Write your name here

Surname	Other names

Pearson Edexcel
International Advanced Level

Centre Number	Candidate Number

Core Mathematics C3
Advanced

Monday 27 January 2014 – Morning
Time: 1 hour 30 minutes

Paper Reference
6665A/01

You must have:
Mathematical Formulae and Statistical Tables (Pink)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions
- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information
- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice
- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

PEARSON

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1.

$$f(x) = \sec x + 3x - 2, \quad -\frac{\pi}{2} < x < \frac{\pi}{2}$$

(a) Show that there is a root of $f(x) = 0$ in the interval $[0.2, 0.4]$

(2)

(b) Show that the equation $f(x) = 0$ can be written in the form

$$x = \frac{2}{3} - \frac{1}{3\cos x}$$

(1)

The solution of $f(x) = 0$ is α, where $\alpha = 0.3$ to 1 decimal place.

(c) Starting with $x_0 = 0.3$, use the iterative formula

$$x_{n+1} = \frac{2}{3} - \frac{1}{3\cos x_n}$$

to calculate the values of x_1, x_2 and x_3, giving your answers to 4 decimal places.

(3)

(d) State the value of α correct to 3 decimal places.

(1)

Question Number	Scheme		Marks
1 (a)	Radians: f(0.2) = - 0.4 , f(0.4) = 0.3 or considers smaller subset of [0.2, 0.4]	Degrees: f(0.2) = - 0.4 , f(0.4) = 0.2 or considers smaller subset of [0.2, 0.4]	M1
	Change of sign implies root		A1 (2)
(b)	$\sec x+3x-2=0 \Rightarrow 3x=2-\sec x$ and so $x=\frac{2}{3}-\frac{1}{3\cos x}$ *		B1 (1)
(c)	Radians: $x_1=0.3177,$ $x_2=0.3158,\quad x_3=0.3160$	Degrees: $x_1=0.3333,$ $x_2=0.3333,\quad x_3=0.3333$	M1, A1, A1 (3)
(d)	0.316 (radians)	0.333 (degrees)	B1 (1)
			[7]

Notes

(a) **M1:** Gives two answers with at least one correct to 1sf. Candidates may work in degrees or in radians in this question, but there is a maximum of 6/7 for those working in degrees. (May choose smaller interval between 0.2 and 0.4 e.g. f(0.3) and f(0.35) but this must span the root which is near to 0.316 in radians and 0.333 in degrees) **If they choose a larger interval then this is M0**
A1: Both their values correct to at least one decimal place, **and** reason given (e.g. change of sign or f(0.2)<0, f(0.4)>0 or product f(0.2)f(0.4)<0 or equivalent) **and** conclusion e.g. root
(b) **B1:** Starts with equation equal to zero, rearranges correctly with **no errors** and at least one intermediate step

(c) **M1**:Substitutes $x_0=0.3$ into $x=\frac{2}{3}-\frac{1}{3\cos x} \Rightarrow x_1=$

This can be implied by $x_1=\frac{2}{3}-\frac{1}{3\cos 0.3}$, or answers which round to 0.32 (rads) or 0.33 (degrees)

A1: x_1 awrt 0.3177 4dp (rads) or to awrt 0.3333 4dp (degrees)
Mark as the first value given. Don't be concerned by the subscript
A1: $x_2=$ awrt 0.3158, $x_3=$ awrt 0.3160 (rads) – NOT just 0.316
NB $x_2=$ awrt 0.3333, $x_3=$ awrt 0.3333 (degrees). **This mark is A0.** They cannot score **A1** if working in degrees
Mark the second and third values given. Don't be concerned by the subscripts Ignore extra values.
(d) **B1**: 0.316 stated to 3dp (independent of part (c)) for radians or 0.333 for degrees
The whole answer must maintain consistent units – either degrees, or radians. Use answer to (c) to determine units being used. NB Degree answers have maximum of M1A1B1M1A1A0B1 ie 6/7

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2.

$$f(x) = \frac{15}{3x+4} - \frac{2x}{x-1} + \frac{14}{(3x+4)(x-1)}, \quad x > 1$$

(a) Express $f(x)$ as a single fraction in its simplest form.

(4)

(b) Hence, or otherwise, find $f'(x)$, giving your answer as a single fraction in its simplest form.

(3)

Question Number	Scheme	Marks
2. **(a)**	Use of common denominator e.g. $\frac{15}{3x+4}-\frac{2x}{x-1}+\frac{14}{(3x+4)(x-1)}=\frac{15(x-1)-2x(3x+4)+14}{(3x+4)(x-1)}$	M1
	$=\frac{-6x^2+7x-1}{(3x+4)(x-1)}$	A1
	$=\frac{-(6x-1)\cancel{(x-1)}}{(3x+4)\cancel{(x-1)}}$	M1
	$=\frac{(1-6x)}{(3x+4)}$ or $=\frac{(-6x+1)}{(3x+4)}$ or $=-\frac{(6x-1)}{(3x+4)}$ o.e.	A1 (4)
First Alternative for (a)	$\frac{15}{3x+4}-\frac{2x}{x-1}+\frac{14}{(3x+4)(x-1)}=\frac{15}{3x+4}+\frac{-2x(3x+4)+14}{(3x+4)(x-1)}$	M1
	$=\frac{15}{3x+4}+\frac{-6x^2-8x+14}{(3x+4)(x-1)}$	A1
	$=\frac{15}{3x+4}+\frac{-2\cancel{(x-1)}(3x+7)}{(3x+4)\cancel{(x-1)}}$	M1
	$=\frac{(1-6x)}{(3x+4)}$ or $=\frac{(-6x+1)}{(3x+4)}$ or $=-\frac{(6x-1)}{(3x+4)}$ o.e.	A1 (4)
Second Alternative for (a)	$\frac{15}{3x+4}-\frac{2x}{x-1}+\frac{14}{(3x+4)(x-1)}=\frac{15(3x+4)(x-1)-2x(3x+4)^2+14(3x+4)}{(3x+4)^2(x-1)}$	M1
	$=\frac{(3x+4)(-6x^2+7x-1)}{(3x+4)^2(x-1)}$ or $=\frac{(x-1)(-18x^2-21x+4)}{(3x+4)^2(x-1)}$	A1
	$=\frac{\cancel{(3x+4)}^2(1-6x)}{\cancel{(3x+4)}^2(x-1)}$, $=\frac{(1-6x)}{(3x+4)}$ or $=\frac{(-6x+1)}{(3x+4)}$ or $=-\frac{(6x-1)}{(3x+4)}$ o.e.	M1, A1 (4)
(b)	$f'(x)=\frac{(3x+4)\times(-6)-(1-6x)\times 3}{(3x+4)^2}$	M1 A1ft
	$=\frac{-27}{(3x+4)^2}$	A1cao (3)
Alternative for (b)	Or $f'(x)=(3x+4)^{-1}\times(-6)+(1-6x)\times(-3)\times(3x+4)^{-2}$	M1 A1ft
	$=\frac{-27}{(3x+4)^2}$	A1cao (3)
Second Alternative for (b)	Or $f(x)=-2+\frac{9}{3x+4}$ so $f'(x)=9\times(-3)\times(3x+4)^{-2}$ $=\frac{-27}{(3x+4)^2}$	M1 A1ft A1cao (3)

Question Number	Scheme	Marks
Third Alternative for (b)	Differentiates original expression: $\frac{-45}{(3x+4)^2}-\left[\frac{2(x-1)-2x}{(x-1)^2}\right]+\frac{-14(6x+1)}{(3x+4)^2(x-1)^2}$	M1 A1
	$=\frac{-27}{(3x+4)^2}$	A1cao **[7]**

Notes

(a) **M**1: Combines two or three fractions into single fraction with **correct** use of common denominator
A1: correct answer with collected terms giving three term quadratic numerator
M1: Factorises their quadratic following usual rules in numerator:
A1 cao (but may be written in different ways – see m-s above)

(b) **M1**: Applies product or quotient rule correctly to **their** fraction (must have x terms in numerator and denominator of their answer to (a) which may be linear, quadratic, or even cubic; **not just constant numerator**) but it should be clear that they are using the correct rule with correct signs and correct term squared (in the case of quotient rule) i.e. using $\frac{vu'-uv'}{v^2}$ and states $u=\ ,\ v=\ ,\frac{du}{dx}=\ ,\frac{dv}{dx}=$ or an answer of the form $\frac{(3x+4)\times A-(1-6x)\times B}{(3x+4)^2}$ implies the method.
Similarly for the product rule : If the formula is quoted it must be correct. There must have been some attempt to differentiate both terms.
If the rule is not quoted nor implied by their working, meaning that term are written out $u="1-6x", v=("3x+4")^{-1}, u'=.., v'=...$ followed by their $vu'+uv'$, then only accept answers of the form $("3x+4")^{-1}\times A\pm"3"("3x+4")^{-2}\times B$.
Condone invisible brackets for the M mark.
For the **third alternative method**, need an attempt at all three differentiations in line with the guidance above. (N.B. the first A1 is not ft for this method).

A1ft: may be **unsimplified** e.g. $\frac{(3x+4)\times(-6)-(1-6x)\times 3}{(3x+4)^2}$ but should be correct for their answer to (a)

A1: correct **simplified** cao but accept $=\frac{-27}{9x^2+24x+16}$, as alternative

So a wrong answer in (a) can only achieve a maximum mark of M1A1A0 in part (b)

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3. (a) By writing $\operatorname{cosec} x$ as $\dfrac{1}{\sin x}$, show that

$$\frac{\mathrm{d}(\operatorname{cosec} x)}{\mathrm{d}x} = -\operatorname{cosec} x \cot x$$

(3)

Given that $y = \mathrm{e}^{3x} \operatorname{cosec} 2x,\ 0 < x < \dfrac{\pi}{2}$,

(b) find an expression for $\dfrac{\mathrm{d}y}{\mathrm{d}x}$.

(3)

The curve with equation $y = \mathrm{e}^{3x} \operatorname{cosec} 2x$, $0 < x < \dfrac{\pi}{2}$, has a single turning point.

(c) Show that the x-coordinate of this turning point is at $x = \dfrac{1}{2}\arctan k$ where the value of the constant k should be found.

(2)

Question Number	Scheme	Marks
3 (a)	Let $y = (\sin x)^{-1}$, then $\dfrac{\mathrm{d}y}{\mathrm{d}x} = -1(\sin x)^{-2} \times \cos x$	M1 A1
	i.e. $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{-1}{\sin x} \times \dfrac{\cos x}{\sin x} = -\operatorname{cosec} x \cot x$ *	B1* (3)
Alternative Method (a)	Use of quotient rule $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{\sin x \times 0 - 1\cos x}{\sin^2 x}$	M1A1
	i.e. $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{-1}{\sin x} \times \dfrac{\cos x}{\sin x} = -\operatorname{cosec} x \cot x$ *	B1* (3)
(b)	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = 3\mathrm{e}^{3x} \operatorname{cosec} 2x + \mathrm{e}^{3x} \times -2\operatorname{cosec} 2x \cot 2x$	M1 A1 A1 (3)
(c)	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{e}^{3x} \operatorname{cosec} 2x(3 - 2\cot 2x) = 0$	M1
	(So cot $2x$ = 1.5) tan $2x$= 2/3 so $x = \dfrac{1}{2}\arctan\dfrac{2}{3}$ (or k = 2/3)	A1 (2) **[8]**

Notes

(a) **M1**: Use of chain rule so $\dfrac{\mathrm{d}y}{\mathrm{d}x} = -1(\sin x)^{-2} \times (\pm\cos x)$

A1: cao

B1: Use of definitions of cosec$x$ and cot$x$ and conclusion, with no errors (need at least intermediate step shown in scheme which may be written $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{-\cos x}{\sin x \sin x}$). This mark is dependent on the M1.

Alternative: **M1**: If quotient rule is used need to see $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{\sin x \times 0 - 1(\pm\cos x)}{\sin^2 x}$, then **A1** is cao

(b) **M1**: If the rule is not quoted nor implied by their working, meaning that terms are written out $u = e^{3x}, v = \cos ec2x, u' = .., v' = ...$followed by their $vu' + uv'$, then only accept answers of the form $\mu\mathrm{e}^{3x} \operatorname{cosec} 2x + \mathrm{e}^{3x} \times \lambda \operatorname{cosec} 2x \cot 2x$.

A1: one term correct, A1 both terms correct (need not simplify isw)

(c) **M1**: Puts $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 0$ and factorises or cancels by $\mathrm{e}^{3x} \operatorname{cosec} 2x$ concluding that $a \pm b\cot 2x = 0$ or $\cot 2x = \pm\dfrac{a}{b}$

A1: Draws correct conclusion $\dfrac{1}{2}\arctan\dfrac{2}{3}$ or k = 2/3

Leave blank

4. A pot of coffee is delivered to a meeting room at 11am. At a time t minutes after 11am the temperature, θ °C, of the coffee in the pot is given by the equation

$$\theta = A + 60e^{-kt}$$

where A and k are positive constants.

Given also that the temperature of the coffee at 11am is 85 °C and that 15 minutes later it is 58 °C,

(a) find the value of A.
(1)

(b) Show that $k = \dfrac{1}{15}\ln\left(\dfrac{20}{11}\right)$
(3)

(c) Find, to the nearest minute, the time at which the temperature of the coffee reaches 50 °C.
(4)

Question Number	Scheme	Marks
4.	(a) When $t = 0$, $\theta = 85$ so $85 = A + 60$, $A = 25$	B1 (1)
	(b) $58 = "25" + 60e^{-k15}$	
	$"33" = 60e^{-k15} \to e^{-k15} = \frac{"33"}{60}$ or $e^{k15} = \frac{60}{"33"}$	M1
	So $-15k = \ln\left(\frac{"11"}{20}\right)$ or $15k = \ln\left(\frac{20}{"11"}\right)$	M1
	$k = -\frac{1}{15}\ln\left(\frac{11}{20}\right) = \frac{1}{15}\ln\left(\frac{20}{11}\right)$ *	A1cso* (3)
	(c) $50 = "25" + 60e^{-kt} \to e^{-kt} =$ or $e^{kt} =$ or $(.96)^t =$	M1
	$(e^{-kt}) = \frac{25}{60}$ (or awrt 0.42) or $(0.96^t) = \frac{25}{60}$ or $(e^{kt}) = \frac{60}{25}$	A1
	$t = \frac{\ln\left(\frac{"25"}{60}\right)}{-k}$ or $t = \frac{\ln\left(\frac{60}{"25"}\right)}{k}$	M1
	$\frac{\ln\left(\frac{25}{60}\right)}{-\frac{1}{15}\ln\left(\frac{20}{11}\right)} = (21.96) =$ 22 mins (approx) or 11.22 or $t = 22$	A1 (4) **[8]**

Notes

(a) **B1**: Gives answer $A = 25$ – any work seen should be correct

(b) **M1**: Uses values 58 and 15 with their A to form equation in k and isolate $e^{-k15} =$ or $e^{k15} =$
M1: Uses logs correctly (*following correct log rules and only applying log to positive quantities)* with their value of A to find k . Need to see line shown in mark scheme.
A1cso: There needs to be a step between $-15k = \ln\left(\frac{"11"}{20}\right)$ and the printed answer. The printed answer needs to be stated. No errors should be seen reaching it. Use of decimals giving 0.03985 **as part of the proof** will result in A0
N.B. This proof must be seen in part (b) to be credited with marks in part (b).

(c) **M1**: Uses 50 with their A and makes their e^{-kt} subject
A1: correct numerical fraction (any correct form- if given as decimal accept awrt 0.42)[ignore LHS]
M1: Uses logs correctly then rearranges correctly to obtain $t = \frac{\ln\left(\frac{"25"}{60}\right)}{-k}$ (Allow 50 – their A instead of 25 in numerator)
A1: awrt 22 minutes. Accept 11.22 i.e. 24 hour clock or $t = 22$ or $t = 22$ minutes but not $t = 22$ degrees C.
Special case: A common error is to reach $0.96t = \frac{25}{60}$; this is a result of log errors- so allow M1A1M0A0
Another common error is to miscopy 15 as 5 (usually part way through the answer). Answer is usually 7.3 and this achieves M1A1M1A0

Leave blank

5.

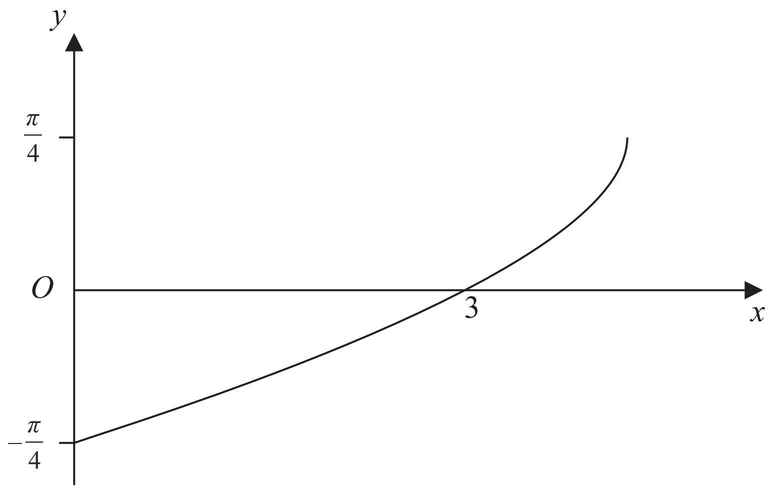

Figure 1

The curve shown in Figure 1 has equation

$$x = 3\sin y + 3\cos y, \qquad -\frac{\pi}{4} < y < \frac{\pi}{4}$$

(a) Express the equation of the curve in the form

$x = R\sin(y + \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < \dfrac{\pi}{2}$

(3)

(b) Find the coordinates of the point on the curve where the value of $\dfrac{dy}{dx}$ is $\dfrac{1}{2}$.

Give your answers to 3 decimal places.

(6)

Question Number	Scheme	Marks
5.	(a) $R\cos\alpha = 3, \quad R\sin\alpha = 3$	
	$R = 3\sqrt{2}$ or $\sqrt{18}$ or awrt 4.24	B1
	$\tan\alpha = 1, \quad \Rightarrow \quad \alpha = \frac{\pi}{4}$ or 0.785	M1, A1 (3)
	(b) $\left(\frac{dx}{dy}\right) = 3\cos y - 3\sin y$ or $3\sqrt{2}\cos(y + \frac{\pi}{4})$	M1 A1
	Puts $3\sqrt{2}\cos(y+\alpha) = 2$ | or puts $-3\sqrt{2}\sin(y-\alpha) = 2$	B1
	So $\cos(y+\alpha) = \frac{\sqrt{2}}{3}$ and $y = "1.0799" - \alpha$ | or $y = "-0.491" + \alpha$	M1
		A1 A1
	y= 0.295 and x = 3.742 (or 3.743)	(6) **[9]**

<u>**Notes**</u>

(a) **B1**: $(R = \sqrt{(3^2 + 3^2)}) = 3\sqrt{2}$ (accept $\pm 3\sqrt{2}$ but not just $-3\sqrt{2}$) No working need be seen. Accept decimal answers which round to 4.24.

M1: For $\tan\alpha = \pm\frac{3}{3}$ If R is used then accept $\sin\alpha = \pm\frac{3}{R}$ or $\cos\alpha = \pm\frac{3}{R}$

A1: Accept awrt 0.785 BUT 45 degrees is A0

(b) **M1**: Attempts differentiation (may be sign errors)

A1: correct in either form shown on scheme – answer is A0 if clearly in degrees.

B1: Obtains equation given in scheme, or $\frac{1}{3\sqrt{2}\cos(y+\alpha)} = \frac{1}{2}$, or equivalent. 3 cos$y$ – 3 sin y = 2 (without further work) is B0 but may be written as $-3\sqrt{2}\sin(y-\alpha) = 2$ which would be B1. **It may also be solved by “*t*” formulae (see below)**

M1: Allow **in degrees or radians** for $\arccos\left(\frac{\pm 2}{R}\right) \pm \alpha$ or for $\arcsin\left(\frac{\pm 2}{R}\right) \pm \alpha$ or for $\arccos\left(\frac{\pm\frac{1}{2}}{R}\right) \pm \alpha$ or for $\arcsin\left(\frac{\pm\frac{1}{2}}{R}\right) \pm \alpha$

A1: one correct answer – allow 3.742 or 3.743 following incorrect y value **A1**: two correct answers (Accept awrt in both cases)

Do not accept mixed units- unless recovery yields a correct final answer.

Special case: Candidate works solely in **degrees:** In part (a) max mark is B1M1A0 In part (b) they can have M1A1 for $\frac{dx}{dy} = 3\cos y - 3\sin y$ or M1A0 for $3\sqrt{2}\cos(y+45)$) then B1 is possible and M1 if solution is completed in degrees. The value for y in degrees is not appropriate but correct work in degrees may lead to correct value for x, so A1, A0 could be earned.

Ignore extra answer outside range. (PTO for little t formula method in part (b))

Question Number	Scheme	Marks
5. Alternative for last four marks in (b)	Contd "little t formula method"	
	(b) $\frac{\mathrm{d}x}{\mathrm{d}y} = 3\cos y - 3\sin y$ (as before)	M1A1
	$3\cos y - 3\sin y = 2$ so $3\frac{1-t^2}{1+t^2} - 3\frac{2t}{1+t^2} = 2$	B1
	Attempt to solve $5t^2 + 6t - 1 = 0$ and use $y = 2 \times \arctan"0.148"$	M1
	y= 0.295 and x = 3.742 (or 3.743)	A1 A1 (6) **[9]**

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6. Given that a and b are constants and that $0 < a < b$,

(a) on separate diagrams, sketch the graph with equation

(i) $y = |2x + a|$,

(ii) $y = |2x + a| - b$.

Show on each sketch the coordinates of each point at which the graph crosses or meets the axes.

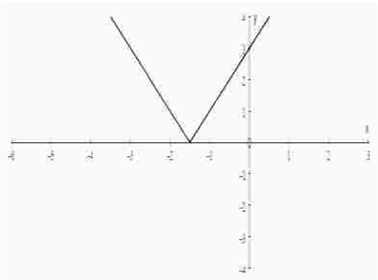
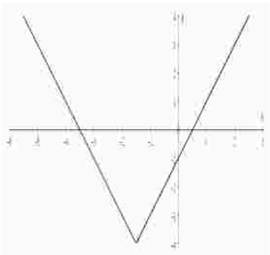
(6)

(b) Solve, for x, the equation

$$|2x + a| - b = \frac{1}{3}x$$

giving any answers in terms of a and b.

(4)

Question Number	Scheme		Marks
6.	(a) (i) 	V shape in correct position i.e. touches –ve x – axis as shown	B1
		(-a/2,0) **and** (0, a)	B1
	(ii) 	Translation down of previous V shape ft or correct position if starts again	B1 ft
		((b –a) /2, 0) and (-(a + b)/2, 0)	B1, B1
		Completely correct graph with y intercept at (0, a - b)	B1 (6)
	(b) $(2x+a)-b=\frac{1}{3}x \rightarrow "\frac{5}{3}x=b-a"$		M1
	So $x=\frac{3}{5}(b-a)$		A1
	And $-(2x+a)-b=\frac{1}{3}x \rightarrow -2x-\frac{1}{3}x=a+b$		M1
	So $x = -\frac{3}{7}(a+b)$		A1
			(4) **[10]**

<u>Notes</u>

(a) (i) **B1**: V shape correct orientation and position. Could be a tick shape (i.e. not whole of V)
B1: (-a/2, 0) **and** (0, a) accept –a/2 and a marked on the correct axes or even (0, -a/2) on x axis and (a, 0) on y axis
There must be a graph for these marks to be awarded in part (a).

(ii) **B1ft:** Translation down of previous V shaped graph by any amount (may be in wrong position) **or** correct V in correct position if candidate starts again and does not relate this to their graph in part (a)
B1: one x coordinate correct **B1**: both correct (may be shown on x axis as ((b –a) /2) and (-(a + b)/2) or even (0, (b –a) /2) and (0, -(a + b)/2)). (May be shown on wrong parts of x – axis, or interchanged) Marks may be given for **correct** coordinates **i.e.** ((b –a) /2, 0) and (-(a + b)/2, 0) **without graph**.
B1: The graph must be completely correct. Intercept m**ust be on negative *y* axis and there should be two *x*-intercepts, one positive and one negative. The** y coordinate must be correct (may be shown on y axis as a – b or even (a – b, 0))

(b) **M1**:Attempts first +ve solution correctly using (2x + a) and obtains equation with multiple of x only on LHS
A1: any equivalent to $x=\frac{3}{5}(b-a)$ e.g. $x=\frac{3}{5}b-\frac{3}{5}a$
M1: Attempts second –ve solution **correctly** using - (2x + a) and obtains equation with multiple of x on LHS
A1: any equivalent to $x = -\frac{3}{7}(a+b)$ e.g. $x=-\frac{3}{7}a-\frac{3}{7}b$

Leave blank

7. (i) (a) Prove that

$$\cos 3\theta \equiv 4\cos^3\theta - 3\cos\theta$$

(You may use the double angle formulae and the identity
$\cos(A + B) \equiv \cos A \cos B - \sin A \sin B$)

(4)

(b) Hence solve the equation

$$2\cos 3\theta + \cos 2\theta + 1 = 0$$

giving answers in the interval $0 \leqslant \theta \leqslant \pi$.
Solutions based entirely on graphical or numerical methods are not acceptable.

(6)

(ii) Given that $\theta = \arcsin x$ and that $0 < \theta < \frac{\pi}{2}$, show that

$$\cot\theta = \frac{\sqrt{(1 - x^2)}}{x}, \quad 0 < x < 1$$

(3)

Question Number	Scheme	Marks
7 (i)(a)	$\cos 3\theta = \cos 2\theta \cos\theta - \sin 2\theta \sin\theta$	M1
	$= (2\cos^2\theta - 1)\cos\theta - 2\sin\theta\cos\theta\sin\theta$	M1
	$= 2\cos^3\theta - \cos\theta - 2(1-\cos^2\theta)\cos\theta$	dM1
	$= 4\cos^3\theta - 3\cos\theta$ *	A1 * (4)
(b)	$8\cos^3\theta - 6\cos\theta + (2\cos^2\theta - 1) + 1 = 0$	M1
	$8\cos^3\theta + 2\cos^2\theta - 6\cos\theta = 0$	A1
	$2\cos\theta(4\cos\theta - 3)(\cos\theta + 1) = 0$ so $\cos\theta =$	dM1
	$\cos\theta = \frac{3}{4}$ (or 0 or -1)	A1
	$\theta = 0.723$ and no extra answers in range , or $\theta = \frac{\pi}{2}$ and π (or 90° and 180°)	A1, B1 (6)
(ii)	($\sin\theta = x$ and so) $\cos\theta = \sqrt{(1-x^2}$ — Or uses right angled triangle with sides 1, x and $\sqrt{(1-x^2}$	M1
	$(\cot\theta) = \dfrac{\cos\theta}{\sin\theta}$ — Indicates θ on diagram and implies $(\cot\theta) = \dfrac{adjacent}{opposite}$	M1
	$= \dfrac{\sqrt{(1-x^2)}}{x}$ *	A1* (3)
		[13]

Notes

(i) (a) **M1**: **Correct** statement for $\cos 3\theta$ as shown using compound angle formula

M1: Uses **correct** double angle formulae for $\sin 2\theta$ and $\cos 2\theta$ (any of the three) – allow invisible brackets

dM1: (dependent on both previous Ms). Uses $\sin^2\theta = (1-\cos^2\theta)$ o.e. to replace all sin terms by cos terms

A1: deduces result with no errors- allow recovery from invisible brackets or from occasional missing θ – need all 3 M marks

(b) **M1**: Replaces $\cos 3\theta$ and $\cos 2\theta$ by expression from (a) and by attempt at double angle formula resulting in expression in cosine only – may do this in one or several steps – allow slips

$8\cos^3\theta - 6\cos\theta + (\cos^2\theta - \sin^2\theta) + 1 = 0$ is not yet enough for M mark- but

$8\cos^3\theta - 6\cos\theta + (\cos^2\theta - (1-\cos^2\theta) + 1 = 0$ would get M1 but not yet the A mark

A1: correct cubic shown with 3 terms

d**M1**: Solves by any valid method (factorising, formula, completion of square or calculator or implied by 3/4) to give at least one non zero value for $\cos\theta =$ **A1**: for ¾

A1: 0.723 or answers which round to this and no extra answers in range. Do not accept degrees.

B1: for $\theta = \dfrac{\pi}{2}$ and π (allow decimals to 3sf 3.14 and 1.57 or degrees)

(ii) **M1:** States $\cos\theta = \sqrt{(1-x^2)}$, **or** see right angled triangle with sides 1, x and $\sqrt{(1-x^2)}$

M1: Implies $(\cot\theta) = \dfrac{\cos\theta}{\sin\theta}$ - not $(\cot\theta) = \dfrac{\cos}{\sin}\theta$ nor $(\cot\theta) = \dfrac{\cos}{\sin}$

or indicates angle on diagram and implies $(\cot\theta) = adjacent \div opposite$

A1: Clear explanation No errors, printed answer achieved. Needs both M marks

Leave blank

8. The function f is defined by

$$\mathrm{f}: x \to 3 - 2\mathrm{e}^{-x}, \qquad x \in \mathbb{R}$$

(a) Find the inverse function, $\mathrm{f}^{-1}(x)$ and give its domain.

(5)

(b) Solve the equation $\mathrm{f}^{-1}(x) = \ln x$.

(4)

The equation $\mathrm{f}(t) = k\mathrm{e}^{t}$, where k is a positive constant, has exactly one real solution.

(c) Find the value of k.

(4)

Question Number	Scheme		Marks
8. (a)	Let $y = 3-2e^{-x}$, then $2e^{-x} = 3-y$	*Or* $2e^{-y} = 3-x$	M1
	$-x = \ln\frac{3-y}{2}$ and $x =$	$-y = \ln\frac{3-x}{2}$ and $y =$	M1
	$x = -\ln\frac{3-y}{2}$	This mark earned with next for correct answer by this method	A1
	$\cancel{f^{-1}(x)} = \ln\frac{2}{3-x}$ or $-\ln\frac{3-x}{2}$ o.e.		A1
	Domain is $x < 3$		B1 (5)
(b)	$\ln\frac{2}{3-x} = \ln x \rightarrow 2 = (3-x)x$		M1
	$x^2 - 3x + 2 = 0$		A1
	$x = 2$ or $x = 1$		M1 A1 (4)
(c)	$3-2e^{-t} = ke^{t} \rightarrow ke^{2t} - 3e^{t} + 2 = 0$ or $\rightarrow ke^{2t} - 3e^{t} = -2$ o.e. (isw)		M1 A1
	Use "$b^2 - 4ac = 0$" or "$b^2 = 4ac$" or attempts $e^{t} = \frac{3 \pm \sqrt{9-8k}}{2k}$		dM1
	So $k = 1.125$ o.e. e.g. $\frac{9}{8}$ or $1\frac{1}{8}$		A1 (4) **[13]**

Notes

(a) **M1**: Puts $y = f(x)$ and makes e^{-x} term subject of formula so $2e^{-x} = 3-y$ or $e^{-x} = \frac{3-y}{2}$ or even

$-2e^{-x} = y-3$ or $-e^{-x} = \frac{y-3}{2}$ - allow sign slips. Allow $f(x)$ instead of y in expression for both Ms

M1: Uses ln to get $x =$ (This mark is for knowing that ln$x$ is inverse of e^{x} so allow sign errors and weak log work. These errors will be penalised in the A mark.)
A1: completely correct log work giving a correct unsimplified answer for $x =$ (then isw for this mark)
A1: any correct answer - do not need to see LHS of equation but variable **must** be x not y

NB Possible answers include $\frac{\log\frac{2}{3-x}}{\log e}$, $-\ln(3-x) + \ln 2$, $-\ln\left(-\frac{1}{2}x + \frac{3}{2}\right)$, or $\ln\frac{-2}{x-3}$ etc

If x and y interchanged at start – see alternative in scheme. Note this method gives A1A1 or A0A0

B1: For $x < 3$ (independent mark); allow $(-\infty, 3)$, but $x \le 3$ is B0

(b) **M1**: Removes ln correctly on both sides and multiplies across
A1: expands bracket to give three term quadratic equation, allow $x^2 - 3x = -2$
M1: Solves quadratic (may be implied by answers)
A1: Need both these correct answers

(c) **M1**: Sets $3-2e^{-t} = ke^{t}$ and attempts to multiply all terms by e^{t} or by e^{-t} (allow use of x instead of t)
A1: three term quadratic – allow x or t so $ke^{2x} - 3e^{x} + 2 = 0$ or $ke^{2x} - 3e^{x} = -2$ or $k = 3e^{-t} - 2e^{-2t}$ etc
dM1: Uses condition for equal roots to give expression in k – may not be simplified- or attempts to solve their quadratic equation in e^{t} using formula or completion of the square **A1**: See scheme