Centre No.						Paper Reference							Surname	Initial(s)
Candidate No.						6	6	6	5	/	0	1	Signature	

Paper Reference(s)

6665/01
Edexcel GCE
Core Mathematics C3
Advanced Level

Monday 12 June 2006 – Afternoon

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
1	
2	
3	
4	
5	
6	
7	
8	
Total	

Materials required for examination	Items included with question papers
Mathematical Formulae (Green)	Nil

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initial(s) and signature.
Check that you have the correct question paper.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.
You must write your answer for each question in the space following the question.
If you need more space to complete your answer to any question, use additional answer sheets.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
Full marks may be obtained for answers to ALL questions.
There are 8 questions in this question paper. The total mark for this paper is 75.
There are 24 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

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1. (a) Simplify $\dfrac{3x^2 - x - 2}{x^2 - 1}$.

(3)

(b) Hence, or otherwise, express $\dfrac{3x^2 - x - 2}{x^2 - 1} - \dfrac{1}{x(x+1)}$ as a single fraction in its simplest form.

(3)

June 2006
6665 Core Mathematics C3
Mark Scheme

Question number	Scheme	Marks
1. (a)	$\dfrac{(3x+2)(x-1)}{(x+1)(x-1)}, \quad = \dfrac{3x+2}{x+1}$	M1B1, A1 (3)
	Notes M1 attempt to factorise numerator, *usual rules* B1 factorising denominator seen anywhere in (a), A1 given answer If factorisation of denom. not seen, correct answer implies B1	
(b)	Expressing over common denominator	
	$\dfrac{3x+2}{x+1} - \dfrac{1}{x(x+1)} = \dfrac{x(3x+2)-1}{x(x+1)}$	M1
	[Or "Otherwise" : $\dfrac{(3x^2-x-2)x-(x-1)}{x(x^2-1)}$]	
	Multiplying out numerator and attempt to factorise	M1
	$[3x^2+2x-1 \equiv (3x-1)(x+1)]$	
	Answer: $\dfrac{3x-1}{x}$	A1 (3)
		Total 6 marks
2. (*a*)	$\dfrac{dy}{dx} = 3e^{3x} + \dfrac{1}{x}$	B1M1A1 (3)
	Notes B1 $3e^{3x}$ M1 : $\dfrac{a}{bx}$ A1: $3e^{3x} + \dfrac{1}{x}$	
(b)	$(5+x^2)^{\frac{1}{2}}$	B1
	$\dfrac{3}{2}(5+x^2)^{\frac{1}{2}} \; . \, 2x \quad = 3x(5+x^2)^{\frac{1}{2}}$ M1 for $kx(5+x^2)^m$	M1 A1 (3)
		Total 6 marks

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2. Differentiate, with respect to x,

(a) $e^{3x} + \ln 2x$,

(3)

(b) $(5 + x^2)^{\frac{3}{2}}$.

(3)

Q2

(Total 6 marks)

June 2006
6665 Core Mathematics C3
Mark Scheme

Question number	Scheme	Marks
1. (a)	$\frac{(3x+2)(x-1)}{(x+1)(x-1)}, \quad = \frac{3x+2}{x+1}$	M1B1, A1 (3)
	Notes	
	M1 attempt to factorise numerator, *usual rules*	
	B1 factorising denominator seen anywhere in (a),	
	A1 given answer	
	If factorisation of denom. not seen, correct answer implies B1	
(b)	Expressing over common denominator	
	$\frac{3x+2}{x+1} - \frac{1}{x(x+1)} = \frac{x(3x+2)-1}{x(x+1)}$	M1
	[Or "Otherwise" : $\frac{(3x^2-x-2)x-(x-1)}{x(x^2-1)}$]	
	Multiplying out numerator and attempt to factorise	M1
	$[3x^2+2x-1 \equiv (3x-1)(x+1)]$	
	Answer: $\frac{3x-1}{x}$	A1 (3)
		Total 6 marks
2. (*a*)	$\frac{dy}{dx} = 3e^{3x} + \frac{1}{x}$	B1M1A1 (3)
	Notes	
	B1 $3e^{3x}$	
	M1 : $\frac{a}{bx}$ A1: $3e^{3x} + \frac{1}{x}$	
(b)	$(5+x^2)^{\frac{1}{2}}$	B1
	$\frac{3}{2}(5+x^2)^{\frac{1}{2}} \cdot 2x \quad = 3x(5+x^2)^{\frac{1}{2}}$ M1 for	M1 A1 (3)
	$kx(5+x^2)^m$	
		Total 6 marks

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3.

Figure 1

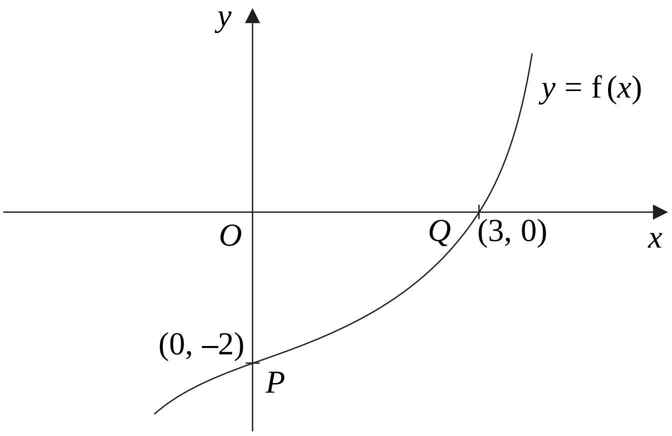


Figure 1 shows part of the curve with equation $y = \mathrm{f}(x)$, $x \in \mathbb{R}$, where f is an increasing function of x. The curve passes through the points $P(0, -2)$ and $Q(3, 0)$ as shown.

In separate diagrams, sketch the curve with equation

(a) $y = |\mathrm{f}(x)|$,

(3)

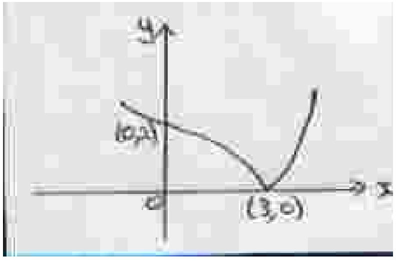
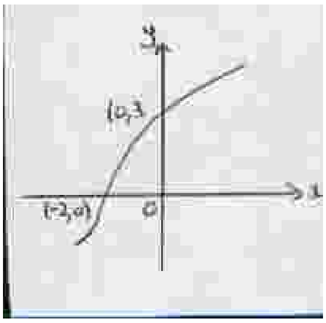
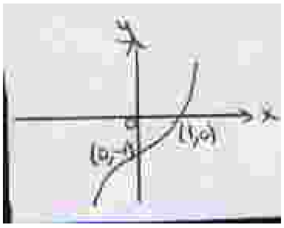
(b) $y = \mathrm{f}^{-1}(x)$,

(3)

(c) $y = \frac{1}{2}\mathrm{f}(3x)$.

(3)

Indicate clearly on each sketch the coordinates of the points at which the curve crosses or meets the axes.

Question Number	Scheme		Marks
3. (*a*)		Mod graph, reflect for $y < 0$ (0, 2), (3, 0) or marked on axes Correct shape, including cusp	M1 A1 A1 (3)
(*b*)		Attempt at reflection in $y = x$ Curvature correct –2, 0), (0, 3) or equiv.	M1 A1 B1 (3)
(*c*)		Attempt at 'stretches' (0, –1) or equiv. (1, 0)	M1 B1 B1 (3)
			Total 9 marks

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4. A heated metal ball is dropped into a liquid. As the ball cools, its temperature, T °C, t minutes after it enters the liquid, is given by

$$T = 400\,e^{-0.05t} + 25, \quad t \geqslant 0.$$

(a) Find the temperature of the ball as it enters the liquid.

(1)

(b) Find the value of t for which $T = 300$, giving your answer to 3 significant figures.

(4)

(c) Find the rate at which the temperature of the ball is decreasing at the instant when $t = 50$. Give your answer in °C per minute to 3 significant figures.

(3)

(d) From the equation for temperature T in terms of t, given above, explain why the temperature of the ball can never fall to 20 °C.

(1)

Question Number	Scheme	Marks
4. (*a*)	425 ºC	B1 (1)
(*b*)	$300 = 400\mathrm{e}^{-0.05t} + 25 \quad \Rightarrow 400\mathrm{e}^{-0.05t} = 275$	
	sub. $T = 300$ and attempt to rearrange to $\mathrm{e}^{-0.05t} = a$, where $a \in \mathbb{Q}$	M1
	$e^{-0.05t} = \frac{275}{400}$	A1
	M1 correct application of logs	M1
	$t = 7.49$	A1 (4)
(*c*)	$\frac{\mathrm{d}T}{\mathrm{d}t} = -20\ \mathrm{e}^{-0.05t}$ (M1 for $k\mathrm{e}^{-0.05t}$)	M1 A1
	At $t = 50$, rate of decrease = ($\pm$) 1.64 ºC/min	A1 (3)
(*d*)	$T > 25$, (since $\mathrm{e}^{-0.05t} \to 0$ as $t \to \infty$)	B1 (1)
		Total 9 marks

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5. **Figure 2**

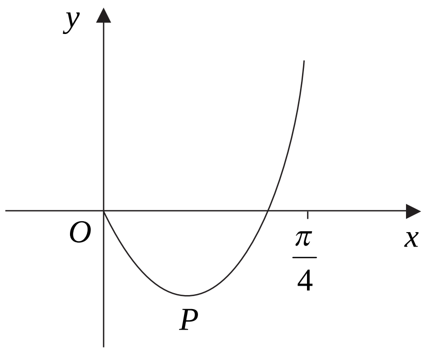

Figure 2 shows part of the curve with equation

$$y = (2x - 1)\tan 2x, \quad 0 \leqslant x < \frac{\pi}{4}.$$

The curve has a minimum at the point P. The x-coordinate of P is k.

(a) Show that k satisfies the equation

$$4k + \sin 4k - 2 = 0.$$

(6)

The iterative formula

$$x_{n+1} = \frac{1}{4}(2 - \sin 4x_n), \quad x_0 = 0.3,$$

is used to find an approximate value for k.

(b) Calculate the values of x_1, x_2, x_3 and x_4, giving your answers to 4 decimal places.

(3)

(c) Show that $k = 0.277$, correct to 3 significant figures.

(2)

Question Number	Scheme	Marks
5. (*a*)	Using product rule: $\frac{dy}{dx} = 2\tan 2x + 2(2x-1)\sec^2 2x$	M1 A1 A1
	Use of "$\tan 2x = \frac{\sin 2x}{\cos 2x}$" and "$\sec 2x = \frac{1}{\cos 2x}$"	M1
	$[= 2\frac{\sin 2x}{\cos 2x} + 2(2x-1)\frac{1}{\cos^2 2x}]$	
	Setting $\frac{dy}{dx} = 0$ and multiplying through to eliminate fractions	M1
	$[\Rightarrow \ 2\sin 2x\cos 2x + 2(2x-1) = 0]$	
	Completion: producing $4k + \sin 4k - 2 = 0$ with no wrong working seen and at least previous line seen. AG	A1* (6)
(*b*)	$x_1 = 0.2670$, $x_2 = 0.2809$, $x_3 = 0.2746$, $x_4 = 0.2774$, **Note:** M1 for first correct application, first A1 for two correct, second A1 for all four correct Max –1 deduction, if ALL correct to > 4 d.p. M1 A0 A1 SC: degree mode: M1 $x_1 = 0.4948$, A1 for $x_2 = 0.4914$, then A0; max 2	M1 A1 A1 (3)
(*c*)	Choose suitable interval for *k*: e.g. [0.2765, 0.2775] and evaluate f(*x*) at these values	M1
	Show that $4k + \sin 4k - 2$ changes sign and deduction	A1 (2)
	[f(0.2765) = –0.000087.., f(0.2775) = +0.0057]	
	Note: Continued iteration: (no marks in degree mode) Some evidence of further iterations leading to 0.2765 or better M1; Deduction A1	**(11 marks)**

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6. (a) Using $\sin^2\theta + \cos^2\theta \equiv 1$, show that $\text{cosec}^2\theta - \cot^2\theta \equiv 1$.

(2)

(b) Hence, or otherwise, prove that

$$\text{cosec}^4\theta - \cot^4\theta \equiv \text{cosec}^2\theta + \cot^2\theta.$$

(2)

(c) Solve, for $90° < \theta < 180°$,

$$\text{cosec}^4\theta - \cot^4\theta = 2 - \cot\,\theta.$$

(6)

Question Number	Scheme	Marks
6. (*a*)	Dividing $\sin^2\theta + \cos^2\theta \equiv 1$ by $\sin^2\theta$ to give $\frac{\sin^2\theta}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} \equiv \frac{1}{\sin^2\theta}$	M1
	Completion: $1 + \cot^2\theta \equiv \operatorname{cosec}^2\theta \Rightarrow \operatorname{cosec}^2\theta - \cot^2\theta \equiv 1$ AG	A1* (2)
(*b*)	$\operatorname{cosec}^4\theta - \cot^4\theta \equiv (\operatorname{cosec}^2\theta - \cot^2\theta)(\operatorname{cosec}^2\theta + \cot^2\theta)$	M1
	$\equiv (\operatorname{cosec}^2\theta + \cot^2\theta)$ using (a) AG	A1* (2)
	Notes: (i) Using LHS = $(1 + \cot^2\theta)^2 - \cot^4\theta$, using (a) & elim. $\cot^4\theta$ M1, conclusion {using (a) again} A1* (ii) Conversion to sines and cosines: needs $\frac{(1-\cos^2\theta)(1+\cos^2\theta)}{\sin^4\theta}$ for M1	
(*c*)	Using (b) to form $\operatorname{cosec}^2\theta + \cot^2\theta \equiv 2 - \cot\theta$	M1
	Forming quadratic in $\cot\theta$	M1
	$\Rightarrow 1 + \cot^2\theta + \cot^2\theta \equiv 2 - \cot\theta$ {using (a)}	
	$2\cot^2\theta + \cot\theta - 1 = 0$	A1
	Solving: $(2\cot\theta - 1)(\cot\theta + 1) = 0$ to $\cot\theta =$	M1
	$\left(\cot\theta = \frac{1}{2}\right)$ or $\cot\theta = -1$	A1
	$\theta = 135°$ (or correct value(s) for candidate dep. on 3Ms)	A1√ (6)
	Note: Ignore solutions outside range Extra "solutions" in range loses A1√, but candidate may possibly have more than one "correct" solution.	
		(10 marks)

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7. For the constant k, where $k > 1$, the functions f and g are defined by

$$\mathrm{f}:\ x \mapsto \ln(x + k), \quad x > -k,$$
$$\mathrm{g}:\ x \mapsto |2x - k|, \quad x \in \mathbb{R}.$$

(a) On separate axes, sketch the graph of f and the graph of g.

On each sketch state, in terms of k, the coordinates of points where the graph meets the coordinate axes.

(5)

(b) Write down the range of f.

(1)

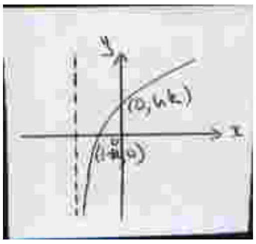
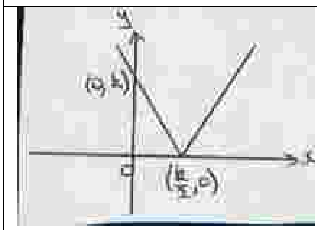
(c) Find $\mathrm{fg}\left(\frac{k}{4}\right)$ in terms of k, giving your answer in its simplest form.

(2)

The curve C has equation $y = \mathrm{f}(x)$. The tangent to C at the point with x-coordinate 3 is parallel to the line with equation $9y = 2x + 1$.

(d) Find the value of k.

(4)

Question Number	Scheme		Marks	
7. (*a*)		Log graph: Shape	B1	
		Intersection with –ve *x*-axis	dB1	
		$(0, \ln k)$, $(1 - k, 0)$	B1	
		Mod graph :V shape, vertex on +ve *x*-axis	B1	
		$(0, k)$ and $\left(\frac{k}{2}, 0\right)$	B1	(5)
(*b*)	$f(x) \in R$, $-\infty < f(x) < \infty$, $-\infty < y < \infty$		B1	(1)
(*c*)	$fg\left(\frac{k}{4}\right) = \ln\{k + \lvert\frac{2k}{4} - k\rvert\}$ or $f\left(\lvert -\frac{k}{2}\rvert\right)$		M1	
	$= \ln\left(\frac{3k}{2}\right)$		A1	(2)
(*d*)	$\frac{dy}{dx} = \frac{1}{x + k}$		B1	
	Equating (with $x = 3$) to grad. of line; $\frac{1}{3 + k} = \frac{2}{9}$		M1; A1	
	$k = 1\frac{1}{2}$		A1√	(4)
				(12 marks)

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8. (a) Given that $\cos A = \frac{3}{4}$, where $270° < A < 360°$, find the exact value of $\sin 2A$.
(5)

(b) (i) Show that $\cos\left(2x + \frac{\pi}{3}\right) + \cos\left(2x - \frac{\pi}{3}\right) \equiv \cos 2x$.
(3)

Given that

$$y = 3\sin^2 x + \cos\left(2x + \frac{\pi}{3}\right) + \cos\left(2x - \frac{\pi}{3}\right),$$

(ii) show that $\frac{dy}{dx} = \sin 2x$.
(4)

Question Number	Scheme	Marks
8. (*a*)	Method for finding sin *A*	M1
	$\sin A = -\dfrac{\sqrt{7}}{4}$	A1 A1
	Note: First A1 for $\dfrac{\sqrt{7}}{4}$, exact. Second A1 for sign (even if dec. answer given)	
	Use of $\sin 2A \equiv 2\sin A\cos A$	M1
	$\sin 2A = -\dfrac{3\sqrt{7}}{8}$ or equivalent exact	A1√ (5)
	Note: ± f.t. Requires exact value, dependent on 2nd M	
(*b*)(i)	$\cos\left(2x+\dfrac{\pi}{3}\right)+\cos\left(2x-\dfrac{\pi}{3}\right) \equiv \cos 2x\cos\dfrac{\pi}{3} - \sin 2x\sin\dfrac{\pi}{3} + \cos 2x\cos\dfrac{\pi}{3} + \sin 2x\sin\dfrac{\pi}{3}$	M1
	$\equiv 2\cos 2x\cos\dfrac{\pi}{3}$	A1
	[This can be just written down (using factor formulae) for M1A1]	
	$\equiv \cos 2x$ AG	A1* (3)
	Note: M1A1 earned, if $\equiv 2\cos 2x\cos\dfrac{\pi}{3}$ just written down, using factor theorem. Final A1* requires some working after first result.	
(*b*)(ii)	$\dfrac{dy}{dx} = 6\sin x\cos x - 2\sin 2x$	B1 B1
	or $6\sin x\cos x - 2\sin\left(2x+\dfrac{\pi}{3}\right) - 2\sin\left(2x-\dfrac{\pi}{3}\right)$	
	$= 3\sin 2x - 2\sin 2x$	M1
	$= \sin 2x$ AG	A1* (4)
	Note: First B1 for $6\sin x\cos x$; second B1 for remaining term(s)	
		(12 marks)