Centre No.						Paper Reference							Surname	Initial(s)
Candidate No.						6	6	6	5	/	0	1	Signature	

Paper Reference(s)

6665/01

Edexcel GCE

Core Mathematics C3

Advanced

Monday 23 January 2012 – Morning

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
1	
2	
3	
4	
5	
6	
7	
8	
Total	

Materials required for examination
Mathematical Formulae (Pink)

Items included with question papers
Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer for each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.
There are 24 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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1. Differentiate with respect to x, giving your answer in its simplest form,

(a) $x^2 \ln(3x)$

(4)

(b) $\dfrac{\sin 4x}{x^3}$

(5)

Question No	Scheme	Marks
1	(a)	
	$\frac{d}{dx}(\ln(3x)) \to \frac{B}{x}$ for any constant B	M1
	Applying vu'+uv' , $\ln(3x) \times 2x + x$	M1, A1 A1 (4)
	(b)	
	Applying $\frac{vu'-uv'}{v^2}$	
	$\frac{x^3 \times 4cos(4x) - \sin(4x) \times 3x^2}{x^6}$	M1 A1+A1 A1
	$= \frac{4xcos(4x) - 3\sin(4x)}{x^4}$	A1
		(5)
		(9 MARKS)

(a) M1 Differentiates the $\ln(3x)$ term to $\frac{B}{x}$. Note that $\frac{1}{3x}$ is fine for this mark.

M1 Applies the product rule to $x^2\ln(3x)$. If the rule is quoted it must be correct.
There must have been some attempt to differentiate both terms.
If the rule is **not quoted (or implied by their working)** only accept answers of the form
$\ln(3x) \times Ax + x^2 \times \frac{B}{x}$ where A and B are non- zero constants

A1 One term correct and simplified, either $2xln(3x)$ or x. $\ln 3x^{2x}$ and $\ln(3x)\,2x$ are acceptable forms

A1 Both terms correct and simplified on the same line. $2xln(3x) + x$, $\ln(3x) \times 2x + x$, $x(2ln3x + 1)$ oe

(b) M1 Applies the quotient rule. A version of this appears in the formula booklet. If the formula is quoted it must be correct. There must have been some attempt to differentiate both terms.
If the formula is **not quoted (nor implied by their working)** only accept answers of the form

$$\frac{x^3 \times \pm Acos(4x) - \sin(4x) \times Bx^2}{(x^3)^2 \text{ or } x^6 \text{ or } x^5 \text{ or } x^9} \text{ with } B > 0$$

A1 Correct first term on numerator $x^3 \times 4cos(4x)$

A1 Correct second term on numerator $-\sin(4x) \times 3x^2$

A1 Correct denominator x^6, the $(x^3)^2$ needs to be simplified

A1 Fully correct simplified expression $\frac{4xcos(4x) - 3\sin(4x)}{x^4}$, $\frac{cos(4x)4x - \sin(4x)3}{x^4}$ oe .
Accept $4x^{-3}\cos(4x) - 3x^{-4}\sin(4x)$ oe

Alternative method using the product rule.

M1,A1 **Writes** $\frac{sin(4x)}{x^3}$ **as** $\sin(4x) \times x^{-3}$ and applies the product rule. They will score both of these marks or neither of them. If the formula is quoted it must be correct. There must have been some attempt to differentiate both terms. If the formula is **not quoted (nor implied by their working)** only accept answers of the form $x^{-3} \times Acos(4x) + \sin(4x) \times \pm Bx^{-4}$

A1 One term correct, either $x^{-3} \times 4cos(4x)$ or $\sin(4x) \times -3x^{-4}$

A1 Both terms correct,Eg. $x^{-3} \times 4cos(4x) + \sin(4x) \times -3x^{-4}$.

A1 Fully correct expression. $4x^{-3}cos(4x) - 3x^{-4}\sin(4x)$ or $4cos(4x)x^{-3} - 3\sin(4x)x^{-4}$ oe
The negative must have been dealt with for the final mark.

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2.

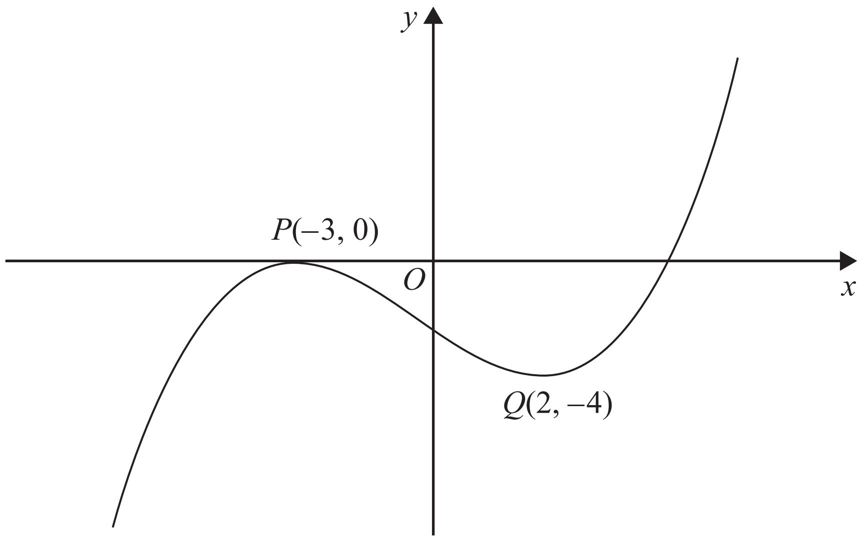


Figure 1

Figure 1 shows the graph of equation $y = \text{f}(x)$.

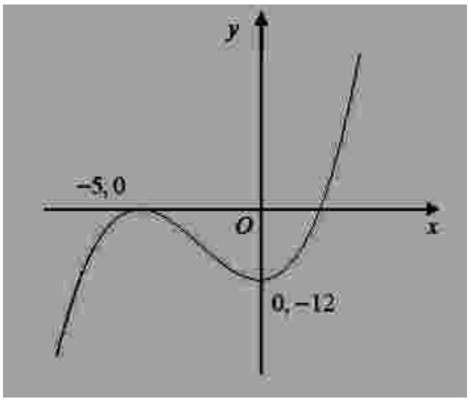
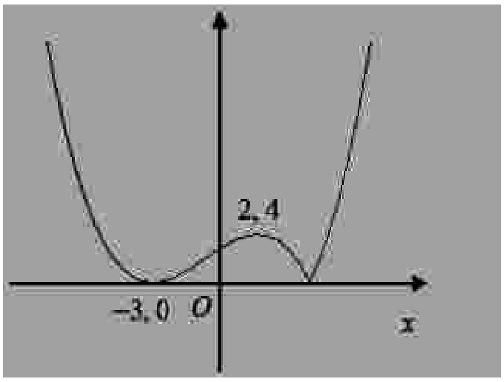
The points $P(-3, 0)$ and $Q(2, -4)$ are stationary points on the graph.

Sketch, on separate diagrams, the graphs of

(a) $y = 3\text{f}(x+2)$ **(3)**

(b) $y = |\text{f}(x)|$ **(3)**

On each diagram, show the coordinates of any stationary points.

Question No	Scheme	Marks
2	(a)  Shape *x* coordinates correct y coordinates correct	B1 B1 B1 (3)
	(b)  Shape Max at (2,4) Min at (-3,0)	B1 B1 B1 (3)
		6 marks

(a)

B1 Shape unchanged. The positioning of the curve is not significant for this mark. The right hand section of the curve does not have to cross *x* axis.

B1 The x- coordinates of P' and Q' are -5 and 0 respectively. This is for translating the curve 2 units left. The minimum point Q' must be on the y axis. Accept if -5 is marked on the x axis for P' with Q' on the *y* axis (marked -12).

B1 The y- coordinates of P' and Q' are 0 and -12 respectively. This is for the stretch ×3 parallel to the *y* axis. The maximum P' must be on the *x* axis. Accept if -12 is marked on the y axis for Q' with P' on the x axis (marked -5)

(b)

B1 The curve below the x axis reflected in the x axis and the curve above the x axis is unchanged. Do not accept if the curve is clearly rounded off with a zero gradient at the x axis but allow small curvature issues. Use the same principles on the lhs- do not accept if this is a cusp.

B1 Both the x- and y- coordinates of Q', (2,4) given correctly and associated with the maximum point in the first quadrant. To gain this mark there must be a graph and it must only have one maximum.
Accept as 2 and 4 marked on the correct axes or in the script as long as there is no ambiguity.

B1 Both the x- and y- coordinates of P', (-3,0) given correctly and associated with the minimum point in the second quadrant. To gain this mark there must be a graph. Tolerate two cusps if this mark has been lost earlier in the question. Accept (0, -3) marked on the correct axis.

Question No	Scheme	Marks
3	(a) 20 (mm^2)	**B1** **(1)** **M1**

Leave blank

3. The area, A mm^2, of a bacterial culture growing in milk, t hours after midday, is given by

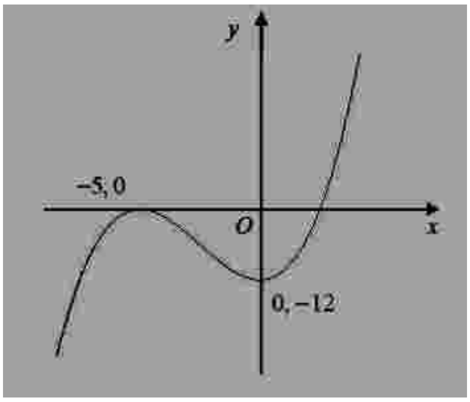
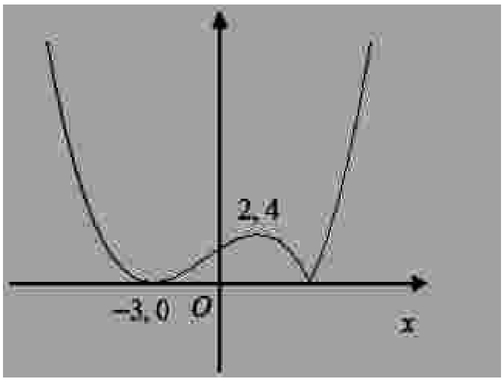
$$A = 20e^{1.5t}, \quad t \geqslant 0$$

(a) Write down the area of the culture at midday.

(1)

(b) Find the time at which the area of the culture is twice its area at midday. Give your answer to the nearest minute.

(5)

Question No	Scheme	Marks
2	(a)  Shape *x* coordinates correct y coordinates correct	B1 B1 B1 (3)
	(b)  Shape Max at (2,4) Min at (-3,0)	B1 B1 B1 (3)
		6 marks

(a)

B1 Shape unchanged. The positioning of the curve is not significant for this mark. The right hand section of the curve does not have to cross *x* axis.

B1 The x- coordinates of P' and Q' are -5 and 0 respectively. This is for translating the curve 2 units left. The minimum point Q' must be on the y axis. Accept if -5 is marked on the x axis for P' with Q' on the *y* axis (marked -12).

B1 The y- coordinates of P' and Q' are 0 and -12 respectively. This is for the stretch ×3 parallel to the *y* axis. The maximum P' must be on the *x* axis. Accept if -12 is marked on the y axis for Q' with P' on the x axis (marked -5)

(b)

B1 The curve below the x axis reflected in the x axis and the curve above the x axis is unchanged. Do not accept if the curve is clearly rounded off with a zero gradient at the x axis but allow small curvature issues. Use the same principles on the lhs- do not accept if this is a cusp.

B1 Both the x- and y- coordinates of Q', (2,4) given correctly and associated with the maximum point in the first quadrant. To gain this mark there must be a graph and it must only have one maximum.
Accept as 2 and 4 marked on the correct axes or in the script as long as there is no ambiguity.

B1 Both the x- and y- coordinates of P', (-3,0) given correctly and associated with the minimum point in the second quadrant. To gain this mark there must be a graph. Tolerate two cusps if this mark has been lost earlier in the question. Accept (0, -3) marked on the correct axis.

Question No	Scheme	Marks
3	(a) 20 (mm^2)	**B1** **(1)** **M1**

	(b) $'40' = 20\ e^{1.5t} \quad \rightarrow \quad e^{1.5t} = c$	
	$e^{1.5t} = \frac{40}{20} = (2)$	**A1**
	Correct order $1.5t = ln'2' \rightarrow \quad t = \frac{lnc}{1.5}$	**M1**
	$t = \frac{ln2}{1.5} = (\ awrt\ 0.46)$	**A1**
	12.28 or 28 (minutes}	**A1** **(5)**
		(6 marks)

(a)

B1 Sight of 20 relating to the value of A at t=0. Do not worry about (incorrect) units. Accept its sight in (b)

(b)

M1 Substitutes A=40 or twice their answer to (a) **and** proceeds to $e^{1.5t} = constant$. Accept non numerical answers.

A1 $e^{1.5t} = \frac{40}{20}\ or\ 2$

M1 Correct ln work to find t. Eg $e^{1.5t} = constant \rightarrow 1.5t = \ln(constant) \rightarrow t =$
The order must be correct. Accept non numerical answers**. See below for correct alternatives**

A1 Achieves either $\frac{\ln(2)}{1.5}$ or awrt 0.46 2sf

A1 Either 12:28 or 28 (minutes). Cao

Alternatives in (b)

Alt 1- taking ln's of both sides on line 1

M1 Substitutes A=40, or twice (a) takes ln's of both sides **and** proceeds to $\ln('40') = ln20 + lne^{1.5t}$

A1 $\ln(40) = ln20 + 1.5t$

M1 Make t the subject with correct *ln* work.

$$\ln('40') - ln20 = 1.5t\ \ or\ \ \ln\left(\frac{'40'}{20}\right) = 1.5t \rightarrow\ t =$$

A1,A1 are the same

Alt 2- trial and improvement-hopefully seen rarely

M1 Substitutes t= 0.46 and t=0.47 into $20e^{1.5t}$ to obtain A at both values. Must be to at least 2dp but you may accept tighter interval but the interval must span the correct value of 0.46209812

A1 Obtains A(0.46)=39.87 AND A(0.47)=40.47 or equivalent

M1 Substitutes t=0.462 and t=0.4625 into $40e^{1.5t}$

A1 Obtains A(0.462)=39.99 AND A(0.4625)=40.02 or equivalent and states t=0.462 (3sf)

A1 AS ABOVE

No working leading to fully correct accurate answer (3sf or better) send/escalate to team leader

Leave blank

4. The point P is the point on the curve $x = 2\tan\left(y + \frac{\pi}{12}\right)$ with y-coordinate $\frac{\pi}{4}$.

Find an equation of the normal to the curve at P.

(7)

Question No	Scheme	Marks
4	$\left(\frac{dx}{dy}\right) = 2sec^2\left(y+\frac{\pi}{12}\right)$	M1,A1
	substitute $y=\frac{\pi}{4}$ into their $\frac{dx}{dy} = 2sec^2\left(\frac{\pi}{4}+\frac{\pi}{12}\right) = 8$	M1, A1
	When $y=\frac{\pi}{4}$. $x=2\sqrt{3}$ awrt 3.46	B1
	$\left(y-\frac{\pi}{4}\right) = their\ m(x - their\ 2\sqrt{3})$	M1
	$\left(y-\frac{\pi}{4}\right) = -8(x-2\sqrt{3})$ oe	A1 (7 marks)

M1 For differentiation of $2\tan\left(\boldsymbol{y}+\frac{\pi}{12}\right) \rightarrow 2sec^2\left(\boldsymbol{y}+\frac{\pi}{12}\right)$. There is no need to identify this with $\frac{dx}{dy}$

A1 For correctly writing $\frac{dx}{dy} = 2sec^2\left(y+\frac{\pi}{12}\right)$ or $\frac{dy}{dx} = \frac{1}{2sec^2\left(y+\frac{\pi}{12}\right)}$

M1 Substitute $y=\frac{\pi}{4}$ into their $\frac{dx}{dy}$. Accept if $\frac{dx}{dy}$ is inverted and $y=\frac{\pi}{4}$ substituted into $\frac{dy}{dx}$.

A1 $\frac{dx}{dy} = 8$ or $\frac{dy}{dx} = \frac{1}{8}$ oe

B1 Obtains the value of $x=2\sqrt{3}$ corresponding to $y=\frac{\pi}{4}$. Accept awrt 3.46

M1 This mark requires **all of the necessary elements for** finding **a numerical equation** of the **normal.**

Either Invert their value of $\frac{dx}{dy}$, to find $\frac{dy}{dx}$, then use $m_1 \times m_2=-1$ to find the numerical gradient of the normal

Or use their numerical value of $-\frac{dx}{dy}$

Having done this then use $\left(y-\frac{\pi}{4}\right) = their\ m(x - their\ 2\sqrt{3})$

The $2\sqrt{3}$ could appear as awrt 3.46, the $\frac{\pi}{4}$ as awrt 0.79,

This cannot be awarded for finding the equation of a tangent.

Watch for candidates who correctly use $(x - their\ 2\sqrt{3}) = -their\ numerical\ \frac{dy}{dx}\left(y-\frac{\pi}{4}\right)$

If they use 'y=mx+c' it must be a full method to find c.

A1 Any correct form of the answer. It does not need to be simplified and the question does not ask for an exact answer.

$\left(y-\frac{\pi}{4}\right) = -8(x-2\sqrt{3})$, $\frac{y-\frac{\pi}{4}}{x-2\sqrt{3}} = -8$, $y = -8x + \frac{\pi}{4} + 16\sqrt{3}$, y=-8x+ (awrt) 28.5

Alternatives using arctan (first 3 marks)

M1 Differentiates $y = \arctan\left(\frac{x}{2}\right) - \frac{\pi}{12}$ to get $\frac{1}{1+(\frac{x}{2})^2} \times constant$. Don't worry about the lhs

A1 Achieves $\frac{dy}{dx} = \frac{1}{1+(\frac{x}{2})^2} \times \frac{1}{2}$

M1 This method mark requires *x* to be found, which then needs to be substituted into $\frac{dy}{dx}$

The rest of the marks are then the same.

Or implicitly (first 2 marks)

M1 Differentiates implicitly to get $1 = 2\,sec^2\left(y + \frac{\pi}{12}\right) \times \frac{dy}{dx}$

A1 Rearranges to get $\frac{dy}{dx}$ or $\frac{dx}{dy}$ in terms of y

The rest of the marks are the same

Or by compound angle identities

$x = 2\tan\left(y + \frac{\pi}{12}\right) = \frac{2tany + 2\tan(\frac{\pi}{12})}{1 - tany\, tan\frac{\pi}{12}}$ oe

M1 Differentiates using quotient rule-see question 1 in applying this. Additionally the tany **must** have been differentiated to $sec^2 y$. There is no need to assign to $\frac{dx}{dy}$

A1 The correct answer for $\frac{dx}{dy} = \frac{\left(1 - tany\, tan\frac{\pi}{12}\right) \times 2sec^2 y - \left(2tany + 2\tan\left(\frac{\pi}{12}\right)\right) \times -sec^2 y tan\frac{\pi}{12}}{(1 - tany\, tan\frac{\pi}{12})^2}$

The rest of the marks are as the main scheme

Question No	Scheme	Marks
5.	Uses the identity $cot^2(3\theta) = cosec^2(3\theta) - 1$ *in* $2cot^2(3\theta) = 7cosec(3\theta) - 5$	**M1**

Leave blank

5. Solve, for $0 \leqslant \theta \leqslant 180°$,

$$2\cot^2 3\theta = 7\operatorname{cosec} 3\theta - 5$$

Give your answers in degrees to 1 decimal place.

(10)

Alternatives using arctan (first 3 marks)

M1 Differentiates $y = \arctan\left(\frac{x}{2}\right) - \frac{\pi}{12}$ to get $\frac{1}{1+(\frac{x}{2})^2} \times constant$. Don't worry about the lhs

A1 Achieves $\frac{dy}{dx} = \frac{1}{1+(\frac{x}{2})^2} \times \frac{1}{2}$

M1 This method mark requires *x* to be found, which then needs to be substituted into $\frac{dy}{dx}$

The rest of the marks are then the same.

Or implicitly (first 2 marks)

M1 Differentiates implicitly to get $1 = 2\, sec^2\left(y + \frac{\pi}{12}\right) \times \frac{dy}{dx}$

A1 Rearranges to get $\frac{dy}{dx}$ or $\frac{dx}{dy}$ in terms of y

The rest of the marks are the same

Or by compound angle identities

$x = 2\tan\left(y + \frac{\pi}{12}\right) = \frac{2tany + 2\tan(\frac{\pi}{12})}{1 - tany\ tan\frac{\pi}{12}}$ oe

M1 Differentiates using quotient rule-see question 1 in applying this. Additionally the tany **must** have been differentiated to $sec^2 y$. There is no need to assign to $\frac{dx}{dy}$

A1 The correct answer for $\frac{dx}{dy} = \frac{\left(1 - tany\ tan\frac{\pi}{12}\right) \times 2sec^2 y - \left(2tany + 2\tan\left(\frac{\pi}{12}\right)\right) \times -sec^2 y tan\frac{\pi}{12}}{(1 - tany\ tan\frac{\pi}{12})^2}$

The rest of the marks are as the main scheme

Question No	Scheme	Marks
5.	Uses the identity $cot^2(3\theta) = cosec^2(3\theta) - 1$ *in* $2cot^2(3\theta) = 7cosec(3\theta) - 5$	**M1**

	$2cosec^2(3\theta) - 7cosec(3\theta) + 3 = 0$	**A1**
	$(2cosec3\theta - 1)(cosec3\theta - 3) = 0$	**dM1**
	$cosec3\theta = 3$	**A1**
	$\theta = \dfrac{invsin(\frac{1}{3})}{3}, \ \dfrac{19.5°}{3} = awrt\ 6.5°$	**ddM1, A1**
	$\theta = \dfrac{180° - invsin(\frac{1}{3})}{3}, 53.5°$ Correct 2nd value	**ddM1,A1**
	$\theta = \dfrac{360° + invsin(\frac{1}{3})}{3}$ Correct 3rd value	**ddM1**
	All 4 correct answers awrt $6.5^0, 53.5^0, 126.5\ ^0$or 173.5^0	**A1** **(10 marks)**

M1 Uses the substitution $cot^2(3\theta) = \pm 1 \pm cosec^2(3\theta)$ to produce a quadratic equation in *cosec*(3θ)
Accept 'invisible' brackets in which $2cot^2(3\theta)$ is replaced by $2cosec^2(3\theta) - 1$
A (longer) but acceptable alternative is to convert everything to $sin(3\theta)$.
For this to be scored $cot^2 3\theta$ must be replaced by $\frac{cos^2(3\theta)}{sin^2(3\theta)}$, $cosec(3\theta)$ must be replaced by $\frac{1}{\sin 3\theta}$.
An attempt must be made to multiply by $sin^2(3\theta)$ and finally $cos^2(3\theta)$replaced by $= \pm 1 \pm sin^2(3\theta)$

A1 A correct equation (=0) written or implied by working is obtained. Terms must be collected together on one side of the equation. The usual alternatives are
$2cosec^2(3\theta) - 7cosec(3\theta) + 3 = 0$ or $3sin^2(3\theta) - 7sin(3\theta) + 2 = 0$

dM1 Either an attempt to factorise a 3 term quadratic **in *cosec*(3θ) or *sin*(3θ)** with the usual rules
Or use of a correct formula to produce a solution in $cosec(3\theta)$ or $sin(3\theta)$

A1 Obtaining the correct value of $cosec(3\theta) = 3$ or $\sin(3\theta) = \frac{1}{3}$. Ignore other values

ddM1 Correct method to produce the principal value of θ. It is dependent upon the two M's being scored.
Look for $\theta = \dfrac{invsin(their\ \frac{1}{3})}{3}$

A1 Awrt 6.5

ddM1 Correct method to produce a secondary value. This is dependent upon the candidate having scored the first 2 M's. Usually you look for $\frac{180 - \text{their } 19.5}{3}$ or $\frac{360 + \text{their } 19.5}{3}$ or $\frac{540 - \text{their } 19.5}{3}$
Note 180-their 6.5 must be marked correct BUT 360+their 6.5 is incorrect

A1 Any other correct answer. Awrt 6.5,53.5,126.5 or 173.5

ddM1 Correct method to produce a THIRD value. This is dependent upon the candidate having scored the first 2 M's . See above for alternatives

A1 All 4 correct answers awrt 6.5,53.5,126.5 or 173.5 and no extras inside the range. Ignore any answers outside the range.

Radian answers: awrt 0.11, 0.93, 2.21, 3.03. Accuracy must be to 2dp.
Lose the first mark that could have been scored. Fully correct radian answer scores 1,1,1,1,1,0,1,1,1,1=9 marks
Candidates cannot mix degrees and radians for method marks.

Special case: Some candidates solve the equation in $cosec(\theta\ or\ x), \sin(\theta\ or\ x)$ to produce $cosec(\theta\ or\ x) = 3$

$$\mathbf{\sin(\theta\ or\ x\ ) = \frac{1}{3}.}$$

Leave blank

6.

$$f(x) = x^2 - 3x + 2\cos\left(\tfrac{1}{2}x\right), \quad 0 \leqslant x \leqslant \pi$$

(a) Show that the equation $f(x) = 0$ has a solution in the interval $0.8 < x < 0.9$

(2)

The curve with equation $y = f(x)$ has a minimum point P.

(b) Show that the x-coordinate of P is the solution of the equation

$$x = \frac{3 + \sin\left(\frac{1}{2}x\right)}{2}$$

(4)

(c) Using the iteration formula

$$x_{n+1} = \frac{3 + \sin\left(\frac{1}{2}x_n\right)}{2}, \quad x_0 = 2$$

find the values of x_1, x_2 and x_3, giving your answers to 3 decimal places.

(3)

(d) By choosing a suitable interval, show that the x-coordinate of P is 1.9078 correct to 4 decimal places.

(3)

Question No	Scheme	Marks
6	(a) f(0.8) = 0.082, f(0.9)= -0.089	M1
	Change of sign $\Rightarrow$ root (0.8,0.9)	A1
		(2)
	(b)	
	$f'(x) = 2x - 3 - \sin\left(\frac{1}{2}x\right)$	M1 A1
	Sets $f'(x) = 0 \Rightarrow \quad x = \frac{3+\sin\left(\frac{1}{2}x\right)}{2}$	M1A1*
		(4)
	(c) Sub x_0=2 into $x_{n+1} = \frac{3+\sin\left(\frac{1}{2}x_n\right)}{2}$	M1
	x_1=awrt 1.921, x_2=awrt 1.91(0) and x_3=awrt 1.908	A1,A1
		(3)
	(d) [1.90775,1.90785]	M1
	f'(1.90775)=-0.00016.. AND f'(1.90785)= 0.0000076..	M1
	Change of sign $\Rightarrow$ x=1.9078	A1
		(3)
		(12 marks)

(a)

M1 Calculates both f(0.8) and f(0.9). Evidence of this mark could be, either, seeing both '*x*' substitutions written out in the expression, or, one value correct to 1 sig fig, or the appearance of incorrect values of f(0.8)=awrt 0.2 or f(0.9)=awrt 0.1 from use of degrees

A1 This requires both values to be correct as well as a reason and a conclusion.
Accept f(0.8)= 0.08 truncated or rounded (2dp) or 0.1 rounded (1dp) and f(0.9)=-0.08 truncated or rounded as -0.09 (2dp) or -0.1(1dp)
Acceptable reasons are change of sign, <0 >0, +ve –ve, f(0.8)f(0.9)<0. Acceptable conclusion is hence root or □

(b)

M1 Attempts to differentiate f(x). Seeing any of 2x,3 or ±Asin(½x) is sufficient evidence.

A1 f'(x) correct. Accept $\frac{dy}{dx} = 2x - 3 - \sin\left(\frac{1}{2}x\right)$

M1 Sets their f'(x)=0 and proceeds to x=….. You must be sure that they are setting what they think is f'(x)=0.
Accept $2x = 3 + \sin\left(\frac{1}{2}x\right)$ going to x=..only if f'(x) =0 is stated first

A1 * $x = \frac{3+\sin\left(\frac{1}{2}x\right)}{2}$. This is a given answer so don't accept just the sight of this answer. It is cso

(c) M1 Substitutes x_0=2 into $x_{n+1} = \frac{3+\sin\left(\frac{1}{2}x_n\right)}{2}$. Evidence of this mark could be awrt 1.9 or 1.5 (from degrees)

A1 x_1=awrt 1.921

A1 x_2=awrt 1.91(0) and x_3=awrt 1.908

(d) Continued iteration is not acceptable for this part. Question states 'By choosing a suitable interval…'

M1 Chooses the interval [1.90775,1.90785] or tighter containing the root= 1.907845522

M1 Calculates f'(1.90775) and f'(1.90785) or tighter with at least one correct, rounded or truncated
f'(1.90775)=-0.0001 truncated or awrt -0.0002 rounded
f'(1.90785)= 0.000007 truncated or awrt 0.000008 rounded

Accept versions of g(*x*)-*x* where g(*x*) $=\frac{3+\sin\left(\frac{1}{2}x\right)}{2}$.
When x= 1.90775, $g(x) - x = 8 \times 10^{-5}$ rounded and truncated
When x= 1.90785, $g(x) - x = -3 \times 10^{-6}$ truncated or $= -4 \times 10^{-6}$ rounded

A1 Both values correct, rounded or truncated, a valid reason (see part a) and a minimal conclusion (see part a). Saying hence root is acceptable. There is no need to refer to the 'turning point'.

Leave blank

7. The function f is defined by

$$f : x \mapsto \frac{3(x+1)}{2x^2+7x-4} - \frac{1}{x+4}, \qquad x \in \mathbb{R}, x > \frac{1}{2}$$

(a) Show that $f(x) = \frac{1}{2x-1}$ **(4)**

(b) Find $f^{-1}(x)$ **(3)**

(c) Find the domain of f^{-1} **(1)**

$$g(x) = \ln(x+1)$$

(d) Find the solution of $fg(x) = \frac{1}{7}$, giving your answer in terms of e. **(4)**

Question No	Scheme	Marks
7	(a) $2x^2+7x-4=(2x-1)(x+4)$	B1
	$\dfrac{3(x+1)}{(2x-1)(x+4)}-\dfrac{1}{(x+4)}=\dfrac{3(x+1)-(2x-1)}{(2x-1)(x+4)}$	M1
	$=\frac{x+4}{(2x-1)(x+4)}$	M1
	$=\frac{1}{2x-1}$	A1*
		(4)
	(b) $y=\frac{1}{2x-1}\Rightarrow y(2x-1)=1\Rightarrow 2xy-y=1$	
	$2xy=1+y\Rightarrow x=\dfrac{1+y}{2y}$	M1M1
	$y\ OR\ f^{-1}(x)=\dfrac{1+x}{2x}$	A1
		(3)
	(c) x>0	B1
		(1)
	(d) $\frac{1}{2\ln(x+1)-1}=\frac{1}{7}$	M1
	$\ln(x+1)=4$	A1
	$x=e^4-1$	M1A1
		(4)
		12 Marks

(a)

B1 Factorises the expression $2x^2+7x-4=(2x-1)(x+4)$. This may not be on line 1

M1 Combines the two fractions to form a single fraction with a common denominator. Cubic denominators are fine for this mark. Allow slips on the numerator but one must have been adapted. Allow 'invisible' brackets. Accept two separate fractions with the same denominator. Amongst many possible options are

Correct $\frac{3(x+1)-(2x-1)}{(2x-1)(x+4)}$,Invisible bracket $\frac{3x+1-2x-1}{(2x-1)(x+4)}$,

Cubic and separate $\frac{3(x+1)(x+4)}{(2x^2+7x-4)(x+4)} - \frac{2x^2+7x-4}{(2x^2+7x-4)(x+4)}$

M1 Simplifies the (now) single fraction to one with a linear numerator divided by a quadratic factorised denominator. Any cubic denominator must have been fully factorised (check first and last terms) and cancelled with terms on a fully factorised numerator (check first and last terms).

A1* Cso. This is a given solution and it must be fully correct. All bracketing/algebra must have been correct. You can however accept $\frac{x+4}{(2x-1)(x+4)}$ going to $\frac{1}{2x-1}$ without the need for 'seeing' the cancelling

For example $\frac{3(x+1)-2x-1}{(2x-1)(x+4)} = \frac{x+4}{(2x-1)(x+4)} = \frac{1}{2x-1}$ scores B1,M1,M1,A0. Incorrect line leading to solution.

Whereas $\frac{3(x+1)-(2x-1)}{(2x-1)(x+4)} = \frac{x+4}{(2x-1)(x+4)} = \frac{1}{2x-1}$ scores B1,M1,M1,A1

(b)

M1 This is awarded for an attempt to make x or a swapped y the subject of the formula. The minimum criteria is that they start by multiplying by (2*x*-1) and finish with x= or swapped y=. Allow 'invisible' brackets.

M1 For applying the order of operations correctly. Allow maximum of one 'slip'. Examples of this are

$y = \frac{1}{2x-1} \to y(2x-1) = 1 \to 2x - 1 = \frac{1}{y} \to x = \frac{\frac{1}{y} \pm 1}{2}$ (allow slip on sign)

$y = \frac{1}{2x-1} \to y(2x-1) = 1 \to 2xy - y = 1 \to 2xy = 1 \pm y \to x = \frac{1 \pm y}{2y}$ (allow slip on sign)

$y = \frac{1}{2x-1} \to 2x - 1 = \frac{1}{y} \to 2x = \frac{1}{y} + 1 \to x = \frac{1}{2y} + 1$ (allow slip on ÷2)

A1 Must be written in terms of x but can be $y = \frac{1+x}{2x}$ or equivalent inc $y = \frac{\frac{1}{x}+1}{2}, y = \frac{x^{-1}+1}{2}, y = \frac{1}{2x} + \frac{1}{2}$

(c)

B1 Accept x>0, (0,∞), domain is all values more than 0. **Do not accept** x≥ 0 , y>0, [0, ∞], $f^{-1}(x) > 0$

(d)

M1 Attempt to write down fg(x) and set it equal to 1/7.
The order must be correct but accept incorrect or lack of bracketing. Eg $\frac{1}{2lnx+1-1} = \frac{1}{7}$

A1 Achieving correctly the line $\ln(x+1) = 4$. Accept also $ln(x+1)^2 = 8$

M1 Moving from $\ln(x \pm A) = c \quad A \neq 0 \; to \; x =$ The ln work must be correct
Alternatively moving from $ln(x+1)^2 = c$ to $x = \cdots$
Full solutions to calculate *x* leading from $gf(x) = \frac{1}{7}$, that is $\ln\left(\frac{1}{2x-1} + 1\right) = \frac{1}{7}$ can score this mark.

A1 Correct answer only $= e^4 - 1$. Accept $e^4 - e^0$

Question No	Scheme	Marks
8	(a) $\tan(A+B) = \frac{\sin(A+B)}{\cos(A+B)} = \frac{sinAcosB+cosAsinB}{cosAcosB-sinAsinB}$	M1A1
	$= \frac{\frac{sinA}{cosA}+\frac{sinB}{cosB}}{1-\frac{sinAsinB}{cosAcosB}}$ $(\div cosAcosB)$	M1

Leave blank

8. (a) Starting from the formulae for $\sin(A+B)$ and $\cos(A+B)$, prove that

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

(4)

(b) Deduce that

$$\tan\left(\theta + \frac{\pi}{6}\right) = \frac{1 + \sqrt{3}\tan\theta}{\sqrt{3} - \tan\theta}$$

(3)

(c) Hence, or otherwise, solve, for $0 \leqslant \theta \leqslant \pi$,

$$1 + \sqrt{3}\tan\theta = (\sqrt{3} - \tan\theta)\tan(\pi - \theta)$$

Give your answers as multiples of π.

(6)

M1 Combines the two fractions to form a single fraction with a common denominator. Cubic denominators are fine for this mark. Allow slips on the numerator but one must have been adapted. Allow 'invisible' brackets. Accept two separate fractions with the same denominator. Amongst many possible options are

Correct $\frac{3(x+1)-(2x-1)}{(2x-1)(x+4)}$,Invisible bracket $\frac{3x+1-2x-1}{(2x-1)(x+4)}$,

Cubic and separate $\frac{3(x+1)(x+4)}{(2x^2+7x-4)(x+4)} - \frac{2x^2+7x-4}{(2x^2+7x-4)(x+4)}$

M1 Simplifies the (now) single fraction to one with a linear numerator divided by a quadratic factorised denominator. Any cubic denominator must have been fully factorised (check first and last terms) and cancelled with terms on a fully factorised numerator (check first and last terms).

A1* Cso. This is a given solution and it must be fully correct. All bracketing/algebra must have been correct. You can however accept $\frac{x+4}{(2x-1)(x+4)}$ going to $\frac{1}{2x-1}$ without the need for 'seeing' the cancelling

For example $\frac{3(x+1)-2x-1}{(2x-1)(x+4)} = \frac{x+4}{(2x-1)(x+4)} = \frac{1}{2x-1}$ scores B1,M1,M1,A0. Incorrect line leading to solution.

Whereas $\frac{3(x+1)-(2x-1)}{(2x-1)(x+4)} = \frac{x+4}{(2x-1)(x+4)} = \frac{1}{2x-1}$ scores B1,M1,M1,A1

(b)

M1 This is awarded for an attempt to make x or a swapped y the subject of the formula. The minimum criteria is that they start by multiplying by (2*x*-1) and finish with x= or swapped y=. Allow 'invisible' brackets.

M1 For applying the order of operations correctly. Allow maximum of one 'slip'. Examples of this are

$y = \frac{1}{2x-1} \to y(2x-1) = 1 \to 2x - 1 = \frac{1}{y} \to x = \frac{\frac{1}{y} \pm 1}{2}$ (allow slip on sign)

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$y = \frac{1}{2x-1} \to 2x - 1 = \frac{1}{y} \to 2x = \frac{1}{y} + 1 \to x = \frac{1}{2y} + 1$ (allow slip on ÷2)

A1 Must be written in terms of x but can be $y = \frac{1+x}{2x}$ or equivalent inc $y = \frac{\frac{1}{x}+1}{2}, y = \frac{x^{-1}+1}{2}, y = \frac{1}{2x} + \frac{1}{2}$

(c)

B1 Accept x>0, (0,∞), domain is all values more than 0. **Do not accept** x≥ 0 , y>0, [0, ∞], $f^{-1}(x) > 0$

(d)

M1 Attempt to write down fg(x) and set it equal to 1/7.
The order must be correct but accept incorrect or lack of bracketing. Eg $\frac{1}{2lnx+1-1} = \frac{1}{7}$

A1 Achieving correctly the line $\ln(x+1) = 4$. Accept also $ln(x+1)^2 = 8$

M1 Moving from $\ln(x \pm A) = c \quad A \neq 0 \; to \; x =$ The ln work must be correct
Alternatively moving from $ln(x+1)^2 = c$ to $x = \cdots$
Full solutions to calculate *x* leading from $gf(x) = \frac{1}{7}$, that is $\ln\left(\frac{1}{2x-1} + 1\right) = \frac{1}{7}$ can score this mark.

A1 Correct answer only $= e^4 - 1$. Accept $e^4 - e^0$

Question No	Scheme	Marks
8	(a) $\tan(A+B) = \frac{\sin(A+B)}{\cos(A+B)} = \frac{sinAcosB + cosAsinB}{cosAcosB - sinAsinB}$	M1A1
	$= \frac{\frac{sinA}{cosA} + \frac{sinB}{cosB}}{1 - \frac{sinAsinB}{cosAcosB}}$ $(\div cosAcosB)$	M1

	$= \dfrac{tanA + tanB}{1 - tanAtanB}$	A1 *
		(4)
	(b) $\tan\left(\theta + \frac{\pi}{6}\right) = \frac{tan\theta + tan\frac{\pi}{6}}{1 - tan\theta tan\frac{\pi}{6}}$	M1
	$= \frac{tan\theta + \frac{1}{\sqrt{3}}}{1 - tan\theta\frac{1}{\sqrt{3}}}$	M1
	$= \frac{\sqrt{3}tan\theta + 1}{\sqrt{3} - tan\theta}$	A1 *
		(3)
	(c) $\tan\left(\theta + \frac{\pi}{6}\right) = \tan(\pi - \theta)$.	M1
	$\left(\theta + \frac{\pi}{6}\right) = (\pi - \theta)$	dM1
	$\theta = \frac{5}{12}\pi$	ddM1 A1
	$\tan\left(\theta + \frac{\pi}{6}\right) = \tan(2\pi - \theta)$	dddM1
	$\theta = \frac{11}{12}\pi$	A1
		(6)
		(13 MARKS)

(a)

M1 Uses the identity $\{\tan(A + B) = \frac{\sin(A+B)}{\cos(A+B)}\} = \frac{sinAcosB \pm cosAsinB}{cosAcosB \mp sinAsinB}$. Accept incorrect signs for this. Just the right hand side is acceptable.

A1 Fully correct statement in terms of cos and sin $\{\tan(A + B)\} = \frac{sinAcosB + cosAsinB}{cosAcosB - sinAsinB}$

M1 Divide **both** numerator and denominator by cosAcosB. This can be stated or implied by working. If implied you must have seen at least one term modified on both the numerator and denominator.

A1* This is a given solution. The last two principal's reports have highlighted lack of evidence in such questions. Both sides of the identity must be seen or implied. Eg lhs=
The minimum expectation for full marks is

$$\tan(A+B)=\frac{\sin(A+B)}{\cos(A+B)}=\frac{sinAcosB+cosAsinB}{cosAcosB-sinAsinB}=\frac{\frac{sinA}{cosA}+\frac{sinB}{cosB}}{1-\frac{sinAsinB}{cosAcosB}}=\frac{tanA+tanB}{1-tanAtanB}$$

The solution $\tan(A+B)=\frac{\sin(A+B)}{\cos(A+B)}=\frac{sinAcosB+cosAsinB}{cosAcosB-sinAsinB}=\frac{tanA+tanB}{1-tanAtanB}$ scores M1A1M0A0

The solution $\tan(A+B)=\frac{\sin(A+B)}{\cos(A+B)}=\frac{sinAcosB+cosAsinB}{cosAcosB-sinAsinB}\ (\div\ cosAcosB)=\frac{tanA+tanB}{1-tanAtanB}$ scores M1A1M1A0

(b)

M1 An attempt to use part (a) with A=θ and B=$\frac{\pi}{6}$. Seeing $\frac{tan\theta+tan\frac{\pi}{6}}{1-tan\theta tan\frac{\pi}{6}}$ is enough evidence. Accept sign slips

M1 Uses the identity $\tan\left(\frac{\pi}{6}\right)=\frac{1}{\sqrt{3}}\ or\ \frac{\sqrt{3}}{3}$ in the rhs of the identity on both numerator and denominator

A1* cso. This is a given solution. Both sides of the identity must be seen. All steps must be correct with no unreasonable jumps. Accept

$$\tan\left(\theta+\frac{\pi}{6}\right)=\frac{tan\theta+tan\frac{\pi}{6}}{1-tan\theta tan\frac{\pi}{6}}=\frac{tan\theta+\frac{1}{\sqrt{3}}}{1-tan\theta\frac{1}{\sqrt{3}}}=\frac{\sqrt{3}tan\theta+1}{\sqrt{3}-tan\theta}$$

However the following is only worth 2 out of 3 as the last step is an unreasonable jump without further explanation
.

$$\tan\left(\theta+\frac{\pi}{6}\right)=\frac{tan\theta+tan\frac{\pi}{6}}{1-tan\theta tan\frac{\pi}{6}}=\frac{tan\theta+\frac{\sqrt{3}}{3}}{1-tan\theta\frac{\sqrt{3}}{3}}=\frac{\sqrt{3}tan\theta+1}{\sqrt{3}-tan\theta}$$

(c)

M1 Use the given identity in (b) to obtain $\tan\left(\theta+\frac{\pi}{6}\right)=\tan(\pi-\theta)$. Accept sign slips

dM1 Writes down an equation that will give one value of θ , usually $\theta+\frac{\pi}{6}=\pi-\theta$. This is dependent upon the first M mark. Follow through on slips

ddM1 Attempts to solve their equation in θ. It must end θ= and the first two marks must have been scored.

A1 Cso θ=$\frac{5}{12}\pi$ or $\frac{11}{12}\pi$

dddM1 Writes down an equation that would produce a second value of θ . Usually $\theta+\frac{\pi}{6}=2\pi-\theta$

A1 cso θ= $\frac{5}{12}\pi\ (accept\ \frac{\pi}{2.4})$and $\frac{11}{12}\pi$ with no extra solutions in the range. Ignore extra solutions outside the range.

Note that under this method one correct solution would score 4 marks. A small number of candidates find the second solution only. They would score 1,1,1,1,0,0

Alternative to (a) starting from rhs

M1 Uses correct identities for both *tan*A and *tan*B in the rhs expression. Accept only errors in signs

A1 $\frac{tanA+tanB}{1-tanAtanB}=\frac{\frac{sinA}{cosA}+\frac{sinB}{cosB}}{1-\frac{sinAsinB}{cosAcosB}}$

M1 Multiplies both numerator and denominator by *cos*A*cos*B. This can be stated or implied by working. If implied you must have seen at least one term modified on both the numerator and denominator

A1 This is a given answer. Correctly completes proof. All three expressions must be seen or implied.

$$\frac{sinAcosB+cosAsinB}{cosAcosB-sinAsinB}=\frac{\sin(A+B)}{\cos(A+B)}=\tan(A+B)$$

Alternative to (a) starting from both sides

The usual method can be marked like this

M1 Uses correct identities for both *tan*A and *tan*B in the rhs expression. Accept only errors in signs

A1 $\frac{tanA+tanB}{1-tanAtanB}=\frac{\frac{sinA}{cosA}+\frac{sinB}{cosB}}{1-\frac{sinAsinB}{cosAcosB}}$

M1 Multiplies both numerator and denominator by *cos*A*cos*B. This can be stated or implied by working. If implied you must have seen at least one term modified on both the numerator and denominator

A1 Completes proof. Starting now from the lhs writes $\tan(A+B)=\frac{\sin(A+B)}{\cos(A+B)}=\frac{sinAcosB+cosAsinB}{cosAcosB-sinAsinB}$
And then states that the lhs is equal to the rhs **Or** hence proven. There must be a statement of closure

Alternative to (b) from sin and cos

M1 Writes $\tan\left(\theta+\frac{\pi}{6}\right)=\frac{\sin(\theta+\frac{\pi}{6})}{\cos(\theta+\frac{\pi}{6})}=\frac{sin\theta cos\frac{\pi}{6}+cos\theta sin\frac{\pi}{6}}{cos\theta cos\frac{\pi}{6}-sin\theta sin\frac{\pi}{6}}$

M1 Uses the identities $\sin\left(\frac{\pi}{6}\right)=\frac{1}{2}$ and $cos\left(\frac{\pi}{6}\right)=\frac{\sqrt{3}}{2}$ oe in the rhs of the identity on both numerator and denominator and divides both numerator and denominator by $\cos\theta$ to produce an identity in $\tan\theta$

A1 As in original scheme

<u>**Alternative solution for c.**</u> Starting with $1+\sqrt{3}tan\theta=(\sqrt{3}-\tan\theta)\,tan(\pi-\theta)$

Let $\tan\theta=t$

$$1+\sqrt{3}t=\left(\sqrt{3}-t\right)\left(-t\right)$$

$$t^2-2\sqrt{3}t-1=0$$

$$t=\frac{2\sqrt{3}\pm\sqrt{(12+4)}}{2}$$

$$=\sqrt{3}\pm2$$ Must find an exact surd

$$\theta = \frac{5\pi}{12}, \frac{11\pi}{12}$$

Accept the use of a calculator for the A marks as long as there is an exact surd for the solution of the quadratic and exact answers are given.

M1 Starting with $1+\sqrt{3}tan\theta = (\sqrt{3} - \tan\theta)\, tan(\pi - \theta)$ expand $\tan(\pi - \theta)$ by the correct compound angle identity (or otherwise) and substitute tan π=0 to produce an equation in tanθ

dM1 Collect terms and produce a 3 term quadratic in tanθ

ddM1 Correct use of quadratic formula to produce exact solutions to tanθ. All previous marks must have been scored.

dddM1 All 3 previous marks must have been scored. This is for producing two exact values for θ

A1 One solution $\frac{5}{12}\pi\ (accept\ \frac{\pi}{2.4})$ or $\frac{11}{12}\pi$

A1 Both solutions $\frac{5}{12}\pi\ (accept\ \frac{\pi}{2.4})$ and $\frac{11}{12}\pi$ and no extra solutions inside the range. Ignore extra solutions outside the range.

Special case: Watch for candidates who write $tan(\pi - \theta) = tan(\pi) - tan(\theta) = -tan(\theta)$ and proceed correctly. They will lose the first mark but potentially can score the others.

Solutions in degrees

Apply as before. Lose the first correct mark that would have been scored-usually 75^0