Centre No.						Paper Reference	Surname	Initial(s)
Candidate No.						6 6 6 5 / 0 1	Signature	

Paper Reference(s)

6665/01

Edexcel GCE

Core Mathematics C3

Advanced

Monday 16 June 2014 – Morning

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
1	
2	
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8	
9	
Total	

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer for each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 9 questions in this question paper. The total mark for this paper is 75.
There are 32 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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1. The curve C has equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = \frac{4x+1}{x-2}, \quad x > 2$$

(a) Show that

$$\mathrm{f}'(x) = \frac{-9}{(x-2)^2}$$

(3)

Given that P is a point on C such that $\mathrm{f}'(x) = -1$,

(b) find the coordinates of P.

(3)

Question Number	Scheme	Marks
1.(a)	$\mathrm{f}(x)=\dfrac{4x+1}{x-2},\quad x>2$	
	Applies $\dfrac{vu'-uv'}{v^2}$ to get $\dfrac{(x-2)\times 4-(4x+1)\times 1}{(x-2)^2}$	M1A1
	$=\dfrac{-9}{(x-2)^2}$	A1*
		(3)
(b)	$\dfrac{-9}{(x-2)^2}=-1\Rightarrow x=..$	M1
	$(5,7)$	A1,A1
		(3)
		6 marks
Alt 1.(a)	$\mathrm{f}(x)=\dfrac{4x+1}{x-2}=4+\dfrac{9}{x-2}$	
	Applies chain rule to get $\mathrm{f}'(x)=A(x-2)^{-2}$	M1
	$=-9(x-2)^{-2}=\dfrac{-9}{(x-2)^2}$	A1, A1*
		(3)

(a)

M1 Applies the quotient rule to $\mathrm{f}(x)=\dfrac{4x+1}{x-2}$ with $u=4x+1$ and $v=x-2$. If the rule is quoted it must be correct. It may be implied by their $u=4x+1, v=x-2, u'=.., v'=..$ followed by $\dfrac{vu'-uv'}{v^2}$.

If neither quoted nor implied only accept expressions of the form $\dfrac{(x-2)\times A-(4x+1)\times B}{(x-2)^2}$ $A, B>0$ allowing for a sign slip inside the brackets.

Condone missing brackets for the method mark but not the final answer mark.

Alternatively they could apply the product rule with $u=4x+1$ and $v=(x-2)^{-1}$. If the rule is quoted it must be correct. It may be implied by their $u=4x+1, v=(x-2)^{-1}, u'=.., v'=..$ followed by $vu'+uv'$.

If it is neither quoted nor implied only accept expressions of the form/ or equivalent to the form

$$(x-2)^{-1}\times C+(4x+1)\times D(x-2)^{-2}$$

A third alternative is to use the Chain rule. For this to score there must have been some attempt to divide first to achieve $\mathrm{f}(x)=\dfrac{4x+1}{x-2}=..+\dfrac{..}{x-2}$ before applying the chain rule to get $\mathrm{f}'(x)=A(x-2)^{-2}$

A1 A correct and unsimplified form of the answer.

Accept $\dfrac{(x-2)\times 4-(4x+1)\times 1}{(x-2)^2}$ from the quotient rule

Accept $\dfrac{4x-8-4x-1}{(x-2)^2}$ from the quotient rule even if the brackets were missing in line 1

Accept $(x-2)^{-1}\times 4+(4x+1)\times -1(x-2)^{-2}$ or equivalent from the product rule

Accept $9\times -1(x-2)^{-2}$ from the chain rule

A1* Proceeds to achieve the given answer $=\dfrac{-9}{(x-2)^2}$. Accept $-9(x-2)^{-2}$

All aspects must be correct including the bracketing.

If they differentiated using the product rule the intermediate lines must be seen.

Eg. $(x-2)^{-1}\times 4+(4x+1)\times -1(x-2)^{-2}=\dfrac{4}{(x-2)}-\dfrac{4x+1}{(x-2)^2}=\dfrac{4(x-2)-(4x+1)}{(x-2)^2}=\dfrac{-9}{(x-2)^2}$

(b)

M1 Sets $\dfrac{-9}{(x-2)^2}=-1$ and proceeds to $x=\ldots$.

The minimum expectation is that they multiply by $(x-2)^2$ and then either, divide by -1 before square rooting or multiply out before solving a 3TQ equation.

A correct answer of $x=5$ would also score this mark following $\dfrac{-9}{(x-2)^2}=-1$ as long as no incorrect work is seen.

A1 $x=5$

A1 (5, 7) or $x=5$, $y=7$. Ignore any reference to $x=-1$ (and $y=1$). Do not accept 21/3 for 7

If there is an extra solution, $x>2$, then withhold this final mark.

Leave blank

2. Find the exact solutions, in their simplest form, to the equations

(a) $2\ln(2x+1) - 10 = 0$

(2)

(b) $3^x e^{4x} = e^7$

(4)

Question Number	Scheme	Marks
2.(a)	$2\ln(2x+1)-10=0 \Rightarrow \ln(2x+1)=5 \quad \Rightarrow 2x+1=e^5 \Rightarrow x=..$	M1
	$\Rightarrow x=\dfrac{e^5-1}{2}$	A1
		(2)
(b)	$3^x e^{4x}=e^7 \Rightarrow \ln(3^x e^{4x})=\ln e^7$	
	$\ln 3^x+\ln e^{4x}=\ln e^7 \Rightarrow x\ln 3+4x\ln e=7\ln e$	M1,M1
	$x(\ln 3+4)=7 \Rightarrow x=...$	dM1
	$x=\dfrac{7}{(\ln 3+4)}$ oe	A1
		(4) **6 marks**
Alt 1 2(b)	$3^x e^{4x}=e^7 \Rightarrow 3^x=\dfrac{e^7}{e^{4x}}$	
	$3^x=e^{7-4x} \Rightarrow x\ln 3=(7-4x)\ln e$	M1,M1
	$x(\ln 3+4)=7 \Rightarrow x=...$	dM1
	$x=\dfrac{7}{(\ln 3+4)}$	A1
		(4)
Alt 2 2(b) Using logs	$3^x e^{4x}=e^7 \Rightarrow \log(3^x e^{4x})=\log e^7$	
	$\log 3^x+\log e^{4x}=\log e^7 \Rightarrow x\log 3+4x\log e=7\log e$	M1, M1
	$x(\log 3+4\log e)=7\log e \Rightarrow x=...$	dM1
	$x=\dfrac{7\log e}{(\log 3+4\log e)}$	A1
		(4)
Alt 3 2(b) Using $\log_3$	$3^x e^{4x}=e^7 \Rightarrow 3^x=\dfrac{e^7}{e^{4x}}$	
	$3^x=e^{7-4x} \Rightarrow x=(7-4x)\log_3 e$	M1,M1
	$x(1+4\log_3 e)=7\log_3 e \Rightarrow x=...$	dM1
	$x=\dfrac{7\log_3 e}{(1+4\log_3 e)}$	A1
		(4)
Alt 4 2(b) Using $3^x=e^{x\ln 3}$	$3^x e^{4x}=e^7 \Rightarrow e^{x\ln 3}e^{4x}=e^7$	
	$\Rightarrow e^{x\ln 3+4x}=e^7, \Rightarrow x\ln 3+4x=7$	M1,M1
	$x(\ln 3+4)=7 \Rightarrow x=... \qquad x=\dfrac{7}{(\ln 3+4)}$	dM1 A1
		(4)

(a)

M1 Proceeds from $2\ln(2x+1)-10=0$ to $\ln(2x+1)=5$ before taking exp's to achieve x in terms of e^5

Accept for M1 $2\ln(2x+1)-10=0 \Rightarrow \ln(2x+1)=5 \Rightarrow x=f(e^5)$

Alternatively they could use the power law before taking exp's to achieve x in terms of $\sqrt{e^{10}}$

$2\ln(2x+1)=10 \Rightarrow \ln(2x+1)^2=10 \Rightarrow (2x+1)^2=e^{10} \Rightarrow x=g\left(\sqrt{e^{10}}\right)$

A1 cso. Accept $x=\frac{e^5-1}{2}$ or other exact simplified alternatives such as $x=\frac{e^5}{2}-\frac{1}{2}$. Remember to isw.

The decimal answer of 73.7 will score M1A0 unless the exact answer has also been given.

The answer $\frac{\sqrt{e^{10}}-1}{2}$ does not score this mark unless simplified. $x=\frac{\pm e^5-1}{2}$ is M1A0

(b)

M1 Takes ln's or logs of both sides and applies the addition law.

$\ln(3^x e^{4x})=\ln 3^x+\ln e^{4x}$ or $\ln(3^x e^{4x})=\ln 3^x+4x$ is evidence for the addition law

If the e^{4x} was 'moved' over to the right hand side score for either e^{7-4x} or the subtraction law.

$\ln\frac{e^7}{e^{4x}}=\ln e^7-\ln e^{4x}$ or $3^x e^{4x}=e^7 \Rightarrow 3^x=\frac{e^7}{e^{4x}} \Rightarrow 3^x=e^{7-4x}$ is evidence of the subtraction law

M1 Uses the power law of logs (seen at least once in a term with x as the index Eg $3^x, e^{4x}$ or e^{7-4x}).

$\ln 3^x+\ln e^{4x}=\ln e^7 \Rightarrow x\ln 3+4x\ln e=7\ln e$ is an example after the addition law

$3^x=e^{7-4x} \Rightarrow x\log 3=(7-4x)\log e$ is an example after the subtraction law.

It is possible to score M0M1 by applying the power law after an incorrect addition/subtraction law

For example $3^x e^{4x}=e^7 \Rightarrow \ln(3^x)\times\ln(e^{4x})=\ln e^7 \Rightarrow x\ln 3\times 4x\ln e=7\ln e$

dM1 This is dependent upon **both** previous M's. Collects/factorises out term in x and proceeds to $x=$.
Condone sign slips for this mark. An unsimplified answer can score this mark.

A1 If the candidate has taken ln's then they must use $\ln e=1$ and achieve $x=\frac{7}{(\ln 3+4)}$ or equivalent.

If the candidate has taken log's they must be writing log as oppose to ln and achieve

$x=\frac{7\log e}{(\log 3+4\log e)}$ or other exact equivalents such as $x=\frac{7\log e}{\log 3e^4}$.

Leave blank

3. The curve C has equation $x = 8y \tan 2y$

The point P has coordinates $\left(\pi, \frac{\pi}{8}\right)$

(a) Verify that P lies on C.

(1)

(b) Find the equation of the tangent to C at P in the form $ay = x + b$, where the constants a and b are to be found in terms of π.

(7)

Question Number	Scheme	Marks
3.(a)	$x = 8\dfrac{\pi}{8}\tan\left(2\times\dfrac{\pi}{8}\right) = \pi$	B1*
		(1)
(b)	$\dfrac{dx}{dy} = 8\tan 2y + 16y\sec^2(2y)$	M1A1A1
	At P $\dfrac{dx}{dy} = 8\tan 2\dfrac{\pi}{8} + 16\dfrac{\pi}{8}\sec^2\left(2\times\dfrac{\pi}{8}\right) = \{8+4\pi\}$	M1
	$\dfrac{y-\frac{\pi}{8}}{x-\pi} = \dfrac{1}{8+4\pi}$, accept $y - \dfrac{\pi}{8} = 0.049(x-\pi)$	M1A1
	$\Rightarrow (8+4\pi)y = x + \dfrac{\pi^2}{2}$	A1
		(7)
		(8 marks)

(a)

B1* Either sub $y = \dfrac{\pi}{8}$ into $x = 8y\tan(2y) \Rightarrow x = 8\times\dfrac{\pi}{8}\tan\left(2\times\dfrac{\pi}{8}\right) = \pi$

Or sub $x = \pi$, $y = \dfrac{\pi}{8}$ into $x = 8y\tan(2y) \Rightarrow \pi = 8\times\dfrac{\pi}{8}\tan\left(2\times\dfrac{\pi}{8}\right) = \pi\times 1 = \pi$

This is a proof and therefore an expectation that at least one intermediate line must be seen, including a term in tangent.

Accept as a minimum $y = \dfrac{\pi}{8} \Rightarrow x = \pi\tan\left(\dfrac{\pi}{4}\right) = \pi$

Or $\pi = \pi\times\tan\left(\dfrac{\pi}{4}\right) = \pi$ ✓

This is a given answer however, and as such there can be no errors.

(b)

M1 Applies the product rule to $8y\tan 2y$ achieving $A\tan 2y + By\sec^2(2y)$

A1 One term correct. Either $8\tan 2y$ or $+16y\sec^2(2y)$. There is no requirement for $\frac{dx}{dy} =$

A1 Both lhs and rhs correct. $\frac{dx}{dy} = 8\tan 2y + 16y\sec^2(2y)$

It is an intermediate line and the expression does not need to be simplified.

Accept $\frac{dx}{dy} = \tan 2y \times 8 + 8y \times 2\sec^2(2y)$ or $\frac{dy}{dx} = \frac{1}{\tan 2y \times 8 + 8y \times 2\sec^2(2y)}$ or using implicit differentiation $1 = \tan 2y \times 8\frac{dy}{dx} + 8y \times 2\sec^2(2y)\frac{dy}{dx}$

M1 For fully substituting $y = \frac{\pi}{8}$ into their $\frac{dx}{dy}$ or $\frac{dy}{dx}$ to find a 'numerical' value

Accept $\frac{dx}{dy} =$ awrt 20.6 or $\frac{dy}{dx} =$ awrt 0.05 as evidence

M1 For a correct attempt at an equation of the tangent at the point $\left(\pi, \frac{\pi}{8}\right)$.

The gradient must be an inverted numerical value of their $\frac{dx}{dy}$

Look for $\frac{y - \frac{\pi}{8}}{x - \pi} = \frac{1}{\text{numerical } \frac{dx}{dy}}$,

Watch for negative reciprocals which is M0

If the form $y = mx + c$ is used it must be a full method to find a 'numerical' value to c.

A1 A correct equation of the tangent.

Accept $\frac{y - \frac{\pi}{8}}{x - \pi} = \frac{1}{8 + 4\pi}$ or if $y = mx + c$ is used accept $m = \frac{1}{8 + 4\pi}$ and $c = \frac{\pi}{8} - \frac{\pi}{8 + 4\pi}$

Watch for answers like this which are correct $x - \pi = (8 + 4\pi)\left(y - \frac{\pi}{8}\right)$

Accept the decimal answers awrt 2sf $y = 0.049x + 0.24$, awrt 2sf $21y = x + 4.9$, $\frac{y - 0.39}{x - 3.1} = 0.049$

Accept a mixture of decimals and π's for example $20.6\left(y - \frac{\pi}{8}\right) = x - \pi$

A1 Correct answer and solution only. $(8 + 4\pi)y = x + \frac{\pi^2}{2}$

Accept exact alternatives such as $4(2 + \pi)y = x + 0.5\pi^2$ and because the question does not ask for a and b to be simplified in the form $ay = x + b$, accept versions like

$(8 + 4\pi)y = x + \frac{\pi}{8}(8 + 4\pi) - \pi$ and $(8 + 4\pi)y = x + (8 + 4\pi)\left(\frac{\pi}{8} - \frac{\pi}{8 + 4\pi}\right)$

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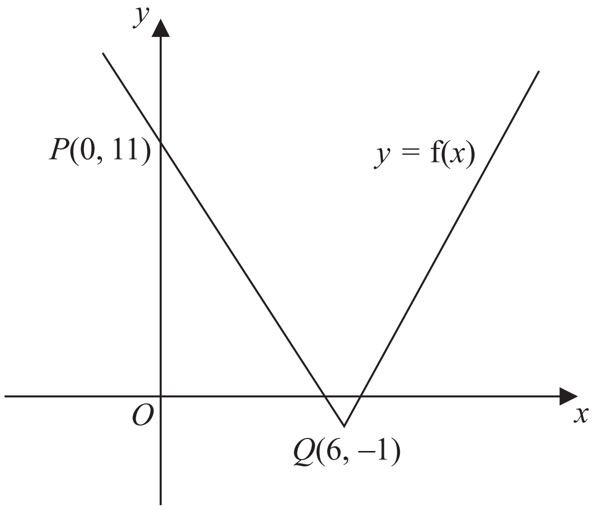


Figure 1

Figure 1 shows part of the graph with equation $y = \mathrm{f}(x)$, $x \in \mathbb{R}$.

The graph consists of two line segments that meet at the point $Q(6, -1)$.

The graph crosses the y-axis at the point $P(0, 11)$.

Sketch, on separate diagrams, the graphs of

(a) $y = |\mathrm{f}(x)|$

(2)

(b) $y = 2\mathrm{f}(-x) + 3$

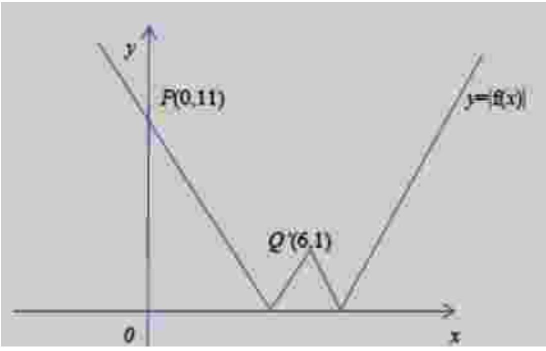
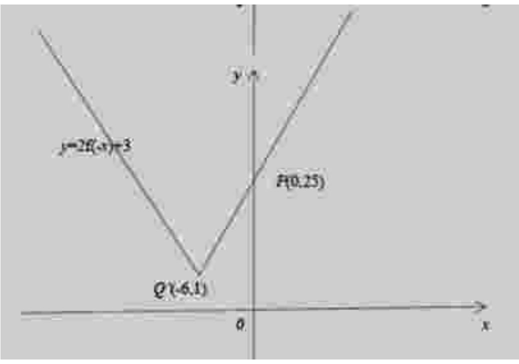
(3)

On each diagram, show the coordinates of the points corresponding to P and Q.

Given that $\mathrm{f}(x) = a|x - b| - 1$, where a and b are constants,

(c) state the value of a and the value of b.

(2)

Question Number	Scheme	Marks
4.(a)		
	'W' Shape	B1
	(0, 11) and (6, 1)	B1
		(2)
(b)		
	'V' shape	B1
	(-6,1)	B1
	(0,25)	B1
		(3)
(c)	One of $a = 2$ or $b = 6$	B1
	$a = 2$ and $b = 6$	B1
		(2)
		(7 marks)

(a)

B1 A W shape in any position. The arms of the W do not need to be symmetrical but the two bottom points must appear to be at the same height. Do not accept rounded W's.
A correct sketch of $y = \mathrm{f}(|x|)$ would score this mark.

B1 A W shape in quadrants 1 and 2 sitting on the x axis with $P' = (0,11)$ **and** $Q' = (6,1)$. It is not necessary to see them labelled. Accept 11 being marked on the y axis for P'. Condone $P' = (11,0)$ marked on the correct axis, but $Q' = (1,6)$ is B0

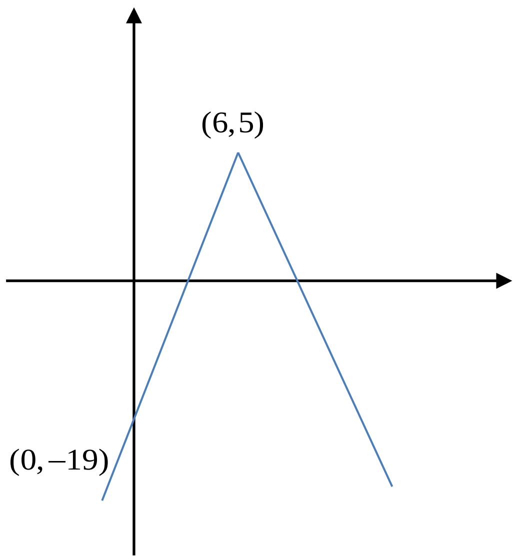
(b)

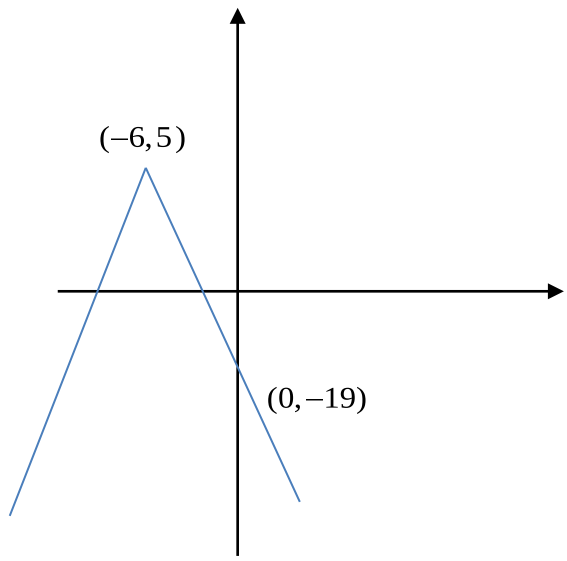
B1 Score for a V shape in any position on the grid. The arms of the V do not need to be symmetrical. Do not accept rounded or upside down V's for this mark.

B1 $Q' = (-6,1)$. It does not need to be labelled but it must correspond to the minimum point on the curve and be in the correct quadrant.

B1 $P' = (0,25)$. It does not need to be labelled but it must correspond to the y intercept and the line must cross the axis. Accept 25 marked on the correct axis. Condone $P' = (25,0)$ marked on the positive y axis.

Special case: A candidate who mistakenly sketches $y = -2\mathrm{f}(x) + 3$ or $y = -2\mathrm{f}(-x) + 3$ will arrive at one of the following. They can be awarded SC B1B0B0





(c)

B1 Either states a = 2 **or** b = 6.
This can be implied (if there are no stated answers given) by the candidate writing that $y = ..|x-6|-1$ or $y = 2|x-..|-1$. If they are both stated and written, the stated answer takes precedence.

B1 States both a = 2 **and** b = 6
This can be implied by the candidate stating that $y = 2|x-6|-1$
If they are both stated and written, the stated answer takes precedence.

Leave blank

5.

$$g(x) = \frac{x}{x+3} + \frac{3(2x+1)}{x^2+x-6}, \quad x > 3$$

(a) Show that $g(x) = \dfrac{x+1}{x-2}, \quad x > 3$

(4)

(b) Find the range of g.

(2)

(c) Find the exact value of a for which $g(a) = g^{-1}(a)$.

(4)

Question Number	Scheme	Marks
5.(a)	$x^2+x-6=(x+3)(x-2)$	B1
	$\frac{x}{x+3}+\frac{3(2x+1)}{(x+3)(x-2)}=\frac{x(x-2)+3(2x+1)}{(x+3)(x-2)}$	M1
	$=\frac{x^2+4x+3}{(x+3)(x-2)}$	A1
	$=\frac{(x+3)(x+1)}{(x+3)(x-2)}$	
	$=\frac{(x+1)}{(x-2)}$ cso	A1*
		(4)
(b)	One end either $(y)>1,(y)\geqslant 1$ or $(y)<4,(y)\leqslant 4$	B1
	$1<y<4$	B1
		(2)
(c)	Attempt to set Either $g(x)=x$ or $g(x)=g^{-1}(x)$ or $g^{-1}(x)=x$ or $g^2(x)=x$	
	$\frac{(x+1)}{(x-2)}=x$ $\quad \frac{x+1}{x-2}=\frac{2x+1}{x-1}$ $\quad \frac{2x+1}{x-1}=x$ $\quad \frac{\frac{x+1}{x-2}+1}{\frac{x+1}{x-2}-2}=x$	M1
	$x^2-3x-1=0\Rightarrow x=...$	A1, dM1
	$a=\frac{3+\sqrt{13}}{2}$ oe $\left(1.5+\sqrt{3.25}\right)$ cso	A1
		(4) (10 marks)

(a)

B1 $x^2+x-6=(x+3)(x-2)$ This can occur anywhere in the solution.

M1 For combining the two fractions with a common denominator. The denominator must be correct for their fractions and at least one numerator must have been adapted. Accept as separate fractions. Condone missing brackets.

Accept $\dfrac{x}{x+3}+\dfrac{3(2x+1)}{x^2+x-6}=\dfrac{x(x^2+x-6)+3(2x+1)(x+3)}{(x+3)(x^2+x-6)}$

Condone $\dfrac{x}{x+3}+\dfrac{3(2x+1)}{(x+3)(x-2)}=\dfrac{x\times x-2}{(x+3)(x-2)}+\dfrac{3(2x+1)}{(x+3)(x-2)}$

A1 A correct intermediate form of $\dfrac{\text{simplified quadratic}}{\text{simplified quadratic}}$

Accept $\dfrac{x^2+4x+3}{(x+3)(x-2)}, \dfrac{x^2+4x+3}{x^2+x-6}$, OR $\dfrac{x^3+7x^2+15x+9}{(x+3)(x^2+x-6)} \rightarrow \dfrac{(x+1)(x+3)\cancel{(x+3)}}{\cancel{(x+3)}(x^2+x-6)}$

As in question one they can score this mark having 'invisible' brackets on line 1.

A1* Further factorises and cancels (which may be implied) to complete the proof to reach the given answer $=\dfrac{(x+1)}{(x-2)}$. All aspects including bracketing must be correct. If a cubic is formed then it needs to be correct.

(b)

B1 States either end of the range. Accept either $y<4,\ y\leqslant 4$ or $y>1,\ y\geqslant 1$ with or without the y's.

B1 Correct range. Accept $1<y<4,\ 1<\text{g}<4,\ y>1$ and $y<4,\ (1,4),\ 1<\text{Range}<4,$ even $1<\text{f}<4,$

Do not accept $1<x<4,\ 1<y\le 4,\ [1,4)$ etc.

Special case, allow B1B0 for $1<x<4$

(c)

M1 Attempting to set $g(x)=x,\ g^{-1}(x)=x$ or $g(x)=g^{-1}(x)$ or $g^2(x)=x$.

If $g^{-1}(x)$ has been used then a full attempt must have been made to make x the subject of the formula. A full attempt would involve cross multiplying, collecting terms, factorising and ending with division.

As a result, it must be in the form $g^{-1}(x)=\dfrac{\pm 2x\pm 1}{\pm x\pm 1}$

Accept as evidence $\dfrac{(x+1)}{(x-2)}=x$ OR $\dfrac{x+1}{x-2}=\dfrac{\pm 2x\pm 1}{\pm x\pm 1}$ OR $\dfrac{\pm 2x\pm 1}{\pm x\pm 1}=x$ OR $\dfrac{\frac{x+1}{x-2}+1}{\frac{x+1}{x-2}-2}=x$

A1 $x^2-3x-1=0$ or exact equivalent. The =0 may be implied by subsequent work.

dM1 For solving a 3TQ=0. It is dependent upon the first M being scored. Do not accept a method using factors unless it clearly factorises. Allow the answer written down awrt 3.30 (from a graphical calculator).

A1 a or $x=\dfrac{3+\sqrt{13}}{2}$. Ignore any reference to $\dfrac{3-\sqrt{13}}{2}$

Withhold this mark if additional values are given for $x,\ x>3$

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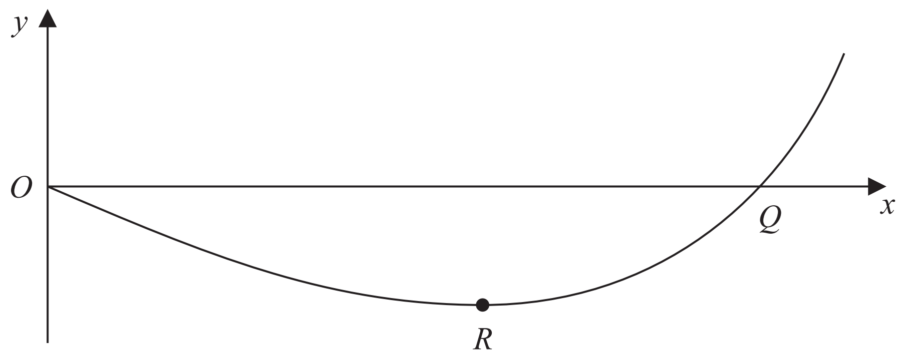

Figure 2

Figure 2 shows a sketch of part of the curve with equation

$$y = 2\cos\left(\frac{1}{2}x^2\right) + x^3 - 3x - 2$$

The curve crosses the x-axis at the point Q and has a minimum turning point at R.

(a) Show that the x coordinate of Q lies between 2.1 and 2.2

(2)

(b) Show that the x coordinate of R is a solution of the equation

$$x = \sqrt{1 + \frac{2}{3}x\sin\left(\frac{1}{2}x^2\right)}$$

(4)

Using the iterative formula

$$x_{n+1} = \sqrt{1 + \frac{2}{3}x_n\sin\left(\frac{1}{2}x_n^{\ 2}\right)}, \quad x_0 = 1.3$$

(c) find the values of x_1 and x_2 to 3 decimal places.

(2)

Question Number	Scheme	Marks
6.(a)	$y_{2.1} = -0.224$, $y_{2.2} = (+)0.546$	M1
	Change of sign$\Rightarrow Q$ lies between	A1
		(2)
(b)	At R $\dfrac{dy}{dx} = -2x\sin\left(\dfrac{1}{2}x^2\right) + 3x^2 - 3$	M1A1
	$-2x\sin\left(\dfrac{1}{2}x^2\right) + 3x^2 - 3 = 0 \Rightarrow$ $x = \sqrt{1 + \dfrac{2}{3}x\sin\left(\dfrac{1}{2}x^2\right)}$ cso	M1A1*
		(4)
(c)	$x_1 = \sqrt{1 + \dfrac{2}{3}\times 1.3\sin\left(\dfrac{1}{2}\times 1.3^2\right)}$	M1
	$x_1 = \text{awrt}\ 1.284$ $x_2 = \text{awrt}\ 1.276$	A1
		(2)
		(8 marks)

(a)

M1 Sub both x = 2.1 and x =2.2 into y and achieve at least one correct to 1 sig fig

In radians $y_{2.1} = \text{awrt} -0.2$ $y_{2.2} = \text{awrt/truncating to } 0.5$

In degrees $y_{2.1} = \text{awrt } 3$ $y_{2.2} = \text{awrt } 4$

A1 Both values correct to 1 sf with a reason and a minimal conclusion.

$y_{2.1} = \text{awrt} -0.2$ $y_{2.2} = \text{awrt/truncating to } 0.5$

Accept change of sign, positive and negative, $y_{2.1} \times y_{2.2} = -1$ as reasons and hence root, Q lies between 2.1 and 2.2, QED as a minimal conclussion.

Accept a smaller interval spanning the root of 2.131528, say 2.13 and 2.14, but the A1 can only be scored when the candidate refers back to the question, stating that as root lies between 2.13 and 2.14 it lies between 2.1 and 2.2

(b)

M1 Differentiating to get $\frac{dy}{dx} = ...\sin\left(\frac{1}{2}x^2\right) + 3x^2 - 3$ where ... is a constant, or a linear function in x.

A1 $\frac{dy}{dx} = -2x\sin\left(\frac{1}{2}x^2\right) + 3x^2 - 3$

M1 Sets their $\frac{dy}{dx} = 0$ and proceeds to make the x of their $3x^2$ the subject of the formula

Alternatively they could state $\frac{dy}{dx} = 0$ and write a line such as

$2x\sin\left(\frac{1}{2}x^2\right) = 3x^2 - 3$, before making the x of $3x^2$ the subject of the formula

A1* Correct given solution. $x = \sqrt{1 + \frac{2}{3}x\sin\left(\frac{1}{2}x^2\right)}$

Watch for missing x's in their formula

(c)

M1 Subs x= 1.3 into the iterative formula to find at least x_1.

This can be implied by $x_1 = \text{awrt} 1.3$ (not just 1.3)

or $x_1 = \sqrt{1 + \frac{2}{3} \times 1.3\sin\left(\frac{1}{2} \times 1.3^2\right)}$ or $x_1 = \text{awrt} 1.006$ (degrees)

A1 Both answers correct (awrt 3 decimal places). The subscripts are not important. Mark as the first and second values seen. $x_1 = \text{awrt } 1.284$ $x_2 = \text{awrt } 1.276$

Leave blank

7. (a) Show that

$$\operatorname{cosec} 2x + \cot 2x = \cot x, \qquad x \neq 90n°, \qquad n \in \mathbb{Z}$$

(5)

(b) Hence, or otherwise, solve, for $0 \leqslant \theta < 180°$,

$$\operatorname{cosec}(4\theta + 10°) + \cot(4\theta + 10°) = \sqrt{3}$$

You must show your working.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(5)

Question Number	Scheme	Marks
7.(a)	$\text{cosec}2x+\cot 2x=\dfrac{1}{\sin 2x}+\dfrac{\cos 2x}{\sin 2x}$	M1
	$=\dfrac{1+\cos 2x}{\sin 2x}$	M1
	$=\dfrac{1+2\cos^2 x-1}{2\sin x\cos x}$	
	$=\dfrac{2\cos^2 x}{2\sin x\cos x}$	M1 A1
	$=\dfrac{\cos x}{\sin x}=\cot x$	A1*
		(5)
(b)	$\text{cosec}(4\theta+10°)+\cot(4\theta+10°)=\sqrt{3}$	
	$\cot(2\theta\pm..°)=\sqrt{3}$	M1
	$2\theta\pm...=30°\Rightarrow\theta=12.5°$	dM1, A1
	$2\theta\pm...=180+PV°\Rightarrow\theta=..°$	dM1
	$\theta=102.5°$	A1
		(5)
		(10 marks)

(a)

M1 Writing $\text{cosec}2x = \frac{1}{\sin 2x}$ **and** $\cot 2x = \frac{\cos 2x}{\sin 2x}$ *or* $\frac{1}{\tan 2x}$

M1 Writing the lhs as a single fraction $\frac{a+b}{c}$. The denominator must be correct for their terms.

M1 Uses the appropriate double angle formulae/trig identities to produce a fraction in a form containing no addition or subtraction signs. A form $\frac{p \times q}{s \times t}$ or similar

A1 A correct intermediate line. Accept $\frac{2\cos^2 x}{2\sin x \cos x}$ or $\frac{2\sin x \cos x}{2\sin x \cos x \tan x}$ or similar

This cannot be scored if errors have been made

A1* Completes the proof by cancelling and using either $\frac{\cos x}{\sin x} = \cot x$ or $\frac{1}{\tan x} = \cot x$

The cancelling could be implied by seeing $\frac{2}{2}\frac{\cos x}{\sin x}\frac{\cos x}{\cos x} = \cot x$

The proof cannot rely on expressions like $\cot = \frac{\cos}{\sin}$ (with missing x's) for the final A1

(b)

M1 Attempt to use the solution to part (a) with $2x = 4\theta + 10 \Rightarrow$ to write or imply $\cot(2\theta \pm ...°) = \sqrt{3}$

Watch for attempts which start $\cot\alpha = \sqrt{3}$. The method mark here is not scored until the α has been replaced by $2\theta \pm ...°$

Accept a solution from $\cot(2x \pm ...°) = \sqrt{3}$ where θ has been replaced by another variable.

dM1 Proceeds from the previous method and uses $\tan .. = \frac{1}{\cot ..}$ and $\arctan\left(\frac{1}{\sqrt{3}}\right) = 30°$ to solve $2\theta \pm ...° = 30° \Rightarrow \theta = ..$

A1 $\theta = 12.5°$ or exact equivalent. Condone answers such as $x = 12.5°$

dM1 This mark is for the correct method to find a second solution to θ. It is dependent upon the first M only.

Accept $2\theta \pm ... = 180 + PV° \Rightarrow \theta = ..°$

A1 $\theta = 102.5°$ or exact equivalent. Condone answers such as $x = 102.5°$

Ignore any solutions outside the range. This mark is withheld for any extra solutions within the range.

If radians appear they could just lose the answer marks. So for example

$2\theta \pm ... = \frac{\pi}{6}(0.524) \Rightarrow \theta = ..$is M1dM1A0 followed by

$2\theta \pm ... = \pi + '\frac{\pi}{6}' \Rightarrow \theta = ..$dM1A0

Special case 1: For candidates in (b) who solve $\cot(4\theta \pm ...°) = \sqrt{3}$ the mark scheme is severe, so we are awarding a special case solution, scoring 00011.

$\cot(4\theta + \beta°) = \sqrt{3} \Rightarrow 4\theta + \beta = 30° \Rightarrow \theta = ..$ is M0M0A0 where $\beta = 5°$ or $10°$

$\Rightarrow 4\theta + \beta = 210° \Rightarrow \theta = ..$ can score M1A1 Special case.

If $\beta = 5°, \theta = 51.25$ If $\beta = 10°, \theta = 50$

Special case 2: Just answers in (b) **with no working** scores 1 1 0 0 0 for 12.5 and 102.5

BUT $\cot(2\theta \pm 5°) = \sqrt{3} \Rightarrow \theta = 12.5°, 102.5°$ scores all available marks.

Question Number	Scheme	Marks
7.(a)Alt 1	$\text{cosec}2x + \cot 2x = \frac{1}{\sin 2x} + \frac{1}{\tan 2x}$	1^{ST} M1
	$= \frac{1}{2\sin x \cos x} + \frac{1 - \tan^2 x}{2\tan x}$	
	$= \frac{\tan x + (1-\tan^2 x)\sin x \cos x}{2\sin x \cos x \tan x}$ or $= \frac{2\tan x + 2(1-\tan^2 x)\sin x \cos x}{4\sin x \cos x \tan x}$	2^{nd} M1
	$= \frac{\tan x + \sin x \cos x - \tan^2 x \sin x \cos x}{2\sin x \cos x \tan x}$	
	$= \frac{\tan x + \sin x \cos x - \tan x \sin^2 x}{2\sin x \cos x \tan x}$	
	$= \frac{\tan x(1-\sin^2 x) + \sin x \cos x}{2\sin x \cos x \tan x}$	
	$= \frac{\tan x \cos^2 x + \sin x \cos x}{2\sin x \cos x \tan x}$	
	$= \frac{\sin x \cos x + \sin x \cos x}{2\sin x \cos x \tan x}$	
	$= \frac{2\sin x \cos x}{2\sin x \cos x \tan x}$ oe	3^{rd} M1A1
	$= \frac{1}{\tan x} = \cot x$	A1* (5)
7.(a)Alt 2	**Example of how main scheme could work in a roundabout route**	
	$\text{cosec}2x + \cot 2x = \cot x \Leftrightarrow \frac{1}{\sin 2x} + \frac{1}{\tan 2x} = \frac{1}{\tan x}$	1^{st} M1
	$\Leftrightarrow \tan 2x \tan x + \sin 2x \tan x = \sin 2x \tan 2x$	2^{nd} M1
	$\Leftrightarrow \frac{2\tan x}{1-\tan^2 x} \times \tan x + 2\sin x \cancel{\cos x} \times \frac{\sin x}{\cancel{\cos x}} = 2\sin x \cos x \times \frac{2\tan x}{1-\tan^2 x}$	

Question Number	Scheme	Marks
	$\Leftrightarrow \dfrac{2\tan^2 x}{1-\tan^2 x} + 2\sin^2 x = \dfrac{4\sin^2 x}{1-\tan^2 x}$	
	$\times(1-\tan^2 x) \Leftrightarrow 2\tan^2 x + 2\sin^2 x(1-\tan^2 x) = 4\sin^2 x$	
	$\Leftrightarrow 2\tan^2 x - 2\sin^2 x\tan^2 x = 2\sin^2 x$	
	$\Leftrightarrow 2\tan^2 x(1-\sin^2 x) = 2\sin^2 x$	3rd M1
	$\div 2\tan^2 x \Leftrightarrow 1-\sin^2 x = \cos^2 x$	A1
	As this is true, initial statement is true	A1*
		(5)

Leave blank

8. A rare species of primrose is being studied. The population, P, of primroses at time t years after the study started is modelled by the equation

$$P = \frac{800e^{0.1t}}{1 + 3e^{0.1t}}, \qquad t \geqslant 0, \quad t \in \mathbb{R}$$

(a) Calculate the number of primroses at the start of the study.

(2)

(b) Find the exact value of t when $P = 250$, giving your answer in the form $a \ln(b)$ where a and b are integers.

(4)

(c) Find the exact value of $\frac{dP}{dt}$ when $t = 10$. Give your answer in its simplest form.

(4)

(d) Explain why the population of primroses can never be 270

(1)

Question Number	Scheme	Marks
8.(a)	$P=\dfrac{800e^{0}}{1+3e^{0}},=\dfrac{800}{1+3}=200$	M1,A1
		(2)
(b)	$250=\dfrac{800e^{0.1t}}{1+3e^{0.1t}}$	
	$250(1+3e^{0.1t})=800e^{0.1t}\Rightarrow 50e^{0.1t}=250,\ \Rightarrow e^{0.1t}=5$	M1,A1
	$t=\dfrac{1}{0.1}\ln(5)$	M1
	$t=10\ln(5)$	A1
		(4)
(c)	$P=\dfrac{800e^{0.1t}}{1+3e^{0.1t}}\Rightarrow\dfrac{dP}{dt}=\dfrac{(1+3e^{0.1t})\times 800\times 0.1e^{0.1t}-800e^{0.1t}\times 3\times 0.1e^{0.1t}}{(1+3e^{0.1t})^{2}}$	M1,A1
	At t=10	
	$\dfrac{dP}{dt}=\dfrac{(1+3e)\times 80e-240e^{2}}{(1+3e)^{2}}=\dfrac{80e}{(1+3e)^{2}}$	M1,A1
		(4)
(d)	$P=\dfrac{800e^{0.1t}}{1+3e^{0.1t}}=\dfrac{800}{e^{-0.1t}+3}\Rightarrow P_{max}=\dfrac{800}{3}=266$. Hence P cannot be 270	B1
		(1)
		(11 marks)

(a)

M1 Sub t = 0 into P **and** use $e^{0}=1$ in at least one of the two cases. Accept $P=\dfrac{800}{1+3}$ as evidence

A1 200. Accept this for both marks as long as no incorrect working is seen.

(b)

M1 Sub P=250 into $P=\dfrac{800e^{0.1t}}{1+3e^{0.1t}}$, cross multiply, collect terms in $e^{0.1t}$ **and** proceed to $Ae^{0.1t}=B$

Condone bracketing issues and slips in arithmetic.

If they divide terms by $e^{0.1t}$ you should expect to see $Ce^{-0.1t}=D$

A1 $e^{0.1t}=5$ or $e^{-0.1t}=0.2$

M1 Dependent upon gaining $e^{0.1t} = E$, for taking ln's of both sides and proceeding to $t=\dots$

Accept $e^{0.1t} = E \Rightarrow 0.1t = \ln E \Rightarrow t = \dots$ It could be implied by $t = \text{awrt } 16.1$

A1 $t = 10\ln(5)$

Accept exact equivalents of this as long as a and b are integers. Eg. $t = 5\ln(25)$ is fine.

(c)

M1 Scored for a full application of the quotient rule and knowing that $\frac{d}{dt}e^{0.1t} = ke^{0.1t}$ and NOT $kte^{0.1t}$

If the rule is quoted it must be correct.

It may be implied by their $u = 800e^{0.1t}, v = 1+3e^{0.1t}, u' = pe^{0.1t}, v' = qe^{0.1t}$ followed by $\frac{vu'-uv'}{v^2}$.

If it is neither quoted nor implied only accept expressions of the form

$$\frac{(1+3e^{0.1t})\times pe^{0.1t} - 800e^{0.1t}\times qe^{0.1t}}{(1+3e^{0.1t})^2}$$

Condone missing brackets.

You may see the chain or product rule applied to

For applying the product rule see question 1 but still insist on $\frac{d}{dt}e^{0.1t} = ke^{0.1t}$

For the chain rule look for

$$P = \frac{800e^{0.1t}}{1+3e^{0.1t}} = \frac{800}{e^{-0.1t}+3} \Rightarrow \frac{dP}{dt} = 800\times\left(e^{-0.1t}+3\right)^{-2}\times -0.1e^{-0.1t}$$

A1 A correct unsimplified answer to

$$\frac{dP}{dt} = \frac{(1+3e^{0.1t})\times 800\times 0.1e^{0.1t} - 800e^{0.1t}\times 3\times 0.1e^{0.1t}}{(1+3e^{0.1t})^2}$$

M1 For substituting $t = 10$ into their $\frac{dP}{dt}$, NOT P

Accept numerical answers for this. 2.59 is the numerical value if $\frac{dP}{dt}$ was correct

A1 $\frac{dP}{dt} = \frac{80e}{(1+3e)^2}$ or equivalent such as $\frac{dP}{dt} = 80e(1+3e)^{-2}, \frac{80e}{1+6e+9e^2}$

Note that candidates who substitute $t = 10$ before differentiation will score 0 marks

(d)

B1 Accept solutions from substituting P=270 and showing that you get an unsolvable equation

Eg. $270 = \frac{800e^{0.1t}}{1+3e^{0.1t}} \Rightarrow -27 = e^{0.1t} \Rightarrow 0.1t = \ln(-27)$ which has no answers.

Eg. $270 = \frac{800e^{0.1t}}{1+3e^{0.1t}} \Rightarrow -27 = e^{0.1t} \Rightarrow e^{0.1t} / e^x$ is never negative

Accept solutions where it implies the max value is 266.6 or 267. For example accept sight of $\frac{800}{3}$, with a comment 'so it cannot reach 270', or a large value of t ($t > 99$) being substituted in to get 266.6 or 267 with a similar statement, or a graph drawn with an asymptote marked at 266.6 or 267

Do not accept exp's cannot be negative or you cannot ln a negative number without numerical evidence.

Look for both a statement and a comment

Leave blank

9. (a) Express $2\sin\theta - 4\cos\theta$ in the form $R\sin(\theta - \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the value of α to 3 decimal places.

(3)

$$\text{H}(\theta) = 4 + 5(2\sin 3\theta - 4\cos 3\theta)^2$$

Find

(b) (i) the maximum value of $\text{H}(\theta)$,

(ii) the smallest value of θ, for $0 \leqslant \theta < \pi$, at which this maximum value occurs.

(3)

Find

(c) (i) the minimum value of $\text{H}(\theta)$,

(ii) the largest value of θ, for $0 \leqslant \theta < \pi$, at which this minimum value occurs.

(3)

Question Number	Scheme	Marks
9.(a)	$R=\sqrt{20}$	B1
	$\tan\alpha=\frac{4}{2}\Rightarrow\alpha=\text{awrt }1.107$	M1A1
		(3)
(b)(i)	$'4+5R^2'=104$	B1ft
(ii)	$3\theta-'1.107'=\frac{\pi}{2}\Rightarrow\theta=\text{awrt }0.89$	M1A1
		(3)
(c)(i)	4	B1
(ii)	$3\theta-'1.107'=2\pi\Rightarrow\theta=\text{awrt }2.46$	M1A1
		(3)
		(9 marks)

(a)

B1 Accept $R=\sqrt{20}$ or $2\sqrt{5}$ or awrt 4.47

Do not accept $R=\pm\sqrt{20}$

This could be scored in parts (b) or (c) as long as you are certain it is R

M1 for sight of $\tan\alpha=\pm\frac{4}{2}, \tan\alpha=\pm\frac{2}{4}$. Condone $\sin\alpha=4, \cos\alpha=2 \Rightarrow \tan\alpha=\frac{4}{2}$

If R is found first only accept $\sin\alpha=\pm\frac{4}{R}, \cos\alpha=\pm\frac{2}{R}$

A1 $\alpha=$ awrt 1.107. The degrees equivalent $63.4°$ is A0.

If a candidate does all the question in degrees they will lose just this mark.

(b)(i)

B1ft Either 104 or if R was incorrect allow for the numerical value of their $'4+5R^2'$.

Allow a tolerance of 1 dp on decimal R's.

(b)(ii)

M1 Using $3\theta \pm \text{their}'1.107'=\frac{\pi}{2} \Rightarrow \theta=..$

Accept $3\theta \pm \text{their}'1.107'=(2n+1)\frac{\pi}{2} \Rightarrow \theta=..$ where n is an integer

Allow slips on the lhs with an extra bracket such as

$3(\theta \pm \text{their}'1.107')=\frac{\pi}{2} \Rightarrow \theta=..$

The degree equivalent is acceptable $3\theta-\text{their}'63.4°'=90° \Rightarrow \theta=$

Do not allow mixed units in this question

A1 awrt 0.89 radians or $51.1°$. Do not allow multiple solutions for this mark.

(c)(i)

B1 4

(c)(ii)

M1 Using $3\theta \pm \text{their}'1.107'=2\pi \Rightarrow \theta=...$

Accept $3\theta \pm \text{their}'1.107'=n\pi \Rightarrow \theta=..$ where n is an integer , including 0

Allow slips on the lhs with an extra bracket such as

$3(\theta \pm \text{their}'1.107')=2\pi \Rightarrow \theta=..$

The degree equivalent is acceptable $3\theta-\text{their}'63.4°'=360° \Rightarrow \theta=$ but

Do not allow mixed units in this question

A1 $\theta=$ awrt 2.46 radians or $141.1°$ Do not allow multiple solutions for this mark.