Centre No.						Paper Reference							Surname	Initial(s)
Candidate No.						6	6	6	5	/	0	1	Signature	

Paper Reference(s)

6665/01

Edexcel GCE

Core Mathematics C3

Advanced Level

Monday 20 June 2005 – Morning

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
1	
2	
3	
4	
5	
6	
7	
Total	

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Green)	Nil

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initial(s) and signature.
Check that you have the correct question paper.
You must write your answer for each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 7 questions in this question paper. The total mark for this paper is 75.
There are 20 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

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1. (a) Given that $\sin^2\theta + \cos^2\theta \equiv 1$, show that $1 + \tan^2\theta \equiv \sec^2\theta$.

(2)

(b) Solve, for $0 \leqslant \theta < 360°$, the equation

$$2\tan^2\theta + \sec\theta = 1,$$

giving your answers to 1 decimal place.

(6)

6665 Core C3

Mark Scheme (Post standardisation)

Question Number	Scheme	Marks
1 (a)	Dividing by $\cos^2\theta$: $\dfrac{\sin^2\theta}{\cos^2\theta} + \dfrac{\cos^2\theta}{\cos^2\theta} \equiv \dfrac{1}{\cos^2\theta}$	M1
	Completion : $1 + \tan^2\theta \equiv \sec^2\theta$ (no errors seen)	A1 (2)
(b)	Use of $1 + \tan^2\theta = \sec^2\theta$: $2(\sec^2\theta - 1) + \sec\theta = 1$	M1
	[$2\sec^2\theta + \sec\theta - 3 = 0$]	
	Factorising or solving: $(2\sec\theta + 3)(\sec\theta - 1) = 0$	M1
	[$\sec\theta = -\frac{3}{2}$ or $\sec\theta = 1$]	
	$\theta = 0$	B1
	$\cos\theta = -\frac{2}{3}$; $\theta_1 = 131.8°$	M1 A1
	$\theta_2 = 228.2°$	A1√ (6)
	[A1ft for $\theta_2 = 360° - \theta_1$]	
		[8]

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2. (a) Differentiate with respect to x

(i) $3\sin^2 x + \sec 2x$,

(3)

(ii) $\{x + \ln(2x)\}^3$.

(3)

Given that $y = \dfrac{5x^2 - 10x + 9}{(x-1)^2}$, $x \neq 1$,

(b) show that $\dfrac{dy}{dx} = -\dfrac{8}{(x-1)^3}$.

(6)

6665 Core C3
Mark Scheme (Post standardisation)

Question Number	Scheme	Marks
2 (a)	(i) $6\sin x\cos x + 2\sec 2x\tan 2x$	M1A1A1 (3)
	or $3\sin 2x + 2\sec 2x\tan 2x$ [M1 for 6 sin *x*]	
(ii)	$3(x + \ln 2x)^2(1 + \frac{1}{x})$	B1M1A1 (3)
	[B1 for $3(x+\ln 2x)^2$]	
(b)	Differentiating numerator to obtain $10x - 10$	
	Differentiating denominator to obtain $2(x-1)$	
	Using quotient rule formula correctly:	
	To obtain $\frac{dy}{dx} = \frac{(x-1)^2(10x-10) - (5x^2 - 10x + 9)2(x-1)}{(x-1)^4}$	
	Simplifying to form $\frac{2(x-1)[5(x-1)^2 - (5x^2 - 10x + 9)}{(x-1)^4}$	
	$= -\frac{8}{(x-1)^3}$ * (c.s.o.)	
3 (a)	$\frac{5x+1}{(x+2)(x-1)} - \frac{3}{x+2}$	B1
	$= \frac{5x + 1 - 3(x-1)}{(x+2)(x-1)}$	M1
	M1 for combining fractions even if the denominator is not lowest common	
	$= \frac{2x+4}{(x+2)(x-1)} = \frac{2(x+2)}{(x+2)(x-1)} = \frac{2}{x-1}$ *	M1 A1 cso (4)
	M1 must have linear numerator	
(b)	$y = \frac{2}{x-1} \Rightarrow xy - y = 2 \Rightarrow xy = 2 + y$	M1A1
	$f^{-1}(x) = \frac{2+x}{x}$ o.e.	A1 (3)
	$fg(x) = \frac{2}{x^2+4}$ (attempt) $[\frac{2}{"g"-1}]$	M1
	Setting $\frac{2}{x^2+4} = \frac{1}{4}$ and finding $x^2 = \ldots$; $x = \pm 2$	M1; A1 (3)
		[10]

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3. The function f is defined by

$$\mathrm{f}: x \rightarrow \frac{5x+1}{x^2+x-2} - \frac{3}{x+2}, \quad x > 1.$$

(a) Show that $\mathrm{f}(x) = \dfrac{2}{x-1}, \quad x > 1.$

(4)

(b) Find $\mathrm{f}^{-1}(x)$.

(3)

The function g is defined by

$$\mathrm{g}: x \rightarrow x^2 + 5, \quad x \in \mathbb{R}.$$

(c) Solve $\mathrm{fg}(x) = \frac{1}{4}$.

(3)

6665 Core C3
Mark Scheme (Post standardisation)

Question Number	Scheme	Marks
2 (a)	(i) $6\sin x\cos x + 2\sec 2x\tan 2x$	M1A1A1 (3)
	or $3\sin 2x + 2\sec 2x\tan 2x$ [M1 for 6 sin *x*]	
(ii)	$3(x+\ln 2x)^2(1+\frac{1}{x})$	B1M1A1 (3)
	[B1 for $3(x+\ln 2x)^2$]	
(b)	Differentiating numerator to obtain $10x - 10$	
	Differentiating denominator to obtain $2(x-1)$	
	Using quotient rule formula correctly:	
	To obtain $\frac{dy}{dx} = \frac{(x-1)^2(10x-10)-(5x^2-10x+9)2(x-1)}{(x-1)^4}$	
	Simplifying to form $\frac{2(x-1)[5(x-1)^2-(5x^2-10x+9)}{(x-1)^4}$	
	$= -\frac{8}{(x-1)^3}$ * (c.s.o.)	
3 (a)	$\frac{5x+1}{(x+2)(x-1)} - \frac{3}{x+2}$	B1
	$= \frac{5x+1-3(x-1)}{(x+2)(x-1)}$	M1
	M1 for combining fractions even if the denominator is not lowest common	
	$= \frac{2x+4}{(x+2)(x-1)} = \frac{2(x+2)}{(x+2)(x-1)} = \frac{2}{x-1}$ *	M1 A1 cso (4)
	M1 must have linear numerator	
(b)	$y = \frac{2}{x-1} \Rightarrow xy - y = 2 \Rightarrow xy = 2 + y$	M1A1
	$f^{-1}(x) = \frac{2+x}{x}$ o.e.	A1 (3)
	$fg(x) = \frac{2}{x^2+4}$ (attempt) $[\frac{2}{"g"-1}]$	M1
	Setting $\frac{2}{x^2+4} = \frac{1}{4}$ and finding $x^2 = \ldots$; $x = \pm 2$	M1; A1 (3)
		[10]

Leave blank

4. $$f(x) = 3e^x - \tfrac{1}{2}\ln x - 2, \quad x > 0.$$

(a) Differentiate to find $f'(x)$.

(3)

The curve with equation $y = f(x)$ has a turning point at P. The x-coordinate of P is α.

(b) Show that $\alpha = \tfrac{1}{6}e^{-\alpha}$.

(2)

The iterative formula

$$x_{n+1} = \tfrac{1}{6}e^{-x_n}, \quad x_0 = 1,$$

is used to find an approximate value for α.

(c) Calculate the values of x_1, x_2, x_3 and x_4, giving your answers to 4 decimal places.

(2)

(d) By considering the change of sign of $f'(x)$ in a suitable interval, prove that $\alpha = 0.1443$ correct to 4 decimal places.

(2)

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6665 Core C3
Mark Scheme (Post standardisation)

Question Number	Scheme	Marks
4 (a)	$f'(x) = 3e^x - \frac{1}{2x}$	M1A1A1 (3)
	$3e^x - \frac{1}{2x} = 0$	M1
	$\Rightarrow 6\alpha e^{\alpha} = 1 \quad \Rightarrow \alpha = \frac{1}{6}e^{-\alpha}$ (*)	A1 cso (2)
(c)	$x_1 = 0.0613...,\ x_2 = 0.1568..,\ x_3 = 0.1425...,\ x_4 = 0.1445....$	M1 A1 (2)
	[M1 at least x_1 correct, A1 all correct to 4 d.p.]	
(d)	Using $f'(x) = 3e^x - \frac{1}{2x}$ with suitable interval	
	e.g. $f'(0.14425) = -0.0007$	M1
	$f'(0.14435) = +0.002(1)$	
	Accuracy (change of sign and correct values)	A1 (2)
		[9]

Leave blank

5. (a) Using the identity $\cos(A + B) \equiv \cos A \cos B - \sin A \sin B$, prove that

$$\cos 2A \equiv 1 - 2\sin^2 A.$$

(2)

(b) Show that

$$2\sin 2\theta - 3\cos 2\theta - 3\sin\theta + 3 \equiv \sin\theta\,(4\cos\theta + 6\sin\theta - 3).$$

(4)

(c) Express $4\cos\theta + 6\sin\theta$ in the form $R\sin(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$.

(4)

(d) Hence, for $0 \leqslant \theta < \pi$, solve

$$2\sin 2\theta = 3(\cos 2\theta + \sin\theta - 1),$$

giving your answers in radians to 3 significant figures, where appropriate.

(5)

6665 Core C3
Mark Scheme (Post standardisation)

Question Number	Scheme	Marks
5 (a)	$\cos 2A = \cos^2 A - \sin^2 A$ (+ use of $\cos^2 A + \sin^2 A \equiv 1$)	M1
	$= (1 - \sin^2 A); - \sin^2 A = 1 - 2\sin^2 A$ (*)	A1 (2)
(b)	$2\sin 2\theta - 3\cos 2\theta - 3\sin\theta + 3 \equiv 4\sin\theta\cos\theta; -3(1-2\sin^2\theta) - 3\sin\theta + 3$	B1; M1
	$\equiv 4\sin\theta\cos\theta + 6\sin^2\theta - 3\sin\theta$	M1
	$\equiv \sin\theta(4\cos\theta + 6\sin\theta - 3)$ (*)	A1 (4)
(c)	$4\cos\theta + 6\sin\theta \equiv R\sin\theta\cos\alpha + R\cos\theta\sin\alpha$	
	Complete method for R (may be implied by correct answer)	
	[$R^2 = 4^2 + 6^2$, $R\sin\alpha = 4$, $R\cos\alpha = 6$]	M1
	$R = \sqrt{52}$ or 7.21	A1
	Complete method for α ; $\alpha = 0.588$ (allow 33.7º)	M1 A1 (4)
(d)	$\sin\theta\ (4\cos\theta + 6\sin\theta - 3) = 0$	M1
	$\theta = 0$	B1
	$\sin(\theta + 0.588) = \frac{3}{\sqrt{52}} = 0.4160..$ (24.6º)	M1
	$\theta + 0.588 = (0.4291),\ 2.7125$ [or $\theta + 33.7° = (24.6º),\ 155.4°$]	dM1
	$\theta = 2.12$ cao	A1 (5)
		[15]

Leave blank

6.

Figure 1

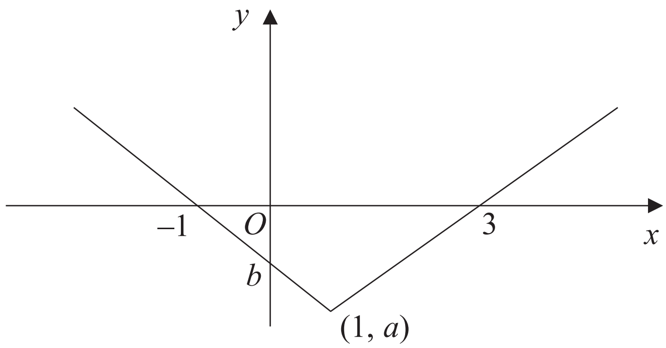


Figure 1 shows part of the graph of $y = \mathrm{f}(x)$, $x \in \mathbb{R}$. The graph consists of two line segments that meet at the point $(1, a)$, $a < 0$. One line meets the x-axis at $(3, 0)$. The other line meets the x-axis at $(–1, 0)$ and the y-axis at $(0, b)$, $b < 0$.

In separate diagrams, sketch the graph with equation

(a) $y = \mathrm{f}(x + 1)$,

(2)

(b) $y = \mathrm{f}(|x|)$.

(3)

Indicate clearly on each sketch the coordinates of any points of intersection with the axes.

Given that $\mathrm{f}(x) = |x – 1| – 2$, find

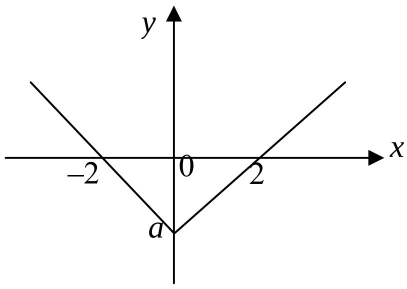
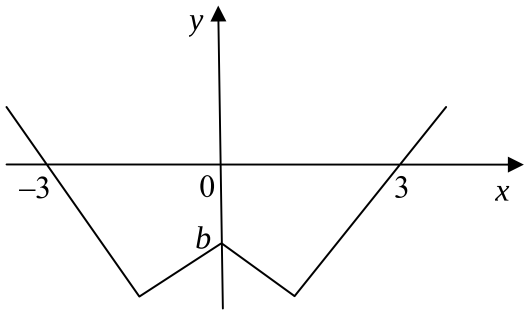
(c) the value of a and the value of b,

(2)

(d) the value of x for which $\mathrm{f}(x) = 5x$.

(4)

6665 Core C3
Mark Scheme (Post standardisation)

Question Number	Scheme	Marks
6. (a)	 Translation ← by 1	M1
	Intercepts correct	A1 (2)
(b)	 $x \geq 0$, correct "shape"	B1
	[provided not just original]	
	Reflection in y-axis	B1√
	Intercepts correct	B1 (3)
(c)	$a = -2$, $b = -1$	B1 B1 (2)
(d)	Intersection of $y = 5x$ with $y = -x - 1$	M1A1
	Solving to give $x = -\frac{1}{6}$	M1A1 (4)
		[11]
	[Notes: (i) If both values found for $5x = -x - 1$ and $5x = x - 3$, or solved algebraically, can score 3 out of 4 for $x = -\frac{1}{6}$ and $x = -\frac{3}{4}$; required to eliminate $x = -\frac{3}{4}$ for final mark. (ii) Squaring approach: M1 correct method, $24x^2 + 22x + 3 = 0$ (correct 3 term quadratic, any form) A1 Solving M1, Final correct answer A1.]	

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7. A particular species of orchid is being studied. The population p at time t years after the study started is assumed to be

$$p = \frac{2800a\,e^{0.2t}}{1 + a\,e^{0.2t}}, \text{ where } a \text{ is a constant.}$$

Given that there were 300 orchids when the study started,

(a) show that $a = 0.12$,

(3)

(b) use the equation with $a = 0.12$ to predict the number of years before the population of orchids reaches 1850.

(4)

(c) Show that $p = \dfrac{336}{0.12 + e^{-0.2t}}$.

(1)

(d) Hence show that the population cannot exceed 2800.

(2)

6665 Core C3
Mark Scheme (Post standardisation)

Question Number	Scheme	Marks
7 (a)	Setting $p = 300$ at $t = 0 \Rightarrow \; 300 = \dfrac{2800a}{1+a}$	M1
	$(300 = 2500a)$; $\quad a = 0.12$ (c.s.o) *	dM1A1 (3)
(b)	$1850 = \dfrac{2800(0.12)e^{0.2t}}{1+0.12e^{0.2t}}$; $\quad e^{0.2t} = 16.2...$	M1A1
	Correctly taking logs to $0.2\,t = \ln k$	M1
	$t = 14 \quad (13.9..)$	A1 (4)
(c)	Correct derivation:	
	(Showing division of num. and den. by $e^{0.2t}$; using a)	B1 (1)
(d)	Using $t \to \infty$, $e^{-0.2t} \to 0$,	M1
	$p \to \dfrac{336}{0.12} = 2800$	A1 (2)
		[10]