

January 2007
6666 Core Mathematics C4
Mark Scheme

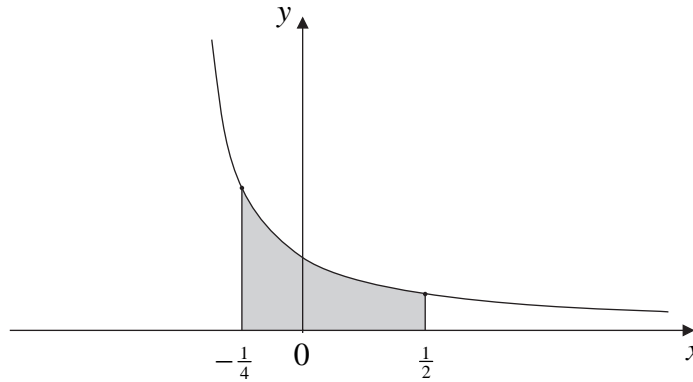
Question Number	Scheme	Marks
1.	<p>** represents a constant</p> $f(x) = (2 - 5x)^{-2} = \underline{(2)^{-2}} \left(1 - \frac{5x}{2}\right)^{-2} = \underline{\frac{1}{4}} \left(1 - \frac{5x}{2}\right)^{-2}$ $= \frac{1}{4} \left\{ 1 + (-2)(**x) + \frac{(-2)(-3)}{2!} (**x)^2 + \frac{(-2)(-3)(-4)}{3!} (**x)^3 + \dots \right\}$ $= \frac{1}{4} \left\{ 1 + (-2)\left(\frac{-5x}{2}\right) + \frac{(-2)(-3)}{2!} \left(\frac{-5x}{2}\right)^2 + \frac{(-2)(-3)(-4)}{3!} \left(\frac{-5x}{2}\right)^3 + \dots \right\}$ $= \frac{1}{4} \left\{ 1 + 5x + \frac{75x^2}{4} + \frac{125x^3}{2} + \dots \right\}$ $= \frac{1}{4} + \frac{5x}{4} + \frac{75x^2}{16} + \frac{125x^3}{8} + \dots$ $= \frac{1}{4} + 1\frac{1}{4}x + 4\frac{11}{16}x^2 + 15\frac{5}{8}x^3 + \dots$	<p>Takes 2 outside the bracket to give any of $(2)^{-2}$ or $\frac{1}{4}$. B1</p> <p>Expands $(1 + **x)^{-2}$ to give an unsimplified $1 + (-2)(**x)$; M1</p> <p>A correct unsimplified {.....} expansion with candidate's $(**x)$ A1</p> <p>Anything that cancels to $\frac{1}{4} + \frac{5x}{4}$; A1;</p> <p>Simplified $\frac{75x^2}{16} + \frac{125x^3}{8}$ A1</p> <p>[5]</p> <p>5 marks</p>

Question Number	Scheme	Marks
<p>Aliter</p> <p>1.</p> <p>Way 2</p>	<p>$f(x) = (2 - 5x)^{-2}$</p> $= \left\{ \begin{aligned} &(2)^{-2} + (-2)(2)^{-3}(**x); + \frac{(-2)(-3)}{2!}(2)^{-4}(**x)^2 \\ &+ \frac{(-2)(-3)(-4)}{3!}(2)^{-5}(**x)^3 + \dots \end{aligned} \right\}$ $= \left\{ \begin{aligned} &(2)^{-2} + (-2)(2)^{-3}(-5x); + \frac{(-2)(-3)}{2!}(2)^{-4}(-5x)^2 \\ &+ \frac{(-2)(-3)(-4)}{3!}(2)^{-5}(-5x)^3 + \dots \end{aligned} \right\}$ $= \left\{ \begin{aligned} &\frac{1}{4} + (-2)(\frac{1}{8})(-5x); + (3)(\frac{1}{16})(25x^2) \\ &+ (-4)(\frac{1}{16})(-125x^3) + \dots \end{aligned} \right\}$ $= \frac{1}{4} + \frac{5x}{4}; + \frac{75x^2}{16} + \frac{125x^3}{8} + \dots$ $= \frac{1}{4} + 1\frac{1}{4}x; + 4\frac{11}{16}x^2 + 15\frac{5}{8}x^3 + \dots$	<p>$\frac{1}{4}$ or $(2)^{-2}$ B1</p> <p>Expands $(2 - 5x)^{-2}$ to give an unsimplified $(2)^{-2} + (-2)(2)^{-3}(**x)$; M1</p> <p>A correct unsimplified $\{.....\}$ expansion with candidate's $(**x)$ A1</p> <p>Anything that cancels to $\frac{1}{4} + \frac{5x}{4}$; A1;</p> <p>Simplified $\frac{75x^2}{16} + \frac{125x^3}{8}$ A1</p> <p>[5]</p> <p>5 marks</p>

Attempts using Maclaurin expansions need to be referred to your team leader.

2.

Figure 1



The curve with equation $y = \frac{1}{3(1+2x)}$, $x > -\frac{1}{2}$, is shown in Figure 1.

The region bounded by the lines $x = -\frac{1}{4}$, $x = \frac{1}{2}$, the x -axis and the curve is shown shaded in Figure 1.

This region is rotated through 360 degrees about the x -axis.

(a) Use calculus to find the exact value of the volume of the solid generated.

(5)

Figure 2

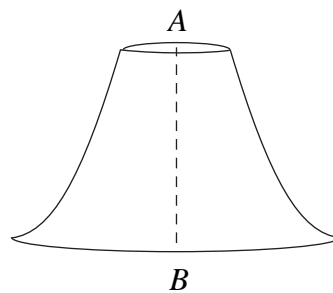


Figure 2 shows a paperweight with axis of symmetry AB where $AB = 3$ cm. A is a point on the top surface of the paperweight, and B is a point on the base of the paperweight. The paperweight is geometrically similar to the solid in part (a).

(b) Find the volume of this paperweight.

(2)



Question Number	Scheme	Marks	
2. (a)	$\text{Volume} = \pi \int_{-\frac{1}{4}}^{\frac{1}{2}} \left(\frac{1}{3(1+2x)} \right)^2 dx = \frac{\pi}{9} \int_{-\frac{1}{4}}^{\frac{1}{2}} \frac{1}{(1+2x)^2} dx$ $= \left(\frac{\pi}{9} \right) \int_{-\frac{1}{4}}^{\frac{1}{2}} (1+2x)^{-2} dx$ $= \left(\frac{\pi}{9} \right) \left[\frac{(1+2x)^{-1}}{(-1)(2)} \right]_{-\frac{1}{4}}^{\frac{1}{2}}$ $= \left(\frac{\pi}{9} \right) \left[-\frac{1}{2}(1+2x)^{-1} \right]_{-\frac{1}{4}}^{\frac{1}{2}}$ $= \left(\frac{\pi}{9} \right) \left[\left(\frac{-1}{2(2)} \right) - \left(\frac{-1}{2(\frac{1}{2})} \right) \right]$ $= \left(\frac{\pi}{9} \right) \left[-\frac{1}{4} - (-1) \right]$ $= \frac{\pi}{12}$	<p>Use of $V = \pi \int y^2 dx$. Can be implied. Ignore limits.</p> <p>Moving their power to the top. (Do not allow power of -1.) Can be implied. Ignore limits and $\frac{\pi}{9}$</p> <p>Integrating to give $\frac{\pm p(1+2x)^{-1}}{-\frac{1}{2}(1+2x)^{-1}}$</p> <p>Use of limits to give exact values of $\frac{\pi}{12}$ or $\frac{3\pi}{36}$ or $\frac{2\pi}{24}$ or aef</p>	<p>B1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>A1 aef</p> <p>[5]</p>
(b)	<p>From Fig.1, $AB = \frac{1}{2} - (-\frac{1}{4}) = \frac{3}{4}$ units</p> <p>As $\frac{3}{4}$ units \equiv 3cm</p> <p>then scale factor $k = \frac{3}{(\frac{3}{4})} = 4$.</p> <p>Hence Volume of paperweight = $(4)^3 \left(\frac{\pi}{12} \right)$</p> <p>$V = \frac{16\pi}{3} \text{ cm}^3 = 16.75516... \text{ cm}^3$</p>	<p>$(4)^3 \times$ (their answer to part (a))</p> <p>$\frac{16\pi}{3}$ or awrt 16.8 or $\frac{64\pi}{12}$ or aef</p>	<p>M1</p> <p>A1</p> <p>[2]</p>
7 marks			

Note: $\frac{\pi}{9}$ (or implied) is not needed for the middle three marks of question 2(a).

Question Number	Scheme	Marks
<p>Aliter</p> <p>2. (a)</p> <p>Way 2</p>	$\text{Volume} = \pi \int_{-\frac{1}{4}}^{\frac{1}{2}} \left(\frac{1}{3(1+2x)} \right)^2 dx = \pi \int_{-\frac{1}{4}}^{\frac{1}{2}} \frac{1}{(3+6x)^2} dx$ $= (\pi) \int_{-\frac{1}{4}}^{\frac{1}{2}} (3+6x)^{-2} dx$ $= (\pi) \left[\frac{(3+6x)^{-1}}{(-1)(6)} \right]_{-\frac{1}{4}}^{\frac{1}{2}}$ $= (\pi) \left[\frac{-1}{6} (3+6x)^{-1} \right]_{-\frac{1}{4}}^{\frac{1}{2}}$ $= (\pi) \left[\left(\frac{-1}{6(6)} \right) - \left(\frac{-1}{6(\frac{3}{2})} \right) \right]$ $= (\pi) \left[-\frac{1}{36} - \left(-\frac{1}{9}\right) \right]$ $= \frac{\pi}{12}$	<p>Use of $V = \pi \int y^2 dx$.</p> <p>Can be implied. Ignore limits.</p> <p>Moving their power to the top. (Do not allow power of -1.) Can be implied. Ignore limits and π</p> <p>Integrating to give $\frac{\pm p(3+6x)^{-1}}{-\frac{1}{6}(3+6x)^{-1}}$</p> <p>Use of limits to give exact values of $\frac{\pi}{12}$ or $\frac{3\pi}{36}$ or $\frac{2\pi}{24}$ or aef</p> <p>B1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>A1 aef</p> <p>[5]</p>

Note: π is not needed for the middle three marks of question 2(a).

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3. A curve has parametric equations

$$x = 7 \cos t - \cos 7t, \quad y = 7 \sin t - \sin 7t, \quad \frac{\pi}{8} < t < \frac{\pi}{3}.$$

(a) Find an expression for $\frac{dy}{dx}$ in terms of t . You need not simplify your answer. (3)

(b) Find an equation of the normal to the curve at the point where $t = \frac{\pi}{6}$.
Give your answer in its simplest exact form. (6)



Question Number	Scheme	Marks
3. (a)	$x = 7 \cos t - \cos 7t, \quad y = 7 \sin t - \sin 7t,$ $\frac{dx}{dt} = -7 \sin t + 7 \sin 7t, \quad \frac{dy}{dt} = 7 \cos t - 7 \cos 7t$ $\therefore \frac{dy}{dx} = \frac{7 \cos t - 7 \cos 7t}{-7 \sin t + 7 \sin 7t}$	<p>Attempt to differentiate x and y with respect to t to give $\frac{dx}{dt}$ in the form $\pm A \sin t \pm B \sin 7t$ M1</p> <p>$\frac{dy}{dt}$ in the form $\pm C \cos t \pm D \cos 7t$ A1</p> <p>Correct $\frac{dx}{dt}$ and $\frac{dy}{dt}$ A1</p> <p>Candidate's $\frac{dy}{dx}$ B1 $\sqrt{\quad}$</p> <p>[3]</p>
(b)	<p>When $t = \frac{\pi}{6}$, $m(T) = \frac{dy}{dx} = \frac{7 \cos \frac{\pi}{6} - 7 \cos \frac{7\pi}{6}}{-7 \sin \frac{\pi}{6} + 7 \sin \frac{7\pi}{6}}$,</p> $= \frac{\frac{7\sqrt{3}}{2} - \left(-\frac{7\sqrt{3}}{2}\right)}{-\frac{7}{2} - \frac{7}{2}} = \frac{7\sqrt{3}}{-7} = -\sqrt{3} = \text{awrt } -1.73$ <p>Hence $m(N) = \frac{-1}{-\sqrt{3}}$ or $\frac{1}{\sqrt{3}} = \text{awrt } 0.58$</p> <p>When $t = \frac{\pi}{6}$,</p> $x = 7 \cos \frac{\pi}{6} - \cos \frac{7\pi}{6} = \frac{7\sqrt{3}}{2} - \left(-\frac{\sqrt{3}}{2}\right) = \frac{8\sqrt{3}}{2} = 4\sqrt{3}$ $y = 7 \sin \frac{\pi}{6} - \sin \frac{7\pi}{6} = \frac{7}{2} - \left(-\frac{1}{2}\right) = \frac{8}{2} = 4$ <p>N: $y - 4 = \frac{1}{\sqrt{3}}(x - 4\sqrt{3})$</p> <p>N: $y = \frac{1}{\sqrt{3}}x$ or $y = \frac{\sqrt{3}}{3}x$ or $3y = \sqrt{3}x$</p> <p>or $4 = \frac{1}{\sqrt{3}}(4\sqrt{3}) + c \Rightarrow c = 4 - 4 = 0$</p> <p>Hence N: $y = \frac{1}{\sqrt{3}}x$ or $y = \frac{\sqrt{3}}{3}x$ or $3y = \sqrt{3}x$</p>	<p>Substitutes $t = \frac{\pi}{6}$ or 30° into their $\frac{dy}{dx}$ expression; M1</p> <p>to give any of the four underlined expressions oe A1 cso</p> <p>(must be correct solution only)</p> <p>Uses $m(T)$ to 'correctly' find $m(N)$. Can be ft from "their tangent gradient". A1 $\sqrt{\quad}$ oe.</p> <p>The point $(4\sqrt{3}, 4)$ or (awrt 6.9, 4) B1</p> <p>Finding an equation of a normal with their point and their normal gradient or finds c by using $y = (\text{their gradient})x + "c"$. M1</p> <p>Correct simplified EXACT equation of <u>normal</u>. This is dependent on candidate using correct $(4\sqrt{3}, 4)$ A1 oe</p> <p>[6] 9 marks</p>

Question Number	Scheme	Marks
<p>Aliter 3. (a) Way 2</p> <p>(b)</p>	<p>$x = 7 \cos t - \cos 7t, y = 7 \sin t - \sin 7t,$</p> <p>$\frac{dx}{dt} = -7 \sin t + 7 \sin 7t, \frac{dy}{dt} = 7 \cos t - 7 \cos 7t$</p> <p>$\frac{dy}{dx} = \frac{7 \cos t - 7 \cos 7t}{-7 \sin t + 7 \sin 7t} = \frac{-7(-2 \sin 4t \sin 3t)}{-7(2 \cos 4t \sin 3t)} = \tan 4t$</p> <p>When $t = \frac{\pi}{6}, m(T) = \frac{dy}{dx} = \tan \frac{4\pi}{6};$</p> <p>$= \frac{2(\frac{\sqrt{3}}{2})(1)}{2(-\frac{1}{2})(1)} = -\sqrt{3} = \underline{\text{awrt } -1.73}$</p> <p>Hence $m(N) = \frac{-1}{-\sqrt{3}}$ or $\frac{1}{\sqrt{3}} = \underline{\text{awrt } 0.58}$</p> <p>When $t = \frac{\pi}{6},$</p> <p>$x = 7 \cos \frac{\pi}{6} - \cos \frac{7\pi}{6} = \frac{7\sqrt{3}}{2} - (-\frac{\sqrt{3}}{2}) = \frac{8\sqrt{3}}{2} = 4\sqrt{3}$</p> <p>$y = 7 \sin \frac{\pi}{6} - \sin \frac{7\pi}{6} = \frac{7}{2} - (-\frac{1}{2}) = \frac{8}{2} = 4$</p> <p>N: $y - 4 = \frac{1}{\sqrt{3}}(x - 4\sqrt{3})$</p> <p>N: $y = \frac{1}{\sqrt{3}}x$ or $y = \frac{\sqrt{3}}{3}x$ or $3y = \sqrt{3}x$</p> <p>or $4 = \frac{1}{\sqrt{3}}(4\sqrt{3}) + c \Rightarrow c = 4 - 4 = 0$</p> <p>Hence N: $y = \frac{1}{\sqrt{3}}x$ or $y = \frac{\sqrt{3}}{3}x$ or $3y = \sqrt{3}x$</p>	<p>Attempt to differentiate x and y with respect to t to give $\frac{dx}{dt}$ in the form $\pm A \sin t \pm B \sin 7t$</p> <p>$\frac{dy}{dt}$ in the form $\pm C \cos t \pm D \cos 7t$</p> <p>Correct $\frac{dx}{dt}$ and $\frac{dy}{dt}$</p> <p>Candidate's $\frac{dy}{dx}$</p> <p>Substitutes $t = \frac{\pi}{6}$ or 30° into their $\frac{dy}{dx}$ expression;</p> <p>to give any of the three underlined expressions oe</p> <p>(must be correct solution only)</p> <p>Uses $m(T)$ to 'correctly' find $m(N)$. Can be ft from "their tangent gradient".</p> <p>The point $(4\sqrt{3}, 4)$ or $(\underline{\text{awrt } 6.9}, 4)$</p> <p>Finding an equation of a normal with their point and their normal gradient or finds c by using $y = (\text{their gradient})x + "c"$.</p> <p>Correct simplified EXACT equation of <u>normal</u>. This is dependent on candidate using correct $(4\sqrt{3}, 4)$</p> <p style="text-align: right;">[3]</p> <p style="text-align: right;">[6]</p>
		9 marks

Beware: A candidate finding an $m(\mathbf{T}) = 0$ can obtain A1 ft for $m(\mathbf{N}) \rightarrow \infty$, but obtains M0 if they write $y - 4 = \infty(x - 4\sqrt{3})$. If they write, however, $\mathbf{N}: x = 4\sqrt{3}$, then they can score M1.

Beware: A candidate finding an $m(\mathbf{T}) = \infty$ can obtain A1 ft for $m(\mathbf{N}) = 0$, and also obtains M1 if they write $y - 4 = 0(x - 4\sqrt{3})$ or $y = 4$.

Question Number	Scheme	Marks
4. (a)	$\frac{2x-1}{(x-1)(2x-3)} \equiv \frac{A}{x-1} + \frac{B}{2x-3}$ $2x-1 \equiv A(2x-3) + B(x-1)$ <p>Let $x = \frac{3}{2}$, $2 = B(\frac{1}{2}) \Rightarrow B = 4$</p> <p>Let $x = 1$, $1 = A(-1) \Rightarrow A = -1$</p> <p>giving $\frac{-1}{x-1} + \frac{4}{2x-3}$</p>	<p>Forming this identity. NB: A & B are not assigned in this question</p> <p>M1</p> <p>either one of $A = -1$ or $B = 4$. both correct for their A, B.</p> <p>A1 A1</p> <p>[3]</p>
(b) & (c)	$\int \frac{dy}{y} = \int \frac{(2x-1)}{(2x-3)(x-1)} dx$ $= \int \frac{-1}{x-1} + \frac{4}{2x-3} dx$ <p>$\therefore \ln y = -\ln(x-1) + 2\ln(2x-3) + c$</p> <p>$y = 10, x = 2$ gives $c = \ln 10$</p> <p>$\therefore \ln y = -\ln(x-1) + 2\ln(2x-3) + \ln 10$</p> <p>$\ln y = -\ln(x-1) + \ln(2x-3)^2 + \ln 10$</p> <p>$\ln y = \ln\left(\frac{(2x-3)^2}{x-1}\right) + \ln 10$ or</p> $\ln y = \ln\left(\frac{10(2x-3)^2}{x-1}\right)$ $y = \frac{10(2x-3)^2}{x-1}$	<p>Separates variables as shown Can be implied</p> <p>B1</p> <p>Replaces RHS with their partial fraction to be integrated.</p> <p>M1 $\sqrt{\quad}$</p> <p><i>At least</i> two terms in \ln's <i>At least</i> two \ln terms correct All three terms correct and '+ c'</p> <p>M1 A1 $\sqrt{\quad}$ A1</p> <p>[5]</p> <p>$c = \ln 10$</p> <p>B1</p> <p>Using the power law for logarithms</p> <p>M1</p> <p>Using the product and/or quotient laws for logarithms to obtain a single RHS logarithmic term with/without constant c.</p> <p>M1</p> <p>$y = \frac{10(2x-3)^2}{x-1}$ or aef. isw</p> <p>A1 aef</p> <p>[4]</p>
		12 marks

Question Number	Scheme	Marks
<p><i>Aliter</i> 4. (b) & (c) Way 2</p>	$\int \frac{dy}{y} = \int \frac{(2x-1)}{(2x-3)(x-1)} dx$ $= \int \frac{-1}{(x-1)} + \frac{4}{(2x-3)} dx$ <p>$\therefore \ln y = -\ln(x-1) + 2\ln(2x-3) + c$</p> <p><i>See below for the award of B1</i></p> $\ln y = -\ln(x-1) + \ln(2x-3)^2 + c$ $\ln y = \ln\left(\frac{(2x-3)^2}{x-1}\right) + c$ $\ln y = \ln\left(\frac{A(2x-3)^2}{x-1}\right) \quad \text{where } c = \ln A$ <p>or $e^{\ln y} = e^{\ln\left(\frac{(2x-3)^2}{x-1}\right) + c} = e^{\ln\left(\frac{(2x-3)^2}{x-1}\right)} e^c$</p> $y = \frac{A(2x-3)^2}{(x-1)}$ <p>$y = 10, x = 2$ gives $A = 10$</p> $\underline{y = \frac{10(2x-3)^2}{(x-1)}}$	<p>Separates variables as shown Can be implied B1</p> <p>Replaces RHS with their partial fraction to be integrated. M1 $\sqrt{\quad}$</p> <p><i>At least</i> two terms in ln's M1 <i>At least</i> two ln terms correct A1 $\sqrt{\quad}$ All three terms correct and '+ c' A1</p> <p><i>decide to award B1 here!!</i> B1</p> <p>Using the power law for logarithms M1</p> <p>Using the product and/or quotient laws for logarithms to obtain a single RHS logarithmic term with/without constant c. M1</p> <p>A = 10 for B1 award above</p> <p>$y = \frac{10(2x-3)^2}{(x-1)}$ or aef & isw A1 aef</p> <p>[5] & [4]</p>

Note: The B1 mark (part (c)) should be awarded in the same place on ePEN as in the Way 1 approach.

Question Number	Scheme	Marks
<p><i>Aliter</i></p> <p>(b) & (c)</p> <p>Way 3</p>	$\int \frac{dy}{y} = \int \frac{(2x-1)}{(2x-3)(x-1)} dx$ $= \int \frac{-1}{(x-1)} + \frac{2}{(x-\frac{3}{2})} dx$ <p>$\therefore \ln y = -\ln(x-1) + 2\ln(x-\frac{3}{2}) + c$</p>	<p>Separates variables as shown Can be implied B1</p> <p>Replaces RHS with their partial fraction to be integrated. M1 $\sqrt{\quad}$</p> <p><i>At least</i> two terms in ln's M1 <i>At least</i> two ln terms correct A1 $\sqrt{\quad}$ All three terms correct and '+ c' A1</p> <p>[5]</p>
	<p>$y = 10, x = 2$ gives $c = \ln 10 - 2\ln(\frac{1}{2}) = \ln 40$</p> <p>$\therefore \ln y = -\ln(x-1) + 2\ln(x-\frac{3}{2}) + \ln 40$</p> <p>$\ln y = -\ln(x-1) + \ln(x-\frac{3}{2})^2 + \ln 10$</p> <p>$\ln y = \ln\left(\frac{(x-\frac{3}{2})^2}{(x-1)}\right) + \ln 40$ or</p> <p>$\ln y = \ln\left(\frac{40(x-\frac{3}{2})^2}{(x-1)}\right)$</p> <p>$y = \frac{40(x-\frac{3}{2})^2}{(x-1)}$</p>	<p>$c = \ln 10 - 2\ln(\frac{1}{2})$ or $c = \ln 40$ B1 oe</p> <p>Using the power law for logarithms M1</p> <p>Using the product and/or quotient laws for logarithms to obtain a single RHS logarithmic term with/without constant c. M1</p> <p>$y = \frac{40(x-\frac{3}{2})^2}{(x-1)}$ or aef. isw A1 aef</p> <p>[4]</p>

Note: Please mark parts (b) and (c) together for any of the three ways.

Question Number	Scheme	Marks
5. (a)	<p style="text-align: center;">$\sin x + \cos y = 0.5$ (eqn *)</p> <p>$\frac{dy}{dx}$ $\cos x - \sin y \frac{dy}{dx} = 0$ (eqn #)</p> <p style="text-align: center;">$\frac{dy}{dx} = \frac{\cos x}{\sin y}$</p> <p>(b) $\frac{dy}{dx} = 0 \Rightarrow \frac{\cos x}{\sin y} = 0 \Rightarrow \cos x = 0$</p> <p>giving $x = -\frac{\pi}{2}$ or $x = \frac{\pi}{2}$</p> <p>When $x = -\frac{\pi}{2}$, $\sin(-\frac{\pi}{2}) + \cos y = 0.5$ When $x = \frac{\pi}{2}$, $\sin(\frac{\pi}{2}) + \cos y = 0.5$</p> <p>$\Rightarrow \cos y = 1.5 \Rightarrow y$ has no solutions $\Rightarrow \cos y = -0.5 \Rightarrow y = \frac{2\pi}{3}$ or $-\frac{2\pi}{3}$</p> <p>In specified range $(x, y) = (\frac{\pi}{2}, \frac{2\pi}{3})$ and $(\frac{\pi}{2}, -\frac{2\pi}{3})$</p>	<p>M1</p> <p>A1 cso</p> <p>[2]</p> <p>M1 $\sqrt{\quad}$</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>Do not award this mark if candidate states other coordinates inside the required range.</p> <p>[5]</p> <hr/> <p>7 marks</p>

Question Number	Scheme	Marks
<p>6.</p> <p>(a)</p> <p>Way 1</p> <p><i>Aliter</i></p> <p>(a)</p> <p>Way 2</p> <p>(b)</p>	<p>$y = 2^x = e^{x \ln 2}$</p> <p>$\frac{dy}{dx} = \ln 2 \cdot e^{x \ln 2}$</p> <p>Hence $\frac{dy}{dx} = \ln 2 \cdot (2^x) = 2^x \ln 2$ AG</p> <p>$\ln y = \ln(2^x)$ leads to $\ln y = x \ln 2$</p> <p>$\frac{1}{y} \frac{dy}{dx} = \ln 2$</p> <p>Hence $\frac{dy}{dx} = y \ln 2 = 2^x \ln 2$ AG</p> <p>$y = 2^{(x^2)} \Rightarrow \frac{dy}{dx} = 2x \cdot 2^{(x^2)} \cdot \ln 2$</p> <p>When $x = 2$, $\frac{dy}{dx} = 2(2)2^4 \ln 2$</p> <p>$\frac{dy}{dx} = \underline{64 \ln 2} = 44.3614\dots$</p>	<p>$\frac{dy}{dx} = \ln 2 \cdot e^{x \ln 2}$ M1</p> <p>$2^x \ln 2$ AG A1 cso</p> <p>[2]</p> <p>Takes logs of both sides, then uses the power law of logarithms... ... and differentiates implicitly to give $\frac{1}{y} \frac{dy}{dx} = \ln 2$ M1</p> <p>$2^x \ln 2$ AG A1 cso</p> <p>[2]</p> <p>$Ax 2^{(x^2)}$ M1</p> <p>$2x \cdot 2^{(x^2)} \cdot \ln 2$ A1</p> <p>or $2x \cdot y \cdot \ln 2$ if y is defined</p> <p>Substitutes $x = 2$ into their $\frac{dy}{dx}$ which is of the form $\pm k 2^{(x^2)}$ or $Ax 2^{(x^2)}$ M1</p> <p>$\underline{64 \ln 2}$ or awrt 44.4 A1</p> <p>[4]</p> <p>6 marks</p>

Question Number	Scheme	Marks
<p><i>Aliter</i></p> <p>6. (b)</p> <p>Way 2</p>	<p>$\ln y = \ln(2^{x^2})$ leads to $\ln y = x^2 \ln 2$</p> <p>$\frac{1}{y} \frac{dy}{dx} = 2x \ln 2$</p> <p>When $x = 2$, $\frac{dy}{dx} = 2(2)2^4 \ln 2$</p> <p>$\frac{dy}{dx} = \underline{64 \ln 2} = 44.3614\dots$</p>	<p>$\frac{1}{y} \frac{dy}{dx} = Ax \ln 2$ M1</p> <p>$\frac{1}{y} \frac{dy}{dx} = 2x \ln 2$ A1</p> <p>Substitutes $x = 2$ into their $\frac{dy}{dx}$ which is of the form $\pm k 2^{(x^2)}$ or $Ax 2^{(x^2)}$ M1</p> <p>$\underline{64 \ln 2}$ or awrt 44.4 A1</p> <p style="text-align: right;">[4]</p>

Question Number	Scheme	Marks
7.	$\mathbf{a} = \overline{OA} = 2\mathbf{i} + 2\mathbf{j} + \mathbf{k} \Rightarrow \overline{OA} = 3$ $\mathbf{b} = \overline{OB} = \mathbf{i} + \mathbf{j} - 4\mathbf{k} \Rightarrow \overline{OB} = \sqrt{18}$ $\overline{BC} = \pm(2\mathbf{i} + 2\mathbf{j} + \mathbf{k}) \Rightarrow \overline{BC} = 3$ $\overline{AC} = \pm(\mathbf{i} + \mathbf{j} - 4\mathbf{k}) \Rightarrow \overline{AC} = \sqrt{18}$	
(a)	$\mathbf{c} = \overline{OC} = \underline{3\mathbf{i} + 3\mathbf{j} - 3\mathbf{k}}$	$\underline{3\mathbf{i} + 3\mathbf{j} - 3\mathbf{k}}$ <p>B1 cao [1]</p>
(b)	$\overline{OA} \cdot \overline{OB} = \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ -4 \end{pmatrix} = \underline{2+2-4} = 0 \quad \text{or...}$ $\overline{BO} \cdot \overline{BC} = \begin{pmatrix} -1 \\ -1 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} = \underline{-2-2+4} = 0 \quad \text{or...}$ $\overline{AC} \cdot \overline{BC} = \begin{pmatrix} 1 \\ 1 \\ -4 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} = \underline{2+2-4} = 0 \quad \text{or...}$ $\overline{AO} \cdot \overline{AC} = \begin{pmatrix} -2 \\ -2 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ -4 \end{pmatrix} = \underline{-2-2+4} = 0$ <p>and therefore OA is perpendicular to OB and hence OACB is a rectangle.</p> $\text{Area} = 3 \times \sqrt{18} = 3\sqrt{18} = 9\sqrt{2}$	<p>An attempt to take the dot product between either \overline{OA} and \overline{OB}, \overline{OA} and \overline{AC}, \overline{AC} and \overline{BC} or \overline{OB} and \overline{BC}</p> <p>M1</p> <p>Showing the result is equal to zero. A1</p> <p><u>perpendicular and OACB is a rectangle</u> A1 cso</p> <p>Using distance formula to find either the correct height or width. M1</p> <p>Multiplying the rectangle's height by its width. M1</p> <p>exact value of $3\sqrt{18}$, $9\sqrt{2}$, $\sqrt{162}$ or aef A1</p> <p>[6]</p>
(c)	$\overline{OD} = \mathbf{d} = \frac{1}{2}(3\mathbf{i} + 3\mathbf{j} - 3\mathbf{k})$	$\underline{\frac{1}{2}(3\mathbf{i} + 3\mathbf{j} - 3\mathbf{k})}$ <p>B1 [1]</p>

Question Number	Scheme	Marks
<p>(d)</p> <p>Way 1</p> <p><i>Aliter</i></p> <p>(d)</p> <p>Way 2</p>	<p><i>using dot product formula</i></p> <p>$\overline{DA} = \pm \left(\frac{1}{2}\mathbf{i} + \frac{1}{2}\mathbf{j} + \frac{5}{2}\mathbf{k}\right)$ & $\overline{DC} = \pm \left(\frac{3}{2}\mathbf{i} + \frac{3}{2}\mathbf{j} - \frac{3}{2}\mathbf{k}\right)$</p> <p>or $\overline{BA} = \pm(\mathbf{i} + \mathbf{j} + 5\mathbf{k})$ & $\overline{OC} = \pm(3\mathbf{i} + 3\mathbf{j} - 3\mathbf{k})$</p> $\cos D = (\pm) \frac{\begin{pmatrix} 0.5 \\ 0.5 \\ 2.5 \end{pmatrix} \cdot \begin{pmatrix} 1.5 \\ 1.5 \\ -1.5 \end{pmatrix}}{\frac{\sqrt{27}}{2} \cdot \frac{\sqrt{27}}{2}} = (\pm) \frac{\frac{3}{4} + \frac{3}{4} - \frac{15}{4}}{\frac{27}{4}} = (\pm) \frac{1}{3}$ $D = \cos^{-1}\left(-\frac{1}{3}\right)$ <p>$D = 109.47122\dots^\circ$</p> <p><i>using dot product formula and direction vectors</i></p> <p>$d\overline{BA} = \pm(\mathbf{i} + \mathbf{j} + 5\mathbf{k})$ & $d\overline{OC} = \pm(\mathbf{i} + \mathbf{j} - \mathbf{k})$</p> $\cos D = (\pm) \frac{\begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ 5 \end{pmatrix}}{\sqrt{3} \cdot \sqrt{27}} = (\pm) \frac{1+1-5}{\sqrt{3} \cdot \sqrt{27}} = (\pm) \frac{1}{3}$ $D = \cos^{-1}\left(-\frac{1}{3}\right)$ <p>$D = 109.47122\dots^\circ$</p>	<p>Identifies a set of two relevant vectors Correct vectors \pm</p> <p>M1 A1</p> <p>Applies dot product formula on multiples of these vectors. <u>Correct ft. application of dot product formula.</u></p> <p>dM1 A1 $\sqrt{}$</p> <p>Attempts to find the correct angle D rather than $180^\circ - D$.</p> <p>ddM1 $\sqrt{}$</p> <p>109.5° or awrt 109° or 1.91°</p> <p>A1 [6]</p> <p>Identifies a set of two direction vectors Correct vectors \pm</p> <p>M1 A1</p> <p>Applies dot product formula on multiples of these vectors. <u>Correct ft. application of dot product formula.</u></p> <p>dM1 A1 $\sqrt{}$</p> <p>Attempts to find the correct angle D rather than $180^\circ - D$.</p> <p>ddM1 $\sqrt{}$</p> <p>109.5° or awrt 109° or 1.91°</p> <p>A1 [6]</p>

Question Number	Scheme	Marks
<p>Aliter</p> <p>(d)</p> <p>Way 3</p>	<p><i>using dot product formula and similar triangles</i></p> <p>$d\overline{OA} = (2\mathbf{i} + 2\mathbf{j} + \mathbf{k})$ & $d\overline{OC} = (\mathbf{i} + \mathbf{j} - \mathbf{k})$</p> $\cos\left(\frac{1}{2}D\right) = \frac{\begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}}{\sqrt{9} \cdot \sqrt{3}} = \frac{2 + 2 - 1}{\sqrt{9} \cdot \sqrt{3}} = \frac{1}{\sqrt{3}}$ $D = 2 \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ $D = 109.47122\dots^\circ$	<p>Identifies a set of two direction vectors Correct vectors M1 A1</p> <p>Applies dot product formula on multiples of these vectors. <u>Correct ft. application of dot product formula.</u> dM1 A1 $\sqrt{}$</p> <p>Attempts to find the correct angle D by doubling their angle for $\frac{1}{2}D$. 109.5° or awrt 109° or 1.91° ddM1 $\sqrt{}$ A1</p> <p>[6]</p>
<p>Aliter</p> <p>(d)</p> <p>Way 4</p>	<p><i>using cosine rule</i></p> <p>$\overline{DA} = \frac{1}{2}\mathbf{i} + \frac{1}{2}\mathbf{j} + \frac{5}{2}\mathbf{k}$, $\overline{DC} = \frac{3}{2}\mathbf{i} + \frac{3}{2}\mathbf{j} - \frac{3}{2}\mathbf{k}$, $\overline{AC} = \mathbf{i} + \mathbf{j} - 4\mathbf{k}$</p> $ \overline{DA} = \frac{\sqrt{27}}{2}, \overline{DC} = \frac{\sqrt{27}}{2}, \overline{AC} = \sqrt{18}$ $\cos D = \frac{\left(\frac{\sqrt{27}}{2}\right)^2 + \left(\frac{\sqrt{27}}{2}\right)^2 - (\sqrt{18})^2}{2\left(\frac{\sqrt{27}}{2}\right)\left(\frac{\sqrt{27}}{2}\right)} = -\frac{1}{3}$ $D = \cos^{-1}\left(-\frac{1}{3}\right)$ $D = 109.47122\dots^\circ$	<p>Attempts to find all the lengths of all three edges of $\triangle ADC$ All Correct M1 A1</p> <p>Using the cosine rule formula with correct 'subtraction'. <u>Correct ft application of the cosine rule formula</u> dM1 A1 $\sqrt{}$</p> <p>Attempts to find the correct angle D rather than $180^\circ - D$. 109.5° or awrt 109° or 1.91° ddM1 $\sqrt{}$ A1</p> <p>[6]</p>

Question Number	Scheme	Marks
<p>Aliter (d) Way 5</p>	<p><i>using trigonometry on a right angled triangle</i> $\overline{DA} = \frac{1}{2}\mathbf{i} + \frac{1}{2}\mathbf{j} + \frac{5}{2}\mathbf{k}$ $\overline{OA} = 2\mathbf{i} + 2\mathbf{j} + \mathbf{k}$ $\overline{AC} = \mathbf{i} + \mathbf{j} - 4\mathbf{k}$</p> <p>Let X be the midpoint of AC</p> <p>$\overline{DA} = \frac{\sqrt{27}}{2}$, $\overline{DX} = \frac{1}{2} \overline{OA} = \frac{3}{2}$, $\overline{AX} = \frac{1}{2} \overline{AC} = \frac{1}{2}\sqrt{18}$ (hypotenuse), (adjacent) , (opposite)</p> <p>$\sin(\frac{1}{2}D) = \frac{\frac{\sqrt{18}}{2}}{\frac{\sqrt{27}}{2}}$, $\cos(\frac{1}{2}D) = \frac{\frac{3}{2}}{\frac{\sqrt{27}}{2}}$ or $\tan(\frac{1}{2}D) = \frac{\frac{\sqrt{18}}{2}}{\frac{3}{2}}$</p> <p>eg. $D = 2 \tan^{-1}\left(\frac{\frac{\sqrt{18}}{2}}{\frac{3}{2}}\right)$</p> <p>$D = 109.47122\dots^\circ$</p>	<p>Attempts to find two out of the three lengths in $\triangle ADX$ M1</p> <p>Any two correct A1</p> <p>Uses correct sohcahtoa to find $\frac{1}{2}D$ dM1</p> <p>Correct ft application of sohcahtoa A1 $\sqrt{}$</p> <p>Attempts to find the correct angle D by doubling their angle for $\frac{1}{2}D$. ddM1 $\sqrt{}$</p> <p>109.5° or awrt 109° or 1.91° A1</p> <p style="text-align: right;">[6]</p>
<p>Aliter (d) Way 6</p>	<p><i>using trigonometry on a right angled similar triangle OAC</i> $\overline{OC} = 3\mathbf{i} + 3\mathbf{j} - 3\mathbf{k}$ $\overline{OA} = 2\mathbf{i} + 2\mathbf{j} + \mathbf{k}$ $\overline{AC} = \mathbf{i} + \mathbf{j} - 4\mathbf{k}$</p> <p>$\overline{OC} = \sqrt{27}$, $\overline{OA} = 3$, $\overline{AC} = \sqrt{18}$ (hypotenuse), (adjacent), (opposite)</p> <p>$\sin(\frac{1}{2}D) = \frac{\sqrt{18}}{\sqrt{27}}$, $\cos(\frac{1}{2}D) = \frac{3}{\sqrt{27}}$ or $\tan(\frac{1}{2}D) = \frac{\sqrt{18}}{3}$</p> <p>eg. $D = 2 \tan^{-1}\left(\frac{\sqrt{18}}{3}\right)$</p> <p>$D = 109.47122\dots^\circ$</p>	<p>Attempts to find two out of the three lengths in $\triangle OAC$ M1</p> <p>Any two correct A1</p> <p>Uses correct sohcahtoa to find $\frac{1}{2}D$ dM1</p> <p>Correct ft application of sohcahtoa A1 $\sqrt{}$</p> <p>Attempts to find the correct angle D by doubling their angle for $\frac{1}{2}D$. ddM1 $\sqrt{}$</p> <p>109.5° or awrt 109° or 1.91° A1</p> <p style="text-align: right;">[6]</p>

Question Number	Scheme	Marks
<p><i>Aliter</i></p> <p>7. (b) (i)</p> <p>Way 2</p> <p><i>Aliter</i></p> <p>7. (b) (i)</p> <p>Way 3</p>	<p>$\mathbf{c} = \overline{OC} = \pm(3\mathbf{i} + 3\mathbf{j} - 3\mathbf{k})$ $\overline{AB} = \pm(-\mathbf{i} - \mathbf{j} - 5\mathbf{k})$</p> <p>$\overline{OC} = \sqrt{(3)^2 + (3)^2 + (-3)^2} = \sqrt{(1)^2 + (1)^2 + (-5)^2} = \overline{AB}$</p> <p>As $\overline{OC} = \overline{AB} = \sqrt{27}$</p> <p>then the <u>diagonals are equal</u>, and OACB is a <u>rectangle</u>.</p> <p>$\mathbf{a} = \overline{OA} = 2\mathbf{i} + 2\mathbf{j} + \mathbf{k} \Rightarrow \overline{OA} = 3$ $\mathbf{b} = \overline{OB} = \mathbf{i} + \mathbf{j} - 4\mathbf{k} \Rightarrow \overline{OB} = \sqrt{18}$ $\overline{BC} = \pm(2\mathbf{i} + 2\mathbf{j} + \mathbf{k}) \Rightarrow \overline{BC} = 3$ $\overline{AC} = \pm(\mathbf{i} + \mathbf{j} - 4\mathbf{k}) \Rightarrow \overline{AC} = \sqrt{18}$ $\mathbf{c} = \overline{OC} = \pm(3\mathbf{i} + 3\mathbf{j} - 3\mathbf{k}) \Rightarrow \overline{OC} = \sqrt{27}$ $\overline{AB} = \pm(-\mathbf{i} - \mathbf{j} - 5\mathbf{k}) \Rightarrow \overline{AB} = \sqrt{27}$</p> <p>$(OA)^2 + (AC)^2 = (OC)^2$ or $(BC)^2 + (OB)^2 = (OC)^2$ or equivalent or $(OA)^2 + (OB)^2 = (AB)^2$ or $(BC)^2 + (AC)^2 = (AB)^2$</p> <p>$\Rightarrow \underline{(3)^2 + (\sqrt{18})^2 = (\sqrt{27})^2}$</p> <p>and therefore OA is <u>perpendicular</u> to OB or AC is <u>perpendicular</u> to BC and hence <u>OACB is a rectangle</u>.</p>	<p>A complete method of proving that the diagonals are equal. M1</p> <p>Correct result. A1</p> <p><u>diagonals are equal</u> and <u>OACB is a rectangle</u> A1 cso</p> <p>[3]</p> <p>A complete method of proving that Pythagoras holds using their values. M1</p> <p><u>Correct result</u> A1</p> <p><u>perpendicular</u> and <u>OACB is a rectangle</u> A1 cso</p> <p>[3]</p> <p>14 marks</p>

Question Number	Scheme	Marks																					
8. (a)	<table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>x</td> <td>0</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> <td>5</td> </tr> <tr> <td>y</td> <td>e^1</td> <td>e^2</td> <td>$e^{\sqrt{7}}$</td> <td>$e^{\sqrt{10}}$</td> <td>$e^{\sqrt{13}}$</td> <td>e^4</td> </tr> <tr> <td>or y</td> <td>2.71828...</td> <td>7.38906...</td> <td>14.09403...</td> <td>23.62434...</td> <td>36.80197...</td> <td>54.59815...</td> </tr> </table> <p style="text-align: right;"> Either $e^{\sqrt{7}}$, $e^{\sqrt{10}}$ and $e^{\sqrt{13}}$ or awrt 14.1, 23.6 and 36.8 or e to the power awrt 2.65, 3.16, 3.61 (or mixture of decimals and e's) At least two correct All three correct </p>	x	0	1	2	3	4	5	y	e^1	e^2	$e^{\sqrt{7}}$	$e^{\sqrt{10}}$	$e^{\sqrt{13}}$	e^4	or y	2.71828...	7.38906...	14.09403...	23.62434...	36.80197...	54.59815...	B1 B1 [2]
x	0	1	2	3	4	5																	
y	e^1	e^2	$e^{\sqrt{7}}$	$e^{\sqrt{10}}$	$e^{\sqrt{13}}$	e^4																	
or y	2.71828...	7.38906...	14.09403...	23.62434...	36.80197...	54.59815...																	
(b)	$I \approx \frac{1}{2} \times 1; \times \left\{ e^1 + 2(e^2 + e^{\sqrt{7}} + e^{\sqrt{10}} + e^{\sqrt{13}}) + e^4 \right\}$ $= \frac{1}{2} \times 221.1352227... = 110.5676113... = \underline{110.6} \text{ (4sf)}$	<p style="text-align: right;"> Outside brackets $\frac{1}{2} \times 1$ <u>For structure of trapezium</u> <u>rule {.....} ;</u> M1 $\sqrt{\quad}$ A1 cao [3] </p>																					

Beware: In part (b) candidates can add up the individual trapezia:

$$(b) I \approx \frac{1}{2} \cdot 1(e^1 + e^2) + \frac{1}{2} \cdot 1(e^2 + e^{\sqrt{7}}) + \frac{1}{2} \cdot 1(e^{\sqrt{7}} + e^{\sqrt{10}}) + \frac{1}{2} \cdot 1(e^{\sqrt{10}} + e^{\sqrt{13}}) + \frac{1}{2} \cdot 1(e^{\sqrt{13}} + e^4)$$

Question Number	Scheme	Marks	
(c)	$t = (3x + 1)^{\frac{1}{2}} \Rightarrow \frac{dt}{dx} = \frac{1}{2} \cdot 3 \cdot (3x + 1)^{-\frac{1}{2}}$	$A(3x + 1)^{-\frac{1}{2}}$ or $t \frac{dt}{dx} = A$	M1
	\dots or $t^2 = 3x + 1 \Rightarrow \underline{2t \frac{dt}{dx} = 3}$	$\underline{\frac{3}{2}(3x + 1)^{-\frac{1}{2}}}$ or $\underline{2t \frac{dt}{dx} = 3}$	A1
	$\text{so } \frac{dt}{dx} = \frac{3}{2 \cdot (3x + 1)^{\frac{1}{2}}} = \frac{3}{2t} \Rightarrow \frac{dx}{dt} = \frac{2t}{3}$	<div style="border: 1px solid black; padding: 5px;"> Candidate obtains either $\frac{dt}{dx}$ or $\frac{dx}{dt}$ in terms of $t \dots$ \dots and moves on to substitute this into I to convert an integral wrt x to an integral wrt t. </div>	
	$\therefore I = \int e^{\sqrt{3x+1}} dx = \int e^t \frac{dx}{dt} \cdot dt = \int e^t \cdot \frac{2t}{3} \cdot dt$		dM1
	$\therefore I = \int \frac{2}{3} te^t dt$	$\int \frac{2}{3} te^t$	A1
change limits: when $x = 0, t = 1$ & when $x = 5, t = 4$	changes limits $x \rightarrow t$ so that $0 \rightarrow 1$ and $5 \rightarrow 4$	B1	
Hence $I = \int_1^4 \frac{2}{3} te^t dt$; where $a = 1, b = 4, k = \frac{2}{3}$		[5]	
(d)	$\left\{ \begin{array}{l} u = t \Rightarrow \frac{du}{dt} = 1 \\ \frac{dv}{dt} = e^t \Rightarrow v = e^t \end{array} \right\}$	Let k be any constant for the first three marks of this part.	
	$k \int te^t dt = k (te^t - \int e^t \cdot 1 dt)$	Use of 'integration by parts' formula in the correct direction. Correct expression with a constant factor k .	M1 A1
	$= k(\underline{te^t - e^t}) + c$	<u>Correct integration</u> with/without a constant factor k	A1
	$\therefore \int_1^4 \frac{2}{3} te^t dt = \frac{2}{3} \{(4e^4 - e^4) - (e^1 - e^1)\}$	Substitutes their changed limits into the integrand and subtracts oe.	dM1 oe
	$= \frac{2}{3}(3e^4) = \underline{2e^4} = 109.1963\dots$	either $2e^4$ or awrt 109.2	A1
		[5]	
		15 marks	

- Note: dM1 denotes a method mark which is dependent upon the award of the previous method mark
- ddM1 denotes a method mark which is dependent upon the award of the previous two method marks.