Centre No.						Paper Reference							Surname	Initial(s)
Candidate No.						6	6	6	6	/	0	1	Signature	

Paper Reference(s)

6666/01

Edexcel GCE

Core Mathematics C4

Advanced Level

Tuesday 23 January 2007 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Green)	Nil

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
1	
2	
3	
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6	
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8	
Total	

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initial(s) and signature.
Check that you have the correct question paper.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.
You must write your answer for each question in the space following the question.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.
There are 20 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the examiner. Answers without working may gain no credit.

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1. $$f(x) = (2-5x)^{-2}, \qquad |x| < \tfrac{2}{5}.$$

Find the binomial expansion of f(x), in ascending powers of x, as far as the term in x^3, giving each coefficient as a simplified fraction.

(5)

January 2007
6666 Core Mathematics C4
Mark Scheme

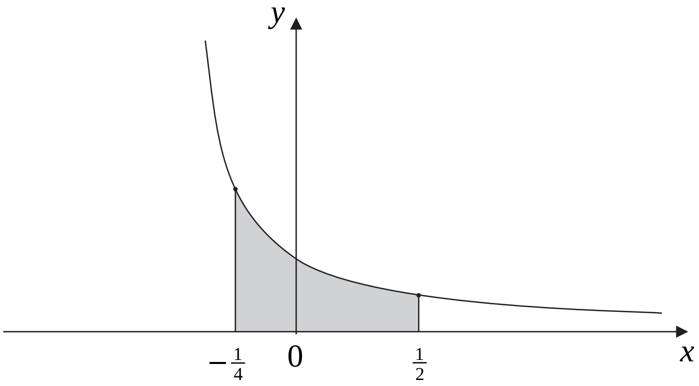
Question Number	Scheme	Marks
	** represents a constant	
1.	$f(x) = (2-5x)^{-2} = \underline{(2)^{-2}}\left(1-\frac{5x}{2}\right)^{-2} = \underline{\frac{1}{4}}\left(1-\frac{5x}{2}\right)^{-2}$ Takes 2 outside the bracket to give any of $(2)^{-2}$ or $\frac{1}{4}$.	B1
	$= \frac{1}{4}\left\{\underline{1+(-2)(**x);+\frac{(-2)(-3)}{2!}(**x)^2+\frac{(-2)(-3)(-4)}{3!}(**x)^3+\dots}\right\}$ Expands $(1+**x)^{-2}$ to give an unsimplified $1+(-2)(**x)$;	M1
	A correct unsimplified $\{\underline{\dots\dots\dots}\}$ expansion with candidate's $(**x)$	A1
	$= \frac{1}{4}\left\{\underline{1+(-2)(\tfrac{-5x}{2});+\frac{(-2)(-3)}{2!}(\tfrac{-5x}{2})^2+\frac{(-2)(-3)(-4)}{3!}(\tfrac{-5x}{2})^3+\dots}\right\}$	
	$= \frac{1}{4}\left\{1+5x;+\frac{75x^2}{4}+\frac{125x^3}{2}+\dots\right\}$	
	$= \frac{1}{4}+\frac{5x}{4};+\frac{75x^2}{16}+\frac{125x^3}{8}+\dots$ Anything that cancels to $\frac{1}{4}+\frac{5x}{4}$; Simplified $\frac{75x^2}{16}+\frac{125x^3}{8}$	A1; A1
	$= \frac{1}{4}+1\frac{1}{4}x;+4\frac{11}{16}x^2+15\frac{5}{8}x^3+\dots$	
		[5]
		5 marks

Question Number	Scheme	Marks
Aliter 1. **Way 2**	$f(x)=(2-5x)^{-2}$	
	$\frac{1}{4}$ or $(2)^{-2}$	B1
	$=\left\{\begin{matrix}(2)^{-2}+(-2)(2)^{-3}(**x);+\frac{(-2)(-3)}{2!}(2)^{-4}(**x)^2\\+\frac{(-2)(-3)(-4)}{3!}(2)^{-5}(**x)^3+\ldots\end{matrix}\right\}$ Expands $(2-5x)^{-2}$ to give an unsimplifed $(2)^{-2}+(-2)(2)^{-3}(**x)$;	M1
	A correct unsimplified $\{\underline{\ldots\ldots\ldots}\}$ expansion with candidate's $(**x)$	A1
	$=\left\{\begin{matrix}(2)^{-2}+(-2)(2)^{-3}(-5x);+\frac{(-2)(-3)}{2!}(2)^{-4}(-5x)^2\\+\frac{(-2)(-3)(-4)}{3!}(2)^{-5}(-5x)^3+\ldots\end{matrix}\right\}$	
	$=\left\{\begin{matrix}\frac{1}{4}+(-2)(\frac{1}{8})(-5x);+(3)(\frac{1}{16})(25x^2)\\+(-4)(\frac{1}{16})(-125x^3)+\ldots\end{matrix}\right\}$	
	$=\frac{1}{4}+\frac{5x}{4};+\frac{75x^2}{16}+\frac{125x^3}{8}+\ldots$ Anything that cancels to $\frac{1}{4}+\frac{5x}{4}$; Simplified $\frac{75x^2}{16}+\frac{125x^3}{8}$	A1; A1
	$=\frac{1}{4}+1\frac{1}{4}x;+4\frac{11}{16}x^2+15\frac{5}{8}x^3+\ldots$	**[5]**
		5 marks

Attempts using Maclaurin expansions need to be referred to your team leader.

2.

Figure 1



The curve with equation $y = \dfrac{1}{3(1+2x)}$, $x > -\frac{1}{2}$, is shown in Figure 1.

The region bounded by the lines $x = -\frac{1}{4}$, $x = \frac{1}{2}$, the x-axis and the curve is shown shaded in Figure 1.

This region is rotated through 360 degrees about the x-axis.

(a) Use calculus to find the exact value of the volume of the solid generated.

(5)

Figure 2

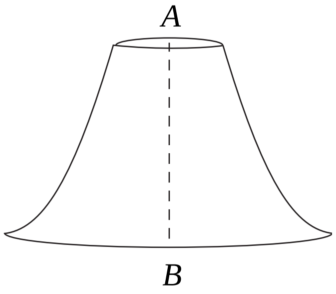


Figure 2 shows a paperweight with axis of symmetry AB where $AB = 3$ cm. A is a point on the top surface of the paperweight, and B is a point on the base of the paperweight. The paperweight is geometrically similar to the solid in part (a).

(b) Find the volume of this paperweight.

(2)

Question Number	Scheme		Marks
2. (a)	Volume $= \underline{\pi \int_{-\frac{1}{4}}^{\frac{1}{2}} \left(\frac{1}{3(1+2x)}\right)^2 dx} = \frac{\pi}{9}\int_{-\frac{1}{4}}^{\frac{1}{2}} \frac{1}{(1+2x)^2}\,dx$	**Use** of $V = \underline{\pi\int y^2\,dx}$. Can be implied. Ignore limits.	B1
	$= \left(\frac{\pi}{9}\right)\int_{-\frac{1}{4}}^{\frac{1}{2}} (1+2x)^{-2}\,dx$	Moving their power to the top. **(Do not allow power of -1.)** Can be implied. Ignore limits and $\frac{\pi}{9}$	M1
	$= \left(\frac{\pi}{9}\right)\left[\frac{(1+2x)^{-1}}{(-1)(2)}\right]_{-\frac{1}{4}}^{\frac{1}{2}}$	Integrating to give $\underline{\pm p(1+2x)^{-1}}$	M1
		$\underline{-\frac{1}{2}(1+2x)^{-1}}$	A1
	$= \left(\frac{\pi}{9}\right)\left[\underline{-\frac{1}{2}(1+2x)^{-1}}\right]_{-\frac{1}{4}}^{\frac{1}{2}}$		
	$= \left(\frac{\pi}{9}\right)\left[\left(\frac{-1}{2(2)}\right) - \left(\frac{-1}{2(\frac{1}{2})}\right)\right]$		
	$= \left(\frac{\pi}{9}\right)\left[-\frac{1}{4} - (-1)\right]$		
	$= \frac{\pi}{12}$	Use of limits to give exact values of $\frac{\pi}{12}$ or $\frac{3\pi}{36}$ or $\frac{2\pi}{24}$ or aef	A1 aef
			[5]
(b)	From Fig.1, AB $= \frac{1}{2} - \left(-\frac{1}{4}\right) = \frac{3}{4}$ units		
	As $\frac{3}{4}$units $\equiv$ 3cm		
	then scale factor $k = \frac{3}{\left(\frac{3}{4}\right)} = 4$.		
	Hence Volume of paperweight $= (4)^3\left(\frac{\pi}{12}\right)$	$(4)^3 \times$ (their answer to part (a))	M1
	$V = \underline{\frac{16\pi}{3}}\ \text{cm}^3 = 16.75516...\ \text{cm}^3$	$\underline{\frac{16\pi}{3}}$ or awrt 16.8 or $\frac{64\pi}{12}$ or aef	A1
			[2]
			7 marks

Note: $\frac{\pi}{9}$ (or implied) is not needed for the middle three marks of question 2(a).

Question Number	Scheme	Marks
Aliter 2. (a) **Way 2**	Volume $= \pi \underline{\int_{-\frac{1}{4}}^{\frac{1}{2}} \left(\frac{1}{3(1+2x)}\right)^2 dx} = \pi \int_{-\frac{1}{4}}^{\frac{1}{2}} \frac{1}{(3+6x)^2} dx$ **Use** of $V = \pi \underline{\int y^2}\, dx$. Can be implied. Ignore limits.	B1
	$= (\pi)\int_{-\frac{1}{4}}^{\frac{1}{2}} (3+6x)^{-2}\, dx$ Moving their power to the top. **(Do not allow power of -1.)** Can be implied. Ignore limits and π	M1
	$= (\pi)\left[\frac{(3+6x)^{-1}}{(-1)(6)}\right]_{-\frac{1}{4}}^{\frac{1}{2}}$ Integrating to give $\underline{\pm p(3+6x)^{-1}}$ $\underline{-\frac{1}{6}(3+6x)^{-1}}$	M1 A1
	$= (\pi)\left[\ \underline{-\frac{1}{6}(3+6x)^{-1}}\ \right]_{-\frac{1}{4}}^{\frac{1}{2}}$	
	$= (\pi)\left[\left(\frac{-1}{6(6)}\right) - \left(\frac{-1}{6(\frac{3}{2})}\right)\right]$	
	$= (\pi)\left[-\frac{1}{36} - (-\frac{1}{9})\right]$	
	$= \frac{\pi}{12}$ Use of limits to give exact values of $\frac{\pi}{12}$ or $\frac{3\pi}{36}$ or $\frac{2\pi}{24}$ or aef	A1 aef **[5]**

Note: π is not needed for the middle three marks of question 2(a).

3. A curve has parametric equations

$$x = 7\cos t - \cos 7t,\quad y = 7\sin t - \sin 7t, \qquad \frac{\pi}{8} < t < \frac{\pi}{3}.$$

(a) Find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of t. You need not simplify your answer.

(3)

(b) Find an equation of the normal to the curve at the point where $t = \frac{\pi}{6}$.

Give your answer in its simplest exact form.

(6)

Question Number	Scheme		Marks
3. (a)	$x = 7\cos t - \cos 7t\ ,\ \ y = 7\sin t - \sin 7t\ ,$		
	$\frac{dx}{dt} = -7\sin t + 7\sin 7t\ ,\ \ \frac{dy}{dt} = 7\cos t - 7\cos 7t$	Attempt to differentiate x **and** y with respect to t to give $\frac{dx}{dt}$ in the form $\pm A\sin t \pm B\sin 7t$ $\frac{dy}{dt}$ in the form $\pm C\cos t \pm D\cos 7t$	M1
		Correct $\frac{dx}{dt}$ and $\frac{dy}{dt}$	A1
	$\therefore \frac{dy}{dx} = \frac{7\cos t - 7\cos 7t}{-7\sin t + 7\sin 7t}$	Candidate's $\frac{\frac{dy}{dt}}{\frac{dx}{dt}}$	B1 √
			[3]
(b)	When $t = \frac{\pi}{6}$, m(**T**) $= \frac{dy}{dx} = \frac{7\cos\frac{\pi}{6} - 7\cos\frac{7\pi}{6}}{-7\sin\frac{\pi}{6} + 7\sin\frac{7\pi}{6}}$;	Substitutes $t = \frac{\pi}{6}$ or 30° into their $\frac{dy}{dx}$ expression;	M1
	$= \underline{\frac{\frac{7\sqrt{3}}{2} - \left(-\frac{7\sqrt{3}}{2}\right)}{-\frac{7}{2} - \frac{7}{2}}} = \underline{\frac{7\sqrt{3}}{-7}} = \underline{-\sqrt{3}} = \underline{\text{awrt } -1.73}$	to give any of the four underlined expressions oe **(must be correct solution only)**	A1 **cso**
	Hence m(**N**) $= \frac{-1}{-\sqrt{3}}$ or $\frac{1}{\sqrt{3}} = \text{awrt } 0.58$	Uses m(**T**) to 'correctly' find m(**N**). Can be ft from "their tangent gradient".	A1√ oe.
	When $t = \frac{\pi}{6}$, $x = 7\cos\frac{\pi}{6} - \cos\frac{7\pi}{6} = \frac{7\sqrt{3}}{2} - \left(-\frac{\sqrt{3}}{2}\right) = \frac{8\sqrt{3}}{2} = 4\sqrt{3}$ $y = 7\sin\frac{\pi}{6} - \sin\frac{7\pi}{6} = \frac{7}{2} - \left(-\frac{1}{2}\right) = \frac{8}{2} = 4$	The point $\underline{\left(4\sqrt{3},\ 4\right)}$ or $\underline{\left(\text{awrt } 6.9,\ 4\right)}$	B1
	N: $y - 4 = \frac{1}{\sqrt{3}}\left(x - 4\sqrt{3}\right)$	Finding an equation of a normal with their point and their normal gradient or finds c by using $y = (\text{their gradient})x + \text{"c"}$.	M1
	N: $\underline{y = \frac{1}{\sqrt{3}}x}$ or $\underline{y = \frac{\sqrt{3}}{3}x}$ or $\underline{3y = \sqrt{3}x}$	Correct simplified EXACT equation of <u>normal</u>. This is dependent on candidate using correct $\left(4\sqrt{3},\ 4\right)$	<u>A1</u> oe
	or $4 = \frac{1}{\sqrt{3}}\left(4\sqrt{3}\right) + c \Rightarrow c = 4 - 4 = 0$		
	Hence **N:** $\underline{y = \frac{1}{\sqrt{3}}x}$ or $\underline{y = \frac{\sqrt{3}}{3}x}$ or $\underline{3y = \sqrt{3}x}$		
			[6] **9 marks**

Question Number	Scheme	Marks
Aliter 3. (a) **Way 2**	$x = 7\cos t - \cos 7t\,,\ \ y = 7\sin t - \sin 7t\,,$	
	$\frac{dx}{dt} = -7\sin t + 7\sin 7t\,,\ \ \frac{dy}{dt} = 7\cos t - 7\cos 7t$ Attempt to differentiate x **and** y with respect to t to give $\frac{dx}{dt}$ in the form $\pm A\sin t \pm B\sin 7t$ $\frac{dy}{dt}$ in theform $\pm C\cos t \pm D\cos 7t$	M1
	Correct $\frac{dx}{dt}$ and $\frac{dy}{dt}$	A1
	$\frac{dy}{dx} = \frac{7\cos t - 7\cos 7t}{-7\sin t + 7\sin 7t} = \frac{-7(-2\sin 4t\sin 3t)}{-7(2\cos 4t\sin 3t)} = \tan 4t$ Candidate's $\frac{\frac{dy}{dt}}{\frac{dx}{dt}}$	B1 $\sqrt{\ }$
		[3]
(b)	When $t = \frac{\pi}{6},\ \ m(\mathbf{T}) = \frac{dy}{dx} = \tan\frac{4\pi}{6}\,;$ Substitutes $t = \frac{\pi}{6}$ or 30° into their $\frac{dy}{dx}$ expression;	M1
	$= \underline{\frac{2\left(\frac{\sqrt{3}}{2}\right)(1)}{2\left(-\frac{1}{2}\right)(1)}} = \underline{-\sqrt{3}} = \underline{\text{awrt } -1.73}$ to give any of the three underlined expressions oe **(must be correct solution only)**	A1 **cso**
	Hence $m(\mathbf{N}) = \frac{-1}{-\sqrt{3}}$ or $\frac{1}{\sqrt{3}}$ = awrt 0.58 Uses m(**T**) to 'correctly' find m(**N**). Can be ft from "their tangent gradient".	A1$\sqrt{\ }$ oe.
	When $t = \frac{\pi}{6}$, $x = 7\cos\frac{\pi}{6} - \cos\frac{7\pi}{6} = \frac{7\sqrt{3}}{2} - \left(-\frac{\sqrt{3}}{2}\right) = \frac{8\sqrt{3}}{2} = 4\sqrt{3}$ $y = 7\sin\frac{\pi}{6} - \sin\frac{7\pi}{6} = \frac{7}{2} - \left(-\frac{1}{2}\right) = \frac{8}{2} = 4$ The point $\underline{\left(4\sqrt{3},\,4\right)}$ or $\underline{(\text{awrt } 6.9,\,4)}$	B1
	N: $y - 4 = \frac{1}{\sqrt{3}}\left(x - 4\sqrt{3}\right)$ Finding an equation of a normal with their point and their normal gradient or finds c by using $y = (\text{their gradient})x + \text{"c"}$.	M1
	N: $\underline{y = \frac{1}{\sqrt{3}}x}$ or $\underline{y = \frac{\sqrt{3}}{3}x}$ or $\underline{3y = \sqrt{3}x}$ Correct simplified EXACT equation of <u>normal</u>. This is dependent on candidate using correct $\left(4\sqrt{3},\,4\right)$	<u>A1</u> oe
	or $4 = \frac{1}{\sqrt{3}}\left(4\sqrt{3}\right) + c \Rightarrow c = 4 - 4 = 0$	
	Hence **N:** $\underline{y = \frac{1}{\sqrt{3}}x}$ or $\underline{y = \frac{\sqrt{3}}{3}x}$ or $\underline{3y = \sqrt{3}x}$	
		[6]
		9 marks

Beware: A candidate finding an m(**T**) = 0 can obtain A1ft for m(**N**) $\to \infty$, but obtains M0 if they write $y - 4 = \infty(x - 4\sqrt{3})$. If they write, however, **N**: $x = 4\sqrt{3}$, then they can score M1.

Beware: A candidate finding an m(**T**) = ∞ can obtain A1ft for m(**N**) = 0, and also obtains M1 if they write $y - 4 = 0(x - 4\sqrt{3})$ or y = 4.

Leave blank

4. (a) Express $\dfrac{2x-1}{(x-1)(2x-3)}$ in partial fractions.

(3)

(b) Given that $x \geqslant 2$, find the general solution of the differential equation

$$(2x-3)(x-1)\frac{\mathrm{d}y}{\mathrm{d}x} = (2x-1)\,y\,.$$

(5)

(c) Hence find the particular solution of this differential equation that satisfies $y = 10$ at $x = 2$, giving your answer in the form $y = \mathrm{f}(x)$.

(4)

Question Number	Scheme	Marks
4. (a)	$\frac{2x-1}{(x-1)(2x-3)} \equiv \frac{A}{(x-1)} + \frac{B}{(2x-3)}$	
	$2x-1 \equiv A(2x-3)+B(x-1)$ Forming this identity. **NB**: A & B are not assigned in this question	M1
	Let $x=\frac{3}{2}$, $2 = B\left(\frac{1}{2}\right) \Rightarrow B=4$	
	Let $x=1$, $1 = A(-1) \Rightarrow A=-1$ either one of $A=-1$ or $B=4$. both correct for their A, B.	A1 A1
	giving $\frac{-1}{(x-1)} + \frac{4}{(2x-3)}$	**[3]**
(b) & (c)	$\int \frac{dy}{y} = \int \frac{(2x-1)}{(2x-3)(x-1)}\,dx$ Separates variables as shown Can be implied	B1
	$= \int \frac{-1}{(x-1)} + \frac{4}{(2x-3)}\,dx$ Replaces RHS with their partial fraction to be integrated.	M1 $\sqrt{}$
	$\therefore \ln y = -\ln(x-1) + 2\ln(2x-3) + c$ ***At least*** two terms in ln's ***At least*** two ln terms correct All three terms correct and '+ c'	M1 A1 $\sqrt{}$ A1 **[5]**
	$y=10$, $x=2$ gives $c=\ln 10$ $c=\ln 10$	B1
	$\therefore \ln y = -\ln(x-1) + 2\ln(2x-3) + \ln 10$	
	$\ln y = -\ln(x-1) + \ln(2x-3)^2 + \ln 10$ Using the power law for logarithms	M1
	$\ln y = \ln\left(\frac{(2x-3)^2}{(x-1)}\right) + \ln 10$ or $\ln y = \ln\left(\frac{10(2x-3)^2}{(x-1)}\right)$ Using the product and/or quotient laws for logarithms to obtain a single RHS logarithmic term with/without constant c.	M1
	$y = \frac{10(2x-3)^2}{(x-1)}$ $y = \frac{10(2x-3)^2}{(x-1)}$ or aef. isw	A1 aef **[4]**
		12 marks

Question Number	Scheme	Marks
Aliter 4. (b) & (c) **Way 2**	$\int \frac{dy}{y} = \int \frac{(2x-1)}{(2x-3)(x-1)}\,dx$ Separates variables as shown Can be implied	B1
	$= \int \frac{-1}{(x-1)} + \frac{4}{(2x-3)}\,dx$ Replaces RHS with their partial fraction to be integrated.	M1 √
	$\therefore \ln y = -\ln(x-1) + 2\ln(2x-3) + c$ ***At least*** two terms in ln's ***At least*** two ln terms correct All three terms correct and '+ c'	M1 A1 √ A1
	See below for the award of B1 ***decide to award B1 here!!***	B1
	$\ln y = -\ln(x-1) + \ln(2x-3)^2 + c$ Using the power law for logarithms	M1
	$\ln y = \ln\left(\frac{(2x-3)^2}{x-1}\right) + c$ Using the product and/or quotient laws for logarithms to obtain a single RHS logarithmic term with/without constant c.	M1
	$\ln y = \ln\left(\frac{A(2x-3)^2}{x-1}\right)$ where $c = \ln A$	
	or $e^{\ln y} = e^{\ln\left(\frac{(2x-3)^2}{x-1}\right) + c} = e^{\ln\left(\frac{(2x-3)^2}{x-1}\right)} e^c$	
	$y = \frac{A(2x-3)^2}{(x-1)}$	
	$y = 10,\ x = 2$ gives $A = 10$ $A = 10$ for B1	award above
	$y = \frac{10(2x-3)^2}{(x-1)}$ $y = \frac{10(2x-3)^2}{(x-1)}$ or aef & isw	A1 aef
		[5] & [4]

Note: The B1 mark (part (c)) should be awarded in the same place on ePEN as in the Way 1 approach.

Question Number	Scheme		Marks
Aliter (b) & (c) **Way 3**	$\int \frac{dy}{y} = \int \frac{(2x-1)}{(2x-3)(x-1)}\,dx$	Separates variables as shown Can be implied	B1
	$= \int \frac{-1}{(x-1)} + \frac{2}{(x-\frac{3}{2})}\,dx$	Replaces RHS with their partial fraction to be integrated.	M1 √
	$\therefore \ln y = -\ln(x-1) + 2\ln(x-\frac{3}{2}) + c$	***At least*** two terms in ln's ***At least*** two ln terms correct All three terms correct and '+ c'	M1 A1 √ A1 **[5]**
	$y = 10,\ x = 2$ gives $c = \underline{\ln 10 - 2\ln(\frac{1}{2})} = \underline{\ln 40}$	$c = \ln 10 - 2\ln(\frac{1}{2})$ or $c = \ln 40$	B1 oe
	$\therefore \ln y = -\ln(x-1) + 2\ln(x-\frac{3}{2}) + \ln 40$		
	$\ln y = -\ln(x-1) + \ln(x-\frac{3}{2})^2 + \ln 10$	Using the power law for logarithms	M1
	$\ln y = \ln\left(\frac{(x-\frac{3}{2})^2}{(x-1)}\right) + \ln 40$ or $\ln y = \ln\left(\frac{40(x-\frac{3}{2})^2}{(x-1)}\right)$	Using the product and/or quotient laws for logarithms to obtain a single RHS logarithmic term with/without constant c.	M1
	$\underline{y = \frac{40(x-\frac{3}{2})^2}{(x-1)}}$	$\underline{y = \frac{40(x-\frac{3}{2})^2}{(x-1)}}$ or aef. isw	A1 aef **[4]**

Note: Please mark parts (b) and (c) together for any of the three ways.

5. A set of curves is given by the equation $\sin x + \cos y = 0.5$.

(a) Use implicit differentiation to find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$.

(2)

For $-\pi < x < \pi$ and $-\pi < y < \pi$,

(b) find the coordinates of the points where $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$.

(5)

Question Number	Scheme	Marks
5. (a)	$\sin x + \cos y = 0.5$ (eqn $*$)	
	$\left\{\frac{\cancel{dy}}{\cancel{dx}}\cancel{\times}\right\}$ $\cos x - \sin y\frac{dy}{dx} = 0$ (eqn #) Differentiates implicitly to include $\pm\sin y\frac{dy}{dx}$. (Ignore $\left(\frac{dy}{dx} =\right)$.)	M1
	$\frac{dy}{dx} = \underline{\frac{\cos x}{\sin y}}$ $\underline{\frac{\cos x}{\sin y}}$	A1 **cso**
		[2]
(b)	$\frac{dy}{dx} = 0 \Rightarrow \frac{\cos x}{\sin y} = 0 \Rightarrow \cos x = 0$ Candidate realises that they need to solve 'their numerator' = 0 …or candidate sets $\frac{dy}{dx} = 0$ in their (eqn #) and attempts to solve the resulting equation.	M1 √
	giving $\underline{x = -\frac{\pi}{2}}$ or $\underline{x = \frac{\pi}{2}}$ both $\underline{x = -\frac{\pi}{2}, \frac{\pi}{2}}$ or $\underline{x = \pm 90^\circ}$ or awrt $\underline{x = \pm 1.57}$ required here	A1
	When $x = -\frac{\pi}{2}$, $\sin\left(-\frac{\pi}{2}\right) + \cos y = 0.5$ When $x = \frac{\pi}{2}$, $\sin\left(\frac{\pi}{2}\right) + \cos y = 0.5$ Substitutes either their $x = \frac{\pi}{2}$ or $x = -\frac{\pi}{2}$ into eqn $*$	M1
	$\Rightarrow \cos y = 1.5 \Rightarrow$ y has no solutions $\Rightarrow \cos y = -0.5 \Rightarrow y = \frac{2\pi}{3}$ or $-\frac{2\pi}{3}$ Only one of $y = \underline{\frac{2\pi}{3}}$ or $\underline{-\frac{2\pi}{3}}$ or $\underline{120^\circ}$ or $\underline{-120^\circ}$ or awrt $\underline{-2.09}$ or awrt $\underline{2.09}$	A1
	In specified range $(x, y) = \underline{\left(\frac{\pi}{2}, \frac{2\pi}{3}\right)}$ and $\underline{\left(\frac{\pi}{2}, -\frac{2\pi}{3}\right)}$ Only exact coordinates of $\underline{\left(\frac{\pi}{2}, \frac{2\pi}{3}\right)}$ and $\underline{\left(\frac{\pi}{2}, -\frac{2\pi}{3}\right)}$ **Do not award this mark if candidate states other coordinates inside the required range.**	A1
		[5]
		7 marks

Leave blank

6. (a) Given that $y = 2^x$, and using the result $2^x = e^{x\ln 2}$, or otherwise, show that $\dfrac{dy}{dx} = 2^x \ln 2$.

(2)

(b) Find the gradient of the curve with equation $y = 2^{(x^2)}$ at the point with coordinates (2,16).

(4)

Question Number	Scheme		Marks
6.	$y = 2^x = e^{x\ln 2}$		
(a) **Way 1**	$\frac{dy}{dx} = \ln 2 . e^{x\ln 2}$	$\frac{dy}{dx} = \ln 2 . e^{x\ln 2}$	M1
	Hence $\frac{dy}{dx} = \ln 2.(2^x) = 2^x \ln 2$ **AG**	$2^x \ln 2$ **AG**	A1 **cso**
			[2]
Aliter (a) **Way 2**	$\ln y = \ln\left(2^x\right)$ leads to $\ln y = x\ln 2$	Takes logs of both sides, then uses the power law of logarithms…	
	$\frac{1}{y}\frac{dy}{dx} = \ln 2$	… and differentiates implicitly to give $\frac{1}{y}\frac{dy}{dx} = \ln 2$	M1
	Hence $\frac{dy}{dx} = y\ln 2 = 2^x \ln 2$ **AG**	$2^x \ln 2$ **AG**	A1 **cso**
			[2]
(b)	$y = 2^{(x^2)} \Rightarrow \frac{dy}{dx} = 2x.\,2^{(x^2)}.\ln 2$	$\text{Ax } 2^{(x^2)}$	M1
		$2x.\,2^{(x^2)}.\ln 2$ or $2x.y.\ln 2$ if y is defined	A1
	When x = 2, $\frac{dy}{dx} = 2(2)2^4 \ln 2$	Substitutes x = 2 into their $\frac{dy}{dx}$ which is of the form $\pm k\,2^{(x^2)}$ or $\text{Ax } 2^{(x^2)}$	M1
	$\frac{dy}{dx} = \underline{64\ln 2} = 44.3614...$	$\underline{64\ln 2}$ or awrt 44.4	A1
			[4]
			6 marks

Question Number	Scheme		Marks
Aliter 6. (b) **Way 2**	$\ln y = \ln\left(2^{x^2}\right)$ leads to $\ln y = x^2 \ln 2$		
	$\frac{1}{y}\frac{dy}{dx} = 2x.\ln 2$	$\frac{1}{y}\frac{dy}{dx} = Ax.\ln 2$ $\frac{1}{y}\frac{dy}{dx} = 2x.\ln 2$	M1 A1
	When x = 2, $\frac{dy}{dx} = 2(2)2^4 \ln 2$	Substitutes x = 2 into their $\frac{dy}{dx}$ which is of the form $\pm k\,2^{(x^2)}$ or $Ax\ 2^{(x^2)}$	M1
	$\frac{dy}{dx} = \underline{64\ln 2} = 44.3614...$	$\underline{64\ln 2}$ or awrt 44.4	A1
			[4]

7. The point A has position vector $\mathbf{a} = 2\mathbf{i} + 2\mathbf{j} + \mathbf{k}$ and the point B has position vector $\mathbf{b} = \mathbf{i} + \mathbf{j} - 4\mathbf{k}$, relative to an origin O.

(a) Find the position vector of the point C, with position vector $\mathbf{c}$, given by

$$\mathbf{c} = \mathbf{a} + \mathbf{b}.$$

(1)

(b) Show that $OACB$ is a rectangle, and find its exact area.

(6)

The diagonals of the rectangle, AB and OC, meet at the point D.

(c) Write down the position vector of the point D.

(1)

(d) Find the size of the angle ADC.

(6)

Question Number	Scheme	Marks
7.	$\mathbf{a} = \overrightarrow{OA} = 2\mathbf{i} + 2\mathbf{j} + \mathbf{k} \quad \Rightarrow \lvert\overrightarrow{OA}\rvert = 3$ $\mathbf{b} = \overrightarrow{OB} = \mathbf{i} + \mathbf{j} - 4\mathbf{k} \quad \Rightarrow \lvert\overrightarrow{OB}\rvert = \sqrt{18}$ $\overrightarrow{BC} = \pm(2\mathbf{i} + 2\mathbf{j} + \mathbf{k}) \Rightarrow \lvert\overrightarrow{BC}\rvert = 3$ $\overrightarrow{AC} = \pm(\mathbf{i} + \mathbf{j} - 4\mathbf{k}) \Rightarrow \lvert\overrightarrow{AC}\rvert = \sqrt{18}$	
(a)	$\mathbf{c} = \overrightarrow{OC} = \underline{3\mathbf{i} + 3\mathbf{j} - 3\mathbf{k}}$ $\underline{3\mathbf{i} + 3\mathbf{j} - 3\mathbf{k}}$	B1 **cao** **[1]**
(b)	$\overrightarrow{OA} \bullet \overrightarrow{OB} = \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 1 \\ -4 \end{pmatrix} = \underline{2+2-4} = 0$ or… $\overrightarrow{BO} \bullet \overrightarrow{BC} = \begin{pmatrix} -1 \\ -1 \\ 4 \end{pmatrix} \bullet \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} = \underline{-2-2+4} = 0$ or… $\overrightarrow{AC} \bullet \overrightarrow{BC} = \begin{pmatrix} 1 \\ 1 \\ -4 \end{pmatrix} \bullet \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} = \underline{2+2-4} = 0$ or… $\overrightarrow{AO} \bullet \overrightarrow{AC} = \begin{pmatrix} -2 \\ -2 \\ -1 \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 1 \\ -4 \end{pmatrix} = \underline{-2-2+4} = 0$	
	An attempt to take the dot product between either $\overrightarrow{OA}$ and $\overrightarrow{OB}$ $\overrightarrow{OA}$ and $\overrightarrow{AC}$, $\overrightarrow{AC}$ and $\overrightarrow{BC}$ or $\overrightarrow{OB}$ and $\overrightarrow{BC}$	<u>M1</u>
	Showing the result is equal to zero.	A1
	and therefore OA is perpendicular to OB and hence OACB is a rectangle. <u>perpendicular</u> and <u>OACB is a rectangle</u>	A1 **cso**
	Using distance formula to find either the correct height or width.	M1
	Area $= 3 \times \sqrt{18} = 3\sqrt{18} = 9\sqrt{2}$ Multiplying the rectangle's height by its width.	M1
	exact value of $3\sqrt{18}$, $9\sqrt{2}$, $\sqrt{162}$ or aef	A1 **[6]**
(c)	$\overrightarrow{OD} = \mathbf{d} = \tfrac{1}{2}(3\mathbf{i} + 3\mathbf{j} - 3\mathbf{k})$ $\underline{\tfrac{1}{2}(3\mathbf{i} + 3\mathbf{j} - 3\mathbf{k})}$	B1 **[1]**

Question Number	Scheme	Marks
(d) **Way 1**	*using dot product formula* $\overrightarrow{DA} = \pm\left(\tfrac{1}{2}\mathbf{i} + \tfrac{1}{2}\mathbf{j} + \tfrac{5}{2}\mathbf{k}\right)$ & $\overrightarrow{DC} = \pm\left(\tfrac{3}{2}\mathbf{i} + \tfrac{3}{2}\mathbf{j} - \tfrac{3}{2}\mathbf{k}\right)$ or $\overrightarrow{BA} = \pm\left(\mathbf{i} + \mathbf{j} + 5\mathbf{k}\right)$ & $\overrightarrow{OC} = \pm\left(3\mathbf{i} + 3\mathbf{j} - 3\mathbf{k}\right)$ Identifies a set of two relevant vectors Correct vectors ±	M1 A1
	$\cos D = (\pm)\dfrac{\begin{pmatrix}0.5\\0.5\\2.5\end{pmatrix}\bullet\begin{pmatrix}1.5\\1.5\\-1.5\end{pmatrix}}{\frac{\sqrt{27}}{2}.\frac{\sqrt{27}}{2}} = (\pm)\underline{\dfrac{\frac{3}{4}+\frac{3}{4}-\frac{15}{4}}{\frac{27}{4}}} = \underline{(\pm)\dfrac{1}{3}}$ Applies dot product formula on multiples of these vectors. Correct ft. application of dot product formula.	dM1 A1 √
	$D = \cos^{-1}\left(-\dfrac{1}{3}\right)$ Attempts to find the correct angle D rather than $180^\circ - D$.	ddM1 √
	$D = 109.47122...^\circ$ 109.5° or awrt 109° or 1.91^c	A1 **[6]**
Aliter (d) **Way 2**	*using dot product formula and direction vectors* $\mathrm{d}\overrightarrow{BA} = \pm\left(\mathbf{i} + \mathbf{j} + 5\mathbf{k}\right)$ & $\mathrm{d}\overrightarrow{OC} = \pm\left(\mathbf{i} + \mathbf{j} - \mathbf{k}\right)$ Identifies a set of two direction vectors Correct vectors ±	M1 A1
	$\cos D = (\pm)\dfrac{\begin{pmatrix}1\\1\\-1\end{pmatrix}\bullet\begin{pmatrix}1\\1\\5\end{pmatrix}}{\sqrt{3}.\sqrt{27}} = \underline{(\pm)\dfrac{1+1-5}{\sqrt{3}.\sqrt{27}}} = \underline{(\pm)\dfrac{1}{3}}$ Applies dot product formula on multiples of these vectors. Correct ft. application of dot product formula.	dM1 A1 √
	$D = \cos^{-1}\left(-\dfrac{1}{3}\right)$ Attempts to find the correct angle D rather than $180^\circ - D$.	ddM1 √
	$D = 109.47122...^\circ$ 109.5° or awrt 109° or 1.91^c	A1 **[6]**

Question Number	Scheme	Marks
Aliter	*using dot product formula and similar triangles*	
(d)	$\mathrm{d}\overrightarrow{OA} = (2\mathbf{i} + 2\mathbf{j} + \mathbf{k})$ & $\mathrm{d}\overrightarrow{OC} = (\mathbf{i} + \mathbf{j} - \mathbf{k})$ Identifies a set of two direction vectors Correct vectors	M1 A1
Way 3	$\cos\left(\tfrac{1}{2}D\right) = \dfrac{\begin{pmatrix}2\\2\\1\end{pmatrix} \bullet \begin{pmatrix}1\\1\\-1\end{pmatrix}}{\sqrt{9}.\sqrt{3}} = \underline{\dfrac{2+2-1}{\sqrt{9}.\sqrt{3}}} = \underline{\dfrac{1}{\sqrt{3}}}$ Applies dot product formula on multiples of these vectors. <u>Correct ft. application of dot product formula.</u>	dM1 A1 √
	$D = 2\cos^{-1}\left(\dfrac{1}{\sqrt{3}}\right)$ Attempts to find the correct angle D by doubling their angle for $\tfrac{1}{2}D$.	ddM1 √
	$D = 109.47122...^{\circ}$ 109.5° or awrt 109° or 1.91^{c}	A1 **[6]**
Aliter	*using cosine rule*	
(d)	$\overrightarrow{DA} = \tfrac{1}{2}\mathbf{i} + \tfrac{1}{2}\mathbf{j} + \tfrac{5}{2}\mathbf{k}$, $\overrightarrow{DC} = \tfrac{3}{2}\mathbf{i} + \tfrac{3}{2}\mathbf{j} - \tfrac{3}{2}\mathbf{k}$, $\overrightarrow{AC} = \mathbf{i} + \mathbf{j} - 4\mathbf{k}$	
Way 4	$\lvert\overrightarrow{DA}\rvert = \dfrac{\sqrt{27}}{2}$, $\lvert\overrightarrow{DC}\rvert = \dfrac{\sqrt{27}}{2}$, $\lvert\overrightarrow{AC}\rvert = \sqrt{18}$ Attempts to find all the lengths of all three edges of ΔADC All Correct	M1 A1
	$\cos D = \underline{\dfrac{\left(\frac{\sqrt{27}}{2}\right)^2 + \left(\frac{\sqrt{27}}{2}\right)^2 - \left(\sqrt{18}\right)^2}{2\left(\frac{\sqrt{27}}{2}\right)\left(\frac{\sqrt{27}}{2}\right)}} = -\dfrac{1}{\underline{3}}$ Using the cosine rule formula with correct 'subtraction'. <u>Correct ft application of the cosine rule formula</u>	dM1 A1 √
	$D = \cos^{-1}\left(-\dfrac{1}{3}\right)$ Attempts to find the correct angle D rather than $180^{\circ} - D$.	ddM1 √
	$D = 109.47122...^{\circ}$ 109.5° or awrt 109° or 1.91^{c}	A1 **[6]**

Question Number	Scheme		Marks
Aliter (d) **Way 5**	*using trigonometry on a right angled triangle* $\overrightarrow{DA} = \frac{1}{2}\mathbf{i} + \frac{1}{2}\mathbf{j} + \frac{5}{2}\mathbf{k}$ $\quad \overrightarrow{OA} = 2\mathbf{i} + 2\mathbf{j} + \mathbf{k}$ $\quad \overrightarrow{AC} = \mathbf{i} + \mathbf{j} - 4\mathbf{k}$		
	Let X be the midpoint of AC $\lvert\overrightarrow{DA}\rvert = \frac{\sqrt{27}}{2}$, $\lvert\overrightarrow{DX}\rvert = \frac{1}{2}\lvert\overrightarrow{OA}\rvert = \frac{3}{2}$, $\lvert\overrightarrow{AX}\rvert = \frac{1}{2}\lvert\overrightarrow{AC}\rvert = \frac{1}{2}\sqrt{18}$	Attempts to find two out of the three lengths in ΔADX	M1
	(hypotenuse), (adjacent) , (opposite)	Any two correct	A1
	$\sin(\frac{1}{2}D) = \frac{\frac{\sqrt{18}}{2}}{\frac{\sqrt{27}}{2}}$, $\cos(\frac{1}{2}D) = \frac{\frac{3}{2}}{\frac{\sqrt{27}}{2}}$ or $\tan(\frac{1}{2}D) = \frac{\frac{\sqrt{18}}{2}}{\frac{3}{2}}$	Uses correct sohcahtoa to find $\frac{1}{2}D$	dM1
		Correct ft application of sohcahtoa	A1 $\sqrt{}$
	eg. $D = 2\tan^{-1}\left(\frac{\frac{\sqrt{18}}{2}}{\frac{3}{2}}\right)$	Attempts to find the correct angle D by doubling their angle for $\frac{1}{2}D$.	ddM1 $\sqrt{}$
	$D = 109.47122...^\circ$	109.5° or awrt 109° or 1.91^c	A1
			[6]
Aliter (d) **Way 6**	*using trigonometry on a right angled similar triangle OAC* $\overrightarrow{OC} = 3\mathbf{i} + 3\mathbf{j} - 3\mathbf{k}$ $\quad \overrightarrow{OA} = 2\mathbf{i} + 2\mathbf{j} + \mathbf{k}$ $\quad \overrightarrow{AC} = \mathbf{i} + \mathbf{j} - 4\mathbf{k}$		
	$\lvert\overrightarrow{OC}\rvert = \sqrt{27}$, $\lvert\overrightarrow{OA}\rvert = 3$, $\lvert\overrightarrow{AC}\rvert = \sqrt{18}$	Attempts to find two out of the three lengths in ΔOAC	M1
	(hypotenuse), (adjacent), (opposite)	Any two correct	A1
	$\sin(\frac{1}{2}D) = \frac{\sqrt{18}}{\sqrt{27}}$, $\cos(\frac{1}{2}D) = \frac{3}{\sqrt{27}}$ or $\tan(\frac{1}{2}D) = \frac{\sqrt{18}}{3}$	Uses correct sohcahtoa to find $\frac{1}{2}D$	dM1
		Correct ft application of sohcahtoa	A1 $\sqrt{}$
	eg. $D = 2\tan^{-1}\left(\frac{\sqrt{18}}{3}\right)$	Attempts to find the correct angle D by doubling their angle for $\frac{1}{2}D$.	ddM1 $\sqrt{}$
	$D = 109.47122...^\circ$	109.5° or awrt 109° or 1.91^c	A1
			[6]

Question Number	Scheme		Marks
Aliter 7. (b) (i) **Way 2**	$\mathbf{c} = \overrightarrow{OC} = \pm(3\mathbf{i} + 3\mathbf{j} - 3\mathbf{k})$ $\overrightarrow{AB} = \pm(-\mathbf{i} - \mathbf{j} - 5\mathbf{k})$		
	$\lvert\overrightarrow{OC}\rvert = \sqrt{(3)^2 + (3)^2 + (-3)^2} = \sqrt{(1)^2 + (1)^2 + (-5)^2} = \lvert\overrightarrow{AB}\rvert$	A **complete** method of proving that the diagonals are equal.	M1
	As $\lvert\overrightarrow{OC}\rvert = \lvert\overrightarrow{AB}\rvert = \sqrt{27}$	Correct result.	A1
	then the diagonals are equal, and OACB is a rectangle.	diagonals are equal and OACB is a rectangle	A1 **cso** **[3]**
	$\mathbf{a} = \overrightarrow{OA} = 2\mathbf{i} + 2\mathbf{j} + \mathbf{k} \quad \Rightarrow \lvert\overrightarrow{OA}\rvert = 3$ $\mathbf{b} = \overrightarrow{OB} = \mathbf{i} + \mathbf{j} - 4\mathbf{k} \quad \Rightarrow \lvert\overrightarrow{OB}\rvert = \sqrt{18}$ $\overrightarrow{BC} = \pm(2\mathbf{i} + 2\mathbf{j} + \mathbf{k}) \Rightarrow \lvert\overrightarrow{BC}\rvert = 3$ $\overrightarrow{AC} = \pm(\mathbf{i} + \mathbf{j} - 4\mathbf{k}) \Rightarrow \lvert\overrightarrow{AC}\rvert = \sqrt{18}$ $\mathbf{c} = \overrightarrow{OC} = \pm(3\mathbf{i} + 3\mathbf{j} - 3\mathbf{k}) \Rightarrow \lvert\overrightarrow{OC}\rvert = \sqrt{27}$ $\overrightarrow{AB} = \pm(-\mathbf{i} - \mathbf{j} - 5\mathbf{k}) \Rightarrow \lvert\overrightarrow{AB}\rvert = \sqrt{27}$		
Aliter 7. (b) (i) **Way 3**	$(OA)^2 + (AC)^2 = (OC)^2$ or $(BC)^2 + (OB)^2 = (OC)^2$ or $(OA)^2 + (OB)^2 = (AB)^2$ or $(BC)^2 + (AC)^2 = (AB)^2$ or equivalent		
	$\Rightarrow \underline{(3)^2 + (\sqrt{18})^2 = \left(\sqrt{27}\right)^2}$	A **complete** method of proving that Pythagoras holds using their values. Correct result	M1 A1
	and therefore OA is perpendicular to OB or AC is perpendicular to BC and hence OACB is a rectangle.	perpendicular and OACB is a rectangle	A1 **cso** **[3]**
			14 marks

8.

$$I = \int_0^5 e^{\sqrt{(3x+1)}}\,dx.$$

(a) Given that $y = e^{\sqrt{(3x+1)}}$, complete the table with the values of y corresponding to $x = 2$, 3 and 4.

x	0	1	2	3	4	5
y	e^1	e^2				e^4

(2)

(b) Use the trapezium rule, with all the values of y in the completed table, to obtain an estimate for the original integral I, giving your answer to 4 significant figures.

(3)

(c) Use the substitution $t = \sqrt{(3x+1)}$ to show that I may be expressed as $\int_a^b kte^t\,dt$, giving the values of a, b and k.

(5)

(d) Use integration by parts to evaluate this integral, and hence find the value of I correct to 4 significant figures, showing all the steps in your working.

(5)

Question Number	Scheme	Marks

8. (a)

x	0	1	2	3	4	5
y	e^1	e^2	$e^{\sqrt{7}}$	$e^{\sqrt{10}}$	$e^{\sqrt{13}}$	e^4
or y	2.71828…	7.38906…	14.09403…	23.62434…	36.80197…	54.59815…

Either $e^{\sqrt{7}}$, $e^{\sqrt{10}}$ and $e^{\sqrt{13}}$
or awrt 14.1, 23.6 and 36.8
or e to the power
awrt 2.65, 3.16, 3.61
(or mixture of decimals and e's)
At least two correct B1
All three correct B1
[2]

(b)

$$I \approx \frac{1}{2}\times 1 ; \times \underline{\left\{ e^1 + 2\left(e^2 + e^{\sqrt{7}} + e^{\sqrt{10}} + e^{\sqrt{13}}\right) + e^4 \right\}}$$

Outside brackets $\frac{1}{2}\times 1$ B1;
For structure of trapezium rule $\underline{\{............\}}$; M1 $\sqrt{\ }$

$$= \frac{1}{2}\times 221.1352227... = 110.5676113... = \underline{110.6} \text{ (4sf)}$$

$\underline{110.6}$ A1 **cao**
[3]

Beware: In part (b) candidates can add up the individual trapezia:

(b) $I \approx \frac{1}{2}.1\left(\underline{e^1 + e^2}\right) + \frac{1}{2}.1\left(\underline{e^2 + e^{\sqrt{7}}}\right) + \frac{1}{2}.1\left(\underline{e^{\sqrt{7}} + e^{\sqrt{10}}}\right) + \frac{1}{2}.1\left(\underline{e^{\sqrt{10}} + e^{\sqrt{13}}}\right) + \frac{1}{2}.1\left(\underline{e^{\sqrt{13}} + e^4}\right)$

Question Number	Scheme		Marks
(c)	$t=(3x+1)^{\frac{1}{2}} \Rightarrow \frac{dt}{dx}=\frac{1}{2}.3.(3x+1)^{-\frac{1}{2}}$	$A(3x+1)^{-\frac{1}{2}}$ or $t\frac{dt}{dx}=A$	M1
	… or $t^2=3x+1 \Rightarrow \underline{2t\frac{dt}{dx}=3}$	$\underline{\frac{3}{2}(3x+1)^{-\frac{1}{2}}}$ or $\underline{2t\frac{dt}{dx}=3}$	A1
	so $\frac{dt}{dx}=\frac{3}{2.(3x+1)^{\frac{1}{2}}}=\frac{3}{2t} \Rightarrow \frac{dx}{dt}=\frac{2t}{3}$	Candidate obtains either $\frac{dt}{dx}$ or $\frac{dx}{dt}$ in terms of t …	
	$\therefore I=\int e^{\sqrt{(3x+1)}}\ dx = \int e^t\frac{dx}{dt}.dt = \int e^t.\frac{2t}{3}.dt$	… and moves on to substitute this into I to convert an integral wrt x to an integral wrt t.	dM1
	$\therefore\ I=\underline{\int \frac{2}{3}te^t}\ dt$	$\underline{\int \frac{2}{3}te^t}$	A1
	change limits: when x = 0, t = 1 & when x = 5, t = 4	changes limits $x \to t$ so that $0 \to 1$ and $5 \to 4$	B1
	Hence $I=\int_1^4 \frac{2}{3}te^t\ dt$; where $a=1,\ b=4,\ k=\frac{2}{3}$		
			[5]
(d)	$\begin{Bmatrix} u=t & \Rightarrow & \frac{du}{dt}=1 \\ \frac{dv}{dt}=e^t & \Rightarrow & v=e^t \end{Bmatrix}$	Let k be any constant for the first three marks of this part.	
	$k\int te^t\ dt = k\left(te^t-\int e^t.1\ dt\right)$	Use of 'integration by parts' formula in the correct direction.	M1
		Correct expression with a constant factor k.	A1
	$= k\left(\underline{te^t-e^t}\right)+c$	<u>Correct integration</u> with/without a constant factor k	A1
	$\therefore\ \int_1^4 \frac{2}{3}te^t\ dt = \frac{2}{3}\left\{\left(4e^4-e^4\right)-\left(e^1-e^1\right)\right\}$	Substitutes their changed limits into the integrand and subtracts oe.	dM1 **oe**
	$=\frac{2}{3}(3e^4)=\underline{2e^4}=109.1963...$	either $2e^4$ or awrt 109.2	A1
			[5] **15 marks**

- Note: dM1 denotes a method mark which is dependent upon the award of the previous method mark
- ddM1 denotes a method mark which is dependent upon the award of the previous two method marks.