Centre No.						Paper Reference							Surname	Initial(s)
Candidate No.						6	6	6	6	/	0	1	Signature	

Paper Reference(s)

6666/01

Edexcel GCE

Core Mathematics C4

Advanced Level

Monday 23 January 2006 – Afternoon

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
1	
2	
3	
4	
5	
6	
7	
8	
Total	

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Green)	Nil

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initial(s) and signature.
Check that you have the correct question paper.
You must write your answer for each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.
There are 20 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

Turn over

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1. A curve C is described by the equation

$$3x^2 + 4y^2 - 2x + 6xy - 5 = 0.$$

Find an equation of the tangent to C at the point (1, –2), giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(7)

<table>
<tr><th>Question Number</th><th>Scheme</th><th>Marks</th></tr>
<tr><td>1.</td><td>Differentiates</td><td>M1</td></tr>
<tr><td></td><td>to obtain : $6x+8y\frac{dy}{dx}-2,$</td><td>A1,</td></tr>
<tr><td></td><td>$......................+(6x\frac{dy}{dx}+6y)=0$</td><td>+(B1)</td></tr>
<tr><td></td><td>$\left[\frac{dy}{dx}=\frac{2-6x-6y}{6x+8y}\right]$</td><td></td></tr>
<tr><td></td><td>Substitutes $x=1, y=-2$ into expression involving $\frac{dy}{dx}$, to give $\frac{dy}{dx}=-\frac{8}{10}$</td><td>M1, A1</td></tr>
<tr><td></td><td>Uses line equation with numerical ‘gradient’ y - (- 2) = (their gradient)(x - 1)
or finds c and uses $y=(their\ gradient)\ x\ +\ "c"$</td><td>M1</td></tr>
<tr><td></td><td>To give $5y+4x+6=0$ (or equivalent = 0)</td><td>A1√ [7]</td></tr>
<tr><td>2.
(a)</td><td>
<table>
<tr><td>x</td><td>0</td><td>$\frac{\pi}{16}$</td><td>$\frac{\pi}{8}$</td><td>$\frac{3\pi}{16}$</td><td>$\frac{\pi}{4}$</td></tr>
<tr><td>y</td><td>1</td><td>1.01959</td><td>1.08239</td><td>1.20269</td><td>1.41421</td></tr>
</table>
M1 for one correct, A1 for all correct</td><td>M1 A1
(2)</td></tr>
<tr><td>(b)</td><td>Integral = $\frac{1}{2}\times\frac{\pi}{16}\times\{1+1.4142+2(1.01959+...+1.20269)\}$</td><td>M1 A1√</td></tr>
<tr><td></td><td>$\left(=\frac{\pi}{32}\,x\,9.02355\right)$ = 0.8859</td><td>A1 cao
(3)</td></tr>
<tr><td>(c)</td><td>Percentage error = $\frac{approx-0.88137}{0.88137}\times100$ = 0.51 % (allow 0.5% to 0.54% for A1)</td><td>M1 A1 (2)</td></tr>
<tr><td></td><td>M1 gained for $(\pm)\ \frac{approx-\ln(1+\sqrt{2})}{\ln(1+\sqrt{2})}$</td><td>[7]</td></tr>
</table>

2. (a) Given that $y = \sec x$, complete the table with the values of y corresponding to $x = \frac{\pi}{16}, \frac{\pi}{8}$ and $\frac{\pi}{4}$.

x	0	$\frac{\pi}{16}$	$\frac{\pi}{8}$	$\frac{3\pi}{16}$	$\frac{\pi}{4}$
y	1			1.20269	

(2)

(b) Use the trapezium rule, with all the values for y in the completed table, to obtain an estimate for $\int_0^{\frac{\pi}{4}} \sec x \, dx$. Show all the steps of your working, and give your answer to 4 decimal places.

(3)

The exact value of $\int_0^{\frac{\pi}{4}} \sec x \, dx$ is $\ln(1 + \sqrt{2})$.

(c) Calculate the % error in using the estimate you obtained in part (b).

(2)

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[7]</td></tr>
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3. Using the substitution $u^2 = 2x - 1$, or otherwise, find the exact value of

$$\int_1^5 \frac{3x}{\sqrt{(2x-1)}}\,dx.$$

(8)

Question Number	Scheme	Marks
3.	Uses substitution to obtain $x = \mathrm{f}(u)$ $\left[\frac{u^2+1}{2}\right]$,	M1
	and to obtain $u\frac{du}{dx}$ = const. or equiv.	M1
	Reaches $\int \frac{3(u^2+1)}{2u}udu$ or equivalent	A1
	Simplifies integrand to $\int\left(3u^2+\frac{3}{2}\right)\mathrm{d}u$ or equiv.	M1
	Integrates to $\frac{1}{2}u^3+\frac{3}{2}u$	M1 A1√
	A1√ dependent on all previous Ms	
	Uses new limits 3 and 1 substituting and subtracting (or returning to function of x with old limits)	M1
	To give 16 cso	A1 **[8]**
	"By Parts" Attempt at " right direction" by parts M1	
	[$3x\left(2x-1)^{\frac{1}{2}}\right)$ – { $\int 3(2x-1)^{\frac{1}{2}}\,\mathrm{d}x$ }] M1{M1A1}	
	……………. – $(2x-1)^{\frac{3}{2}}$ M1A1√	
	Uses limits 5 and 1 correctly; [42 – 26] 16 M1A1	

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4.

Figure 1

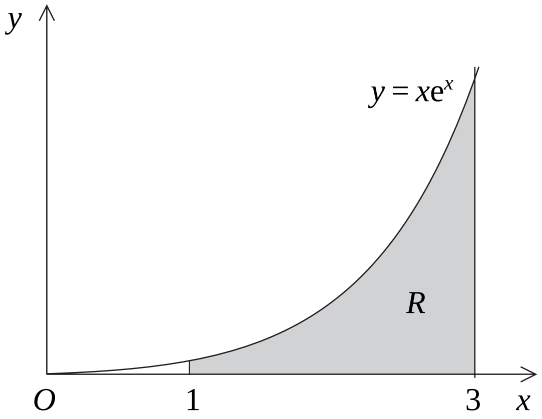

Figure 1 shows the finite shaded region, R, which is bounded by the curve $y = xe^x$, the line $x = 1$, the line $x = 3$ and the x-axis.

The region R is rotated through 360 degrees about the x-axis.

Use integration by parts to find an exact value for the **volume** of the solid generated.

(8)

4.	Attempts $V = \pi\int x^2e^{2x}\mathrm{d}x$	M1
	$= \pi\left[\frac{x^2e^{2x}}{2} - \int xe^{2x}dx\right]$ (M1 needs parts in the correct direction)	M1 A1
	$= \pi[\frac{x^2e^{2x}}{2} - \left(\frac{xe^{2x}}{2} - \int\frac{e^{2x}}{2}dx\right)]$ (M1 needs second application of parts)	M1 A1√
	M1A1√ refers to candidates $\int xe^{2x}\mathrm{d}x$, but dependent on prev. M1	
	$= \pi[\frac{x^2e^{2x}}{2} - \left(\frac{xe^{2x}}{2} - \frac{e^{2x}}{4}\right)]$	A1 cao
	Substitutes limits 3 and 1 and subtracts to give… [dep. on second and third Ms]	dM1
	$= \pi\left[\frac{13}{4}e^6 - \frac{1}{4}e^2\right]$ or any correct exact equivalent.	A1 **[8]**
	[Omission of π loses first and last marks only]	

5.

$$\mathrm{f}(x) = \frac{3x^2 + 16}{(1-3x)(2+x)^2} = \frac{A}{(1-3x)} + \frac{B}{(2+x)} + \frac{C}{(2+x)^2}, \qquad |x| < \tfrac{1}{3}.$$

(a) Find the values of A and C and show that $B = 0$.

(4)

(b) Hence, or otherwise, find the series expansion of f(x), in ascending powers of x, up to and including the term in x^3. Simplify each term.

(7)

Question Number	Scheme	Marks
5. (a)	Considers $3x^2+16 = A(2+x)^2 + B(1-3x)(2+x) + C(1-3x)$	
	and substitutes x = –2 , or x = 1/3 ,	M1
	or compares coefficients and solves simultaneous equations	
	To obtain A = 3, and C = 4	A1, A1
	Compares coefficients or uses simultaneous equation to show B = 0.	B1 (4)
(b)	Writes $3(1-3x)^{-1} + 4(2+x)^{-2}$	M1
	$= 3(1+3x, +9x^2+27x^3+......)$ +	(M1, A1)
	$\frac{4}{4}(1+\frac{(-2)}{1}\left(\frac{x}{2}\right)+\frac{(-2)(-3)}{1.2}\left(\frac{x}{2}\right)^2+\frac{(-2)(-3)(-4)}{1.2.3}\left(\frac{x}{2}\right)^3$ +....)	(M1 A1)
	$= 4+8x, \ +27\tfrac{3}{4}x^2+80\tfrac{1}{2}x^3+...$	A1, A1 (7)
	Or uses $(3x^2+16)(1-3x)^{-1}(2+x)^{-2}$	M1
	$(3x^2+16)\ (1+3x, +9x^2+27x^3+)\ \times$	(M1A1)×
	$\tfrac{1}{4}(1+\frac{(-2)}{1}\left(\frac{x}{2}\right)+\frac{(-2)(-3)}{1.2}\left(\frac{x}{2}\right)^2+\frac{(-2)(-3)(-4)}{1.2.3}\left(\frac{x}{2}\right)^3)$	(M1A1)
	$= 4+8x, +27\tfrac{3}{4}x^2+80\tfrac{1}{2}x^3+...$	A1, A1 (7) **[11]**

6. The line l_1 has vector equation

$$\mathbf{r} = 8\mathbf{i} + 12\mathbf{j} + 14\mathbf{k} + \lambda(\mathbf{i} + \mathbf{j} - \mathbf{k}),$$

where λ is a parameter.

The point A has coordinates $(4, 8, a)$, where a is a constant. The point B has coordinates $(b, 13, 13)$, where b is a constant. Points A and B lie on the line l_1.

(a) Find the values of a and b.

(3)

Given that the point O is the origin, and that the point P lies on l_1 such that OP is perpendicular to l_1,

(b) find the coordinates of P.

(5)

(c) Hence find the distance OP, giving your answer as a simplified surd.

(2)

6. (a)	$\lambda=-4\to a=18, \qquad \mu=1\to b=9$	M1 A1, A1 (3)
(b)	$\begin{pmatrix}8+\lambda\\12+\lambda\\14-\lambda\end{pmatrix}\bullet\begin{pmatrix}1\\1\\-1\end{pmatrix}=0$	M1
	$\therefore 8+\lambda+12+\lambda-14+\lambda=0$	A1
	Solves to obtain λ ($\lambda=-2$)	dM1
	Then substitutes value for λ to give P at the point (6, 10, 16) (any form)	M1, A1 (5)
(c)	OP = $\sqrt{36+100+256}$	M1
	(= $\sqrt{392}$) $=14\sqrt{2}$	A1 cao (2) **[10]**
7. (a)	$\frac{dV}{dr}=4\pi r^2$	B1 (1)
(b)	Uses $\frac{dr}{dt}=\frac{dV}{dt}.\frac{dr}{dV}$ in any form, $=\frac{1000}{4\pi r^2(2t+1)^2}$	M1,A1 (2)
(c)	$V=\int 1000(2t+1)^{-2}dt$ and integrate to $p\,(2t+1)^{-1}$, $=-500(2t+1)^{-1}(+c)$	M1, A1
	Using V=0 when t=0 to find c , (c = 500 , or equivalent)	M1
	$\therefore V=500(1-\frac{1}{2t+1})$ (any form)	A1 (4)
(d)	(i) Substitute t = 5 to give V,	M1,
	then use $r=\sqrt[3]{\left(\frac{3V}{4\pi}\right)}$ to give *r* , = *4.77*	M1, A1 (3)
	(ii) Substitutes t = 5 and r = ‘their value’ into ‘their’ part (b)	M1
	$\frac{dr}{dt}=0.0289$ $(\approx 2.90x10^{-2})$ (cm/s) $*$ AG	A1 (2) **[12]**

7. The volume of a spherical balloon of radius r cm is $V\,\text{cm}^3$, where $V = \frac{4}{3}\pi r^3$.

(a) Find $\dfrac{\mathrm{d}V}{\mathrm{d}r}$.

(1)

The volume of the balloon increases with time t seconds according to the formula

$$\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{1000}{(2t+1)^2}, \qquad t \geqslant 0.$$

(b) Using the chain rule, or otherwise, find an expression in terms of r and t for $\dfrac{\mathrm{d}r}{\mathrm{d}t}$.

(2)

(c) Given that $V = 0$ when $t = 0$, solve the differential equation $\dfrac{\mathrm{d}V}{\mathrm{d}t} = \dfrac{1000}{(2t+1)^2}$, to obtain V in terms of t.

(4)

(d) Hence, at time $t = 5$,

(i) find the radius of the balloon, giving your answer to 3 significant figures,

(3)

(ii) show that the rate of increase of the radius of the balloon is approximately $2.90 \times 10^{-2}\,\text{cm s}^{-1}$.

(2)

6. (a)	$\lambda = -4 \to a = 18, \qquad \mu = 1 \to b = 9$	M1 A1, A1 (3)
(b)	$\begin{pmatrix} 8+\lambda \\ 12+\lambda \\ 14-\lambda \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} = 0$	M1
	$\therefore 8+\lambda+12+\lambda-14+\lambda = 0$	A1
	Solves to obtain λ ($\lambda = -2$)	dM1
	Then substitutes value for λ to give P at the point (6, 10, 16) (any form)	M1, A1 (5)
(c)	OP = $\sqrt{36+100+256}$	M1
	(= $\sqrt{392}$) $= 14\sqrt{2}$	A1 cao (2) **[10]**
7. (a)	$\frac{dV}{dr} = 4\pi r^2$	B1 (1)
(b)	Uses $\frac{dr}{dt} = \frac{dV}{dt} . \frac{dr}{dV}$ in any form, $= \frac{1000}{4\pi r^2 (2t+1)^2}$	M1,A1 (2)
(c)	$V = \int 1000(2t+1)^{-2} dt$ and integrate to $p\,(2t+1)^{-1}$, $= -500(2t+1)^{-1}(+c)$	M1, A1
	Using V=0 when t=0 to find c , (c = 500 , or equivalent)	M1
	$\therefore V = 500(1 - \frac{1}{2t+1})$ (any form)	A1 (4)
(d)	(i) Substitute t = 5 to give V,	M1,
	then use $r = \sqrt[3]{\left(\frac{3V}{4\pi}\right)}$ to give *r* , = *4.77*	M1, A1 (3)
	(ii) Substitutes t = 5 and r = 'their value' into 'their' part (b)	M1
	$\frac{dr}{dt} = 0.0289 \quad (\approx 2.90 x 10^{-2})$ (cm/s) $*$ AG	A1 (2) **[12]**

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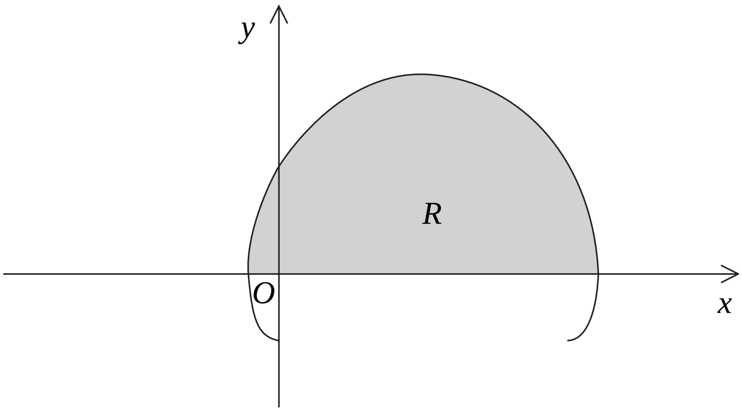
8.

Figure 2



The curve shown in Figure 2 has parametric equations

$$x = t - 2\sin t, \qquad y = 1 - 2\cos t, \qquad 0 \leqslant t \leqslant 2\pi.$$

(a) Show that the curve crosses the x-axis where $t = \dfrac{\pi}{3}$ and $t = \dfrac{5\pi}{3}$.

(2)

The finite region R is enclosed by the curve and the x-axis, as shown shaded in Figure 2.

(b) Show that the area of R is given by the integral

$$\int_{\frac{\pi}{3}}^{\frac{5\pi}{3}} (1 - 2\cos t)^2 \, \mathrm{d}t.$$

(3)

(c) Use this integral to find the exact value of the shaded area.

(7)

8. (a) Solves y = 0 $\Rightarrow$ $\cos t = \frac{1}{2}$ to obtain $t = \frac{\pi}{3}\ or\ \frac{5\pi}{3}$ (need both for A1)

Or substitutes **both** values of *t* and shows that $y = 0$

(b)

$$\frac{dx}{dt} = 1 - 2\cos t$$

Area= $\int y dx = \int_{\pi/3}^{5\pi/3} (1 - 2\cos t)(1 - 2\cos t) dt$ = $\int_{\frac{\pi}{3}}^{\frac{5\pi}{3}} (1 - 2\cos t)^2 dt$ $*$ AG

(c)

Area = $\int 1 - 4\cos t + 4\cos^2 t dt$ 3 terms

= $\int 1 - 4\cos t + 2(\cos 2t + 1) dt$ (use of correct double angle formula)

= $\int 3 - 4\cos t + 2\cos 2t dt$

= $\left[3t - 4\sin t + \sin 2t\right]$

Substitutes the two correct limits $t = \frac{5\pi}{3}$ and $\frac{\pi}{3}$ and subtracts.

$= 4\pi + 3\sqrt{3}$