Centre No.						Paper Reference	Surname	Initial(s)
Candidate No.						6 6 6 5 / 0 1	Signature	

Paper Reference(s)

6665/01

Edexcel GCE

Core Mathematics C3

Advanced

Friday 25 January 2013 – Afternoon

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
1	
2	
3	
4	
5	
6	
7	
8	
Total	

Materials required for examination
Mathematical Formulae (Pink)

Items included with question papers
Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer for each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.
There are 28 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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1. The curve C has equation

$$y = (2x - 3)^5$$

The point P lies on C and has coordinates $(w, -32)$.

Find

(a) the value of w,

(2)

(b) the equation of the tangent to C at the point P in the form $y = mx + c$, where m and c are constants.

(5)

January 2013
6665 Core Mathematics C3
Mark Scheme

Question Number	Scheme	Marks
1.	(a) $-32=(2w-3)^5 \Rightarrow w=\frac{1}{2}$ oe	M1A1
		(2)
	(b) $\frac{dy}{dx}=5\times(2x-3)^4\times 2$ or $10(2x-3)^4$	M1A1
	When $x=\frac{1}{2}$, Gradient = 160	M1
	Equation of tangent is $'160'=\frac{y-(-32)}{x-'\frac{1}{2}'}$ oe	dM1
	$y=160x-112$ cso	A1
		(5)
		(7 marks)

(a) M1 Substitute ***y***=-32 into $y=(2w-3)^5$ **and** proceed to *w*=….. [Accept positive sign used of *y*, ie *y*=+32]

A1 Obtains w or $x=\frac{1}{2}$ oe with no incorrect working seen. Accept alternatives such as 0.5.

Sight of just the answer would score both marks as long as no incorrect working is seen.

(b) M1 Attempts to differentiate $y=(2x-3)^5$ using the chain rule.

Sight of $\pm A(2x-3)^4$ where *A* is a non- zero constant is sufficient for the method mark.

A1 A correct (un simplified) form of the differential.

Accept $\frac{dy}{dx}=5\times(2x-3)^4\times 2$ or $\frac{dy}{dx}=10(2x-3)^4$

M1 This is awarded for an attempt to find the gradient of the tangent to the curve at *P*
Award for substituting their numerical value to part (a) into their differential to find the numerical gradient of the tangent

dM1 Award for a correct method to find an equation of the tangent to the curve at *P*. It is dependent upon the previous M mark being awarded.

Award for $'their\ 160'=\frac{y-(-32)}{x-their\ '\frac{1}{2}'}$

If they use $y=mx+c$ it must be a full method, using m= 'their 160', their '$\frac{1}{2}$' and -32.

An attempt must be seen to find *c*=…

A1 cso $y = 160x - 112$. The question is specific and requires the answer in this form.
You may isw in this question after a correct answer.

Leave blank

2. $$g(x) = e^{x-1} + x - 6$$

(a) Show that the equation $g(x) = 0$ can be written as

$$x = \ln(6 - x) + 1, \qquad x < 6$$

(2)

The root of $g(x) = 0$ is α.

The iterative formula

$$x_{n+1} = \ln(6 - x_n) + 1, \qquad x_0 = 2$$

is used to find an approximate value for α.

(b) Calculate the values of x_1, x_2 and x_3 to 4 decimal places.

(3)

(c) By choosing a suitable interval, show that $\alpha = 2.307$ correct to 3 decimal places.

(3)

Question Number	Scheme	Marks
2.	(a) $0 = e^{x-1} + x - 6 \Rightarrow x = \ln(6-x)+1$	M1A1*
		(2)
	(b) Sub $x_0 = 2$ into $x_{n+1} = \ln(6-x_n)+1 \Rightarrow x_1 = 2.3863$	M1, A1
	AWRT 4 dp. $x_2 = 2.2847$ $x_3 = 2.3125$	A1
		(3)
	(c) Chooses interval [2.3065,2.3075]	M1
	$g(2.3065)$=-0.0002(7), $g(2.3075)$=0.004(4)	dM1
	Sign change , hence root (correct to 3dp)	A1
		(3)
		(8 marks)

(a) M1 Sets g(x)=0,and using correct *ln* work, makes the x of the e^{x-1} term the subject of the formula.
Look for $e^{x-1} + x - 6 = 0 \Rightarrow e^{x-1} = \pm 6 \pm x \Rightarrow x = \ln(\pm 6 \pm x) \pm 1$
Do not accept $e^{x-1} = 6 - x$ **without firstly seeing** $e^{x-1} + x - 6 = 0$ **or a statement that g(x)=0 ⇒**

A1* cso. $x = \ln(6-x)+1$ **Note that this is a given answer (and a proof).**
'Invisible' brackets are allowed for the M but not the A
Do not accept recovery from earlier errors for the A mark. The solution below scores 0 marks.
$0 = e^{x-1} + x - 6 \Rightarrow \quad 0 = x - 1 + \ln(x-6) \Rightarrow x = \ln(6-x)+1$

(b) M1 Sub $x_0 = 2$ into $x_{n+1} = \ln(6-x_n)+1$ to produce a numerical value for x_1.
Evidence for the award could be any of $\ln(6-2)+1$, $\ln 4 + 1$, 2.3….. or awrt 2.4

A1 Answer correct to 4 dp $x_1 = 2.3863$.
The subscript is not important. Mark as the first value given/found.

A1 Awrt 4 dp. $x_2 = 2.2847$ *and* $x_3 = 2.3125$
The subscripts are not important. Mark as the second and third values given/found

(c) M1 Chooses the interval [2.3065,2.3075] or smaller containing the root 2.306558641

dM1 Calculates $g(2.3065)$ and $g(2.3075)$ with at least one of these correct to 1sf.
The answers can be rounded or truncated
$g(2.3065)$ = -0.0003 rounded, $g(2.3065)$= -0.0002 truncated
$g(2.3075)$ = (+) 0.004 rounded and truncated

A1 Both values correct (rounded or truncated),
A reason which could include change of sign, >0 <0, $g(2.3065) \times g(2.3075) < 0$
AND a minimal conclusion such as hence root, α=2.307 or □
Do not accept continued iteration as question demands an interval to be chosen.

Alternative solution to (a) working backwards

M1 Proceeds from $x = \ln(6-x)+1$ using correct exp work to …….=0

A1 **Arrives correctly** at $e^{x-1} + x - 6 = 0$ **and** makes a statement to the effect that this is g(x)=0

Alternative solution to (c) using $\mathrm{f}(x) = \ln(6-x)+1-x$ **{Similarly** $\mathrm{h}(x) = x - 1 - \ln(6-x)$ **}**

M1 Chooses the interval [2.3065,2.3075] or smaller containing the root 2.306558641

dM1 Calculates f(2.3065) and f(2.3075) with at least 1 correct rounded or truncated
f(2.3065) = 0.000074. Accept 0.00007 rounded or truncated. Also accept 0.0001

f(2.3075) = - 0.0011.. Accept -0.001 rounded or truncated

Leave blank

3.

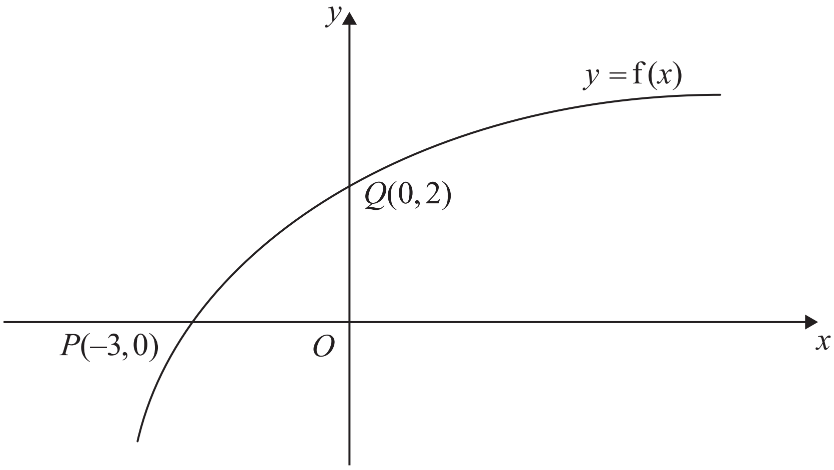


Figure 1

Figure 1 shows part of the curve with equation $y = \mathrm{f}(x)$, $x \in \mathbb{R}$.

The curve passes through the points $Q(0, 2)$ and $P(-3, 0)$ as shown.

(a) Find the value of $\mathrm{ff}(-3)$.

(2)

On separate diagrams, sketch the curve with equation

(b) $y = \mathrm{f}^{-1}(x)$,

(2)

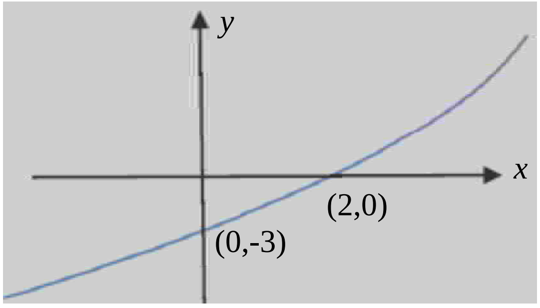
(c) $y = \mathrm{f}(|x|) - 2$,

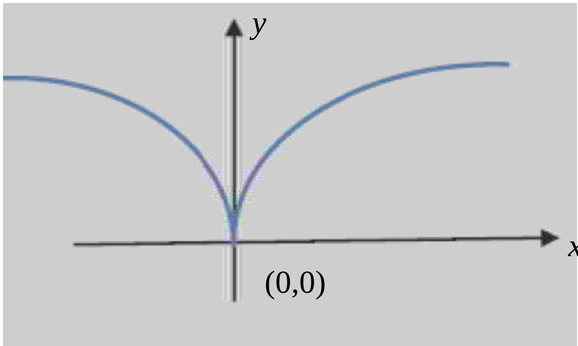
(2)

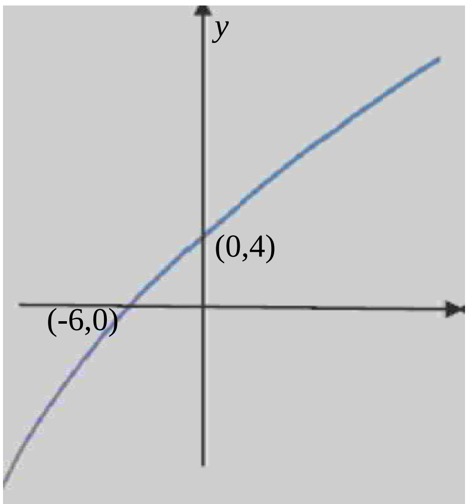
(d) $y = 2\mathrm{f}\left(\frac{1}{2}x\right)$.

(3)

Indicate clearly on each sketch the coordinates of the points at which the curve crosses or meets the axes.

<table>
<tr><th>Question Number</th><th>Scheme</th><th>Marks</th></tr>
<tr><td>3.</td><td>(a) ff(-3)= f(0),=2</td><td>M1,A1
(2)</td></tr>
<tr><td></td><td>(b) 

$y = \mathrm{f}^{-1}(x)$
Shape
(0,-3) and (2,0)</td><td>B1
B1
(2)</td></tr>
<tr><td></td><td>(c) 

y=f(|x|)-2
Shape
(0,0)</td><td>B1
B1
(2)</td></tr>
<tr><td></td><td>(d) 

x
Shape
(-6,0) or (0,4)
(-6,0) and (0,4)</td><td>B1
B1
B1
(3)</td></tr>
<tr><td></td><td></td><td>(9 marks)</td></tr>
</table>

(a) M1 A full method of finding ff(-3). f(0) is acceptable but f(-3)=0 is not.
Accept a solution obtained from two substitutions into the equation $y = \frac{2}{3}x + 2$ as the line passes through both points. Do not allow for $y = \ln(x+4)$, which only passes through one of the points.
A1 Cao ff(-3)=2. Writing down 2 on its own is enough for both marks provided no incorrect working is seen.

(b)
B1 For the correct shape. Award this mark for an increasing function in quadrants 3, 4 and 1 only. Do not award if the curve bends back on itself or has a clear minimum
B1 This is independent to the first mark and for the graph passing through (0,-3) and (2, 0)

Accept -3 and 2 marked on the correct axes.
Accept (-3,0) and (0,2) instead of (0,-3) and (2,0) as long as they are on the correct axes
Accept P'=(0,-3), Q'=(2,0) stated elsewhere as long as P'and Q' are marked in the correct place on the graph
There must be a graph for this to be awarded

(c)

B1 Award for a correct shape 'roughly' symmetrical about the *y*- axis. It must have a cusp and a gradient that 'decreases' either side of the cusp. Do not award if the graph has a clear maximum

B1 (0,0) lies on their graph. Accept the graph passing through the origin without seeing (0, 0) marked

(d) B1 Shape. The position is not important. The gradient should be always positive but decreasing There should not be a clear maximum point.

B1 The graph passes through (0,4) **or** (-6,0). See part (b) for allowed variations

B1 The graph passes through (0,4) **and** (-6,0). See part (b) for allowed variations

Leave blank

4. (a) Express $6\cos\theta + 8\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

Give the value of α to 3 decimal places.

(4)

(b)
$$\mathrm{p}(\theta) = \frac{4}{12 + 6\cos\theta + 8\sin\theta}, \qquad 0 \leqslant \theta \leqslant 2\pi$$

Calculate

(i) the maximum value of $\mathrm{p}(\theta)$,

(ii) the value of θ at which the maximum occurs.

(4)

Question Number	Scheme	Marks
4.	(a) $R^2 = 6^2 + 8^2 \Rightarrow R = 10$	M1A1
	$\tan\alpha = \frac{8}{6} \Rightarrow \alpha = \text{awrt } 0.927$	M1A1
		(4)
	(b)(i) $\text{p}(x) = \frac{4}{12 + 10\cos(\theta - 0.927)}$	
	$\text{p}(x) = \frac{4}{12 - 10}$	M1
	Maximum = 2	A1
		(2)
	(b)(ii) $\theta - \text{'}\mathit{their}\ \alpha\text{'} = \pi$	M1
	$\theta = \text{awrt } 4.07$	A1
		(2)
		(8 marks)

(a) M1 Using Pythagoras' Theorem with 6 and 8 to find *R*. Accept $R^2 = 6^2 + 8^2$

If α has been found first accept $R = \pm\frac{8}{\sin\text{'}\alpha\text{'}}$ or $R = \pm\frac{6}{\cos\text{'}\alpha\text{'}}$

A1 $R = 10$. Many candidates will just write this down which is fine for the 2 marks.
Accept ± 10 but not -10

M1 For $\tan\alpha = \pm\frac{8}{6}$ or $\tan\alpha = \pm\frac{6}{8}$

If *R* is used then only accept $\sin\alpha = \pm\frac{8}{R}$ or $\cos\alpha = \pm\frac{6}{R}$

A1 $\alpha = \text{awrt } 0.927$. Note that 53.1^0 is A0

(b) Note that (b)(i) and (b)(ii) can be marked together

(i) M1 Award for $\text{p}(x) = \frac{4}{12 - \text{'}R\text{'}}$.

A1 Cao $\text{p}(x)_{\max} = 2$.
The answer is acceptable for both marks as long as no incorrect working is seen

(ii) M1 For setting $\theta - \text{'}\mathit{their}\ \alpha\text{'} = \pi$ and proceeding to θ=..
If working exclusively in degrees accept $\theta - \text{'}\mathit{their}\ \alpha\text{'} = 180$
Do not accept mixed units

A1 $\theta = \text{awrt } 4.07$. If the final A mark in part (a) is lost for 53.1, then accept awrt 233.1

Leave blank

5. (i) Differentiate with respect to x

(a) $y = x^3 \ln 2x$

(b) $y = (x + \sin 2x)^3$

(6)

Given that $x = \cot y$,

(ii) show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{-1}{1+x^2}$

(5)

Question Number	Scheme	Marks
5.	(i)(a) $\frac{dy}{dx}=3x^2\times\ln 2x+x^3\times\frac{1}{2x}\times 2$	M1A1A1
	$=3x^2\ln 2x+x^2$	
		(3)
	(i)(b) $\frac{dy}{dx}=3(\boldsymbol{x+\sin 2x})^2\times(1+2\cos 2x)$	**B1**M1A1
		(3)
	(ii) $\frac{dx}{dy}=-\text{cosec}^2 y$	M1A1
	$\frac{dy}{dx}=-\frac{1}{\text{cosec}^2 y}$	M1
	Uses $\text{cosec}^2 y=1+\cot^2 y$ and $x=\cot y$ in $\frac{dy}{dx}$ or $\frac{dx}{dy}$ to get an expression in x	
	$\frac{dy}{dx}=-\frac{1}{\text{cosec}^2 y}=-\frac{1}{1+\cot^2 y}=-\frac{1}{1+x^2}$ cso	M1, A1*
		(5)
		(11 marks)

(i)(a) M1 Applies the product rule vu'+uv' to $x^3\ln 2x$.
If the rule is quoted it must be correct. There must have been some attempt to differentiate both terms. If the rule is not quoted (nor implied by their working, with terms written out u=…,u'=….,v=….,v'=….followed by their vu'+uv') then only accept answers of the form

$$Ax^2\times\ln 2x+x^3\times\frac{B}{x}\qquad \text{where } A, B \text{ are constants}\neq 0$$

A1 One term correct, either $3x^2\times\ln 2x$ or $x^3\times\frac{1}{2x}\times 2$

A1 Cao. $\frac{dy}{dx}=3x^2\times\ln 2x+x^3\times\frac{1}{2x}\times 2$. The answer does not need to be simplified.

For reference the simplified answer is $\frac{dy}{dx}=3x^2\ln 2x+x^2=x^2(3\ln 2x+1)$

(i)(b) B1 Sight of $(x+\sin 2x)^2$

M1 For applying the chain rule to $(x+\sin 2x)^3$. If the rule is quoted it must be correct. If it is not quoted possible forms of evidence could be sight of $C(x+\sin 2x)^2\times(1\pm D\cos 2x)$ where C and D are non- zero constants.
Alternatively accept $u=x+\sin 2x$, u'= followed by $Cu^2\times$ their u'
Do not accept $C(x+\sin 2x)^2\times 2\cos 2x$ unless you have evidence that this is their u'
Allow 'invisible' brackets for this mark, ie. $C(x+\sin 2x)^2\times 1\pm D\cos 2x$

A1 Cao $\frac{dy}{dx}=3(x+\sin 2x)^2\times(1+2\cos 2x)$. There is no requirement to simplify this.

You may ignore subsequent working (isw) after a correct answer in part (i)(a) and (b)

(ii) M1 Writing the derivative of cot***y*** as -cosec^2***y***. It must be in terms of *y*

A1 $\frac{dx}{dy} = -\text{cosec}^2 y$ or $1 = -\text{cosec}^2 y \frac{dy}{dx}$. Both lhs and rhs must be correct.

M1 Using $\frac{dy}{dx} = \frac{1}{dx/dy}$

M1 Using $\text{cosec}^2 y = 1 + \cot^2 y$ **and** $x = \cot y$ to get $\frac{dy}{dx}$ or $\frac{dx}{dy}$ just in terms of x.

A1 cso $\frac{dy}{dx} = -\frac{1}{1+x^2}$

Alternative to (a)(i) when ln(2x) is written lnx+ln2

M1 Writes $x^3 \ln 2x$ as $x^3 \ln 2 + x^3 \ln x$.
Achieves Ax^2 for differential of $x^3 \ln 2$ and applies the product rule vu'+uv' to $x^3 \ln x$.

A1 Either $3x^2 \times \ln 2 + 3x^2 \ln x$ or $x^3 \times \frac{1}{x}$

A1 A correct (un simplified) answer. Eg $3x^2 \times \ln 2 + 3x^2 \ln x + x^3 \times \frac{1}{x}$

Alternative to 5(ii) using quotient rule

M1 Writes $\cot y$ as $\frac{\cos y}{\sin y}$ and applies the quotient rule, a form of which appears in the formula book. If the rule is quoted it must be correct. There must have been some attempt to differentiate both terms. If the rule is not quoted (nor implied by their working, meaning terms are written out u=…,u'=….,v=….,v'=….followed by their $\frac{vu'-uv'}{v^2}$)
only accept answers of the form $\frac{\sin y \times \pm \sin y - \cos y \times \pm \cos y}{(\sin y)^2}$

A1 Correct un simplified answer with both lhs and rhs correct.

$$\frac{dx}{dy} = \frac{\sin y \times -\sin y - \cos y \times \cos y}{(\sin y)^2} = \{-1 - \cot^2 y\}$$

M1 Using $\frac{dy}{dx} = \frac{1}{dx/dy}$

M1 Using $\sin^2 y + \cos^2 y = 1$, $\frac{1}{\sin^2 y} = \text{cosec}^2 y$ and $\text{cosec}^2 y = 1 + \cot^2 y$ to get $\frac{dy}{dx}$ or $\frac{dx}{dy}$ in x

A1 cso $\frac{dy}{dx} = -\frac{1}{1+x^2}$

Alternative to 5(ii) using the chain rule, first two marks

M1 Writes $\cot y$ as $(\tan y)^{-1}$ and applies the chain rule (or quotient rule).
Accept answers of the form $-(\tan y)^{-2} \times \sec^2 y$

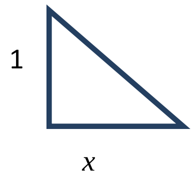
A1 Correct un simplified answer with both lhs and rhs correct.

$$\frac{dx}{dy} = -(\tan y)^{-2} \times \sec^2 y$$

Alternative to 5(ii) using a triangle – last M1

M1 Uses triangle with $\tan y = \frac{1}{x}$ to find siny and get $\frac{dy}{dx}$ or $\frac{dx}{dy}$ just in terms of x

$x = \cot y \Rightarrow \tan y = \frac{1}{x}$



$\sin y = \frac{1}{\sqrt{1+x^2}}$

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6. (i) Without using a calculator, find the exact value of

$$(\sin 22.5° + \cos 22.5°)^2$$

You must show each stage of your working.

(5)

(ii) (a) Show that $\cos 2\theta + \sin\theta = 1$ may be written in the form

$$k\sin^2\theta - \sin\theta = 0, \text{ stating the value of } k.$$

(2)

(b) Hence solve, for $0 \leqslant \theta < 360°$, the equation

$$\cos 2\theta + \sin\theta = 1$$

(4)

Question Number	Scheme	Marks
6.	(i) $(\sin 22.5+\cos 22.5)^2=\sin^2 22.5+\cos^2 22.5+......$	M1
	$=\sin^2 22.5+\cos^2 22.5+2\sin 22.5\cos 22.5$	
	States or uses $\sin^2 22.5+\cos^2 22.5=1$	B1
	Uses $2\sin x\cos x=\sin 2x \Rightarrow 2\sin 22.5\cos 22.5=\sin 45$	M1
	$(\sin 22.5+\cos 22.5)^2=1+\sin 45$	A1
	$=1+\frac{\sqrt{2}}{2}$ or $1+\frac{1}{\sqrt{2}}$ cso	A1
		(5)
	(ii) (a) $\cos 2\theta+\sin\theta=1 \Rightarrow 1-2\sin^2\theta+\sin\theta=1$	M1
	$\sin\theta-2\sin^2\theta=0$	
	$2\sin^2\theta-\sin\theta=0$ or $k=2$	A1*
		(2)
	(b) $\sin\theta(2\sin\theta-1)=0$	M1
	$\sin\theta=0, \quad \sin\theta=\frac{1}{2}$	A1
	Any two of 0,30,150,180	B1
	All four answers 0,30,150,180	A1
		(4)
		(11 marks)

(i) M1 Attempts to expand $(\sin 22.5+\cos 22.5)^2$. Award if you see $\sin^2 22.5+\cos^2 22.5+......$ There must be > two terms. Condone missing brackets ie $\sin 22.5^2+\cos 22.5^2+......$

B1 Stating or using $\sin^2 22.5+\cos^2 22.5=1$. Accept $\sin 22.5^2+\cos 22.5^2=1$ as the intention is clear. Note that this may also come from using the double angle formula
$\sin^2 22.5+\cos^2 22.5=(\frac{1-\cos 45}{2})+(\frac{1+\cos 45}{2})=1$

M1 Uses $2\sin x\cos x=\sin 2x$ to write $2\sin 22.5\cos 22.5$ as sin 45 or sin(2×22.5)

A1 Reaching the intermediate answer $1+\sin 45$

A1 Cso $1+\frac{\sqrt{2}}{2}$ or $1+\frac{1}{\sqrt{2}}$. Be aware that both 1.707 and $\frac{2+\sqrt{2}}{2}$ can be found by using a calculator for 1+sin45. Neither can be accepted on their own without firstly seeing one of the two answers given above. **Each stage should be shown as required by the mark scheme.**
Note that if the candidates use $(\sin\theta+\cos\theta)^2$ they can pick up the first M and B marks, but no others until they use $\theta=22.5$. All other marks then become available.

(iia) M1 Substitutes $\cos 2\theta=1-2\sin^2\theta$ in $\cos 2\theta+\sin\theta=1$ to produce an equation in $\sin\theta$ only.
It is acceptable to use $\cos 2\theta=2\cos^2\theta-1$ *or* $\cos^2\theta-\sin^2\theta$ as long as the $\cos^2\theta$ is subsequently replaced by $1-\sin^2\theta$

A1* Obtains the correct simplified equation in $\sin\theta$.
$\sin\theta-2\sin^2\theta=0$ or $\sin\theta=2\sin^2\theta$ must be written in the form $2\sin^2\theta-\sin\theta=0$ as required by the question. Also accept $k=2$ as long as no incorrect working is seen.

(iib) M1 Factorises or divides by $\sin\theta$. For this mark $1='k'\sin\theta$ is acceptable. If they have a 3 TQ in $\sin\theta$ this can be scored for correct factorisation

A1 **Both** $\sin\theta=0$, and $\sin\theta=\frac{1}{2}$

B1 Any two answers from 0, 30, 150, 180.

A1 All four answers 0, 30, 150, 180 with no extra solutions inside the range. Ignore solutions outside the range.

Question Number	Scheme	Marks
6.alt 1	(i) $(\sin 22.5+\cos 22.5)^2=\sin^2 22.5+\cos^2 22.5+......$	M1
	$=\sin^2 22.5+\cos^2 22.5+2\sin 22.5\cos 22.5$	
	States or uses $\sin^2 22.5+\cos^2 22.5=1$	B1
	Uses $2\sin x\cos x=2\sqrt{\frac{1-\cos 2x}{2}}\sqrt{\frac{\cos 2x+1}{2}}\Rightarrow\sqrt{1-\cos 45}\sqrt{1+\cos 45}$	M1
	$=\sqrt{1-\cos^2 45}$	A1
	Hence $(\sin 22.5+\cos 22.5)^2=1+\frac{\sqrt{2}}{2}$ or $1+\frac{1}{\sqrt{2}}$	A1
		(5)

Question Number	Scheme	Marks
6.alt 2	(i) Uses Factor Formula $(\sin 22.5+\sin 67.5)^2=\left(2\sin 45\cos 22.5\right)^2$	M1,A1
	Reaching the stage $=2\cos^2 22.5$	B1
	Uses the double angle formula $=2\cos^2 22.5=1+\cos 45$	M1
	$=1+\frac{\sqrt{2}}{2}$ or $1+\frac{1}{\sqrt{2}}$	A1
		(5)

Question Number	Scheme	Marks
6.alt 3	(i) Uses Factor Formula $(\cos 67.5+\cos 22.5)^2=\left(2\cos 45\cos 22.5\right)^2$	M1,A1
	Reaching the stage $=2\cos^2 22.5$	B1
	Uses the double angle formula $=2\cos^2 22.5=1+\cos 45$	M1
	$=1+\frac{\sqrt{2}}{2}$ or $1+\frac{1}{\sqrt{2}}$	A1
		(5)

Leave blank

7. $$h(x) = \frac{2}{x+2} + \frac{4}{x^2+5} - \frac{18}{(x^2+5)(x+2)}, \qquad x \geqslant 0$$

(a) Show that $h(x) = \dfrac{2x}{x^2+5}$ **(4)**

(b) Hence, or otherwise, find $h'(x)$ in its simplest form. **(3)**

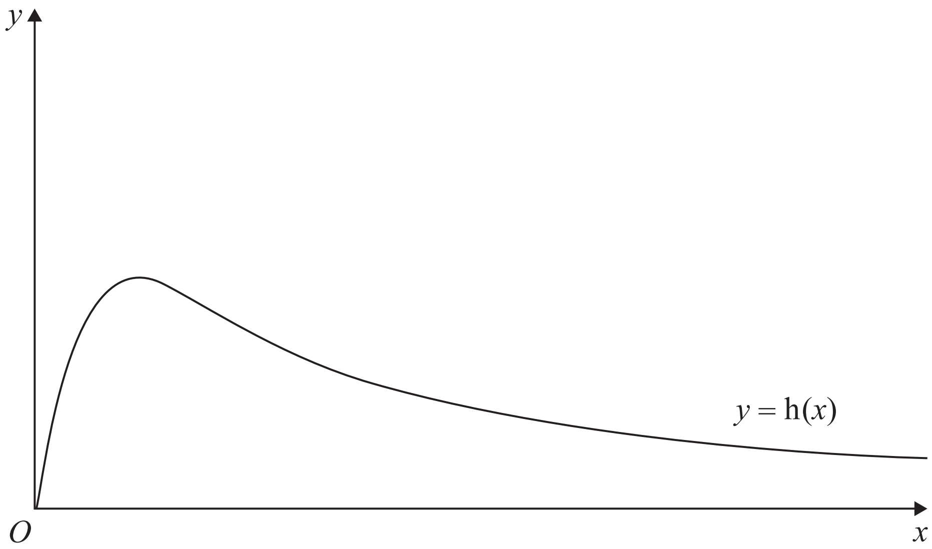

Figure 2

Figure 2 shows a graph of the curve with equation $y = h(x)$.

(c) Calculate the range of $h(x)$. **(5)**

Question Number	Scheme	Marks
7.	(a) $\dfrac{2}{x+2}+\dfrac{4}{x^2+5}-\dfrac{18}{(x+2)(x^2+5)}=\dfrac{2(x^2+5)+4(x+2)-18}{(x+2)(x^2+5)}$	M1A1
	$=\dfrac{2x(x+2)}{(x+2)(x^2+5)}$	M1
	$=\dfrac{2x}{(x^2+5)}$	A1*
		(4)
	(b) $\text{h}'(x)=\dfrac{(x^2+5)\times 2-2x\times 2x}{(x^2+5)^2}$	M1A1
	$\text{h}'(x)=\dfrac{10-2x^2}{(x^2+5)^2}$ cso	A1
		(3)
	(c) Maximum occurs when $\text{h}'(x)=0\Rightarrow 10-2x^2=0\Rightarrow x=..$	M1
	$\Rightarrow x=\sqrt{5}$	A1
	When $x=\sqrt{5}\Rightarrow \text{h}(x)=\dfrac{\sqrt{5}}{5}$	M1,A1
	Range of h(x) is $0\le h(x)\le\dfrac{\sqrt{5}}{5}$	A1ft
		(5) **(12 marks)**

(a) M1 Combines the three fractions to form a single fraction with a common denominator.
Allow errors on the numerator but at least one must have been adapted.
Condone 'invisible' brackets for this mark.
Accept three separate fractions with the same denominator.
Amongst possible options allowed for this method are

$\dfrac{2x^2+5+4x+2-18}{(x+2)(x^2+5)}$ Eg 1 An example of 'invisible' brackets

$\dfrac{2(x^2+5)}{(x+2)(x^2+5)}+\dfrac{4}{(x+2)(x^2+5)}-\dfrac{18}{(x+2)(x^2+5)}$ Eg 2An example of an error (on middle term), 1st term has been adapted

$\dfrac{2(x^2+5)^2(x+2)+4(x+2)^2(x^2+5)-18(x^2+5)(x+2)}{(x+2)^2(x^2+5)^2}$ Eg 3 An example of a correct fraction with a different denominator

A1 Award for a correct un simplified fraction with the correct (lowest) common denominator.

$\dfrac{2(x^2+5)+4(x+2)-18}{(x+2)(x^2+5)}$

Accept if there are three separate fractions with the correct (lowest) common denominator.

Eg $\dfrac{2(x^2+5)}{(x+2)(x^2+5)}+\dfrac{4(x+2)}{(x+2)(x^2+5)}-\dfrac{18}{(x+2)(x^2+5)}$

Note, Example 3 would score M1A0 as it does not have the correct lowest common denominator

M1 There must be a single denominator. Terms must be collected on the numerator. A factor of (x+2) must be taken out of the numerator and then cancelled with one in the denominator. The cancelling may be assumed if the term 'disappears'

A1* Cso $\dfrac{2x}{(x^2+5)}$ This is a given solution and this mark should be withheld if there are any errors

(b) M1 Applies the quotient rule to $\dfrac{2x}{(x^2+5)}$, a form of which appears in the formula book.

If the rule is quoted it must be correct. There must have been some attempt to differentiate both terms. If the rule is not quoted (nor implied by their working, meaning terms are written out u=…,u'=….,v=….,v'=….followed by their $\dfrac{vu'-uv'}{v^2}$) then only accept answers of the form

$$\frac{(x^2+5)\times A-2x\times Bx}{(x^2+5)^2} \quad \text{where } A,B>0$$

A1 Correct unsimplified answer $\mathrm{h}'(x)=\dfrac{(x^2+5)\times 2-2x\times 2x}{(x^2+5)^2}$

A1 $\mathrm{h}'(x)=\dfrac{10-2x^2}{(x^2+5)^2}$ The correct simplified answer. Accept $\dfrac{2(5-x^2)}{(x^2+5)^2}$ $\dfrac{-2(x^2-5)}{(x^2+5)^2}$, $\dfrac{10-2x^2}{(x^4+10x^2+25)}$

DO NOT ISW FOR PART (b). INCORRECT SIMPLIFICATION IS A0

(c) M1 Sets their h'(x)=0 and proceeds with a correct method to find x. There must have been an attempt to differentiate. Allow numerical errors but do not allow solutions from 'unsolvable' equations.

A1 Finds the correct x value of the maximum point $x=\sqrt{5}$. Ignore the solution $x=-\sqrt{5}$ but withhold this mark if other positive values found.

M1 Substitutes their answer into their h'(x)=0 in h(x) to determine the maximum value

A1 Cso-the maximum value of h(x) = $\dfrac{\sqrt{5}}{5}$. Accept equivalents such as $\dfrac{2\sqrt{5}}{10}$ **but not** 0.447

A1ft Range of h(x) is $0\le \mathrm{h}(x)\le \dfrac{\sqrt{5}}{5}$. Follow through on their maximum value if the M's have been scored. Allow $0\le y\le\dfrac{\sqrt{5}}{5}$, $0\le Range\le\dfrac{\sqrt{5}}{5}$, $\left[0,\dfrac{\sqrt{5}}{5}\right]$ but not $0\le x\le\dfrac{\sqrt{5}}{5}$, $\left(0,\dfrac{\sqrt{5}}{5}\right)$

If a candidate attempts to work out $h^{-1}(x)$ in (b) and does all that is required for (b) in (c), then allow. Do not allow $h^{-1}(x)$ to be used for h'(x) in part (c). For this question (b) and (c) can be scored together.

Alternative to (b) using the product rule

M1 Sets $\mathrm{h}(x)=2x(x^2+5)^{-1}$ and applies the product rule vu'+uv' with terms being 2x and $(x^2+5)^{-1}$
If the rule is quoted it must be correct. There must have been some attempt to differentiate both terms. If the rule is not quoted (nor implied by their working, meaning terms are written out u=…,u'=….,v=….,v'=….followed by their vu'+uv') then only accept answers of the form

$$(x^2+5)^{-1}\times A+2x\times\pm Bx(x^2+5)^{-2}$$

A1 Correct un simplified answer $(x^2+5)^{-1}\times 2+2x\times -2x(x^2+5)^{-2}$

A1 The question asks for h'(x) to be put in its simplest form. Hence in this method the terms need to be combined to form a single correct expression.

For a correct simplified answer accept

$$\mathrm{h}'(x) = \frac{10-2x^2}{(x^2+5)^2} = \frac{2(5-x^2)}{(x^2+5)^2} = \frac{-2(x^2-5)}{(x^2+5)^2} = (10-2x^2)(x^2+5)^{-2}$$

Leave blank

8. The value of Bob's car can be calculated from the formula

$$V = 17000e^{-0.25t} + 2000e^{-0.5t} + 500$$

where V is the value of the car in pounds (£) and t is the age in years.

(a) Find the value of the car when $t = 0$

(1)

(b) Calculate the exact value of t when $V = 9500$

(4)

(c) Find the rate at which the value of the car is decreasing at the instant when $t = 8$. Give your answer in pounds per year to the nearest pound.

(4)

Question Number	Scheme	Marks
8.	(a) (£) 19500	B1
		(1)
	(b) $9500 = 17000e^{-0.25t} + 2000e^{-0.5t} + 500$	
	$17e^{-0.25t} + 2e^{-0.5t} = 9$	
	$(\times e^{0.5t}) \Rightarrow 17e^{0.25t} + 2 = 9e^{0.5t}$	
	$0 = 9e^{0.5t} - 17e^{0.25t} - 2$	M1
	$0 = (9e^{0.25t} + 1)(e^{0.25t} - 2)$	M1
	$e^{0.25t} = 2$	A1
	$t = 4\ln(2)$ *oe*	A1
		(4)
	(c)	
	$(\frac{dV}{dt}) = -4250e^{-0.25t} - 1000e^{-0.5t}$	M1A1
	When t=8 Decrease = 593 (£/year)	M1A1
		(4)
		(9 marks)

(a) B1 19500. The £ sign is not important for this mark

(b) M1 Substitute V=9500, collect terms and set on 1 side of an equation =0. Indices must be correct
Accept $17000e^{-0.25t} + 2000e^{-0.5t} - 9000 = 0$ and $17000x + 2000x^2 - 9000 = 0$ where $x = e^{-0.25t}$

M1 Factorise the quadratic in $e^{0.25t}$ or $e^{-0.25t}$
For your information the factorised quadratic in $e^{-0.25t}$ is $(2e^{-0.25t} - 1)(e^{-0.25t} + 9) = 0$
Alternatively let $'x' = e^{0.25t}$ or otherwise and factorise a quadratic equation in x

A1 Correct solution of the quadratic. Either $e^{0.25t} = 2$ or $e^{-0.25t} = \frac{1}{2}$ oe.

A1 Correct exact value of t. Accept variations of $4\ln(2)$, such as $\ln(16)$, $\frac{\ln(\frac{1}{2})}{-0.25}$, $\frac{\ln(2)}{0.25}$, $-4\ln(\frac{1}{2})$

.(c) M1 Differentiates $V = 17000e^{-0.25t} + 2000e^{-0.5t} + 500$ by the chain rule.
Accept answers of the form $(\frac{dV}{dt}) = \pm Ae^{-0.25t} \pm Be^{-0.5t}$ A, B are constants $\neq 0$

A1 Correct derivative $(\frac{dV}{dt}) = -4250e^{-0.25t} - 1000e^{-0.5t}$.
There is no need for it to be simplified so accept

$$(\frac{dV}{dt}) = 17000 \times -0.25e^{-0.25t} + 2000 \times -0.5e^{-0.5t} \quad oe$$

M1 Substitute t=8 into their $\frac{dV}{dt}$.
This is not dependent upon the first M1 but there must have been some attempt to differentiate.
Do not accept t=8 in V

A1 ±593. Ignore the sign and the units. If the candidate then divides by 8, withhold this mark. **This would not be isw. Be aware that sub t=8 into V first and then differentiating can achieve 593. This is M0A0M0A0.**