| Centre No. | | | | | | Paper Reference | | | | | | | Surname | Initial(s) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Candidate No. | | | | | | 6 | 6 | 6 | 5 | / | 0 | 1 | Signature | |

Paper Reference(s)

**6665/01**

# Edexcel GCE

## Core Mathematics C3

## Advanced

Thursday 11 June 2009 – Morning

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

| Question Number | Leave Blank |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |
| 7 | |
| 8 | |
| Total | |

| **Materials required for examination** | **Items included with question papers** |
|---|---|
| Mathematical Formulae (Orange or Green) | Nil |

**Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

### Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer for each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

### Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.
There are 28 pages in this question paper. Any blank pages are indicated.

### Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.



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**1.**

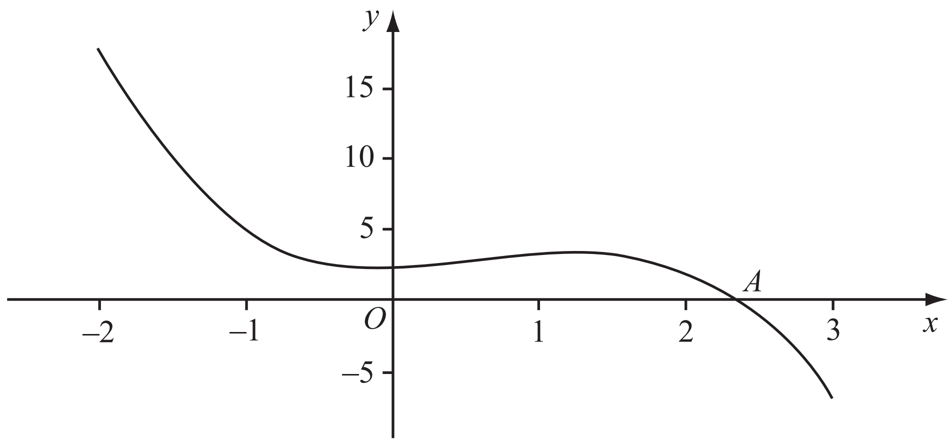


**Figure 1**

Figure 1 shows part of the curve with equation $y = -x^3 + 2x^2 + 2$, which intersects the $x$-axis at the point $A$ where $x = \alpha$.

To find an approximation to $\alpha$, the iterative formula

$$x_{n+1} = \frac{2}{(x_n)^2} + 2$$

is used.

(a) Taking $x_0 = 2.5$, find the values of $x_1$, $x_2$, $x_3$ and $x_4$.
Give your answers to 3 decimal places where appropriate.

**(3)**

(b) Show that $\alpha = 2.359$ correct to 3 decimal places.

**(3)**

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# June 2009
# 6665 Core Mathematics C3
# Mark Scheme

| Question Number | Scheme | | Marks |
|---|---|---|---|
| Q1 (a) | Iterative formula: $x_{n+1} = \dfrac{2}{(x_n)^2} + 2$ , $x_0 = 2.5$ | | |
| | $x_1 = \dfrac{2}{(2.5)^2} + 2$ | An attempt to substitute $x_0 = 2.5$ into the iterative formula. Can be implied by $x_1 = 2.32$ or 2.320 | M1 |
| | $x_1 = 2.32$<br>$x_2 = 2.371581451...$ | Both $x_1 = 2.32(0)$ and $x_2 =$ awrt 2.372 | A1 |
| | $x_3 = 2.355593575...$<br>$x_4 = 2.360436923...$ | Both $x_3 =$ awrt 2.356 and $x_4 =$ awrt 2.360 or 2.36 | A1 cso |
| | | | (3) |
| (b) | Let $f(x) = -x^3 + 2x^2 + 2 = 0$ | | |
| | $f(2.3585) = 0.00583577...$ | Choose suitable interval for $x$, e.g. [2.3585, 2.3595] or tighter | M1 |
| | $f(2.3595) = -0.00142286...$ | any one value awrt 1 sf or truncated 1 sf | dM1 |
| | Sign change (and $f(x)$ is continuous) therefore a root $\alpha$ is such that $\alpha \in (2.3585, 2.3595) \Rightarrow \alpha = 2.359$ (3 dp) | both values correct, sign change and conclusion | A1 |
| | | At a minimum, both values must be correct to 1sf or truncated 1sf, candidate states "change of sign, hence root". | (3) |
| | | | **[6]** |

Leave blank

**2.** (a) Use the identity $\cos^2\theta + \sin^2\theta = 1$ to prove that $\tan^2\theta = \sec^2\theta - 1$.

**(2)**

(b) Solve, for $0 \leqslant \theta < 360°$, the equation

$$2\tan^2\theta + 4\sec\theta + \sec^2\theta = 2$$

**(6)**



H 3 4 2 6 4 A 0 4 2 8

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| Question Number | Scheme | | Marks |
|---|---|---|---|
| Q2 (a) | $\cos^2\theta + \sin^2\theta = 1 \quad (\div \cos^2\theta)$ | | |
| | $\dfrac{\cos^2\theta}{\cos^2\theta} + \dfrac{\sin^2\theta}{\cos^2\theta} = \dfrac{1}{\cos^2\theta}$ | Dividing $\cos^2\theta + \sin^2\theta = 1$ by $\cos^2\theta$ to give underlined equation. | M1 |
| | $1 + \tan^2\theta = \sec^2\theta$ | | |
| | $\tan^2\theta = \sec^2\theta - 1$ (as required) **AG** | Complete proof. No errors seen. | A1 cso |
| | | | (2) |
| (b) | $2\tan^2\theta + 4\sec\theta + \sec^2\theta = 2$, (eqn *) $\quad 0 \le \theta < 360^\circ$ | | |
| | $2(\sec^2\theta - 1) + 4\sec\theta + \sec^2\theta = 2$ | Substituting $\tan^2\theta = \sec^2\theta - 1$ into eqn * to get a quadratic in $\sec\theta$ only | M1 |
| | $2\sec^2\theta - 2 + 4\sec\theta + \sec^2\theta = 2$ | | |
| | $3\sec^2\theta + 4\sec\theta - 4 = 0$ | Forming a three term "one sided" quadratic expression in $\sec\theta$. | M1 |
| | $(\sec\theta + 2)(3\sec\theta - 2) = 0$ | Attempt to factorise or solve a quadratic. | M1 |
| | $\sec\theta = -2$ or $\sec\theta = \frac{2}{3}$ | | |
| | $\dfrac{1}{\cos\theta} = -2$ or $\dfrac{1}{\cos\theta} = \dfrac{2}{3}$ | | |
| | $\cos\theta = -\frac{1}{2}$; or $\cos\theta = \frac{3}{2}$ | $\cos\theta = -\frac{1}{2}$ | A1; |
| | $\alpha = 120^\circ$ or $\alpha$ = no solutions | | |
| | $\theta_1 = 120^\circ$ | $120^\circ$ | A1 |
| | $\theta_2 = 240^\circ$ | $240^\circ$ or $\theta_2 = 360^\circ - \theta_1$ when solving using $\cos\theta = \ldots$ | B1 $\sqrt{}$ |
| | $\theta = \{120^\circ, 240^\circ\}$ | Note the final A1 mark has been changed to a B1 mark. | (6) |
| | | | **[8]** |

Leave blank

**3.** Rabbits were introduced onto an island. The number of rabbits, $P$, $t$ years after they were introduced is modelled by the equation

$$P = 80e^{\frac{1}{5}t}, \qquad t \in \mathbb{R},\ t \geqslant 0$$

(a) Write down the number of rabbits that were introduced to the island.
**(1)**

(b) Find the number of years it would take for the number of rabbits to first exceed 1000.
**(2)**

(c) Find $\frac{dP}{dt}$.
**(2)**

(d) Find $P$ when $\frac{dP}{dt} = 50$.
**(3)**

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| Question Number | Scheme | | Marks |
|---|---|---|---|
| Q3 | $P = 80\,\mathrm{e}^{\frac{t}{5}}$ | | |
| (a) | $t = 0 \Rightarrow P = 80\mathrm{e}^{\frac{0}{5}} = 80(1) = \underline{80}$ | $\underline{80}$ | B1 |
| | | | (1) |
| (b) | $P = 1000 \Rightarrow 1000 = 80\mathrm{e}^{\frac{t}{5}} \Rightarrow \dfrac{1000}{80} = \mathrm{e}^{\frac{t}{5}}$ | Substitutes $P = 1000$ and rearranges equation to make $\mathrm{e}^{\frac{t}{5}}$ the subject. | M1 |
| | $\therefore\ t = 5\ln\left(\dfrac{1000}{80}\right)$ | | |
| | $t = 12.6286...$ | awrt 12.6 or 13 years | A1 |
| | | Note $t = 12$ or $t =$ awrt $12.6 \Rightarrow t = 12$ will score A0 | (2) |
| (c) | $\dfrac{\mathrm{d}P}{\mathrm{d}t} = 16\mathrm{e}^{\frac{t}{5}}$ | $k\mathrm{e}^{\frac{1}{5}t}$ and $k \neq 80$. | M1 |
| | | $16\mathrm{e}^{\frac{1}{5}t}$ | A1 |
| | | | (2) |
| (d) | $50 = 16\mathrm{e}^{\frac{t}{5}}$ | | |
| | $\therefore\ t = 5\ln\left(\dfrac{50}{16}\right) \quad \{= 5.69717...\}$ | Using $50 = \frac{\mathrm{d}P}{\mathrm{d}t}$ and an attempt to solve to find the value of $t$ or $\frac{t}{5}$. | M1 |
| | $P = 80\,\mathrm{e}^{\frac{1}{5}\left(5\ln\left(\frac{50}{16}\right)\right)}$ or $P = 80\,\mathrm{e}^{\frac{1}{5}(5.69717...)}$ | Substitutes their value of $t$ back into the equation for $P$. | dM1 |
| | $P = \dfrac{80(50)}{16} = \underline{250}$ | $\underline{250}$ or awrt 250 | A1 |
| | | | (3) |
| | | | **[8]** |

Leave blank

**4.** (i) Differentiate with respect to $x$

(a) $x^2 \cos 3x$

**(3)**

(b) $\dfrac{\ln(x^2+1)}{x^2+1}$

**(4)**

(ii) A curve $C$ has the equation

$$y = \sqrt{(4x+1)}, \quad x > -\tfrac{1}{4}, \; y > 0$$

The point $P$ on the curve has $x$-coordinate 2. Find an equation of the tangent to $C$ at $P$ in the form $ax + by + c = 0$, where $a$, $b$ and $c$ are integers.

**(6)**

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| Question Number | Scheme | | Marks |
| --- | --- | --- | --- |
| Q4 (i)(a) | $y = x^2 \cos 3x$ | | |
| | Apply product rule: $\left\{ \begin{array}{ll} u = x^2 & v = \cos 3x \\ \frac{du}{dx} = 2x & \frac{dv}{dx} = -3\sin 3x \end{array} \right\}$ | | |
| | $\frac{dy}{dx} = 2x\cos 3x - 3x^2 \sin 3x$ | Applies $vu' + uv'$ correctly for their $u, u', v, v'$ AND gives an expression of the form $\alpha x \cos 3x \pm \beta x^2 \sin 3x$ | M1 |
| | | Any one term correct | A1 |
| | | Both terms correct and no further simplification to terms in $\cos \alpha x^2$ or $\sin \beta x^3$. | A1 |
| | | | (3) |
| (b) | $y = \frac{\ln(x^2+1)}{x^2+1}$ | | |
| | $u = \ln(x^2+1) \Rightarrow \frac{du}{dx} = \frac{2x}{x^2+1}$ | $\ln(x^2+1) \to \frac{\text{something}}{x^2+1}$ | M1 |
| | | $\ln(x^2+1) \to \frac{2x}{x^2+1}$ | A1 |
| | Apply quotient rule: $\left\{ \begin{array}{ll} u = \ln(x^2+1) & v = x^2+1 \\ \frac{du}{dx} = \frac{2x}{x^2+1} & \frac{dv}{dx} = 2x \end{array} \right\}$ | | |
| | $\frac{dy}{dx} = \frac{\left(\frac{2x}{x^2+1}\right)(x^2+1) - 2x\ln(x^2+1)}{(x^2+1)^2}$ | Applying $\frac{vu' - uv'}{v^2}$ | M1 |
| | | Correct differentiation with correct bracketing but allow recovery. | A1 |
| | | | (4) |
| | $\left\{ \frac{dy}{dx} = \frac{2x - 2x\ln(x^2+1)}{(x^2+1)^2} \right\}$ | {Ignore subsequent working.} | |

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| Question Number | Scheme | | Marks |
|---|---|---|---|
| (ii) | $y = \sqrt{4x+1},\ \ x > -\frac{1}{4}$ | | |
| | At $P$, $y = \sqrt{4(2)+1} = \underline{\sqrt{9}} = \underline{3}$ | At $P$, $y = \underline{\sqrt{9}}$ or $\underline{3}$ | B1 |
| | $\frac{\mathrm{d}y}{\mathrm{d}x} = \underline{\frac{1}{2}(4x+1)^{-\frac{1}{2}}(4)}$ | $\pm k(4x+1)^{-\frac{1}{2}}$ | M1* |
| | | $2(4x+1)^{-\frac{1}{2}}$ | A1 aef |
| | $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2}{(4x+1)^{\frac{1}{2}}}$ | | |
| | At $P$, $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2}{(4(2)+1)^{\frac{1}{2}}}$ | Substituting $x = 2$ into an equation involving $\frac{\mathrm{d}y}{\mathrm{d}x}$; | M1 |
| | Hence m(**T**) $= \frac{2}{3}$ | | |
| | Either **T**: $y - 3 = \frac{2}{3}(x-2)$;<br>or $y = \frac{2}{3}x + c$ and<br>$3 = \frac{2}{3}(2) + c \Rightarrow c = 3 - \frac{4}{3} = \frac{5}{3}$; | $y - y_1 = m(x-2)$<br>or $y - y_1 = m(x -$ their stated $x)$ with 'their TANGENT gradient' and their $y_1$;<br>or uses $y = mx + c$ with 'their TANGENT gradient', their $x$ and their $y_1$. | dM1*; |
| | Either **T**: $3y - 9 = 2(x-2)$; | | |
| | **T**: $3y - 9 = 2x - 4$ | | |
| | **T**: $\underline{2x - 3y + 5 = 0}$ | $\underline{2x - 3y + 5 = 0}$<br>Tangent must be stated in the form $ax + by + c = 0$, where $a$, $b$ and $c$ are integers. | A1 |
| | | | (6) |
| | or **T**: $y = \frac{2}{3}x + \frac{5}{3}$ | | |
| | **T**: $3y = 2x + 5$ | | |
| | **T**: $\underline{2x - 3y + 5 = 0}$ | | |
| | | | **[13]** |

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**5.**

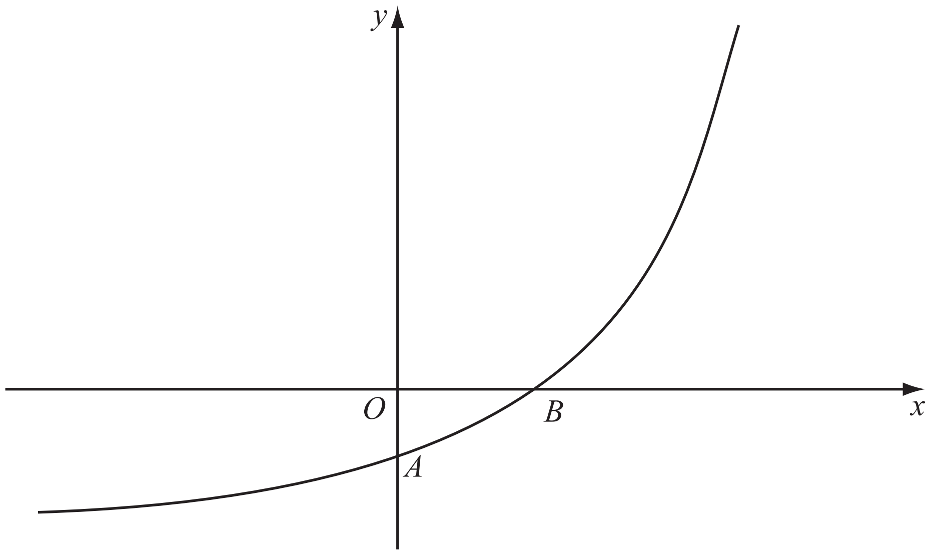


**Figure 2**

Figure 2 shows a sketch of part of the curve with equation $y = \mathrm{f}(x)$, $x \in \mathbb{R}$.

The curve meets the coordinate axes at the points $A(0, 1-k)$ and $B(\frac{1}{2}\ln k, 0)$, where $k$ is a constant and $k > 1$, as shown in Figure 2.

On separate diagrams, sketch the curve with equation

(a) $y = |\mathrm{f}(x)|$,

**(3)**

(b) $y = \mathrm{f}^{-1}(x)$.

**(2)**

Show on each sketch the coordinates, in terms of $k$, of each point at which the curve meets or cuts the axes.

Given that $\mathrm{f}(x) = \mathrm{e}^{2x} - k$,

(c) state the range of f ,

**(1)**

(d) find $\mathrm{f}^{-1}(x)$,

**(3)**

(e) write down the domain of $\mathrm{f}^{-1}$.

**(1)**



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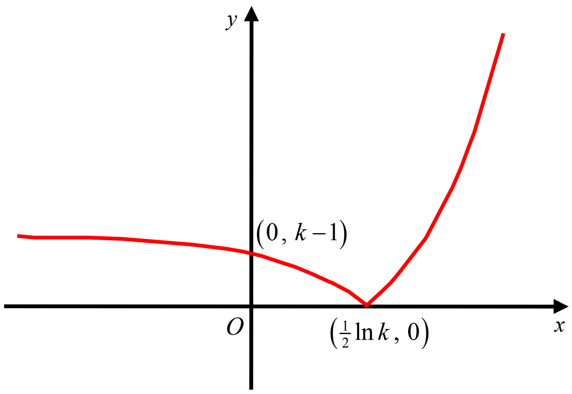
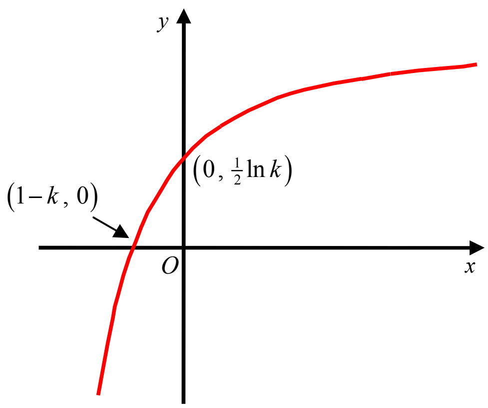
| Question Number | Scheme | | Marks |
|---|---|---|---|
| Q5 (a) |  | Curve retains shape when $x > \frac{1}{2}\ln k$ | B1 |
| | | Curve reflects through the $x$-axis when $x < \frac{1}{2}\ln k$ | B1 |
| | | $(0,\ k-1)$ and $(\frac{1}{2}\ln k,\ 0)$ marked in the correct positions. | B1 |
| | | | (3) |
| (b) |  | Correct shape of curve. The curve should be contained in quadrants 1, 2 and 3 (Ignore asymptote) | B1 |
| | | $(1-k,\ 0)$ and $(0,\ \frac{1}{2}\ln k)$ | B1 |
| | | | (2) |
| (c) | Range of f: $\underline{f(x) > -k}$ or $\underline{y > -k}$ or $\underline{(-k,\ \infty)}$ | Either $\underline{f(x) > -k}$ or $\underline{y > -k}$ or $\underline{(-k,\ \infty)}$ or $\underline{f > -k}$ or $\underline{\text{Range} > -k}$. | B1 |
| | | | (1) |
| (d) | $y = e^{2x} - k \Rightarrow y + k = e^{2x}$<br>$\Rightarrow \ln(y+k) = 2x$<br>$\Rightarrow \frac{1}{2}\ln(y+k) = x$ | Attempt to make $x$ (or swapped $y$) the subject | M1 |
| | | Makes $e^{2x}$ the subject and takes ln of both sides | M1 |
| | Hence $f^{-1}(x) = \underline{\frac{1}{2}\ln(x+k)}$ | $\underline{\frac{1}{2}\ln(x+k)}$ or $\underline{\ln\sqrt{(x+k)}}$ | A1 cao |
| | | | (3) |
| (e) | $f^{-1}(x)$: Domain: $\underline{x > -k}$ or $\underline{(-k,\ \infty)}$ | Either $\underline{x > -k}$ or $\underline{(-k,\ \infty)}$ or Domain $> -k$ or $x$ "ft one sided inequality" their part (c) RANGE answer | B1 $\sqrt{}$ |
| | | | (1) |
| | | | [10] |

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**6.** (a) Use the identity $\cos(A+B) = \cos A \cos B - \sin A \sin B$, to show that

$$\cos 2A = 1 - 2\sin^2 A$$

**(2)**

The curves $C_1$ and $C_2$ have equations

$$C_1: \quad y = 3\sin 2x$$

$$C_2: \quad y = 4\sin^2 x - 2\cos 2x$$

(b) Show that the $x$-coordinates of the points where $C_1$ and $C_2$ intersect satisfy the equation

$$4\cos 2x + 3\sin 2x = 2$$

**(3)**

(c) Express $4\cos 2x + 3\sin 2x$ in the form $R\cos(2x - \alpha)$, where $R > 0$ and $0 < \alpha < 90°$, giving the value of $\alpha$ to 2 decimal places.

**(3)**

(d) Hence find, for $0 \leqslant x < 180°$, all the solutions of

$$4\cos 2x + 3\sin 2x = 2$$

giving your answers to 1 decimal place.

**(4)**

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| Question Number | Scheme | | Marks |
|---|---|---|---|
| Q6 (a) | $A = B \Rightarrow \cos(A + A) = \cos 2A = \underline{\cos A \cos A - \sin A \sin A}$ | Applies $A = B$ to $\cos(A+B)$ to give the underlined equation or $\cos 2A = \underline{\cos^2 A - \sin^2 A}$ | M1 |
| | $\cos 2A = \cos^2 A - \sin^2 A$ and $\cos^2 A + \sin^2 A = 1$ gives | | |
| | $\underline{\cos 2A} = 1 - \sin^2 A - \sin^2 A = \underline{1 - 2\sin^2 A}$ (as required) | Complete proof, with a link between LHS and RHS. No errors seen. | A1 AG |
| | | | (2) |
| (b) | $C_1 = C_2 \Rightarrow 3\sin 2x = 4\sin^2 x - 2\cos 2x$ | Eliminating $y$ correctly. | M1 |
| | $3\sin 2x = 4\left(\frac{1-\cos 2x}{2}\right) - 2\cos 2x$ | Using result in part (a) to substitute for $\sin^2 x$ as $\frac{\pm 1 \pm \cos 2x}{2}$ or $k\sin^2 x$ as $k\left(\frac{\pm 1 \pm \cos 2x}{2}\right)$ to produce an equation in only double angles. | M1 |
| | $3\sin 2x = 2(1 - \cos 2x) - 2\cos 2x$ | | |
| | $3\sin 2x = 2 - 2\cos 2x - 2\cos 2x$ | | |
| | $3\sin 2x + 4\cos 2x = 2$ | Rearranges to give correct result | A1 AG |
| | | | (3) |
| (c) | $3\sin 2x + 4\cos 2x = R\cos(2x - \alpha)$ | | |
| | $3\sin 2x + 4\cos 2x = R\cos 2x \cos\alpha + R\sin 2x \sin\alpha$ | | |
| | Equate $\sin 2x$: $3 = R\sin\alpha$<br>Equate $\cos 2x$: $4 = R\cos\alpha$ | | |
| | $R = \sqrt{3^2 + 4^2}; = \sqrt{25} = 5$ | $R = 5$ | B1 |
| | $\tan\alpha = \frac{3}{4} \Rightarrow \alpha = 36.86989765...^\circ$ | $\tan\alpha = \pm\frac{3}{4}$ or $\tan\alpha = \pm\frac{4}{3}$ or $\sin\alpha = \pm\frac{3}{\text{their } R}$ or $\cos\alpha = \pm\frac{4}{\text{their } R}$ | M1 |
| | | awrt 36.87 | A1 |
| | Hence, $3\sin 2x + 4\cos 2x = 5\cos(2x - 36.87)$ | | |
| | | | (3) |

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| Question Number | Scheme | | Marks |
|---|---|---|---|
| (d) | $3\sin 2x + 4\cos 2x = 2$ | | |
| | $5\cos(2x - 36.87) = 2$ | | |
| | $\cos(2x - 36.87) = \frac{2}{5}$ | $\cos(2x \pm \text{their } \alpha) = \frac{2}{\text{their } R}$ | M1 |
| | $(2x - 36.87) = 66.42182...^\circ$ | awrt 66 | A1 |
| | $(2x - 36.87) = 360 - 66.42182...^\circ$ | | |
| | Hence, $x = 51.64591...^\circ,\ 165.22409...^\circ$ | One of either awrt 51.6 or awrt 51.7 or awrt 165.2 or awrt 165.3 | A1 |
| | | Both awrt 51.6 AND awrt 165.2 | A1 (4) |
| | | If there are any EXTRA solutions inside the range $0 \le x < 180^\circ$ then withhold the final accuracy mark. Also ignore EXTRA solutions outside the range $0 \le x < 180^\circ$. | |
| | | | **[12]** |

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**7.** The function f is defined by

$$f(x) = 1 - \frac{2}{(x+4)} + \frac{x-8}{(x-2)(x+4)}, \quad x \in \mathbb{R},\ x \neq -4,\ x \neq 2$$

(a) Show that $f(x) = \dfrac{x-3}{x-2}$ **(5)**

The function g is defined by

$$g(x) = \frac{e^x - 3}{e^x - 2}, \quad x \in \mathbb{R},\ x \neq \ln 2$$

(b) Differentiate $g(x)$ to show that $g'(x) = \dfrac{e^x}{(e^x - 2)^2}$ **(3)**

(c) Find the exact values of $x$ for which $g'(x) = 1$ **(4)**

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| Question Number | Scheme | | Marks |
|---|---|---|---|
| Q7 | $\mathrm{f}(x) = 1 - \frac{2}{(x+4)} + \frac{x-8}{(x-2)(x+4)}$ <br> $x \in \mathbb{R},\ x \neq -4,\ x \neq 2.$ | | |
| (a) | $\mathrm{f}(x) = \frac{(x-2)(x+4) - 2(x-2) + x - 8}{(x-2)(x+4)}$ | An attempt to combine to one fraction | M1 |
| | | Correct result of combining all three fractions | A1 |
| | $= \frac{x^2 + 2x - 8 - 2x + 4 + x - 8}{(x-2)(x+4)}$ | | |
| | $= \frac{x^2 + x - 12}{[(x+4)(x-2)]}$ | Simplifies to give the correct numerator. Ignore omission of denominator | A1 |
| | $= \frac{(x+4)(x-3)}{[(x+4)(x-2)]}$ | An attempt to factorise the numerator. | dM1 |
| | $= \frac{(x-3)}{(x-2)}$ | Correct result | A1 cso **AG** |
| | | | (5) |
| (b) | $\mathrm{g}(x) = \frac{e^x - 3}{e^x - 2} \quad x \in \mathbb{R},\ x \neq \ln 2.$ | | |
| | Apply quotient rule: $\left\{ \begin{array}{ll} u = e^x - 3 & v = e^x - 2 \\ \frac{du}{dx} = e^x & \frac{dv}{dx} = e^x \end{array} \right\}$ | | |
| | $\mathrm{g}'(x) = \frac{e^x(e^x - 2) - e^x(e^x - 3)}{(e^x - 2)^2}$ | Applying $\frac{vu' - uv'}{v^2}$ | M1 |
| | | Correct differentiation | A1 |
| | $= \frac{e^{2x} - 2e^x - e^{2x} + 3e^x}{(e^x - 2)^2}$ | | |
| | $= \frac{e^x}{(e^x - 2)^2}$ | Correct result | A1 **AG** cso |
| | | | (3) |

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| Question Number | Scheme | | Marks |
|---|---|---|---|
| (c) | $g'(x) = 1 \Rightarrow \dfrac{e^x}{(e^x - 2)^2} = 1$ | | |
| | $e^x = (e^x - 2)^2$ | Puts their differentiated numerator equal to their denominator. | M1 |
| | $e^x = e^{2x} - 2e^x - 2e^x + 4$ | | |
| | $\underline{e^{2x} - 5e^x + 4} = 0$ | $\underline{e^{2x} - 5e^x + 4}$ | A1 |
| | $(e^x - 4)(e^x - 1) = 0$ | Attempt to factorise or solve quadratic in $e^x$ | M1 |
| | $e^x = 4$ or $e^x = 1$ | | |
| | $x = \ln 4$ or $x = 0$ | both $x = 0, \ln 4$ | A1 |
| | | | (4) |
| | | | **[12]** |

Leave blank

**8.** (a) Write down $\sin 2x$ in terms of $\sin x$ and $\cos x$.

**(1)**

(b) Find, for $0 < x < \pi$, all the solutions of the equation

$$\operatorname{cosec} x - 8\cos x = 0$$

giving your answers to 2 decimal places.

**(5)**

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| Question Number | Scheme | | Marks |
|---|---|---|---|
| Q8 (a) | $\sin 2x = \underline{2\sin x \cos x}$ | $\underline{2\sin x \cos x}$ | B1 aef (1) |
| (b) | $\operatorname{cosec} x - 8\cos x = 0, \quad 0 < x < \pi$ | | |
| | $\frac{1}{\sin x} - 8\cos x = 0$ | Using $\operatorname{cosec} x = \frac{1}{\sin x}$ | M1 |
| | $\frac{1}{\sin x} = 8\cos x$ | | |
| | $1 = 8\sin x \cos x$ | | |
| | $1 = 4(2\sin x \cos x)$ | | |
| | $1 = 4\sin 2x$ | | |
| | $\underline{\sin 2x = \tfrac{1}{4}}$ | $\sin 2x = k$, where $-1 < k < 1$ and $k \neq 0$ | M1 |
| | | $\underline{\sin 2x = \tfrac{1}{4}}$ | A1 |
| | Radians $2x = \{0.25268..., 2.88891...\}$<br>Degrees $2x = \{14.4775..., 165.5225...\}$ | | |
| | Radians $x = \{0.12634..., 1.44445...\}$<br>Degrees $x = \{7.23875..., 82.76124...\}$ | Either arwt 7.24 or 82.76 or 0.13 or 1.44 or 1.45 or awrt $0.04\pi$ or awrt $0.46\pi$. | A1 |
| | | Both $\underline{0.13}$ and $\underline{1.44}$ | A1 cao |
| | | Solutions for the final two A marks must be given in $x$ only.<br>If there are any EXTRA solutions inside the range $0 < x < \pi$ then withhold the final accuracy mark.<br>Also ignore EXTRA solutions outside the range $0 < x < \pi$. | (5) |
| | | | **[6]** |