





January 2008  
6666 Core Mathematics C4  
Mark Scheme

| Question Number               | Scheme  | Marks   |                 |                  |                 |                  |       |     |   |                |                |         |   |  |
|-------------------------------|---|---|-----------------|------------------|-----------------|------------------|-------|-----|---|----------------|----------------|---------|---|--|
| 1. (a)                        | <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td style="padding: 5px;"><math>x</math></td> <td style="padding: 5px;">0</td> <td style="padding: 5px;"><math>\frac{\pi}{4}</math></td> <td style="padding: 5px;"><math>\frac{\pi}{2}</math></td> <td style="padding: 5px;"><math>\frac{3\pi}{4}</math></td> <td style="padding: 5px;"><math>\pi</math></td> </tr> <tr> <td style="padding: 5px;"><math>y</math></td> <td style="padding: 5px;">0</td> <td style="padding: 5px;">1.844321332...</td> <td style="padding: 5px;">4.810477381...</td> <td style="padding: 5px;">8.87207</td> <td style="padding: 5px;">0</td> </tr> </table> | $x$   | 0               | $\frac{\pi}{4}$  | $\frac{\pi}{2}$ | $\frac{3\pi}{4}$ | $\pi$ | $y$ | 0 | 1.844321332... | 4.810477381... | 8.87207 | 0 |  |
| $x$                           | 0   | $\frac{\pi}{4}$   | $\frac{\pi}{2}$ | $\frac{3\pi}{4}$ | $\pi$           |                  |       |     |   |                |                |         |   |  |
| $y$                           | 0   | 1.844321332...  | 4.810477381...  | 8.87207          | 0               |                  |       |     |   |                |                |         |   |  |
| (b)<br>Way 1                  | <p style="text-align: center;">0 can be implied</p> <p style="text-align: center;">↓</p> $\text{Area} \approx \frac{1}{2} \times \frac{\pi}{4} \times \{0 + 2(1.84432 + 4.81048 + 8.87207) + 0\}$   | <p>awrt 1.84432 B1<br/>awrt 4.81048 or 4.81047 B1</p> <p>Outside brackets<br/>awrt 0.39 or <math>\frac{1}{2} \times</math> awrt 0.79 B1<br/><math>\frac{1}{2} \times \frac{\pi}{4}</math> or <math>\frac{\pi}{8}</math></p> <p>For structure of trapezium<br/><u>rule</u> {.....}; M1 <math>\sqrt{\quad}</math></p> <p>Correct expression<br/><u>inside brackets</u> which all must<br/>be multiplied by their "outside<br/>constant". A1 <math>\sqrt{\quad}</math></p> <p><math>= \frac{\pi}{8} \times 31.05374... = 12.19477518... = \underline{12.1948}</math> (4dp) A1 <b>cao</b></p> | [2]             |                  |                 |                  |       |     |   |                |                |         |   |  |
| <i>Aliter</i><br>(b)<br>Way 2 | $\text{Area} \approx \frac{\pi}{4} \times \left\{ \frac{0+1.84432}{2} + \frac{1.84432+4.81048}{2} + \frac{4.81048+8.87207}{2} + \frac{8.87207+0}{2} \right\}$ <p>which is equivalent to:</p> $\text{Area} \approx \frac{1}{2} \times \frac{\pi}{4} \times \{0 + 2(1.84432 + 4.81048 + 8.87207) + 0\}$ <p><math>= \frac{\pi}{4} \times 15.52687... = 12.19477518... = \underline{12.1948}</math> (4dp)</p>   | <p><math>\frac{\pi}{4}</math> (or awrt 0.79) and a divisor<br/>of 2 on all terms inside<br/>brackets. B1</p> <p>One of first and last ordinates,<br/>two of the middle ordinates<br/>inside brackets ignoring the 2. M1 <math>\sqrt{\quad}</math></p> <p>Correct expression inside<br/>brackets if <math>\frac{1}{2}</math> was to be<br/>factorised out. A1 <math>\sqrt{\quad}</math></p> <p><math>= \frac{\pi}{4} \times 15.52687... = 12.19477518... = \underline{12.1948}</math> (4dp) A1 <b>cao</b></p>  | [4]             |                  |                 |                  |       |     |   |                |                |         |   |  |
|                               |   | <b>6 marks</b>  |                 |                  |                 |                  |       |     |   |                |                |         |   |  |

Note an expression like  $\text{Area} \approx \frac{1}{2} \times \frac{\pi}{4} + 2(1.84432 + 4.81048 + 8.87207)$  would score B1M1A0A0



| Question Number          | Scheme   | Marks  |
|--------------------------|--|--|
| <p>2. (a)</p> <p>(b)</p> | <p>** represents a constant<br/>(which must be consistent for first accuracy mark)</p> $(8-3x)^{\frac{1}{3}} = \underline{(8)^{\frac{1}{3}}}\left(1-\frac{3x}{8}\right)^{\frac{1}{3}} = \underline{2}\left(1-\frac{3x}{8}\right)^{\frac{1}{3}}$ <p>with <math>** \neq 1</math></p> $= 2 \left\{ 1 + \frac{(\frac{1}{3})(**x)}{1} + \frac{(\frac{1}{3})(-\frac{2}{3})(**x)^2}{2!} + \frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})(**x)^3}{3!} + \dots \right\}$ <p>Award SC M1 if you see <math>\frac{(\frac{1}{3})(-\frac{2}{3})(**x)^2}{2!} + \frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})(**x)^3}{3!}</math></p> $= 2 \left\{ 1 + \frac{(\frac{1}{3})(-\frac{3x}{8})}{1} + \frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{3x}{8})^2}{2!} + \frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})(-\frac{3x}{8})^3}{3!} + \dots \right\}$ $= 2 \left\{ 1 - \frac{1}{8}x; -\frac{1}{64}x^2 - \frac{5}{1536}x^3 - \dots \right\}$ $= 2 - \frac{1}{4}x; -\frac{1}{32}x^2 - \frac{5}{768}x^3 - \dots$ <p>Either <math>2\{1 - \frac{1}{8}x \dots\}</math> or anything that cancels to <math>2 - \frac{1}{4}x</math>;<br/>Simplified <math>-\frac{1}{32}x^2 - \frac{5}{768}x^3</math></p> $(7.7)^{\frac{1}{3}} \approx 2 - \frac{1}{4}(0.1) - \frac{1}{32}(0.1)^2 - \frac{5}{768}(0.1)^3 - \dots$ $= 2 - 0.025 - 0.0003125 - 0.0000065104166\dots$ $= 1.97468099\dots$ <p>awrt 1.9746810</p> | <p>Takes 8 outside the bracket to give any of <math>\underline{(8)^{\frac{1}{3}}}</math> or <math>\underline{2}</math>. <b>B1</b></p> <p>Expands <math>(1+**x)^{\frac{1}{3}}</math> to give a simplified or an un-simplified <math>1 + \frac{1}{3}(**x)</math>; <b>M1</b>;</p> <p>A correct simplified or an un-simplified <math>\{ \dots \}</math> expansion with candidate's followed through <math>(**x)</math> <b>A1</b> ✓</p> <p><b>[5]</b></p> <p><i>Attempt to substitute</i><br/><math>x = 0.1</math> into a candidate's binomial expansion. <b>M1</b></p> <p><b>[2]</b></p> <p><b>7 marks</b></p> |

You would award B1M1A0 for

$$= 2 \left\{ 1 + \frac{(\frac{1}{3})(-\frac{3x}{8})}{1} + \frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{3x}{8})^2}{2!} + \frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})(-\frac{3x}{8})^3}{3!} + \dots \right\}$$

because \*\* is not consistent.

If you see the constant term "2" in a candidate's final binomial expansion, then you can award B1.

Be wary of calculator value of  $(7.7)^{\frac{1}{3}} = 1.974680822\dots$

| Question Number                                 | Scheme  | Marks  |
|---|---|--|
| <p><i>Aliter</i></p> <p>2. (a)</p> <p>Way 2</p> | $(8-3x)^{\frac{1}{3}}$ $= \left\{ \begin{aligned} &(8)^{\frac{1}{3}} + \frac{(\frac{1}{3})(8)^{-\frac{2}{3}}(**x)}{1!} + \frac{(\frac{1}{3})(-\frac{2}{3})(8)^{-\frac{5}{3}}(**x)^2}{2!} \\ &+ \frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})(8)^{-\frac{8}{3}}(**x)^3 + \dots}{3!} \end{aligned} \right\}$ <p><b>with <math>** \neq 1</math></b></p> $= \left\{ \begin{aligned} &(8)^{\frac{1}{3}} + \frac{(\frac{1}{3})(8)^{-\frac{2}{3}}(-3x)}{1!} + \frac{(\frac{1}{3})(-\frac{2}{3})(8)^{-\frac{5}{3}}(-3x)^2}{2!} \\ &+ \frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})(8)^{-\frac{8}{3}}(-3x)^3 + \dots}{3!} \end{aligned} \right\}$ $= \left\{ 2 + \frac{(\frac{1}{3})(1)}{4}(-3x) + \frac{(-\frac{1}{9})(\frac{1}{32})(9x^2)}{2} + \frac{(\frac{5}{81})(\frac{1}{256})(-27x^3)}{6} + \dots \right\}$ $= 2 - \frac{1}{4}x; - \frac{1}{32}x^2 - \frac{5}{768}x^3 - \dots$ | <p>2 or <math>(8)^{\frac{1}{3}}</math> (See note ↓) B1</p> <p>Expands <math>(8-3x)^{\frac{1}{3}}</math> to give an un-simplified or simplified M1;</p> <p><math>(8)^{\frac{1}{3}} + \frac{(\frac{1}{3})(8)^{-\frac{2}{3}}(**x)}{1!}</math>;<br/>A correct un-simplified or simplified<br/>{.....} expansion with A1 ✓<br/>candidate's followed through <math>(**x)</math></p> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> <p>Award SC M1 if you see</p> <math display="block">\frac{(\frac{1}{3})(-\frac{2}{3})(8)^{-\frac{5}{3}}(**x)^2}{2!} + \frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})(8)^{-\frac{8}{3}}(**x)^3}{3!}</math> </div> <p>Anything that cancels to <math>2 - \frac{1}{4}x</math>; A1;</p> <p>or <math>2\{1 - \frac{1}{8}x \dots\}</math></p> <p>Simplified <math>-\frac{1}{32}x^2 - \frac{5}{768}x^3</math> A1</p> <p style="text-align: right;"><b>[5]</b></p> |

Attempts using Maclaurin expansion should be escalated up to your team leader.

Be wary of calculator value of  $(7.7)^{\frac{1}{3}} = 1.974680822\dots$

If you see the constant term "2" in a candidate's final binomial expansion, then you can award B1.



| Question Number | Scheme   | Marks  |
|-----------------|--|--|
| 3.              | $\text{Volume} = \pi \int_a^b \left( \frac{1}{2x+1} \right)^2 dx = \pi \int_a^b \frac{1}{(2x+1)^2} dx$ $= \pi \int_a^b (2x+1)^{-2} dx$ $= (\pi) \left[ \frac{(2x+1)^{-1}}{(-1)(2)} \right]_a^b$ $= (\pi) \left[ \frac{-1}{2(2x+1)} \right]_a^b$ $= (\pi) \left[ \left( \frac{-1}{2(2b+1)} \right) - \left( \frac{-1}{2(2a+1)} \right) \right]$ $= \frac{\pi}{2} \left[ \frac{-2a - 1 + 2b + 1}{(2a+1)(2b+1)} \right]$ $= \frac{\pi}{2} \left[ \frac{2(b-a)}{(2a+1)(2b+1)} \right]$ $= \frac{\pi(b-a)}{(2a+1)(2b+1)}$ | <p>Use of <math>V = \pi \int y^2 dx</math>.<br/>Can be implied. Ignore limits.</p> <p>B1</p> <p>Integrating to give <math>\frac{\pm p(2x+1)^{-1}}{-\frac{1}{2}(2x+1)^{-1}}</math></p> <p>M1<br/>A1</p> <p>Substitutes limits of <math>b</math> and <math>a</math> and subtracts the correct way round.</p> <p>dM1</p> <p><math>\frac{\pi(b-a)}{(2a+1)(2b+1)}</math></p> <p>A1 aef</p> <p>[5]</p> <p><b>5 marks</b></p> |

Allow other equivalent forms such as

$$\frac{\pi b - \pi a}{(2a+1)(2b+1)} \text{ or } \frac{-\pi(a-b)}{(2a+1)(2b+1)} \text{ or } \frac{\pi(b-a)}{4ab+2a+2b+1} \text{ or } \frac{\pi b - \pi a}{4ab+2a+2b+1}$$

Note that  $\pi$  is not required for the middle three marks of this question.



| Question Number                       | Scheme  | Marks   |
|---------------------------------------|---|---|
| <p><i>Aliter</i><br/>3.<br/>Way 2</p> | <p>Volume = <math>\pi \int_a^b \left( \frac{1}{2x+1} \right)^2 dx = \pi \int_a^b \frac{1}{(2x+1)^2} dx</math></p> <p><math>= \pi \int_a^b (2x+1)^{-2} dx</math></p> <p>Applying substitution <math>u = 2x+1 \Rightarrow \frac{du}{dx} = 2</math> and changing limits <math>x \rightarrow u</math> so that <math>a \rightarrow 2a+1</math> and <math>b \rightarrow 2b+1</math>, gives</p> <p><math>= (\pi) \int_{2a+1}^{2b+1} \frac{u^{-2}}{2} du</math></p> <p><math>= (\pi) \left[ \frac{u^{-1}}{(-1)(2)} \right]_{2a+1}^{2b+1}</math></p> <p><math>= (\pi) \left[ \frac{-\frac{1}{2}u^{-1}}{2a+1} \right]_{2a+1}^{2b+1}</math></p> <p><math>= (\pi) \left[ \left( \frac{-1}{2(2b+1)} \right) - \left( \frac{-1}{2(2a+1)} \right) \right]</math></p> <p><math>= \frac{\pi}{2} \left[ \frac{-2a-1+2b+1}{(2a+1)(2b+1)} \right]</math></p> <p><math>= \frac{\pi}{2} \left[ \frac{2(b-a)}{(2a+1)(2b+1)} \right]</math></p> <p><math>= \frac{\pi(b-a)}{(2a+1)(2b+1)}</math></p> | <p>Use of <math>V = \pi \int y^2 dx</math>.<br/>Can be implied. Ignore limits.</p> <p>B1</p> <p>Integrating to give <math>\frac{\pm pu^{-1}}{-\frac{1}{2}u^{-1}}</math></p> <p>M1<br/>A1</p> <p>Substitutes limits of <math>2b+1</math> and <math>2a+1</math> and subtracts the correct way round.</p> <p>dM1</p> <p><math>\frac{\pi(b-a)}{(2a+1)(2b+1)}</math></p> <p>A1 aef</p> <p>[5]</p> <p>5 marks</p> |

Note that  $\pi$  is not required for the middle three marks of this question.

Allow other equivalent forms such as

$$\frac{\pi b - \pi a}{(2a+1)(2b+1)} \text{ or } \frac{-\pi(a-b)}{(2a+1)(2b+1)} \text{ or } \frac{\pi(b-a)}{4ab+2a+2b+1} \text{ or } \frac{\pi b - \pi a}{4ab+2a+2b+1}$$



| Question Number | Scheme  | Marks  |
|-----------------|---|--|
| 4. (i)          | $\int \ln\left(\frac{x}{2}\right) dx = \int 1 \cdot \ln\left(\frac{x}{2}\right) dx \Rightarrow \left\{ \begin{array}{l} u = \ln\left(\frac{x}{2}\right) \Rightarrow \frac{du}{dx} = \frac{1}{2} = \frac{1}{x} \\ \frac{dv}{dx} = 1 \Rightarrow v = x \end{array} \right\}$ $\int \ln\left(\frac{x}{2}\right) dx = x \ln\left(\frac{x}{2}\right) - \int x \cdot \frac{1}{x} dx$ $= x \ln\left(\frac{x}{2}\right) - \int 1 dx$ $= x \ln\left(\frac{x}{2}\right) - x + c$  | <p>Use of 'integration by parts' formula in the correct direction. M1<br/>                 Correct expression. A1<br/>                 An attempt to multiply <math>x</math> by a candidate's <math>\frac{a}{x}</math> or <math>\frac{1}{bx}</math> or <math>\frac{1}{x}</math>. <u>dM1</u><br/>                 Correct integration with <math>+ c</math> A1 aef</p> <p style="text-align: right;"><b>[4]</b></p>   |
| (ii)            | $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin^2 x dx$ <p>[NB: <math>\cos 2x = \pm 1 \pm 2\sin^2 x</math> or <math>\sin^2 x = \frac{1}{2}(\pm 1 \pm \cos 2x)</math>]</p> $= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{1 - \cos 2x}{2} dx = \frac{1}{2} \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (1 - \cos 2x) dx$ $= \frac{1}{2} \left[ x - \frac{1}{2} \sin 2x \right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$ $= \frac{1}{2} \left[ \left( \frac{\pi}{2} - \frac{\sin(\pi)}{2} \right) - \left( \frac{\pi}{4} - \frac{\sin\left(\frac{\pi}{2}\right)}{2} \right) \right]$ $= \frac{1}{2} \left[ \left( \frac{\pi}{2} - 0 \right) - \left( \frac{\pi}{4} - \frac{1}{2} \right) \right]$ $= \frac{1}{2} \left( \frac{\pi}{4} + \frac{1}{2} \right) = \frac{\pi}{8} + \frac{1}{4}$ | <p>Consideration of double angle formula for <math>\cos 2x</math> M1</p> <p>Integrating to give <math>\pm ax \pm b \sin 2x</math>; <math>a, b \neq 0</math> dM1<br/>                 Correct result of anything equivalent to <math>\frac{1}{2}x - \frac{1}{4}\sin 2x</math> A1</p> <p>Substitutes limits of <math>\frac{\pi}{2}</math> and <math>\frac{\pi}{4}</math> and subtracts the correct way round. ddM1</p> <p><math>\frac{1}{2}\left(\frac{\pi}{4} + \frac{1}{2}\right)</math> or <math>\frac{\pi}{8} + \frac{1}{4}</math> or <math>\frac{\pi}{8} + \frac{2}{8}</math> A1 aef, <b>cso</b></p> <p>Candidate must collect their <math>\pi</math> term and constant term together for A1<br/>                 No fluked answers, hence <b>cso</b>.</p> <p style="text-align: right;"><b>[5]</b></p> |
| <b>9 marks</b>  |   |  |

Note:  $\int \ln\left(\frac{x}{2}\right) dx = (\text{their } v)\ln\left(\frac{x}{2}\right) - \int (\text{their } v) \cdot (\text{their } \frac{du}{dx}) dx$  for M1 in part (i).

Note  $\frac{\pi}{8} + \frac{1}{4} = 0.64269\dots$

| Question Number                           | Scheme  | Marks  |
|---|---|--|
| <p><i>Aliter</i><br/>4. (i)<br/>Way 2</p> | $\int \ln\left(\frac{x}{2}\right) dx = \int (\ln x - \ln 2) dx = \int \ln x dx - \int \ln 2 dx$ $\int \ln x dx = \int 1 \cdot \ln x dx \Rightarrow \left\{ \begin{array}{l} u = \ln x \Rightarrow \frac{du}{dx} = \frac{1}{x} \\ \frac{dv}{dx} = 1 \Rightarrow v = x \end{array} \right\}$ $\int \ln x dx = x \ln x - \int x \cdot \frac{1}{x} dx$ $= x \ln x - x + c$ $\int \ln 2 dx = x \ln 2 + c$ <p>Hence, <math>\int \ln\left(\frac{x}{2}\right) dx = x \ln x - x - x \ln 2 + c</math></p> | <p>Use of 'integration by parts' formula in the correct direction. M1</p> <p>Correct integration of <math>\ln x</math> with or without <math>+ c</math> A1</p> <p>Correct integration of <math>\ln 2</math> with or without <math>+ c</math> M1</p> <p>Correct integration with <math>+ c</math> A1 aef</p> <p style="text-align: right;"><b>[4]</b></p> |

Note:  $\int \ln x dx = (\text{their } v) \ln x - \int (\text{their } v) \cdot (\text{their } \frac{du}{dx}) dx$  for M1 in part (i).

| Question Number                           | Scheme   | Marks  |
|---|--|--|
| <p><i>Aliter</i><br/>4. (i)<br/>Way 3</p> | $\int \ln\left(\frac{x}{2}\right) dx$ $u = \frac{x}{2} \Rightarrow \frac{du}{dx} = \frac{1}{2}$ $\int \ln\left(\frac{x}{2}\right) dx = 2 \int \ln u \, du$ $\int \ln u \, dx = \int 1 \cdot \ln u \, du$ $\int \ln u \, dx = u \ln u - \int u \cdot \frac{1}{u} \, du$ $= u \ln u - u + c$ $\int \ln\left(\frac{x}{2}\right) dx = 2(u \ln u - u) + c$ <p>Hence, <math>\int \ln\left(\frac{x}{2}\right) dx = x \ln\left(\frac{x}{2}\right) - x + c</math></p> | <p>Applying substitution correctly to give</p> $\int \ln\left(\frac{x}{2}\right) dx = 2 \int \ln u \, du$ <p><b>Decide to award 2<sup>nd</sup> M1 here!</b></p> <p>Use of ‘integration by parts’ formula in the correct direction. M1</p> <p>Correct integration of <math>\ln u</math> with or without <math>+ c</math> A1</p> <p><b>Decide to award 2<sup>nd</sup> M1 here!</b> M1</p> <p>Correct integration with <math>+ c</math> A1 aef</p> <p style="text-align: right;"><b>[4]</b></p> |

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|---|---|---|
| <p><i>Aliter</i></p> <p>4. (ii)<br/>Way 2</p> | $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin^2 x \, dx = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin x \cdot \sin x \, dx \quad \text{and} \quad I = \int \sin^2 x \, dx$ $\left\{ \begin{array}{l} u = \sin x \Rightarrow \frac{du}{dx} = \cos x \\ \frac{dv}{dx} = \sin x \Rightarrow v = -\cos x \end{array} \right\}$ $\therefore I = \left\{ -\sin x \cos x + \int \cos^2 x \, dx \right\}$ $\therefore I = \left\{ -\sin x \cos x + \int (1 - \sin^2 x) \, dx \right\}$ $\int \sin^2 x \, dx = \left\{ -\sin x \cos x + \int 1 \, dx - \int \sin^2 x \, dx \right\}$ $2 \int \sin^2 x \, dx = \left\{ -\sin x \cos x + \int 1 \, dx \right\}$ $2 \int \sin^2 x \, dx = \left\{ -\sin x \cos x + x \right\}$ $\int \sin^2 x \, dx = \left\{ -\frac{1}{2} \sin x \cos x + \frac{x}{2} \right\}$ $\therefore \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin^2 x \, dx = \left[ \left( -\frac{1}{2} \sin\left(\frac{\pi}{2}\right) \cos\left(\frac{\pi}{2}\right) + \frac{\left(\frac{\pi}{2}\right)}{2} \right) - \left( -\frac{1}{2} \sin\left(\frac{\pi}{4}\right) \cos\left(\frac{\pi}{4}\right) + \frac{\left(\frac{\pi}{4}\right)}{2} \right) \right]$ $= \left[ \left( 0 + \frac{\pi}{4} \right) - \left( -\frac{1}{4} + \frac{\pi}{8} \right) \right]$ $= \frac{\pi}{8} + \frac{1}{4}$ | <p>An attempt to use the correct by parts formula. M1</p> <p>For the LHS becoming 2I dM1</p> <p><u>Correct integration</u> A1</p> <p>Substitutes limits of <math>\frac{\pi}{2}</math> and <math>\frac{\pi}{4}</math> and subtracts the correct way round. ddM1</p> <p><math>\frac{1}{2} \left( \frac{\pi}{4} + \frac{1}{2} \right)</math> or <math>\frac{\pi}{8} + \frac{1}{4}</math> or <math>\frac{\pi}{8} + \frac{2}{8}</math> A1 aef<br/>cso [5]</p> <p>Candidate must collect their <math>\pi</math> term and constant term together for A1</p> <p>No fluked answers, hence cso.</p> |

Note  $\frac{\pi}{8} + \frac{1}{4} = 0.64269\dots$



| Question Number | Scheme  | Marks  |
|-----------------|---|--|
| 5. (a)          | $x^3 - 4y^2 = 12xy \quad (\text{eqn } *)$ $x = -8 \Rightarrow -512 - 4y^2 = 12(-8)y$ $-512 - 4y^2 = -96y$ $4y^2 - 96y + 512 = 0$ $y^2 - 24y + 128 = 0$ $(y - 16)(y - 8) = 0$ $y = \frac{24 \pm \sqrt{576 - 4(128)}}{2}$ $y = 16 \text{ or } y = 8.$   | <p>Substitutes <math>x = -8</math> (at least once) into * to obtain a three term quadratic in <math>y</math>.<br/>Condone the loss of <math>= 0</math>.</p> <p>M1</p> <p>An attempt to solve the quadratic in <math>y</math> by either factorising or by the formula or by <b>completing the square</b>.</p> <p>dM1</p> <p>Both <math>y = 16</math> and <math>y = 8</math>.<br/>or <math>(-8, 8)</math> and <math>(-8, 16)</math>.</p> <p>A1</p> <p>[3]</p>  |
| (b)             | $\left\{ \begin{array}{l} \cancel{\frac{dy}{dx}} \\ \cancel{\frac{dy}{dx}} \end{array} \right\} \times 3x^2 - 8y \frac{dy}{dx} = \left( 12y + 12x \frac{dy}{dx} \right)$ $\left\{ \frac{dy}{dx} = \frac{3x^2 - 12y}{12x + 8y} \right\}$ $\text{@ } (-8, 8), \frac{dy}{dx} = \frac{3(64) - 12(8)}{12(-8) + 8(8)} = \frac{96}{-32} = -3,$ $\text{@ } (-8, 16), \frac{dy}{dx} = \frac{3(64) - 12(16)}{12(-8) + 8(16)} = \frac{0}{32} = 0.$ | <p>Differentiates implicitly to include either <math>\pm ky \frac{dy}{dx}</math> or <math>12x \frac{dy}{dx}</math>. Ignore <math>\frac{dy}{dx} = \dots</math></p> <p>M1</p> <p>Correct LHS equation;<br/><u>Correct application of product rule</u></p> <p>A1;<br/>(B1)</p> <p><i>not necessarily required.</i></p> <p>Substitutes <math>x = -8</math> and <i>at least one</i> of their <math>y</math>-values to attempt to find any one of <math>\frac{dy}{dx}</math>.</p> <p>dM1</p> <p>One gradient found.<br/>Both gradients of <u>-3</u> and <u>0</u> <b>correctly</b> found.</p> <p>A1<br/>A1 cso</p> <p>[6]</p> |
| <b>9 marks</b>  |   |  |



| Question Number                           | Scheme  | Marks   |
|---|---|---|
| <p><i>Aliter</i><br/>5. (b)<br/>Way 2</p> | $\left\{ \begin{array}{l} \cancel{\frac{dx}{dy}} \times \\ \cancel{\frac{dy}{dx}} \times \end{array} \right\} 3x^2 \frac{dx}{dy} - 8y; = \left( 12y \frac{dx}{dy} + 12x \right)$<br>$\left\{ \begin{array}{l} dy \\ dx \end{array} = \frac{3x^2 - 12y}{12x + 8y} \right\}$<br><p>@ (-8, 8), <math>\frac{dy}{dx} = \frac{3(64) - 12(8)}{12(-8) + 8(8)} = \frac{96}{-32} = -3,</math></p> <p>@ (-8, 16), <math>\frac{dy}{dx} = \frac{3(64) - 12(16)}{12(-8) + 8(16)} = \frac{0}{32} = 0.</math></p> | <p>Differentiates implicitly to include either <math>\pm kx^2 \frac{dx}{dy}</math> or <math>12y \frac{dx}{dy}</math>. Ignore <math>\frac{dx}{dy} = \dots</math></p> <p>Correct LHS equation</p> <p><u>Correct application of product rule</u></p> <p><i>not necessarily required.</i></p> <p>Substitutes <math>x = -8</math> and <i>at least one</i> of their <math>y</math>-values to attempt to find any one of <math>\frac{dy}{dx}</math> or <math>\frac{dx}{dy}</math>.</p> <p>One gradient found.</p> <p>Both gradients of <u>-3</u> and <u>0</u> <b>correctly</b> found.</p> <p>M1<br/>A1;<br/>(B1)</p> <p>dM1<br/>A1<br/>A1 <b>cs0</b><br/>[6]</p> |





| Question Number | Scheme  | Marks  |
|-----------------|---|--|
| 6. (a)          | $\overline{OA} = \begin{pmatrix} 2 \\ 6 \\ -1 \end{pmatrix} \quad \& \quad \overline{OB} = \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix}$ $\overline{AB} = \overline{OB} - \overline{OA} = \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix} - \begin{pmatrix} 2 \\ 6 \\ -1 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$ <p style="text-align: right;">Finding the difference between <math>\overline{OB}</math> and <math>\overline{OA}</math>.<br/>Correct answer.</p> $l_1: \mathbf{r} = \begin{pmatrix} 2 \\ 6 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} \quad \text{or} \quad \mathbf{r} = \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$   | <p>M1 ±<br/>A1<br/><b>[2]</b></p>  |
| (b)             | $l_1: \mathbf{r} = \begin{pmatrix} 2 \\ 6 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} \quad \text{or} \quad \mathbf{r} = \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$   | <p>An expression of the form (vector) ± λ(vector)<br/>M1<br/><math>\mathbf{r} = \overline{OA} \pm \lambda(\text{their } \overline{AB})</math> or<br/><math>\mathbf{r} = \overline{OB} \pm \lambda(\text{their } \overline{AB})</math> or<br/><math>\mathbf{r} = \overline{OA} \pm \lambda(\text{their } \overline{BA})</math> or<br/><math>\mathbf{r} = \overline{OB} \pm \lambda(\text{their } \overline{BA})</math><br/>(r is needed.)<br/>A1 √<br/>aef<br/><b>[2]</b></p> |
| (c)             | $l_2: \mathbf{r} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \Rightarrow \mathbf{r} = \mu \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ <p><math>\overline{AB} = \mathbf{d}_1 = \mathbf{i} - 2\mathbf{j} + 2\mathbf{k}</math>, <math>\mathbf{d}_2 = \mathbf{i} + 0\mathbf{j} + \mathbf{k}</math> &amp; <math>\theta</math> is angle</p> $\cos \theta = \frac{\overline{AB} \cdot \mathbf{d}_2}{( \overline{AB}  \cdot  \mathbf{d}_2 )} = \frac{\begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}}{(\sqrt{(1)^2 + (-2)^2 + (2)^2} \cdot \sqrt{(1)^2 + (0)^2 + (1)^2})}$ <p style="text-align: right;">← Considers dot product between <math>\mathbf{d}_2</math> and their <math>\overline{AB}</math>.</p> $\cos \theta = \frac{1 + 0 + 2}{\sqrt{(1)^2 + (-2)^2 + (2)^2} \cdot \sqrt{(1)^2 + (0)^2 + (1)^2}}$ $\cos \theta = \frac{3}{3 \cdot \sqrt{2}} \Rightarrow \theta = 45^\circ \text{ or } \frac{\pi}{4} \text{ or awrt } 0.79.$ | <p>M1 √<br/>A1 √<br/>A1 cao<br/><b>[3]</b></p>   |

Correct followed through expression or equation.

$\theta = 45^\circ \text{ or } \frac{\pi}{4} \text{ or awrt } 0.79$

This means that  $\cos \theta$  does not necessarily have to be the subject of the equation. It could be of the form  $3\sqrt{2} \cos \theta = 3$ .

| Question Number   | Scheme   | Marks  |
|---|--|--|
| <p>6. (d)</p> <p><b>Aliter</b><br/>6. (d)<br/>Way 2</p> | <p>If <math>l_1</math> and <math>l_2</math> intersect then: <math>\begin{pmatrix} 2 \\ 6 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} = \mu \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}</math></p> <p><b>i:</b> <math>2 + \lambda = \mu</math> (1)<br/> <b>j:</b> <math>6 - 2\lambda = 0</math> (2)<br/> <b>k:</b> <math>-1 + 2\lambda = \mu</math> (3)</p> <p>(2) yields <math>\lambda = 3</math><br/> Any two yields <math>\lambda = 3, \mu = 5</math></p> <p><math>l_1: \mathbf{r} = \begin{pmatrix} 2 \\ 6 \\ -1 \end{pmatrix} + 3 \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \\ 5 \end{pmatrix}</math> or <math>\mathbf{r} = 5 \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \\ 5 \end{pmatrix}</math></p> <p>If <math>l_1</math> and <math>l_2</math> intersect then: <math>\begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} = \mu \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}</math></p> <p><b>i:</b> <math>3 + \lambda = \mu</math> (1)<br/> <b>j:</b> <math>4 - 2\lambda = 0</math> (2)<br/> <b>k:</b> <math>1 + 2\lambda = \mu</math> (3)</p> <p>(2) yields <math>\lambda = 2</math><br/> Any two yields <math>\lambda = 2, \mu = 5</math></p> <p><math>l_1: \mathbf{r} = \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix} + 2 \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \\ 5 \end{pmatrix}</math> or <math>\mathbf{r} = 5 \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \\ 5 \end{pmatrix}</math></p> | <p><b>Either</b> seeing equation (2) written down correctly with or without any other equation <b>or</b> seeing equations (1) and (3) written down correctly. M1 <math>\sqrt{\phantom{x}}</math></p> <p>Attempt to solve either equation (2) or simultaneously solve any two of the three equations to find ... either one of <math>\lambda</math> or <math>\mu</math> correct. dM1</p> <p><math>\begin{pmatrix} 5 \\ 0 \\ 5 \end{pmatrix}</math> or <math>5\mathbf{i} + 5\mathbf{k}</math> A1</p> <p>Fully correct solution &amp; no incorrect values of <math>\lambda</math> or <math>\mu</math> seen earlier. A1 <b>cso</b></p> <p><b>Either</b> seeing equation (2) written down correctly with or without any other equation <b>or</b> seeing equations (1) and (3) written down correctly. M1 <math>\sqrt{\phantom{x}}</math></p> <p>Attempt to solve either equation (2) or simultaneously solve any two of the three equations to find ... either one of <math>\lambda</math> or <math>\mu</math> correct. dM1</p> <p><math>\begin{pmatrix} 5 \\ 0 \\ 5 \end{pmatrix}</math> or <math>5\mathbf{i} + 5\mathbf{k}</math> A1</p> <p>Fully correct solution &amp; no incorrect values of <math>\lambda</math> or <math>\mu</math> seen earlier. A1 <b>cso</b></p> <p>[4]</p> <p>[4]</p> <p><b>11 marks</b></p> |

**Note:** Be careful!  $\lambda$  and  $\mu$  are not defined in the question, so a candidate could interchange these or use different scalar parameters.

| Question Number                           | Scheme  | Marks  |
|---|---|--|
| <p><b>Aliter</b><br/>6. (d)<br/>Way 3</p> | <p>If <math>l_1</math> and <math>l_2</math> intersect then: <math>\begin{pmatrix} 2 \\ 6 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 2 \\ -2 \end{pmatrix} = \mu \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}</math></p> <p><b>i:</b> <math>2 - \lambda = \mu</math> (1)<br/> <b>j:</b> <math>6 + 2\lambda = 0</math> (2)<br/> <b>k:</b> <math>-1 - 2\lambda = \mu</math> (3)</p> <p>(2) yields <math>\lambda = -3</math><br/> Any two yields <math>\lambda = -3, \mu = 5</math></p> <p><math>l_1: \mathbf{r} = \begin{pmatrix} 2 \\ 6 \\ -1 \end{pmatrix} - 3 \begin{pmatrix} -1 \\ 2 \\ -2 \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \\ 5 \end{pmatrix}</math> or <math>\mathbf{r} = 5 \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \\ 5 \end{pmatrix}</math></p> | <p><b>Either</b> seeing equation (2) written down correctly with or without any other equation <b>or</b> seeing equations (1) and (3) written down correctly. M1 <math>\sqrt{\phantom{x}}</math></p> <p>Attempt to solve either equation (2) or simultaneously solve any two of the three equations to find ... either one of <math>\lambda</math> or <math>\mu</math> correct. dM1</p> <p>A1</p> <p><math>\begin{pmatrix} 5 \\ 0 \\ 5 \end{pmatrix}</math> or <math>5\mathbf{i} + 5\mathbf{k}</math> A1 <b>cs0</b></p> <p>Fully correct solution &amp; no incorrect values of <math>\lambda</math> or <math>\mu</math> seen earlier.</p> <p>[4]</p> |
| <p><b>Aliter</b><br/>6. (d)<br/>Way 4</p> | <p>If <math>l_1</math> and <math>l_2</math> intersect then: <math>\begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 2 \\ -2 \end{pmatrix} = \mu \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}</math></p> <p><b>i:</b> <math>3 - \lambda = \mu</math> (1)<br/> <b>j:</b> <math>4 + 2\lambda = 0</math> (2)<br/> <b>k:</b> <math>1 - 2\lambda = \mu</math> (3)</p> <p>(2) yields <math>\lambda = -2</math><br/> Any two yields <math>\lambda = -2, \mu = 5</math></p> <p><math>l_1: \mathbf{r} = \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix} - 2 \begin{pmatrix} -1 \\ 2 \\ -2 \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \\ 5 \end{pmatrix}</math> or <math>\mathbf{r} = 5 \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \\ 5 \end{pmatrix}</math></p>    | <p><b>Either</b> seeing equation (2) written down correctly with or without any other equation <b>or</b> seeing equations (1) and (3) written down correctly. M1 <math>\sqrt{\phantom{x}}</math></p> <p>Attempt to solve either equation (2) or simultaneously solve any two of the three equations to find ... either one of <math>\lambda</math> or <math>\mu</math> correct. dM1</p> <p>A1</p> <p><math>\begin{pmatrix} 5 \\ 0 \\ 5 \end{pmatrix}</math> or <math>5\mathbf{i} + 5\mathbf{k}</math> A1 <b>cs0</b></p> <p>Fully correct solution &amp; no incorrect values of <math>\lambda</math> or <math>\mu</math> seen earlier.</p> <p>[4]</p> |
|   |   | <p><b>11 marks</b></p>   |



| Question Number | Scheme  | Marks   |
|-----------------|---|---|
| 7. (a)          | $\left[ x = \ln(t+2), y = \frac{1}{t+1} \right], \Rightarrow \frac{dx}{dt} = \frac{1}{t+2}$ <p style="text-align: right;">Must state <math>\frac{dx}{dt} = \frac{1}{t+2}</math></p> $\text{Area}(R) = \int_{\ln 2}^{\ln 4} \frac{1}{t+1} dx; = \int_0^2 \left( \frac{1}{t+1} \right) \left( \frac{1}{t+2} \right) dt$ <p style="text-align: right;">Area = <math>\int \frac{1}{t+1} dx</math>.<br/>Ignore limits.</p> $\int \left( \frac{1}{t+1} \right) \times \left( \frac{1}{t+2} \right) dt \text{ . Ignore limits.}$ <p>Changing limits, when:<br/> <math>x = \ln 2 \Rightarrow \ln 2 = \ln(t+2) \Rightarrow 2 = t+2 \Rightarrow t = 0</math><br/> <math>x = \ln 4 \Rightarrow \ln 4 = \ln(t+2) \Rightarrow 4 = t+2 \Rightarrow t = 2</math></p> <p style="text-align: right;">changes limits <math>x \rightarrow t</math><br/>so that <math>\ln 2 \rightarrow 0</math> and <math>\ln 4 \rightarrow 2</math></p> <p>Hence, <math>\text{Area}(R) = \int_0^2 \frac{1}{(t+1)(t+2)} dt</math></p>  | <p>B1</p> <p>M1;</p> <p>A1 <b>AG</b></p> <p>B1</p> <p style="text-align: right;"><b>[4]</b></p>   |
| (b)             | $\left( \frac{1}{(t+1)(t+2)} \right) = \frac{A}{(t+1)} + \frac{B}{(t+2)}$ <p style="text-align: right;"><math>\frac{A}{(t+1)} + \frac{B}{(t+2)}</math> with <math>A</math> and <math>B</math> found</p> $1 = A(t+2) + B(t+1)$ <p>Let <math>t = -1, 1 = A(1) \Rightarrow \underline{A = 1}</math></p> <p>Let <math>t = -2, 1 = B(-1) \Rightarrow \underline{B = -1}</math></p> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> <p>Finds both <math>A</math> and <math>B</math> correctly.<br/>Can be implied.<br/>(See note below)</p> </div> $\int_0^2 \frac{1}{(t+1)(t+2)} dt = \int_0^2 \frac{1}{(t+1)} - \frac{1}{(t+2)} dt$ <p style="text-align: right;">Either <math>\pm a \ln(t+1)</math> or <math>\pm b \ln(t+2)</math><br/>Both ln terms correctly ft.</p> $= [\ln(t+1) - \ln(t+2)]_0^2$ <p style="text-align: right;">Substitutes <b>both</b> limits of 2 and 0<br/>and subtracts the correct way round.</p> $= (\ln 3 - \ln 4) - (\ln 1 - \ln 2)$ <p style="text-align: right;"><math>\ln 3 - \ln 4 + \ln 2</math> or <math>\ln\left(\frac{3}{4}\right) - \ln\left(\frac{1}{2}\right)</math><br/>or <math>\ln 3 - \ln 2</math> or <math>\ln\left(\frac{3}{2}\right)</math><br/>(must deal with ln 1)</p> $= \ln 3 - \ln 4 + \ln 2 = \ln 3 - \ln 2 = \ln\left(\frac{3}{2}\right)$ | <p>M1</p> <p>A1</p> <p>dM1</p> <p>A1 <math>\sqrt{\quad}</math></p> <p>ddM1</p> <p>A1 aef isw</p> <p style="text-align: right;"><b>[6]</b></p> |

Takes out brackets.

Writing down  $\frac{1}{(t+1)(t+2)} = \frac{1}{(t+1)} + \frac{1}{(t+2)}$  means first M1A0 in (b).

Writing down  $\frac{1}{(t+1)(t+2)} = \frac{1}{(t+1)} - \frac{1}{(t+2)}$  means first M1A1 in (b).



| Question Number           | Scheme   | Marks                              |
|---------------------------|--|------------------------------------|
| 7. (c)                    | $x = \ln(t+2), \quad y = \frac{1}{t+1}$ $e^x = t+2 \Rightarrow t = e^x - 2$ $y = \frac{1}{e^x - 2 + 1} \Rightarrow y = \frac{1}{e^x - 1}$  | M1<br>A1<br>dM1<br>A1<br>[4]       |
| Aliter<br>7. (c)<br>Way 2 | $t+1 = \frac{1}{y} \Rightarrow t = \frac{1}{y} - 1 \text{ or } t = \frac{1-y}{y}$ $y(t+1) = 1 \Rightarrow yt + y = 1 \Rightarrow yt = 1 - y \Rightarrow t = \frac{1-y}{y}$ $x = \ln\left(\frac{1}{y} - 1 + 2\right) \text{ or } x = \ln\left(\frac{1-y}{y} + 2\right)$ $x = \ln\left(\frac{1}{y} + 1\right)$ $e^x = \frac{1}{y} + 1$ $e^x - 1 = \frac{1}{y}$ $y = \frac{1}{e^x - 1}$ | M1<br>A1<br>dM1<br>A1<br>[4]       |
| (d)                       | Domain : $x > 0$   | $x > 0$ or just $> 0$<br>B1<br>[1] |
| <b>15 marks</b>           |  |                                    |

| Question Number                           | Scheme  | Marks   |
|---|---|---|
| <p><i>Aliter</i><br/>7. (c)<br/>Way 3</p> | $e^x = t + 2 \Rightarrow t + 1 = e^x - 1$<br>$y = \frac{1}{t + 1} \Rightarrow y = \frac{1}{e^x - 1}$  | <p>Attempt to make <math>t + 1 = \dots</math> the subject<br/>giving <math>t + 1 = e^x - 1</math> M1<br/>A1</p> <p>Eliminates <math>t</math> by substituting in <math>y</math><br/>giving <math>y = \frac{1}{e^x - 1}</math> dM1<br/>A1</p> <p style="text-align: right;"><b>[4]</b></p>  |
| <p><i>Aliter</i><br/>7. (c)<br/>Way 4</p> | $t + 1 = \frac{1}{y} \Rightarrow t + 2 = \frac{1}{y} + 1 \text{ or } t + 2 = \frac{1 + y}{y}$<br>$x = \ln\left(\frac{1}{y} + 1\right) \text{ or } x = \ln\left(\frac{1 + y}{y}\right)$<br>$x = \ln\left(\frac{1}{y} + 1\right)$<br>$e^x = \frac{1}{y} + 1 \Rightarrow e^x - 1 = \frac{1}{y}$<br>$y = \frac{1}{e^x - 1}$ | <div style="border: 1px solid black; padding: 5px; margin-bottom: 10px;"> <p>Attempt to make <math>t + 2 = \dots</math> the subject<br/>Either <math>t + 2 = \frac{1}{y} + 1</math> or <math>t + 2 = \frac{1 + y}{y}</math></p> </div> <p>M1<br/>A1</p> <p>Eliminates <math>t</math> by substituting in <math>x</math> dM1</p> <p>giving <math>y = \frac{1}{e^x - 1}</math> A1</p> <p style="text-align: right;"><b>[4]</b></p> |



| Question Number                      | Scheme   | Marks   |
|--------------------------------------|--|---|
| 8. (a)                               | $\frac{dV}{dt} = 1600 - c\sqrt{h} \quad \text{or} \quad \frac{dV}{dt} = 1600 - k\sqrt{h},$ $(V = 4000h \Rightarrow) \frac{dV}{dh} = 4000$ $\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt} = \frac{\frac{dV}{dt}}{\frac{dV}{dh}}$   | <p>Either of these statements M1</p> <p><math>\frac{dV}{dh} = 4000</math> or <math>\frac{dh}{dV} = \frac{1}{4000}</math> M1</p>   |
|                                      | <p>Either, <math>\frac{dh}{dt} = \frac{1600 - c\sqrt{h}}{4000} = \frac{1600}{4000} - \frac{c\sqrt{h}}{4000} = 0.4 - k\sqrt{h}</math></p> <p>or <math>\frac{dh}{dt} = \frac{1600 - k\sqrt{h}}{4000} = \frac{1600}{4000} - \frac{k\sqrt{h}}{4000} = 0.4 - k\sqrt{h}</math></p>       | <div style="border: 1px solid black; padding: 10px; width: fit-content; margin: auto;"> <p>Convincing proof of <math>\frac{dh}{dt}</math></p> </div> <p>A1 AG</p> <p style="text-align: right;">[3]</p>                                   |
| (b)                                  | <p>When <math>h = 25</math> water <i>leaks out such that</i> <math>\frac{dV}{dt} = 400</math></p> $400 = c\sqrt{h} \Rightarrow 400 = c\sqrt{25} \Rightarrow 400 = c(5) \Rightarrow c = 80$ <p>From above; <math>k = \frac{c}{4000} = \frac{80}{4000} = 0.02</math> as required</p> | <p>Proof that <math>k = 0.02</math> B1 AG</p> <p style="text-align: right;">[1]</p>   |
| <i>Aliter</i><br>(b)<br><b>Way 2</b> | $400 = 4000k\sqrt{h}$ $\Rightarrow 400 = 4000k\sqrt{25}$ $\Rightarrow 400 = k(20000) \Rightarrow k = \frac{400}{20000} = 0.02$   | <p>Using 400, 4000 and <math>h = 25</math><br/>or <math>\sqrt{h} = 5</math>. Proof that <math>k = 0.02</math> B1 AG</p> <p style="text-align: right;">[1]</p>   |
| (c)                                  | $\frac{dh}{dt} = 0.4 - k\sqrt{h} \Rightarrow \int \frac{dh}{0.4 - k\sqrt{h}} = \int dt$ $\therefore \text{time required} = \int_0^{100} \frac{1}{0.4 - 0.02\sqrt{h}} dh \quad \frac{\div 0.02}{\div 0.02}$ $\text{time required} = \int_0^{100} \frac{50}{20 - \sqrt{h}} dh$       | <p><i>Separates the variables</i> with <math>\int \frac{dh}{0.4 - k\sqrt{h}}</math> and <math>\int dt</math> on either side with integral signs not necessary. M1 oe</p> <p>Correct proof A1 AG</p> <p style="text-align: right;">[2]</p> |

| Question Number | Scheme   | Marks  |
|-----------------|--|--|
| 8. (d)          | $\int_0^{100} \frac{50}{20-\sqrt{h}} dh \quad \text{with substitution } h = (20-x)^2$ $\frac{dh}{dx} = 2(20-x)(-1) \quad \text{or} \quad \frac{dh}{dx} = -2(20-x)$ $h = (20-x)^2 \Rightarrow \sqrt{h} = 20-x \Rightarrow x = 20-\sqrt{h}$ $\int \frac{50}{20-\sqrt{h}} dh = \int \frac{50}{x} \cdot -2(20-x) dx$ $= 100 \int \frac{x-20}{x} dx$ $= 100 \int \left(1 - \frac{20}{x}\right) dx$ $= 100(x - 20 \ln x) (+c)$ <p>change limits: when <math>h=0</math> then <math>x=20</math><br/>and when <math>h=100</math> then <math>x=10</math></p> $\int_0^{100} \frac{50}{20-\sqrt{h}} dh = [100x - 2000 \ln x]_{20}^{10}$ <p>or</p> $\int_0^{100} \frac{50}{20-\sqrt{h}} dh = [100(20-\sqrt{h}) - 2000 \ln(20-\sqrt{h})]_0^{100}$ $= (1000 - 2000 \ln 10) - (2000 - 2000 \ln 20)$ $= 2000 \ln 20 - 2000 \ln 10 - 1000$ $= 2000 \ln 2 - 1000$ | <p>Correct <math>\frac{dh}{dx}</math> B1 aef</p> <p><math>\pm \lambda \int \frac{20-x}{x} dx</math> or<br/><math>\pm \lambda \int \frac{20-x}{20-(20-x)} dx</math><br/>where <math>\lambda</math> is a constant M1</p> <p><math>\pm \alpha x \pm \beta \ln x ; \alpha, \beta \neq 0</math> M1<br/><math>100x - 2000 \ln x</math> A1</p> <p>Correct use of limits, ie. putting them in the correct way round<br/>Either <math>x=10</math> and <math>x=20</math><br/>or <math>h=100</math> and <math>h=0</math> ddM1</p> <p>Combining logs to give...<br/><math>2000 \ln 2 - 1000</math><br/>or <math>-2000 \ln(\frac{1}{2}) - 1000</math> A1 aef</p> <p>[6]</p> |
| (e)             | <p>Time required = <math>2000 \ln 2 - 1000 = 386.2943611... \text{ sec}</math></p> <p>= 386 seconds (nearest second)</p> <p>= 6 minutes and 26 seconds (nearest second)</p>  | <p><u>6 minutes, 26 seconds</u> B1</p> <p>[1]</p> <p><b>13 marks</b></p>   |