

January 2007
6665 Core Mathematics C3
Mark Scheme

Question Number	Scheme	Marks
1.	<p>(a) $\sin 3\theta = \sin(2\theta + \theta) = \sin 2\theta \cos \theta + \cos 2\theta \sin \theta$ $= 2 \sin \theta \cos^2 \theta + (1 - 2 \sin^2 \theta) \sin \theta$ $= 2 \sin \theta - 2 \sin^3 \theta + \sin \theta - 2 \sin^3 \theta$ $= 3 \sin \theta - 4 \sin^3 \theta$ *</p> <p>(b) $\sin 3\theta = 3 \times \frac{\sqrt{3}}{4} - 4 \left(\frac{\sqrt{3}}{4} \right)^3 = \frac{3\sqrt{3}}{4} - \frac{3\sqrt{3}}{16} = \frac{9\sqrt{3}}{16}$ or exact</p> <p>equivalent</p>	<p>B1 B1 B1 M1 A1 (5)</p> <p>M1 A1 (2)</p> <p>[7]</p>
2.	<p>(a) $f(x) = \frac{(x+2)^2, -3(x+2)+3}{(x+2)^2}$ $= \frac{x^2 + 4x + 4 - 3x - 6 + 3}{(x+2)^2} = \frac{x^2 + x + 1}{(x+2)^2}$ *</p> <p>(b) $x^2 + x + 1 = \left(x + \frac{1}{2}\right)^2 + \frac{3}{4}, > 0$ for all values of x.</p> <p>(c) $f(x) = \frac{\left(x + \frac{1}{2}\right)^2 + \frac{3}{4}}{(x+2)^2}$</p> <p>Numerator is positive from (b) $x \neq -2 \Rightarrow (x+2)^2 > 0$ (Denominator is positive) Hence $f(x) > 0$</p>	<p>M1 A1, A1</p> <p>CSO A1 (4)</p> <p>M1 A1, A1 (3)</p> <p>B1 (1)</p> <p>[8]</p>
	<p><i>Alternative to (b)</i></p> <p>$\frac{d}{dx}(x^2 + x + 1) = 2x + 1 = 0 \Rightarrow x = -\frac{1}{2} \Rightarrow x^2 + x + 1 = \frac{3}{4}$</p> <p>A parabola with positive coefficient of x^2 has a minimum $\Rightarrow x^2 + x + 1 > 0$ Accept equivalent arguments</p>	<p>M1 A1</p> <p>A1 (3)</p>

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2.	<p>(a) $f(x) = \frac{(x+2)^2, -3(x+2)+3}{(x+2)^2}$ $= \frac{x^2 + 4x + 4 - 3x - 6 + 3}{(x+2)^2} = \frac{x^2 + x + 1}{(x+2)^2}$ *</p> <p>(b) $x^2 + x + 1 = \left(x + \frac{1}{2}\right)^2 + \frac{3}{4}, > 0$ for all values of x.</p> <p>(c) $f(x) = \frac{\left(x + \frac{1}{2}\right)^2 + \frac{3}{4}}{(x+2)^2}$</p> <p>Numerator is positive from (b) $x \neq -2 \Rightarrow (x+2)^2 > 0$ (Denominator is positive) Hence $f(x) > 0$</p>	<p>M1 A1, A1</p> <p>cs0 A1 (4)</p> <p>M1 A1, A1 (3)</p> <p>B1 (1)</p> <p>[8]</p>
	<p>Alternative to (b)</p> $\frac{d}{dx}(x^2 + x + 1) = 2x + 1 = 0 \Rightarrow x = -\frac{1}{2} \Rightarrow x^2 + x + 1 = \frac{3}{4}$ <p>A parabola with positive coefficient of x^2 has a minimum $\Rightarrow x^2 + x + 1 > 0$ Accept equivalent arguments</p>	<p>M1 A1</p> <p>A1 (3)</p>

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3.	<p>(a) $y = \frac{\pi}{4} \Rightarrow x = 2 \sin \frac{\pi}{4} = 2 \times \frac{1}{\sqrt{2}} = \sqrt{2} \Rightarrow P \in C$</p> <p>Accept equivalent (reversed) arguments. In any method it must be clear that $\sin \frac{\pi}{4} = \frac{1}{\sqrt{2}}$ or exact equivalent is used.</p> <p>(b) $\frac{dx}{dy} = 2 \cos y \quad \text{or} \quad 1 = 2 \cos y \frac{dy}{dx}$</p> <p>$\frac{dy}{dx} = \frac{1}{2 \cos y}$ May be awarded after substitution</p> <p>$y = \frac{\pi}{4} \Rightarrow \frac{dy}{dx} = \frac{1}{\sqrt{2}} \quad *$ cso</p> <p>(c) $m' = -\sqrt{2}$</p> <p>$y - \frac{\pi}{4} = -\sqrt{2}(x - \sqrt{2})$</p> <p>$y = -\sqrt{2}x + 2 + \frac{\pi}{4}$</p>	<p>B1 (1)</p> <p>M1 A1</p> <p>M1</p> <p>A1 (4)</p> <p>B1</p> <p>M1 A1</p> <p>A1 (4)</p> <p>[9]</p>
4.	<p>(i) $\frac{dy}{dx} = \frac{(9+x^2) - x(2x)}{(9+x^2)^2} \left(= \frac{9-x^2}{(9+x^2)^2} \right)$</p> <p>$\frac{dy}{dx} = 0 \Rightarrow 9 - x^2 = 0 \Rightarrow x = \pm 3$</p> <p>$\left(3, \frac{1}{6} \right), \left(-3, -\frac{1}{6} \right)$ Final two A marks depend on second M only</p> <p>(ii) $\frac{dy}{dx} = \frac{3}{2} (1 + e^{2x})^{\frac{1}{2}} \times 2e^{2x}$</p> <p>$x = \frac{1}{2} \ln 3 \Rightarrow \frac{dy}{dx} = \frac{3}{2} (1 + e^{\ln 3})^{\frac{1}{2}} \times 2e^{\ln 3} = 3 \times 4^{\frac{1}{2}} \times 3 = 18$</p>	<p>M1 A1</p> <p>M1 A1</p> <p>A1, A1 (6)</p> <p>M1 A1 A1</p> <p>M1 A1 (5)</p> <p>[11]</p>

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3.	<p>(a) $y = \frac{\pi}{4} \Rightarrow x = 2 \sin \frac{\pi}{4} = 2 \times \frac{1}{\sqrt{2}} = \sqrt{2} \Rightarrow P \in C$</p> <p>Accept equivalent (reversed) arguments. In any method it must be clear that $\sin \frac{\pi}{4} = \frac{1}{\sqrt{2}}$ or exact equivalent is used.</p> <p>(b) $\frac{dx}{dy} = 2 \cos y \quad \text{or} \quad 1 = 2 \cos y \frac{dy}{dx}$</p> <p>$\frac{dy}{dx} = \frac{1}{2 \cos y}$ May be awarded after substitution</p> <p>$y = \frac{\pi}{4} \Rightarrow \frac{dy}{dx} = \frac{1}{\sqrt{2}} \quad *$ cso</p> <p>(c) $m' = -\sqrt{2}$</p> <p>$y - \frac{\pi}{4} = -\sqrt{2}(x - \sqrt{2})$</p> <p>$y = -\sqrt{2}x + 2 + \frac{\pi}{4}$</p>	<p>B1 (1)</p> <p>M1 A1</p> <p>M1</p> <p>A1 (4)</p> <p>B1</p> <p>M1 A1</p> <p>A1 (4)</p> <p>[9]</p>
4.	<p>(i) $\frac{dy}{dx} = \frac{(9+x^2) - x(2x)}{(9+x^2)^2} \left(= \frac{9-x^2}{(9+x^2)^2} \right)$</p> <p>$\frac{dy}{dx} = 0 \Rightarrow 9 - x^2 = 0 \Rightarrow x = \pm 3$</p> <p>$\left(3, \frac{1}{6} \right), \left(-3, -\frac{1}{6} \right)$ Final two A marks depend on second M only</p> <p>(ii) $\frac{dy}{dx} = \frac{3}{2} (1 + e^{2x})^{\frac{1}{2}} \times 2e^{2x}$</p> <p>$x = \frac{1}{2} \ln 3 \Rightarrow \frac{dy}{dx} = \frac{3}{2} (1 + e^{\ln 3})^{\frac{1}{2}} \times 2e^{\ln 3} = 3 \times 4^{\frac{1}{2}} \times 3 = 18$</p>	<p>M1 A1</p> <p>M1 A1</p> <p>A1, A1 (6)</p> <p>M1 A1 A1</p> <p>M1 A1 (5)</p> <p>[11]</p>

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5.	<p>(a) $R^2 = (\sqrt{3})^2 + 1^2 \Rightarrow R = 2$ $\tan \alpha = \sqrt{3} \Rightarrow \alpha = \frac{\pi}{3}$</p> <p>(b) $\sin(x + \text{their } \alpha) = \frac{1}{2}$ $x + \text{their } \alpha = \frac{\pi}{6} \left(\frac{5\pi}{6}, \frac{13\pi}{6} \right)$ $x = \frac{\pi}{2}, \frac{11\pi}{6}$</p> <p>The use of degrees loses only one mark in this question. Penalise the first time it occurs in an answer and then ignore.</p>	<p>M1 A1 M1 A1 (4) M1 A1 M1 A1 (4) [8]</p> <p>accept awrt 1.05</p> <p>accept awrt 1.57, 5.76</p>

6. The function f is defined by

$$f : x \mapsto \ln(4 - 2x), \quad x < 2 \quad \text{and} \quad x \in \mathbb{R}.$$

(a) Show that the inverse function of f is defined by

$$f^{-1} : x \mapsto 2 - \frac{1}{2}e^x$$

and write down the domain of f^{-1} .

(4)

(b) Write down the range of f^{-1} .

(1)

(c) In the space provided on page 16, sketch the graph of $y = f^{-1}(x)$. State the coordinates of the points of intersection with the x and y axes.

(4)

The graph of $y = x + 2$ crosses the graph of $y = f^{-1}(x)$ at $x = k$.

The iterative formula

$$x_{n+1} = -\frac{1}{2}e^{x_n}, \quad x_0 = -0.3$$

is used to find an approximate value for k .

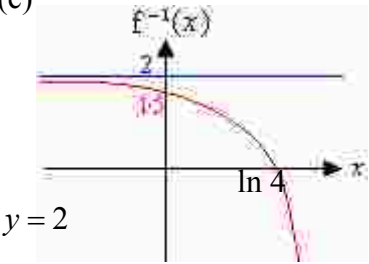
(d) Calculate the values of x_1 and x_2 , giving your answers to 4 decimal places.

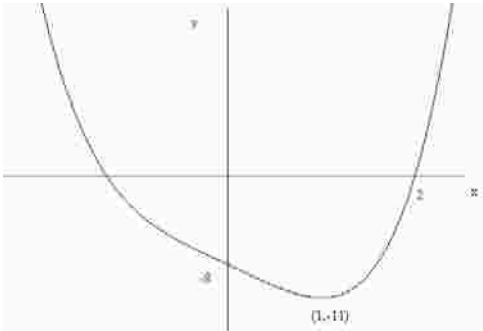
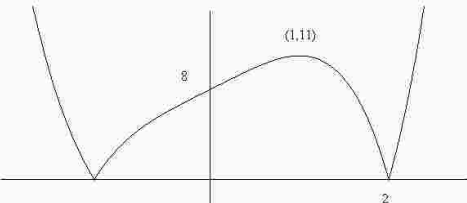
(2)

(e) Find the value of k to 3 decimal places.

(2)



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6.	<p>(a) $y = \ln(4 - 2x)$</p> <p>$e^y = 4 - 2x$ leading to $x = 2 - \frac{1}{2}e^y$ Changing subject and removing ln</p> <p>$y = 2 - \frac{1}{2}e^x \Rightarrow f^{-1} \mapsto 2 - \frac{1}{2}e^x$ *</p> <p>Domain of f^{-1} is \square</p> <p>(b) Range of f^{-1} is $f^{-1}(x) < 2$ (and $f^{-1}(x) \in \square$)</p> <p>(c)</p>  <p>(d) $x_1 \approx -0.3704, x_2 \approx -0.3452$</p> <p>If more than 4 dp given in this part a maximum on one mark is lost. Penalise on the first occasion.</p> <p>(e) $x_3 = -0.354\ 030\ 19 \dots$ $x_4 = -0.350\ 926\ 88 \dots$ $x_5 = -0.352\ 017\ 61 \dots$ $x_6 = -0.351\ 633\ 86 \dots$ Calculating to at least x_6 to at least four dp $k \approx -0.352$</p> <p>Alternative to (e) $k \approx -0.352$ Found in any way</p> <p>Let $g(x) = x + \frac{1}{2}e^x$ $g(-0.3515) \approx +0.0003, g(-0.3525) \approx -0.001$ Change of sign (and continuity) $\Rightarrow k \in (-0.3525, -0.3515)$ $\Rightarrow k = -0.352$ (to 3 dp)</p>	<p>M1 A1</p> <p>cso A1</p> <p>B1 (4)</p> <p>B1 (1)</p> <p>Shape B1 1.5 B1 ln 4 B1</p> <p>B1 (4)</p> <p>cao B1, B1 (2)</p> <p>M1 A1 (2)</p> <p>[13]</p> <p>M1</p> <p>A1 (2)</p>

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7.	<p>(a) $f(-2) = 16 + 8 - 8 (= 16) > 0$ $f(-1) = 1 + 4 - 8 (= -3) < 0$ Change of sign (and continuity) \Rightarrow root in interval $(-2, -1)$ ft their calculation as long as there is a sign change</p> <p>(b) $\frac{dy}{dx} = 4x^3 - 4 = 0 \Rightarrow x = 1$ Turning point is $(1, -11)$</p> <p>(c) $a = 2, b = 4, c = 4$</p> <p>(d) </p> <p>(e) </p>	<p>B1 B1 B1ft (3)</p> <p>M1 A1 A1 (3)</p> <p>B1 B1 B1 (3)</p> <p>Shape ft their turning point in correct quadrant only 2 and -8 B1 (3)</p> <p>Shape B1 (1) [13]</p>

Question Number	Scheme	Marks
8.	<p>(i) $\sec^2 x - \operatorname{cosec}^2 x = (1 + \tan^2 x) - (1 + \cot^2 x)$ $= \tan^2 x - \cot^2 x$ *</p> <p>(ii)(a) $y = \arccos x \Rightarrow x = \cos y$ $x = \sin\left(\frac{\pi}{2} - y\right) \Rightarrow \arcsin x = \frac{\pi}{2} - y$ Accept $\arcsin x = \arcsin \cos y$</p> <p>(b) $\arccos x + \arcsin x = y + \frac{\pi}{2} - y = \frac{\pi}{2}$</p>	<p>M1 A1 A1 (3)</p> <p>B1 B1 (2)</p> <p>B1 (1)</p> <p>[6]</p>

	<p><i>Alternatives for (i)</i></p> <p>$\sec^2 x - \tan^2 x = 1 = \operatorname{cosec}^2 x - \cot^2 x$ Rearranging $\sec^2 x - \operatorname{cosec}^2 x = \tan^2 x - \cot^2 x$ * cso</p> <p>$\left(\text{LHS} = \frac{1}{\cos^2 x} - \frac{1}{\sin^2 x} = \frac{\sin^2 x - \cos^2 x}{\cos^2 x \sin^2 x} \right)$</p> <p>$\text{RHS} = \frac{\sin^2 x}{\cos^2 x} - \frac{\cos^2 x}{\sin^2 x} = \frac{\sin^4 x - \cos^4 x}{\cos^2 x \sin^2 x} = \frac{(\sin^2 x - \cos^2 x)(\sin^2 x + \cos^2 x)}{\cos^2 x \sin^2 x}$ $= \frac{\sin^2 x - \cos^2 x}{\cos^2 x \sin^2 x}$ $= \text{LHS} *$ or equivalent</p>	<p>M1 A1 A1 (3)</p> <p>M1 A1 A1 (3)</p>