Centre No.						Paper Reference	Surname	Initial(s)
Candidate No.						6 6 6 5 / 0 1	Signature	

Paper Reference(s)

6665/01

Edexcel GCE

Core Mathematics C3

Advanced

Friday 6 June 2008 – Afternoon

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
1	
2	
3	
4	
5	
6	
7	
Total	

Materials required for examination	Items included with question papers
Mathematical Formulae (Green)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer for each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 7 questions in this question paper. The total mark for this paper is 75.
There are 24 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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1. The point P lies on the curve with equation

$$y = 4e^{2x+1}.$$

The y-coordinate of P is 8.

(a) Find, in terms of ln 2, the x-coordinate of P.

(2)

(b) Find the equation of the tangent to the curve at the point P in the form $y = ax + b$, where a and b are exact constants to be found.

(4)

June 2008
6665 Core Mathematics C3
Mark Scheme

Question Number	Scheme	Marks
1.	(a) $e^{2x+1} = 2$	
	$2x + 1 = \ln 2$	M1
	$x = \frac{1}{2}(\ln 2 - 1)$	A1 (2)
	(b) $\frac{dy}{dx} = 8e^{2x+1}$	B1
	$x = \frac{1}{2}(\ln 2 - 1) \Rightarrow \frac{dy}{dx} = 16$	B1
	$y - 8 = 16\left(x - \frac{1}{2}(\ln 2 - 1)\right)$	M1
	$y = 16x + 16 - 8\ln 2$	A1 (4)
		[6]

Leave blank

2. $$f(x) = 5\cos x + 12\sin x$$

Given that $f(x) = R\cos(x - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$,

(a) find the value of R and the value of α to 3 decimal places.

(4)

(b) Hence solve the equation

$$5\cos x + 12\sin x = 6$$

for $0 \leqslant x < 2\pi$.

(5)

(c) (i) Write down the maximum value of $5\cos x + 12\sin x$.

(1)

(ii) Find the smallest positive value of x for which this maximum value occurs.

(2)

Question Number	Scheme		Marks	
2.	(a) $R^2 = 5^2 + 12^2$		M1	
	$R = 13$		A1	
	$\tan\alpha = \dfrac{12}{5}$		M1	
	$\alpha \approx 1.176$	cao	A1	(4)
	(b) $\cos(x-\alpha) = \dfrac{6}{13}$		M1	
	$x-\alpha = \arccos\dfrac{6}{13} = 1.091\ \ldots$		A1	
	$x = 1.091\ \ldots + 1.176\ \ldots \approx 2.267\ldots$	awrt 2.3	A1	
	$x-\alpha = -1.091\ \ldots$	accept … = 5.19 … for M	M1	
	$x = -1.091\ \ldots + 1.176\ \ldots \approx 0.0849\ \ldots$	awrt 0.084 or 0.085	A1	(5)
	(c)(i) $R_{\max} = 13$	ft their R	B1 ft	
	(ii) At the maximum, $\cos(x-\alpha) = 1$ *or* $x-\alpha = 0$		M1	
	$x = \alpha = 1.176\ \ldots$	awrt 1.2, ft their α	A1ft	(3)
				[12]

Leave blank

3.

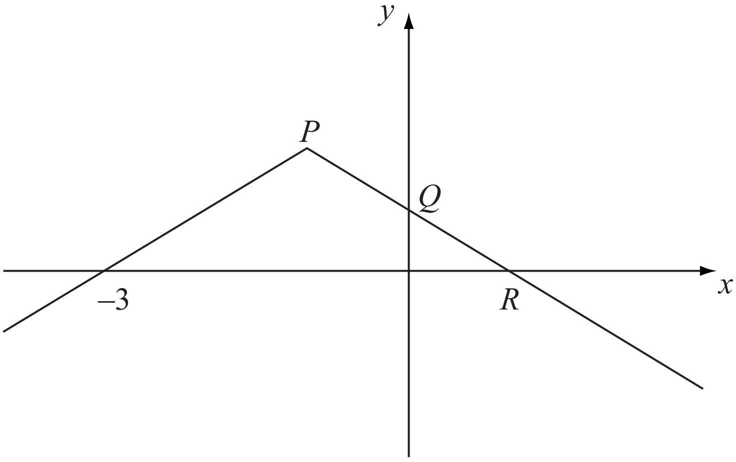


Figure 1

Figure 1 shows the graph of $y = \mathrm{f}(x),\ x \in \mathbb{R}$.
The graph consists of two line segments that meet at the point P.
The graph cuts the y-axis at the point Q and the x-axis at the points (–3, 0) and R.
Sketch, on separate diagrams, the graphs of

(a) $y = |\mathrm{f}(x)|$, **(2)**

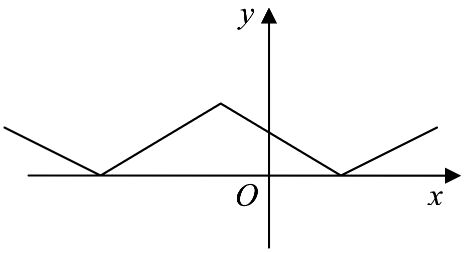
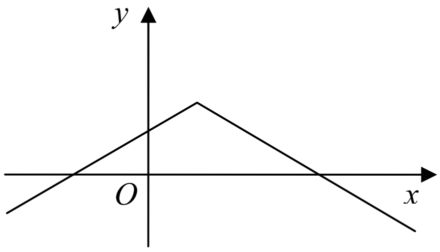
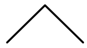
(b) $y = \mathrm{f}(-x)$. **(2)**

Given that $\mathrm{f}(x) = 2 - |x + 1|$,

(c) find the coordinates of the points P, Q and R, **(3)**

(d) solve $\mathrm{f}(x) = \frac{1}{2}x$. **(5)**

N 3 0 7 4 5 A 0 8 2 4

Question Number	Scheme	Marks
3.	(a) 	
	 shape	B1
	Vertices correctly placed	B1 (2)
	(b) 	
	 shape	B1
	Vertex and intersections with axes correctly placed	B1 (2)
	(c) $P:(-1, 2)$	B1
	$Q:(0, 1)$	B1
	$R:(1, 0)$	B1 (3)
	(d) $x > -1$; $2 - x - 1 = \frac{1}{2}x$	M1 A1
	Leading to $x = \frac{2}{3}$	A1
	$x < -1$; $2 + x + 1 = \frac{1}{2}x$	M1
	Leading to $x = -6$	A1 (5)
		[12]

Leave blank

4. The function f is defined by

$$f : x \mapsto \frac{2(x-1)}{x^2-2x-3} - \frac{1}{x-3}, \quad x > 3.$$

(a) Show that $f(x) = \frac{1}{x+1}$, $\quad x > 3$.
(4)

(b) Find the range of f.
(2)

(c) Find $f^{-1}(x)$. State the domain of this inverse function.
(3)

The function g is defined by

$$g : x \mapsto 2x^2 - 3, \quad x \in \mathbb{R}.$$

(d) Solve $fg(x) = \frac{1}{8}$.
(3)

Question Number	Scheme		Marks	
4.	(a) $x^2-2x-3=(x-3)(x+1)$		B1	
	$\text{f}(x)=\dfrac{2(x-1)-(x+1)}{(x-3)(x+1)}$ $\left(or\ \dfrac{2(x-1)}{(x-3)(x+1)}-\dfrac{x+1}{(x-3)(x+1)} \right)$		M1 A1	
	$=\dfrac{x-3}{(x-3)(x+1)}=\dfrac{1}{x+1}$ *	cso	A1	(4)
	(b) $\left(0,\dfrac{1}{4}\right)$ Accept $0<y<\dfrac{1}{4},\ 0<\text{f}(x)<\dfrac{1}{4}$ etc.		B1 B1	(2)
	(c) Let $y=\text{f}(x)$ $\quad y=\dfrac{1}{x+1}$			
	$x=\dfrac{1}{y+1}$			
	$yx+x=1$			
	$y=\dfrac{1-x}{x}$	or $\dfrac{1}{x}-1$	M1 A1	
	$\text{f}^{-1}(x)=\dfrac{1-x}{x}$			
	Domain of f^{-1} is $\left(0,\dfrac{1}{4}\right)$	ft their part (b)	B1 ft	(3)
	(d) $\text{fg}(x)=\dfrac{1}{2x^2-3+1}$			
	$\dfrac{1}{2x^2-2}=\dfrac{1}{8}$		M1	
	$x^2=5$		A1	
	$x=\pm\sqrt{5}$	both	A1	(3) **[12]**

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5. (a) Given that $\sin^2\theta + \cos^2\theta \equiv 1$, show that $1 + \cot^2\theta \equiv \text{cosec}^2\theta$.

(2)

(b) Solve, for $0 \leqslant \theta < 180°$, the equation

$$2\cot^2\theta - 9\,\text{cosec}\,\theta = 3,$$

giving your answers to 1 decimal place.

(6)

Question Number	Scheme	Marks
5.	(a) $\sin^2\theta + \cos^2\theta = 1$	
	$\div\sin^2\theta$ $\quad \dfrac{\sin^2\theta}{\sin^2\theta} + \dfrac{\cos^2\theta}{\sin^2\theta} = \dfrac{1}{\sin^2\theta}$	M1
	$1 + \cot^2\theta = \operatorname{cosec}^2\theta$ ∗ cso	A1 (2)
	Alternative for (a)	
	$1 + \cot^2\theta = 1 + \dfrac{\cos^2\theta}{\sin^2\theta} = \dfrac{\sin^2\theta + \cos^2\theta}{\sin^2\theta} = \dfrac{1}{\sin^2\theta}$	M1
	$= \operatorname{cosec}^2\theta$ ∗ cso	A1
	(b) $2\left(\operatorname{cosec}^2\theta - 1\right) - 9\operatorname{cosec}\theta = 3$	M1
	$2\operatorname{cosec}^2\theta - 9\operatorname{cosec}\theta - 5 = 0$ *or* $5\sin^2\theta + 9\sin\theta - 2 = 0$	M1
	$\left(2\operatorname{cosec}\theta + 1\right)\left(\operatorname{cosec}\theta - 5\right) = 0$ *or* $\left(5\sin\theta - 1\right)\left(\sin\theta + 2\right) = 0$	M1
	$\operatorname{cosec}\theta = 5$ *or* $\sin\theta = \dfrac{1}{5}$	A1
	$\theta = 11.5°, 168.5°$	A1 A1 (6) **[8]**

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6. (a) Differentiate with respect to x,

(i) $e^{3x}(\sin x + 2\cos x)$,

(3)

(ii) $x^3 \ln(5x + 2)$.

(3)

Given that $y = \dfrac{3x^2 + 6x - 7}{(x+1)^2}$, $\quad x \neq -1$,

(b) show that $\dfrac{dy}{dx} = \dfrac{20}{(x+1)^3}$.

(5)

(c) Hence find $\dfrac{d^2y}{dx^2}$ and the real values of x for which $\dfrac{d^2y}{dx^2} = -\dfrac{15}{4}$.

(3)

Question Number	Scheme	Marks
6.	(a)(i) $\frac{\mathrm{d}}{\mathrm{d}x}\left(\mathrm{e}^{3x}\left(\sin x+2\cos x\right)\right)=3\mathrm{e}^{3x}\left(\sin x+2\cos x\right)+\mathrm{e}^{3x}\left(\cos x-2\sin x\right)$	M1 A1 A1 (3)
	$\left(=\mathrm{e}^{3x}\left(\sin x+7\cos x\right)\right)$	
	(ii) $\frac{\mathrm{d}}{\mathrm{d}x}\left(x^3\ln\left(5x+2\right)\right)=3x^2\ln\left(5x+2\right)+\frac{5x^3}{5x+2}$	M1 A1 A1 (3)
	(b) $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\left(x+1\right)^2\left(6x+6\right)-2\left(x+1\right)\left(3x^2+6x-7\right)}{\left(x+1\right)^4}$	M1 $\frac{\text{A1}}{\text{A1}}$
	$=\frac{\left(x+1\right)\left(6x^2+12x+6-6x^2-12x+14\right)}{\left(x+1\right)^4}$	M1
	$=\frac{20}{\left(x+1\right)^3}$ ∗ cso	A1 (5)
	(c) $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=-\frac{60}{\left(x+1\right)^4}=-\frac{15}{4}$	M1
	$\left(x+1\right)^4=16$	M1
	$x=1,-3$ both	A1 (3)
		[14]
	Note: The simplification in part (b) can be carried out as follows	
	$\frac{\left(x+1\right)^2\left(6x+6\right)-2\left(x+1\right)\left(3x^2+6x-7\right)}{\left(x+1\right)^4}$	
	$=\frac{\left(6x^3+18x^2+18x+6\right)-\left(6x^3+18x^2-2x-14\right)}{\left(x+1\right)^4}$	
	$=\frac{20x+20}{\left(x+1\right)^4}=\frac{20\left(x+1\right)}{\left(x+1\right)^4}=\frac{20}{\left(x+1\right)^3}$	M1 A1

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7. $$f(x) = 3x^3 - 2x - 6$$

(a) Show that $f(x) = 0$ has a root, α, between $x = 1.4$ and $x = 1.45$

(2)

(b) Show that the equation $f(x) = 0$ can be written as

$$x = \sqrt{\left(\frac{2}{x} + \frac{2}{3}\right)}, \quad x \neq 0.$$

(3)

(c) Starting with $x_0 = 1.43$, use the iteration

$$x_{n+1} = \sqrt{\left(\frac{2}{x_n} + \frac{2}{3}\right)}$$

to calculate the values of x_1, x_2 and x_3, giving your answers to 4 decimal places.

(3)

(d) By choosing a suitable interval, show that $\alpha = 1.435$ is correct to 3 decimal places.

(3)

Question Number	Scheme	Marks
7.	(a) $\mathrm{f}(1.4) = -0.568 \ldots < 0$	
	$\mathrm{f}(1.45) = 0.245 \ldots > 0$	M1
	Change of sign (and continuity) $\Rightarrow \alpha \in (1.4, 1.45)$	A1 (2)
	(b) $3x^3 = 2x + 6$	
	$x^3 = \frac{2x}{3} + 2$	
	$x^2 = \frac{2}{3} + \frac{2}{x}$	M1 A1
	$x = \sqrt{\left(\frac{2}{x} + \frac{2}{3}\right)}$ ∗ cso	A1 (3)
	(c) $x_1 = 1.4371$	B1
	$x_2 = 1.4347$	B1
	$x_3 = 1.4355$	B1 (3)
	(d) Choosing the interval $(1.4345, 1.4355)$ or appropriate tighter interval.	M1
	$\mathrm{f}(1.4345) = -0.01$ ...	
	$\mathrm{f}(1.4355) = 0.003 \ldots$	M1
	Change of sign (and continuity) $\Rightarrow \alpha \in (1.4345, 1.4355)$	
	$\Rightarrow \alpha = 1.435$, correct to 3 decimal places ∗ cso	A1 (3) **[11]**
	Note: $\alpha = 1.435\,304\,553 \ldots$	