Centre No.						Paper Reference							Surname	Initial(s)
Candidate No.						6	6	6	5	/	0	1	Signature	

Paper Reference(s)

6665/01

Edexcel GCE

Core Mathematics C3

Advanced Level

Thursday 18 January 2007 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Green)	Nil

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
1	
2	
3	
4	
5	
6	
7	
8	
Total	

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initial(s) and signature.
Check that you have the correct question paper.
You must write your answer for each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.
There are 24 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

Printer's Log. No.
N23583A
W850/R6665/57570 3/3/3/3/3/3/21,900

N 2 3 5 8 3 A 0 1 2 4

Turn over

edexcel
advancing learning, changing lives

Leave blank

1. (a) By writing $\sin 3\theta$ as $\sin (2\theta + \theta)$, show that

$$\sin 3\theta = 3\sin\theta - 4\sin^3\theta.$$

(5)

(b) Given that $\sin\theta = \frac{\sqrt{3}}{4}$, find the exact value of $\sin 3\theta$.

(2)

January 2007
6665 Core Mathematics C3
Mark Scheme

Question Number	Scheme	Marks
1.	(a) $\sin 3\theta = \sin(2\theta+\theta) = \sin 2\theta\cos\theta + \cos 2\theta\sin\theta$	B1
	$= 2\sin\theta\cos^2\theta + (1-2\sin^2\theta)\sin\theta$	B1 B1
	$= 2\sin\theta - 2\sin^3\theta + \sin\theta - 2\sin^3\theta$	M1
	$= 3\sin\theta - 4\sin^3\theta$ * cso	A1 **(5)**
	(b) $\sin 3\theta = 3\times\frac{\sqrt{3}}{4} - 4\left(\frac{\sqrt{3}}{4}\right)^3 = \frac{3\sqrt{3}}{4} - \frac{3\sqrt{3}}{16} = \frac{9\sqrt{3}}{16}$ or exact equivalent	M1 A1 **(2)**
		[7]
2.	(a) $f(x) = \frac{(x+2)^2, -3(x+2)+3}{(x+2)^2}$	M1 A1, A1
	$= \frac{x^2+4x+4-3x-6+3}{(x+2)^2} = \frac{x^2+x+1}{(x+2)^2}$ * cso	A1 **(4)**
	(b) $x^2+x+1 = \left(x+\frac{1}{2}\right)^2 + \frac{3}{4},\ >0$ for all values of x.	M1 A1, A1 **(3)**
	(c) $f(x) = \frac{\left(x+\frac{1}{2}\right)^2+\frac{3}{4}}{(x+2)^2}$ Numerator is positive from (b) $x \neq -2 \Rightarrow (x+2)^2 > 0$ (Denominator is positive) Hence $f(x) > 0$	B1 **(1)**
		[8]
	Alternative to (b)	
	$\frac{d}{dx}(x^2+x+1) = 2x+1 = 0 \Rightarrow x = -\frac{1}{2} \Rightarrow x^2+x+1 = \frac{3}{4}$	M1 A1
	A parabola with positive coefficient of x^2 has a minimum $\Rightarrow x^2+x+1>0$ Accept equivalent arguments	A1 **(3)**

Leave blank

2.

$$f(x) = 1 - \frac{3}{x+2} + \frac{3}{(x+2)^2}, \ x \neq -2.$$

(a) Show that $f(x) = \dfrac{x^2 + x + 1}{(x+2)^2}, \ x \neq -2.$

(4)

(b) Show that $x^2 + x + 1 > 0$ for all values of x.

(3)

(c) Show that $f(x) > 0$ for all values of x, $x \neq -2$.

(1)

January 2007
6665 Core Mathematics C3
Mark Scheme

Question Number	Scheme		Marks
1.	(a) $\sin 3\theta = \sin(2\theta+\theta) = \sin 2\theta\cos\theta + \cos 2\theta\sin\theta$		B1
	$= 2\sin\theta\cos^2\theta + (1-2\sin^2\theta)\sin\theta$		B1 B1
	$= 2\sin\theta - 2\sin^3\theta + \sin\theta - 2\sin^3\theta$		M1
	$= 3\sin\theta - 4\sin^3\theta$ *	cso	A1 **(5)**
	(b) $\sin 3\theta = 3\times\frac{\sqrt{3}}{4} - 4\left(\frac{\sqrt{3}}{4}\right)^3 = \frac{3\sqrt{3}}{4} - \frac{3\sqrt{3}}{16} = \frac{9\sqrt{3}}{16}$ or exact equivalent		M1 A1 **(2)**
			[7]
2.	(a) $\mathrm{f}(x) = \frac{(x+2)^2, -3(x+2)+3}{(x+2)^2}$		M1 A1, A1
	$= \frac{x^2+4x+4-3x-6+3}{(x+2)^2} = \frac{x^2+x+1}{(x+2)^2}$ *	cso	A1 **(4)**
	(b) $x^2+x+1 = \left(x+\frac{1}{2}\right)^2 + \frac{3}{4}$, >0 for all values of x.		M1 A1, A1 **(3)**
	(c) $\mathrm{f}(x) = \frac{\left(x+\frac{1}{2}\right)^2+\frac{3}{4}}{(x+2)^2}$		
	Numerator is positive from (b)		
	$x\neq -2 \Rightarrow (x+2)^2 > 0$ (Denominator is positive)		
	Hence $\mathrm{f}(x) > 0$		B1 **(1)**
			[8]
	Alternative to (b)		
	$\frac{\mathrm{d}}{\mathrm{d}x}(x^2+x+1) = 2x+1 = 0 \Rightarrow x = -\frac{1}{2} \Rightarrow x^2+x+1 = \frac{3}{4}$		M1 A1
	A parabola with positive coefficient of x^2 has a minimum $\Rightarrow x^2+x+1>0$		A1 **(3)**
	Accept equivalent arguments		

Leave blank

3. The curve C has equation

$$x = 2\sin y.$$

(a) Show that the point $P\left(\sqrt{2}, \frac{\pi}{4}\right)$ lies on C.

(1)

(b) Show that $\frac{dy}{dx} = \frac{1}{\sqrt{2}}$ at P.

(4)

(c) Find an equation of the normal to C at P. Give your answer in the form $y = mx + c$, where m and c are exact constants.

(4)

Question Number	Scheme	Marks
3.	(a) $y=\frac{\pi}{4} \Rightarrow x=2\sin\frac{\pi}{4}=2\times\frac{1}{\sqrt{2}}=\sqrt{2} \Rightarrow P\in C$	B1 **(1)**
	Accept equivalent (reversed) arguments. In any method it must be clear that $\sin\frac{\pi}{4}=\frac{1}{\sqrt{2}}$ or exact equivalent is used.	
	(b) $\frac{dx}{dy}=2\cos y$ *or* $1=2\cos y\frac{dy}{dx}$	M1 A1
	$\frac{dy}{dx}=\frac{1}{2\cos y}$ May be awarded after substitution	M1
	$y=\frac{\pi}{4} \Rightarrow \frac{dy}{dx}=\frac{1}{\sqrt{2}}$ * cso	A1 **(4)**
	(c) $m'=-\sqrt{2}$	B1
	$y-\frac{\pi}{4}=-\sqrt{2}\left(x-\sqrt{2}\right)$	M1 A1
	$y=-\sqrt{2}x+2+\frac{\pi}{4}$	A1 **(4)**
		[9]
4.	(i) $\frac{dy}{dx}=\frac{(9+x^2)-x(2x)}{(9+x^2)^2}$ $\left(=\frac{9-x^2}{(9+x^2)^2}\right)$	M1 A1
	$\frac{dy}{dx}=0 \Rightarrow 9-x^2=0 \Rightarrow x=\pm3$	M1 A1
	$\left(3,\frac{1}{6}\right)$, $\left(-3,-\frac{1}{6}\right)$ Final two A marks depend on second M only	A1, A1 **(6)**
	(ii) $\frac{dy}{dx}=\frac{3}{2}\left(1+e^{2x}\right)^{\frac{1}{2}}\times2e^{2x}$	M1 A1 A1
	$x=\frac{1}{2}\ln3 \Rightarrow \frac{dy}{dx}=\frac{3}{2}\left(1+e^{\ln3}\right)^{\frac{1}{2}}\times2e^{\ln3}=3\times4^{\frac{1}{2}}\times3=18$	M1 A1 **(5)**
		[11]

Leave blank

4. (i) The curve C has equation

$$y = \frac{x}{9 + x^2}.$$

Use calculus to find the coordinates of the turning points of C.

(6)

(ii) Given that

$$y = (1 + e^{2x})^{\frac{3}{2}},$$

find the value of $\frac{dy}{dx}$ at $x = \frac{1}{2}\ln 3$.

(5)

Question Number	Scheme	Marks
3.	(a) $y=\frac{\pi}{4} \Rightarrow x = 2\sin\frac{\pi}{4} = 2\times\frac{1}{\sqrt{2}} = \sqrt{2} \Rightarrow P\in C$	B1 **(1)**
	Accept equivalent (reversed) arguments. In any method it must be clear that $\sin\frac{\pi}{4}=\frac{1}{\sqrt{2}}$ or exact equivalent is used.	
	(b) $\frac{dx}{dy} = 2\cos y$ *or* $1 = 2\cos y\frac{dy}{dx}$	M1 A1
	$\frac{dy}{dx} = \frac{1}{2\cos y}$ May be awarded after substitution	M1
	$y=\frac{\pi}{4} \Rightarrow \frac{dy}{dx} = \frac{1}{\sqrt{2}}$ * cso	A1 **(4)**
	(c) $m' = -\sqrt{2}$	B1
	$y-\frac{\pi}{4} = -\sqrt{2}\left(x-\sqrt{2}\right)$	M1 A1
	$y = -\sqrt{2}x + 2 + \frac{\pi}{4}$	A1 **(4)**
		[9]
4.	(i) $\frac{dy}{dx} = \frac{\left(9+x^2\right)-x\left(2x\right)}{\left(9+x^2\right)^2}$ $\left(=\frac{9-x^2}{\left(9+x^2\right)^2}\right)$	M1 A1
	$\frac{dy}{dx} = 0 \Rightarrow 9-x^2 = 0 \Rightarrow x = \pm 3$	M1 A1
	$\left(3, \frac{1}{6}\right), \left(-3, -\frac{1}{6}\right)$ Final two A marks depend on second M only	A1, A1 **(6)**
	(ii) $\frac{dy}{dx} = \frac{3}{2}\left(1+e^{2x}\right)^{\frac{1}{2}} \times 2e^{2x}$	M1 A1 A1
	$x = \frac{1}{2}\ln 3 \Rightarrow \frac{dy}{dx} = \frac{3}{2}\left(1+e^{\ln 3}\right)^{\frac{1}{2}} \times 2e^{\ln 3} = 3\times 4^{\frac{1}{2}} \times 3 = 18$	M1 A1 **(5)**
		[11]

Leave blank

5. **Figure 1**

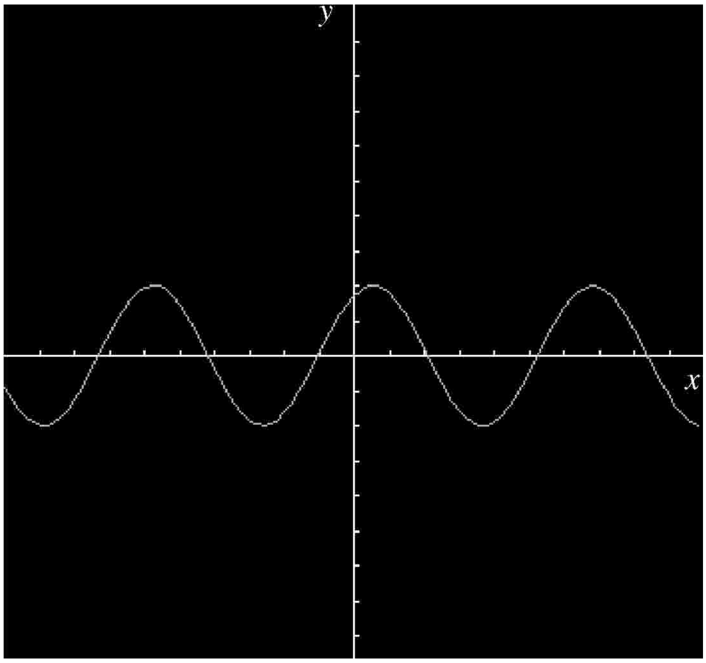


Figure 1 shows an oscilloscope screen.

The curve shown on the screen satisfies the equation

$$y = \sqrt{3}\cos x + \sin x.$$

(a) Express the equation of the curve in the form $y = R\sin(x + \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

(4)

(b) Find the values of x, $0 \leqslant x < 2\pi$, for which $y = 1$.

(4)

Question Number	Scheme	Marks
5.	(a) $R^2 = (\sqrt{3})^2 + 1^2 \Rightarrow R = 2$	M1 A1
	$\tan\alpha = \sqrt{3} \Rightarrow \alpha = \frac{\pi}{3}$ accept awrt 1.05	M1 A1 **(4)**
	(b) $\sin(x + \text{their } \alpha) = \frac{1}{2}$	M1
	$x + \text{their } \alpha = \frac{\pi}{6}\ \left(\frac{5\pi}{6}, \frac{13\pi}{6}\right)$	A1
	$x = \frac{\pi}{2}, \frac{11\pi}{6}$ accept awrt 1.57, 5.76	M1 A1 **(4)**
		[8]
	The use of degrees loses only one mark in this question. Penalise the first time it occurs in an answer and then ignore.	

Leave blank

6. The function f is defined by

$$\mathrm{f} : x \mapsto \ln(4-2x), \quad x < 2 \quad \text{and} \quad x \in \mathbb{R}.$$

(a) Show that the inverse function of f is defined by

$$\mathrm{f}^{-1} : x \mapsto 2 - \frac{1}{2}\mathrm{e}^{x}$$

and write down the domain of f^{-1}.

(4)

(b) Write down the range of f^{-1}.

(1)

(c) In the space provided on page 16, sketch the graph of $y = \mathrm{f}^{-1}(x)$. State the coordinates of the points of intersection with the x and y axes.

(4)

The graph of $y = x + 2$ crosses the graph of $y = \mathrm{f}^{-1}(x)$ at $x = k$.

The iterative formula

$$x_{n+1} = -\frac{1}{2}\mathrm{e}^{x_n}, \;\; x_0 = -0.3$$

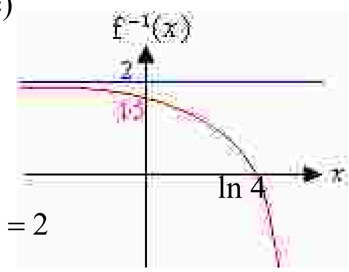
is used to find an approximate value for k.

(d) Calculate the values of x_1 and x_2, giving your answers to 4 decimal places.

(2)

(e) Find the value of k to 3 decimal places.

(2)

Question Number	Scheme	Marks
6.	(a) $y = \ln(4-2x)$	
	$e^y = 4-2x$ leading to $x = 2 - \frac{1}{2}e^y$ Changing subject and removing ln	M1 A1
	$y = 2 - \frac{1}{2}e^x \Rightarrow f^{-1} \mapsto 2 - \frac{1}{2}e^x$ ∗ cso	A1
	Domain of f^{-1} is □	B1 **(4)**
	(b) Range of f^{-1} is $f^{-1}(x) < 2$ (and $f^{-1}(x) \in$ □)	B1 **(1)**
	(c)  $y = 2$	
	Shape	B1
	1.5	B1
	ln 4	B1
		B1 **(4)**
	(d) $x_1 \approx -0.3704$, $x_2 \approx -0.3452$ cao	B1, B1 **(2)**
	If more than 4 dp given in this part a maximum on one mark is lost. Penalise on the first occasion.	
	(e) $x_3 = -0.354\,030\,19\ \ldots$	
	$x_4 = -0.350\,926\,88\ \ldots$	
	$x_5 = -0.352\,017\,61\ \ldots$	
	$x_6 = -0.351\,633\,86\ \ldots$ Calculating to at least x_6 to at least four dp	M1
	$k \approx -0.352$ cao	A1 **(2)** **[13]**
	Alternative to (e)	
	$k \approx -0.352$ Found in any way	
	Let $g(x) = x + \frac{1}{2}e^x$	
	$g(-0.3515) \approx +0.0003$, $g(-0.3525) \approx -0.001$	M1
	Change of sign (and continuity) $\Rightarrow k \in (-0.3525, -0.3515)$	
	$\Rightarrow k = -0.352$ (to 3 dp)	A1 **(2)**

Leave blank

7. $$f(x) = x^4 - 4x - 8.$$

(a) Show that there is a root of $f(x) = 0$ in the interval $[-2, -1]$.

(3)

(b) Find the coordinates of the turning point on the graph of $y = f(x)$.

(3)

(c) Given that $f(x) = (x - 2)(x^3 + ax^2 + bx + c)$, find the values of the constants, a, b and c.

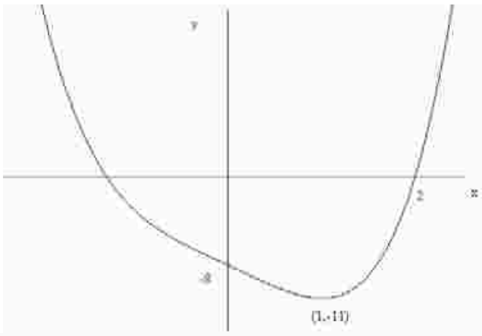
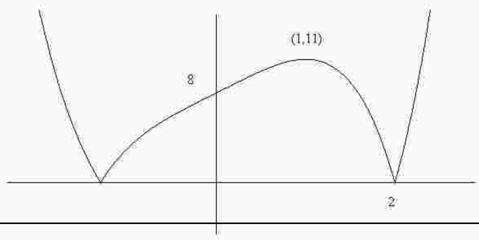
(3)

(d) In the space provided on page 21, sketch the graph of $y = f(x)$.

(3)

(e) Hence sketch the graph of $y = |f(x)|$.

(1)

Question Number	Scheme	Marks
7.	(a) $\mathrm{f}(-2) = 16 + 8 - 8\ (= 16) > 0$	B1
	$\mathrm{f}(-1) = 1 + 4 - 8\ (= -3) < 0$	B1
	Change of sign (and continuity) $\Rightarrow$ root in interval $(-2, -1)$	B1ft **(3)**
	ft their calculation as long as there is a sign change	
	(b) $\frac{\mathrm{d}y}{\mathrm{d}x} = 4x^3 - 4 = 0 \quad \Rightarrow \quad x = 1$	M1 A1
	Turning point is $(1, -11)$	A1 **(3)**
	(c) $a = 2, b = 4, c = 4$	B1 B1 B1 **(3)**
	(d) 	
	Shape	B1
	ft their turning point in correct quadrant only	B1 ft
	2 and −8	B1 **(3)**
	(e) 	
	Shape	B1 **(1)**
		[13]

Leave blank

8. (i) Prove that

$$\sec^2 x - \operatorname{cosec}^2 x \equiv \tan^2 x - \cot^2 x.$$

(3)

(ii) Given that

$$y = \arccos x, \quad -1 \leqslant x \leqslant 1 \text{ and } 0 \leqslant y \leqslant \pi,$$

(a) express $\arcsin x$ in terms of y.

(2)

(b) Hence evaluate $\arccos x + \arcsin x$. Give your answer in terms of π.

(1)

Question Number	Scheme	Marks
8.	(i) $\sec^2 x - \text{cosec}^2 x = (1+\tan^2 x) - (1+\cot^2 x)$	M1 A1
	$= \tan^2 x - \cot^2 x$ ∗ cso	A1 **(3)**
	(ii)(a) $y = \arccos x \Rightarrow x = \cos y$	B1
	$x = \sin\left(\frac{\pi}{2} - y\right) \Rightarrow \arcsin x = \frac{\pi}{2} - y$	B1 **(2)**
	Accept $\arcsin x = \arcsin \cos y$	
	(b) $\arccos x + \arcsin x = y + \frac{\pi}{2} - y = \frac{\pi}{2}$	B1 **(1)**
		[6]
	Alternatives for (i)	
	$\sec^2 x - \tan^2 x = 1 = \text{cosec}^2 x - \cot^2 x$	M1 A1
	Rearranging $\sec^2 x - \text{cosec}^2 x = \tan^2 x - \cot^2 x$ ∗ cso	A1 **(3)**
	$\left(\text{LHS} = \frac{1}{\cos^2 x} - \frac{1}{\sin^2 x} = \frac{\sin^2 x - \cos^2 x}{\cos^2 x \sin^2 x}\right)$	
	RHS $= \frac{\sin^2 x}{\cos^2 x} - \frac{\cos^2 x}{\sin^2 x} = \frac{\sin^4 x - \cos^4 x}{\cos^2 x \sin^2 x} = \frac{(\sin^2 x - \cos^2 x)(\sin^2 x + \cos^2 x)}{\cos^2 x \sin^2 x}$	M1
	$= \frac{\sin^2 x - \cos^2 x}{\cos^2 x \sin^2 x}$	A1
	= LHS ∗ or equivalent	A1 **(3)**