Winter 2013 Past Paper

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Mathematics C3 6665

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1.

Find

www.mystudybro.com This resource was created and owned by Pearson Edexcel Leave blank The curve C has equation $y = (2x - 3)^5$ The point *P* lies on *C* and has coordinates (w, -32). (a) the value of w, (2) (b) the equation of the tangent to C at the point P in the form y = mx + c, where m and c are constants. (5)



January 2013 6665 Core Mathematics C3 Mark Scheme

Question Number	Scheme	Marks
1.	(a) $-32 = (2w-3)^5 \Longrightarrow w = \frac{1}{2} \text{ oe}$	M1A1 (2)
	(b) $\frac{dy}{dx} = 5 \times (2x-3)^4 \times 2$ or $10(2x-3)^4$	M1A1
	When $x = \frac{1}{2}$, Gradient = 160	M1
	Equation of tangent is '160' = $\frac{y - (-32)}{x - \frac{1}{2}}$ oe	dM1
	y = 160x - 112 cso	A1
		(5)
		(7 marks)

(a) M1 Substitute y=-32 into $y = (2w-3)^5$ and proceed to w=... [Accept positive sign used of y, ie y=+32] A1 Obtains w or $x = \frac{1}{2}$ oe with no incorrect working seen. Accept alternatives such as 0.5. Sight of just the answer would score both marks as long as no incorrect working is seen.

(b) M1 Attempts to differentiate $y = (2x-3)^5$ using the chain rule. Sight of $\pm A(2x-3)^4$ where A is a non-zero constant is sufficient for the method mark. A1 A correct (un simplified) form of the differential.

Accept
$$\frac{dy}{dx} = 5 \times (2x-3)^4 \times 2 \text{ or } \frac{dy}{dx} = 10(2x-3)^4$$

- M1 This is awarded for an attempt to find the gradient of the tangent to the curve at *P* Award for substituting their numerical value to part (a) into their differential to find the numerical gradient of the tangent
- dM1 Award for a correct method to find an equation of the tangent to the curve at *P*. It is dependent upon the previous M mark being awarded.

Award for 'their 160' =
$$\frac{y - (-32)}{x - their' \frac{1}{2}}$$

If they use y = mx + c it must be a full method, using m= 'their 160', their ' $\frac{1}{2}$ ' and -32. An attempt must be seen to find c=... A1 cso y = 160x - 112. The question is specific and requires the answer in this form. You may isw in this question after a correct answer.

(2)

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2.

 $g(x) = e^{x-1} + x - 6$

(a) Show that the equation g(x) = 0 can be written as

$$x = \ln(6 - x) + 1, \qquad x < 6$$

The root of g(x) = 0 is α .

The iterative formula

 $x_{n+1} = \ln(6 - x_n) + 1,$ $x_0 = 2$

is used to find an approximate value for α .

(b) Calculate the values of x_1 , x_2 and x_3 to 4 decimal places.

(3)

(c) By choosing a suitable interval, show that $\alpha = 2.307$ correct to 3 decimal places.

(3)



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G N	uestion) Number	Scheme	Marks	
	2.	(a) $0 = e^{x-1} + x - 6 \Rightarrow x = \ln(6-x) + 1$	M1A1*	(2)
		(b) Sub $x_0 = 2$ into $x_{n+1} = \ln(6 - x_n) + 1 \Longrightarrow x_1 = 2.3863$	M1, A1	(-)
		AWRT 4 dp. $x_2 = 2.2847 x_3 = 2.3125$	A1	
		(c) Chooses interval [2.3065,2.3075]	M1	(3)
		g(2.3065)=-0.0002(7), g(2.3075)=0.004(4)	dM1	
		Sign change, hence root (correct to 3dp)	A1	(3)
			(8 marks)	(3)
(a)	M1	Sets g(x)=0, and using correct <i>ln</i> work, makes the x of the e^{x-1} term the subject Look for $e^{x-1} + x - 6 = 0 \Rightarrow e^{x-1} = \pm 6 \pm x \Rightarrow x = \ln(\pm 6 \pm x) \pm 1$	ct of the for	nula.
	A1*	bo not accept $e^{-x} = 6 - x$ without firstly seeing $e^{-x} + x - 6 = 0$ or a statement cso. $x = \ln(6-x) + 1$ Note that this is a given answer (and a proof). 'Invisible' brackets are allowed for the M but not the A Do not accept recovery from earlier errors for the A mark. The solution below $0 = e^{x-1} + x - 6 \Rightarrow 0 = x - 1 + \ln(x - 6) \Rightarrow x = \ln(6 - x) + 1$	w scores 0	0 ⇒ marks
(b)	M1	Sub $x_0 = 2$ into $x_{n+1} = \ln(6 - x_n) + 1$ to produce a numerical value for x_1 . Evidence for the award could be any of $\ln(6-2)+1$, $\ln 4+1$ 2.3 or awrt	2.4	
	A1	Answer correct to 4 dp $x_1 = 2.3863$.		
	A1	The subscript is not important. Mark as the first value given/found. Awrt 4 dp. $x_2 = 2.2847$ and $x_3 = 2.3125$		
		The subscripts are not important. Mark as the second and third values given/f	found	
(c)	M1 dM1	Chooses the interval [2.3065,2.3075] or smaller containing the root 2.306558 Calculates $g(2.3065)$ and $g(2.3075)$ with at least one of these correct to 1sf. The answers can be rounded or truncated g(2.3065) = -0.0003 rounded, $g(2.3065) = -0.0002$ truncated	3641	
	A1	g(2.5075) = (+) 0.004 rounded and truncated Both values correct (rounded or truncated), A reason which could include change of sign, >0 <0, $g(2.3065) \times g(2.3075)$ AND a minimal conclusion such as hence root, $\alpha=2.307$ or \Box Do not accept continued iteration as question demands an interval to be	<0 chosen.	
Alter	native s	olution to (a) working backwards		

M1 Proceeds from $x = \ln(6-x) + 1$ using correct exp work to=0 A1 Arrives correctly at $e^{x-1} + x - 6 = 0$ and makes a statement to the effect that this is g(x)=0

Alternative solution to (c) using $f(x) = \ln(6-x) + 1 - x$ {Similarly $h(x) = x - 1 - \ln(6-x)$ }

- M1 Chooses the interval [2.3065,2.3075] or smaller containing the root 2.306558641
- dM1 Calculates f(2.3065) and f(2.3075) with at least 1 correct rounded or truncated f(2.3065) = 0.000074. Accept 0.00007 rounded or truncated. Also accept 0.0001

f(2.3075) = -0.0011.. Accept -0.001 rounded or truncated





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(a) M1 A full method of finding ff(-3). f(0) is acceptable but f(-3)=0 is not. Accept a solution obtained from two substitutions into the equation $y = \frac{2}{3}x + 2$ as the line passes through both points. Do not allow for $y = \ln(x+4)$, which only passes through one of the points. A1 Cao ff(-3)=2. Writing down 2 on its own is enough for both marks provided no incorrect working is seen.

(b)

- B1 For the correct shape. Award this mark for an increasing function in quadrants 3, 4 and 1 only. Do not award if the curve bends back on itself or has a clear minimum
- B1 This is independent to the first mark and for the graph passing through (0,-3) and (2,0)

Accept -3 and 2 marked on the correct axes.

Accept (-3,0) and (0,2) instead of (0,-3) and (2,0) as long as they are on the correct axes Accept P'=(0,-3), Q'=(2,0) stated elsewhere as long as P' and Q' are marked in the correct place on the graph

There must be a graph for this to be awarded

(c)

- B1 Award for a correct shape 'roughly' symmetrical about the *y* axis. It must have a cusp and a gradient that 'decreases' either side of the cusp. Do not award if the graph has a clear maximum
- B1 (0,0) lies on their graph. Accept the graph passing through the origin without seeing (0, 0) marked
- (d) B1 Shape. The position is not important. The gradient should be always positive but decreasing There should not be a clear maximum point.
 - B1 The graph passes through (0,4) or (-6,0). See part (b) for allowed variations
 - B1 The graph passes through (0,4) **and** (-6,0). See part (b) for allowed variations

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4. (a) Express $6\cos\theta + 8\sin\theta$ in the form $R\cos(\theta - \alpha)$, where R > 0 and $0 < \alpha < \frac{\pi}{2}$. Give the value of α to 3 decimal places. (4) $p(\theta) = \frac{4}{12 + 6\cos\theta + 8\sin\theta}, \qquad 0 \leqslant \theta \leqslant 2\pi$ (b) Calculate (i) the maximum value of $p(\theta)$, (ii) the value of θ at which the maximum occurs. (4) 10 P 4 1 4 8 6 A 0 1 0 2 8

Winter 2013 Past Paper (Mark Scheme)

Question Number		Scheme	Marks
4.	(a)	$R^2 = 6^2 + 8^2 \Longrightarrow R = 10$	M1A1
		$\tan \alpha = \frac{8}{6} \Rightarrow \alpha = \text{awrt } 0.927$	M1A1
			(4)
	(b)(i)	$p(x) = \frac{4}{12 + 10\cos(\theta - 0.927)}$	
		$\mathbf{p}(x) = \frac{4}{12 - 10}$	M1
		Maximum = 2	A1 (2)
	(b)(ii)	θ – 'their α ' = π	(2) M1
		$\theta = $ awrt 4.07	A1
			(2) (8 marks)

- (a) M1 Using Pythagoras' Theorem with 6 and 8 to find *R*. Accept $R^2 = 6^2 + 8^2$ If α has been found first accept $R = \pm \frac{8}{\sin'\alpha'}$ or $R = \pm \frac{6}{\cos'\alpha'}$
 - A1 R = 10. Many candidates will just write this down which is fine for the 2 marks. Accept ± 10 but not -10
 - M1 For $\tan \alpha = \pm \frac{8}{6}$ or $\tan \alpha = \pm \frac{6}{8}$

If *R* is used then only accept $\sin \alpha = \pm \frac{8}{R}$ or $\cos \alpha = \pm \frac{6}{R}$

- A1 $\alpha =$ awrt 0.927 . Note that 53.1° is A0
- (b) Note that (b)(i) and (b)(ii) can be marked together
- (i) M1 Award for $p(x) = \frac{4}{12 R'}$.
 - A1 Cao $p(x)_{max} = 2$. The answer is acceptable for both marks as long as no incorrect working is seen
- (ii) M1 For setting $\theta their \alpha = \pi$ and proceeding to $\theta = ..$ If working exclusively in degrees accept $\theta - their \alpha = 180$ Do not accept mixed units
 - A1 θ = awrt 4.07. If the final A mark in part (a) is lost for 53.1, then accept awrt 233.1

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Winter 2013 Past Paper (Mark Scheme)

Question Number	Scheme	Marks
5.	(i)(a) $\frac{dy}{dx} = 3x^2 \times \ln 2x + x^3 \times \frac{1}{2x} \times 2$	M1A1A1
	$= 3x^2 \ln 2x + x^2$	
	dy dy a solution and	(3)
	(1)(b) $\frac{1}{dx} = 3(x + \sin 2x)^2 \times (1 + 2\cos 2x)$	B1M1A1
	dr	(3)
	(ii) $\frac{dx}{dy} = -\csc^2 y$	M1A1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{1}{\mathrm{cosec}^2 y}$	M1
	Uses $\csc^2 y = 1 + \cot^2 y$ and $x = \cot y$ in $\frac{dy}{dx}$ or $\frac{dx}{dy}$ to get an expression in x	
	$\frac{dy}{dx} = -\frac{1}{\csc^2 y} = -\frac{1}{1 + \cot^2 y} = -\frac{1}{1 + x^2}$ cso	M1, A1*
		(5) (11 marks)
	If the rule is quoted it must be correct. There must have been some at differentiate both terms. If the rule is not quoted (nor implied by their terms written out u=,u'=,v=,v'=followed by their vu'+uv' accept answers of the form $Ax^2 \times \ln 2x + x^3 \times \frac{B}{x} \qquad \text{where } A, B \text{ are constants} \neq 0$	tempt to working, with) then only
	A1 One term correct, either $3x^2 \times \ln 2x$ or $x^3 \times \frac{1}{2x} \times 2$	
	A1 Cao. $\frac{dy}{dx} = 3x^2 \times \ln 2x + x^3 \times \frac{1}{2x} \times 2$. The answer does not need to be si	mplified.
	For reference the simplified answer is $\frac{dy}{dx} = 3x^2 \ln 2x + x^2 = x^2 (3 \ln 2x)$	+1)
(i)(b)	B1 Sight of $(x + \sin 2x)^2$	
	M1 For applying the chain rule to $(x + \sin 2x)^3$. If the rule is quoted it mu not quoted possible forms of evidence could be sight of $C(x + \sin 2x)$ where <i>C</i> and <i>D</i> are non-zero constants. Alternatively accept $u = x + \sin 2x$, $u' =$ followed by $Cu^2 \times$ their <i>u'</i> Do not accept $C(x + \sin 2x)^2 \times 2 \cos 2x$ unless you have evidence that Allow 'invisible' brackets for this mark, ie. $C(x + \sin 2x)^2 \times 1 \pm D \cos^2$	st be correct. If it is $^{2} \times (1 \pm D \cos 2x)$ this is their <i>u</i> ' 2x
	A1 Cao $\frac{dy}{dx} = 3(x + \sin 2x)^2 \times (1 + 2\cos 2x)$. There is no requirement to sir	nplify this.

You may ignore subsequent working (isw) after a correct answer in part (i)(a) and (b)

(ii) M1 Writing the derivative of $\cot y$ as $-\csc^2 y$. It must be in terms of y

- A1 $\frac{dx}{dy} = -\csc^2 y$ or $1 = -\csc^2 y \frac{dy}{dx}$. Both lhs and rhs must be correct.
- M1 Using $\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}$

M1 Using
$$\csc^2 y = 1 + \cot^2 y$$
 and $x = \cot y$ to get $\frac{dy}{dx}$ or $\frac{dx}{dy}$ just in terms of x.

A1 cso
$$\frac{dy}{dx} = -\frac{1}{1+x^2}$$

Alternative to (a)(i) when ln(2x) is written lnx+ln2

M1 Writes $x^3 \ln 2x$ as $x^3 \ln 2 + x^3 \ln x$. Achieves Ax^2 for differential of $x^3 \ln 2$ and applies the product rule vu'+uv' to $x^3 \ln x$.

A1 Either
$$3x^2 \times \ln 2 + 3x^2 \ln x$$
 or $x^3 \times \frac{1}{r}$

A1 A correct (un simplified) answer. Eg $3x^2 \times \ln 2 + 3x^2 \ln x + x^3 \times \frac{1}{x}$

Alternative to 5(ii) using quotient rule

M1 Writes $\cot y$ as $\frac{\cos y}{\sin y}$ and applies the quotient rule, a form of which appears in the formula book. If the rule is quoted it must be correct. There must have been some attempt to differentiate both terms. If the rule is not quoted (nor implied by their working,

meaning terms are written out u=...,v=...,v=...,v'=....followed by their $\frac{vu'-uv'}{v^2}$) only accept answers of the form $\frac{\sin y \times \pm \sin y - \cos y \times \pm \cos y}{(\sin y)^2}$

A1 Correct un simplified answer with both lhs and rhs correct.

$$\frac{\mathrm{d}x}{\mathrm{d}y} = \frac{\sin y \times -\sin y - \cos y \times \cos y}{(\sin y)^2} = \left\{-1 - \cot^2 y\right\}$$

M1 Using
$$\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}$$

M1 Using
$$\sin^2 y + \cos^2 y = 1$$
, $\frac{1}{\sin^2 y} = \csc^2 y$ and $\csc^2 y = 1 + \cot^2 y$ to get $\frac{dy}{dx}$ or $\frac{dx}{dy}$ in x
A1 $\cos \frac{dy}{dx} = -\frac{1}{1+x^2}$

Alternative to 5(ii) using the chain rule, first two marks

- M1 Writes $\cot y$ as $(\tan y)^{-1}$ and applies the chain rule (or quotient rule). Accept answers of the form $-(\tan y)^{-2} \times \sec^2 y$
- A1 Correct un simplified answer with both lhs and rhs correct.

$$\frac{\mathrm{d}x}{\mathrm{d}y} = -(\tan y)^{-2} \times \sec^2 y$$

Alternative to 5(ii) using a triangle – last M1

M1 Uses triangle with $\tan y = \frac{1}{x}$ to find siny

and get
$$\frac{dy}{dx}$$
 or $\frac{dx}{dy}$ just in terms of x



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6. (i) V	Without using a calculator, find the exact value of	Ulalik
	$(\sin 22.5^\circ + \cos 22.5^\circ)^2$	
N. N	You must show each stage of your working.	(5)
(ii) ((a) Show that $\cos 2\theta + \sin \theta = 1$ may be written in the form	
	$k\sin^2\theta - \sin\theta = 0$, stating the value of k.	(2)
((b) Hence solve, for $0 \le \theta < 360^\circ$, the equation	
	$\cos 2\theta + \sin \theta = 1$	(4)

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Q N	uestion lumber	Scheme	Marks
	6.	(i) $(\sin 22.5 + \cos 22.5)^2 = \sin^2 22.5 + \cos^2 22.5 + \dots$	M1
		$=\sin^2 22.5 + \cos^2 22.5 + 2\sin 22.5 \cos 22.5$	
		States or uses $\sin^2 22.5 + \cos^2 22.5 = 1$	B1
		Uses $2\sin x \cos x = \sin 2x \implies 2\sin 22.5 \cos 22.5 = \sin 45$	M1
		$(\sin 22.5 + \cos 22.5)^2 = 1 + \sin 45$	A1
		$=1+\frac{\sqrt{2}}{2} \text{ or } 1+\frac{1}{\sqrt{2}}$ cso	A1
			(5)
		(ii) (a) $\cos 2\theta + \sin \theta = 1 \Rightarrow 1 - 2\sin^2 \theta + \sin \theta = 1$	M1
		$\sin\theta - 2\sin^2\theta = 0$	A 1 4
		$2\sin^2\theta - \sin\theta = 0$ or $k = 2$	$ A1^* $ (2)
		(b) $\sin\theta(2\sin\theta - 1) = 0$	(2) M1
		$\sin\theta = 0, \sin\theta = \frac{1}{2}$	A1
		Any two of 0,30,150,180	B1
		All four answers 0,30,150,180	AI (4)
			(11 marks)
(i)	M1	Attempts to expand $(\sin 22.5 + \cos 22.5)^2$. Award if you see $\sin^2 22.5 + \cos^2 3$	22.5 +
		Note that this may also come from using the double angle formula $\sin^2 22.5 + \cos^2 22.5 = (\frac{1 - \cos 45}{2}) + (\frac{1 + \cos 45}{2}) = 1$	
	M 1	Uses $2 \sin x \cos x = \sin 2x$ to write $2 \sin 22.5 \cos 22.5 \operatorname{as} \sin 45$ or $\sin(2 \times 22.5)$	
	A1	Reaching the intermediate answer 1+sin 45	
	A1	$\operatorname{Cso1} + \frac{\sqrt{2}}{2}$ or $1 + \frac{1}{\sqrt{2}}$. Be aware that both 1.707 and $\frac{2 + \sqrt{2}}{2}$ can be found by	y using a calculato
		for 1+sin45. Neither can be accepted on their own without firstly seeing one given above. Each stage should be shown as required by the mark scheme Note that if the candidates use $(\sin \theta + \cos \theta)^2$ they can pick up the first M as	of the two answer ne. nd B marks, but nc
		others until they use $\theta = 22.5$. All other marks then become available.	
(iia)	M1	Substitutes $\cos 2\theta = 1 - 2\sin^2 \theta$ in $\cos 2\theta + \sin \theta = 1$ to produce an equation in It is acceptable to use $\cos 2\theta = 2\cos^2 \theta - 1$ or $\cos^2 \theta - \sin^2 \theta$ as long as the co	$\sin \theta$ only. $\sin^2 \theta$ is
	A1*	subsequently replaced by $1 - \sin^2 \theta$ Obtains the correct simplified equation in $\sin \theta$. $\sin \theta - 2\sin^2 \theta = 0$ or $\sin \theta = 2\sin^2 \theta$ must be written in the form $2\sin^2 \theta - \sin \theta$ the question. Also accent $k = 2$ as long as no incorrect working is seen.	$\theta = 0$ as required by
(iib)	M1	Factorises or divides by $\sin\theta$. For this mark $1 = k'\sin\theta$ is acceptable. If they $\sin\theta$ this can be scored for correct factorisation	have a 3 TQ in
	A1	Both $\sin \theta = 0$, and $\sin \theta = \frac{1}{2}$	
	B1 A1	Any two answers from 0, 30, 150, 180. All four answers 0, 30, 150, 180 with no extra solutions inside the range. Igr outside the range.	nore solutions

Question Number	Scheme	Marks
6.alt 1	(i) $(\sin 22.5 + \cos 22.5)^2 = \sin^2 22.5 + \cos^2 22.5 + \dots$	M1
	$= \sin^2 22.5 + \cos^2 22.5 + 2\sin 22.5 \cos 22.5$	
	States or uses $\sin^2 22.5 + \cos^2 22.5 = 1$	B1
	Uses $2\sin x \cos x = 2\sqrt{\frac{1-\cos 2x}{2}}\sqrt{\frac{\cos 2x+1}{2}} \Rightarrow \sqrt{1-\cos 45}\sqrt{1+\cos 45}$	M1
	$=\sqrt{1-\cos^2 45}$	A1
	Hence $(\sin 22.5 + \cos 22.5)^2 = 1 + \frac{\sqrt{2}}{2}$ or $1 + \frac{1}{\sqrt{2}}$	A1
		(5)

M1,A1
B1
M1
A1 (5)

Question Number	Scheme	Marks
6.alt 3	(i) Uses Factor Formula $(\cos 67.5 + \cos 22.5)^2 = (2\cos 45\cos 22.5)^2$	M1,A1
	Reaching the stage = $2\cos^2 22.5$	B1
	Uses the double angle formula $= 2\cos^2 22.5 = 1 + \cos 45$	M1
	$=1+\frac{\sqrt{2}}{2} \text{ or } 1+\frac{1}{\sqrt{2}}$	A1 (5)



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Question

Number

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7. (a)
$$\frac{2}{x+2} + \frac{4}{x^2+5} - \frac{18}{(x+2)(x^2+5)} = \frac{2(x^2+5)+4(x+2)-18}{(x+2)(x^2+5)}$$
 M1A1

$$= \frac{2x(x+2)}{(x+2)(x^2+5)}$$
 M1

$$= \frac{2x}{(x^2+5)}$$
 A1*
(b) $h'(x) = \frac{(x^2+5)\times 2-2x\times 2x}{(x^2+5)^2}$ A1*
 $h'(x) = \frac{10-2x^2}{(x^2+5)^2}$ cso A1
(3)
(c) Maximum occurs when $h'(x) = 0 \Rightarrow 10-2x^2 = 0 \Rightarrow x = ..$
 $\Rightarrow x = \sqrt{5}$ M1
A1
When $x = \sqrt{5} \Rightarrow h(x) = \frac{\sqrt{5}}{5}$ M1,A1
Range of $h(x)$ is $0 \le h(x) \le \frac{\sqrt{5}}{5}$ M1,A1
(5)
(12 marks)

(a) M1 Combines the three fractions to form a single fraction with a common denominator. Allow errors on the numerator but at least one must have been adapted. Condone 'invisible' brackets for this mark. Accept three separate fractions with the same denominator.

Amongst possible options allowed for this method are

 $2x^2+5+4x+2-18$ Eg 1 An example of 'invisible' brackets $(x+2)(x^2+5)$ $\frac{2(x^2+5)}{(x+2)(x^2+5)} + \frac{4}{(x+2)(x^2+5)} - \frac{18}{(x+2)(x^2+5)}$ 18 Eg 2An example of an error (on middle term), 1st term has been adapted

$$\frac{2(x^2+5)^2(x+2)+4(x+2)^2(x^2+5)-18(x^2+5)(x+2)}{(x+2)^2(x^2+5)^2}$$
 Eg 3 An example of a correct fraction with a different denominator

A1 Award for a correct un simplified fraction with the correct (lowest) common denominator. $2(x^2+5)+4(x+2)-18$ $(x+2)(x^2+5)$

Accept if there are three separate fractions with the correct (lowest) common denominator. Eg $\frac{2(x^2+5)}{(x+2)(x^2+5)} + \frac{4(x+2)}{(x+2)(x^2+5)} - \frac{18}{(x+2)(x^2+5)}$

- Note, Example 3 would score M1A0 as it does not have the correct lowest common denominator M1 There must be a single denominator. Terms must be collected on the numerator. A factor of (x+2) must be taken out of the numerator and then cancelled with one in the denominator. The cancelling may be assumed if the term 'disappears'
- A1* Cso $\frac{2x}{(x^2+5)}$ This is a given solution and this mark should be withheld if there are any errors

(b) M1 Applies the quotient rule to
$$\frac{2x}{(x^2+5)}$$
, a form of which appears in the formula book.

If the rule is quoted it must be correct. There must have been some attempt to differentiate both terms. If the rule is not quoted (nor implied by their working, meaning terms are written out

u=...,u'=...,v=...,v'=....followed by their $\frac{vu'-uv'}{v^2}$) then only accept answers of the form

$$\frac{(x^2+5) \times A - 2x \times Bx}{(x^2+5)^2} \quad \text{where } A, B > 0$$

A1 Correct unsimplified answer
$$h'(x) = \frac{(x^2+5) \times 2 - 2x \times 2x}{(x^2+5)^2}$$

A1 $h'(x) = \frac{10 - 2x^2}{(x^2 + 5)^2}$ The correct simplified answer. Accept $\frac{2(5 - x^2)}{(x^2 + 5)^2} = \frac{-2(x^2 - 5)}{(x^2 + 5)^2}$, $\frac{10 - 2x^2}{(x^4 + 10x^2 + 25)}$

DO NOT ISW FOR PART (b). INCORRECT SIMPLIFICATION IS A0

- (c) M1 Sets their h'(x)=0 and proceeds with a correct method to find x. There must have been an attempt to differentiate. Allow numerical errors but do not allow solutions from 'unsolvable' equations.
 - A1 Finds the correct x value of the maximum point $x=\sqrt{5}$. Ignore the solution $x=-\sqrt{5}$ but withhold this mark if other positive values found.
 - M1 Substitutes their answer into their h'(x)=0 in h(x) to determine the maximum value

A1 Cso-the maximum value of
$$h(x) = \frac{\sqrt{5}}{5}$$
. Accept equivalents such as $\frac{2\sqrt{5}}{10}$ but not 0.447

A1ft Range of h(x) is $0 \le h(x) \le \frac{\sqrt{5}}{5}$. Follow through on their maximum value if the M's have been

scored. Allow
$$0 \le y \le \frac{\sqrt{5}}{5}$$
, $0 \le Range \le \frac{\sqrt{5}}{5}$, $\left[0, \frac{\sqrt{5}}{5}\right]$ but not $0 \le x \le \frac{\sqrt{5}}{5}$, $\left(0, \frac{\sqrt{5}}{5}\right)$

If a candidate attempts to work out $h^{-1}(x)$ in (b) and does all that is required for (b) in (c), then allow. Do not allow $h^{-1}(x)$ to be used for h'(x) in part (c). For this question (b) and (c) can be scored together. Alternative to (b) using the product rule

M1 Sets $h(x) = 2x(x^2 + 5)^{-1}$ and applies the product rule vu'+uv' with terms being 2x and $(x^2+5)^{-1}$ If the rule is quoted it must be correct. There must have been some attempt to differentiate both terms. If the rule is not quoted (nor implied by their working, meaning terms are written out u=...,u'=...,v=...,v'=....followed by their vu'+uv') then only accept answers of the form

$$(x^{2}+5)^{-1} \times A + 2x \times \pm Bx(x^{2}+5)^{-2}$$

- A1 Correct un simplified answer $(x^2+5)^{-1} \times 2 + 2x \times -2x(x^2+5)^{-2}$
- A1 The question asks for h'(x) to be put in its simplest form. Hence in this method the terms need to be combined to form a single correct expression.

For a correct simplified answer accept

h'(x) =
$$\frac{10-2x^2}{(x^2+5)^2} = \frac{2(5-x^2)}{(x^2+5)^2} = \frac{-2(x^2-5)}{(x^2+5)^2} = (10-2x^2)(x^2+5)^{-2}$$

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		L	.eave
8	The value of Bob's car can be calculated from the formula	b	lank
0.			
	$V = 17000e^{-0.25t} + 2000e^{-0.5t} + 500$		
	where V is the value of the car in pounds (f) and t is the age in years.		
	(a) Find the value of the car when $t = 0$		
		(1)	
	(b) Calculate the exact value of t when $V = 9500$		
		(4)	
	(c) Find the rate at which the value of the car is decreasing at the instan	t when $t = 8$.	
	Give your answer in pounds per year to the nearest pound.	(4)	

	Question Number	Scheme	Marks
	8.	(a) (£) 19500	B1 (1)
		(b) $9500 = 17000e^{-0.25t} + 2000e^{-0.5t} + 500$	~ /
		$17e^{-0.25t} + 2e^{-0.5t} = 9$	
		$(\times e^{0.05}) \Longrightarrow 1/e^{0.05t} + 2 = 9e^{0.05t}$	M1
		$0 = 9e^{-1/e} - 2$ $0 = (9e^{0.25t} + 1)(e^{0.25t} - 2)$	M1
		$e^{0.25t} = 2$	A1
		$t = 4\ln(2) oe$	A1
		(c)	(4)
		$\left(\frac{\mathrm{d}V}{\mathrm{d}t}\right) = -4250e^{-0.25t} - 1000e^{-0.5t}$	M1A1
		When $t=8$ Decrease = 593 (£/year)	M1A1 (4)
			(9 marks)
(a)	B1	19500. The £ sign is not important for this mark	
(b)	M1	Substitute V=9500, collect terms and set on 1 side of an equation =0. Indices must be correct Accept $17000e^{-0.25t} + 2000e^{-0.5t} - 9000 = 0$ and $17000x + 2000x^2 - 9000 = 0$ where $x = e^{-0.25t}$	
	M 1	Factorise the quadratic in $e^{0.25t}$ or $e^{-0.25t}$	
		For your information the factorised quadratic in $e^{-0.25t}$ is $(2e^{-0.25t} - 1)(e^{-0.25t} + 9)$	$(\theta) = 0$
		Alternatively let $x' = e^{0.25t}$ or otherwise and factorise a quadratic equation in x	x
	A1	Correct solution of the quadratic. Either $e^{0.25t} = 2$ or $e^{-0.25t} = \frac{1}{2}$ oe.	
	A1	Correct exact value of t. Accept variations of $4\ln(2)$, such as $\ln(16)$, $\frac{\ln(\frac{1}{2})}{-0.25}$,	$\frac{\ln(2)}{0.25}, -4\ln(\frac{1}{2})$
.(c) M1	Differentiates $V = 17000e^{-0.25t} + 2000e^{-0.5t} + 500$ by the chain rule.	
		Accept answers of the form $\left(\frac{dV}{dt}\right) = \pm Ae^{-0.25t} \pm Be^{-0.5t}$ A, B are constants $\neq 0$	
	A1	Correct derivative $\left(\frac{dV}{dt}\right) = -4250e^{-0.25t} - 1000e^{-0.5t}$.	
		There is no need for it to be simplified so accept	
		$\left(\frac{\mathrm{d}V}{\mathrm{d}t}\right) = 17000 \times -0.25e^{-0.25t} + 2000 \times -0.5e^{-0.5t} oe$	
	M 1	Substitute $t=8$ into their $\frac{dV}{dt}$.	
		This is not dependent upon the first M1 but there must have been some attem Do not accept $t=8$ in V	pt to differentiate

A1 ±593. Ignore the sign and the units. If the candidate then divides by 8, withhold this mark. This would not be isw. Be aware that sub t=8 into V first and then differentiating can achieve 593. This is M0A0M0A0.