cos x - 4\sin x.$$

Given that $f(x) = R\cos(x + \alpha)$, where $R \geqslant 0$ and $0 \leqslant \alpha \leqslant 90°$,

(a) find the value of R and the value of α.

(4)

(b) Hence solve the equation

$$12\cos x - 4\sin x = 7$$

for $0 \leqslant x < 360°$, giving your answers to one decimal place.

(5)

(c) (i) Write down the minimum value of $12\cos x - 4\sin x$.

(1)

(ii) Find, to 2 decimal places, the smallest positive value of x for which this minimum value occurs.

(2)

N 2 3 4 9 5 A 0 1 2 2 0

Question Number	Scheme	Marks
5.	(a) $2x^2 - 1 - \frac{4}{x} = 0$ Dividing equation by x	M1
	$x^2 = \frac{1}{2} + \frac{4}{2x}$ Obtaining $x^2 = \ldots$	M1
	$x = \sqrt{\left(\frac{2}{x} + \frac{1}{2}\right)}$ * cso	A1 **(3)**
	(b) $x_1 = 1.41, x_2 = 1.39, x_3 = 1.39$	B1, B1, B1
	If answers given to more than 2 dp, penalise first time then accept awrt above.	**(3)**
	(c) Choosing $(1.3915, 1.3925)$ or a tighter interval	M1
	$\text{f}(1.3915) \approx -3\times10^{-3}$, $\text{f}(1.3925) \approx 7\times10^{-3}$ Both, awrt	A1
	Change of sign (and continuity) $\Rightarrow \alpha \in (1.3915, 1.3925)$	
	$\Rightarrow \alpha = 1.392$ to 3 decimal places * cso	A1 **(3)** **[9]**
6.	(a) $R\cos\alpha = 12, \quad R\sin\alpha = 4$	
	$R = \sqrt{(12^2 + 4^2)} = \sqrt{160}$ Accept if just written down, awrt 12.6	M1 A1
	$\tan\alpha = \frac{4}{12}, \quad \Rightarrow \quad \alpha \approx 18.43°$ awrt 18.4°	M1, A1**(4)**
	(b) $\cos(x + \text{ their } \alpha) = \frac{7}{\text{their } R}$ (≈ 0.5534)	M1
	$x + \text{their } \alpha = 56.4°$ awrt 56°	A1
	$= \ \ldots \ , \ 303.6°$ 360° – their principal value	M1
	$x = 38.0°, 285.2°$ Ignore solutions out of range	A1, A1 **(5)**
	If answers given to more than 1 dp, penalise first time then accept awrt above.	
	(c)(i) minimum value is $-\sqrt{160}$ ft their R	B1ft
	(ii) $\cos(x + \text{ their } \alpha) = -1$	M1
	$x \approx 161.57°$ cao	A1 **(3)** **[12]**

Leave blank

7. (a) Show that

(i) $\dfrac{\cos 2x}{\cos x + \sin x} \equiv \cos x - \sin x, \qquad x \neq (n - \tfrac{1}{4})\pi,\ n \in \mathbb{Z},$ **(2)**

(ii) $\tfrac{1}{2}(\cos 2x - \sin 2x) \equiv \cos^2 x - \cos x \sin x - \tfrac{1}{2}$. **(3)**

(b) Hence, or otherwise, show that the equation

$$\cos\theta\left(\frac{\cos 2\theta}{\cos\theta + \sin\theta}\right) = \frac{1}{2}$$

can be written as

$$\sin 2\theta = \cos 2\theta.$$

(3)

(c) Solve, for $0 \leqslant \theta < 2\pi$,

$$\sin 2\theta = \cos 2\theta,$$

giving your answers in terms of π.

(4)

Question Number	Scheme	Marks
7.	(a) (i) Use of $\cos 2x = \cos^2 x - \sin^2 x$ in an attempt to prove the identity.	M1
	$\frac{\cos 2x}{\cos x + \sin x} = \frac{\cos^2 x - \sin^2 x}{\cos x + \sin x} = \frac{(\cos x - \sin x)(\cos x + \sin x)}{\cos x + \sin x} = \cos x - \sin x$ ∗ cso	A1 **(2)**
	(ii) Use of $\cos 2x = 2\cos^2 x - 1$ in an attempt to prove the identity.	M1
	Use of $\sin 2x = 2\sin x \cos x$ in an attempt to prove the identity.	M1
	$\frac{1}{2}(\cos 2x - \sin 2x) = \frac{1}{2}(2\cos^2 x - 1 - 2\sin x \cos x) = \cos^2 x - \cos x \sin x - \frac{1}{2}$ ∗ cso	A1 **(3)**
	(b) $\cos\theta(\cos\theta - \sin\theta) = \frac{1}{2}$ Using (a)(i)	M1
	$\cos^2\theta - \cos\theta\sin\theta - \frac{1}{2} = 0$	
	$\frac{1}{2}(\cos 2\theta - \sin 2\theta) = 0$ Using (a)(ii)	M1
	$\cos 2\theta = \sin 2\theta$ ∗	A1 **(3)**
	(c) $\tan 2\theta = 1$	M1
	$2\theta = \frac{\pi}{4}, \left(\frac{5\pi}{4}, \frac{9\pi}{4}, \frac{13\pi}{4}\right)$ any one correct value of 2θ	A1
	$\theta = \frac{\pi}{8}, \frac{5\pi}{8}, \frac{9\pi}{8}, \frac{13\pi}{8}$ Obtaining at least 2 solutions in range	M1
	The 4 correct solutions	A1 **(4)**
	If decimals $(0.393, 1.963, 3.534, 5.105)$ or degrees $(22.5°, 112.5°, 202.5°, 292.5°)$ are given, but all 4 solutions are found, penalise one A mark only.	**[12]**
	Ignore solutions out of range.	

Leave blank

8. The functions f and g are defined by

$$\mathrm{f}: x \to 2x + \ln 2, \quad x \in \mathbb{R},$$

$$\mathrm{g}: x \to \mathrm{e}^{2x}, \quad x \in \mathbb{R}.$$

(a) Prove that the composite function gf is

$$\mathrm{gf}: x \to 4\mathrm{e}^{4x}, \quad x \in \mathbb{R}.$$

(4)

(b) In the space provided on page 19, sketch the curve with equation $y = \mathrm{gf}(x)$, and show the coordinates of the point where the curve cuts the y-axis.

(1)

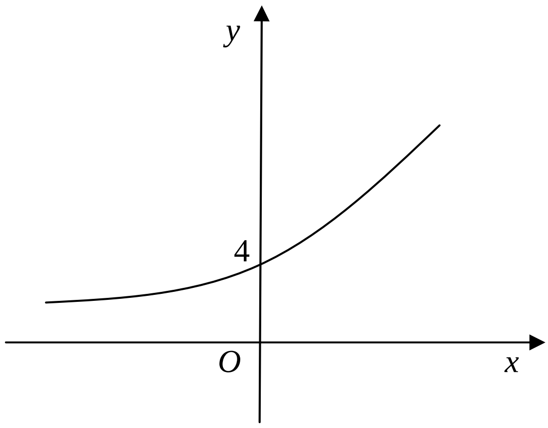
(c) Write down the range of gf.

(1)

(d) Find the value of x for which $\frac{\mathrm{d}}{\mathrm{d}x}[\mathrm{gf}(x)] = 3$, giving your answer to 3 significant figures.

(4)

N 2 3 4 9 5 A 0 1 8 2 0

Question Number	Scheme	Marks
8.	(a) $\text{gf}(x) = e^{2(2x+\ln 2)}$	M1
	$= e^{4x}e^{2\ln 2}$	M1
	$= e^{4x}e^{\ln 4}$	M1
	$= 4e^{4x}$ Give mark at this point, cso	A1 **(4)**
	$\left(\text{Hence gf} : x \mapsto 4e^{4x}, \quad x \in \mathbb{R}\right)$	
	(b)  Shape and point	B1 **(1)**
	(c) Range is $\mathbb{R}_+$ Accept $\text{gf}(x) > 0$, $y > 0$	B1 **(1)**
	(d) $\frac{d}{dx}\left[\text{gf}(x)\right] = 16e^{4x}$	
	$e^{4x} = \frac{3}{16}$	M1 A1
	$4x = \ln\frac{3}{16}$	M1
	$x \approx -0.418$	A1 **(4)**
		[10]