Centre No.						Paper Reference	Surname	Initial(s)
Candidate No.						6 6 6 6 / 0 1	Signature	

Paper Reference(s)

6666/01

Edexcel GCE

Core Mathematics C4

Advanced

Tuesday 22 January 2008 – Afternoon

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
1	
2	
3	
4	
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6	
7	
8	
Total	

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Green)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initial(s) and signature.
Check that you have the correct question paper.
You must write your answer for each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.
There are 24 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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Turn over

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1.

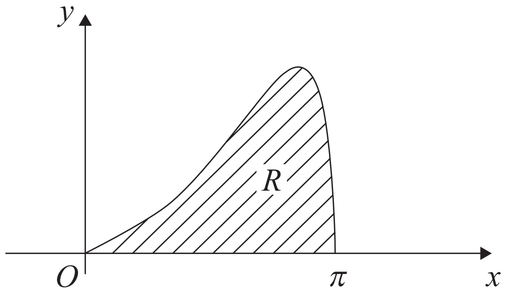

Figure 1

The curve shown in Figure 1 has equation $y = e^x \sqrt{(\sin x)}$, $0 \leqslant x \leqslant \pi$. The finite region R bounded by the curve and the x-axis is shown shaded in Figure 1.

(a) Complete the table below with the values of y corresponding to $x = \frac{\pi}{4}$ and $\frac{\pi}{2}$, giving your answers to 5 decimal places.

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
y	0			8.87207	0

(2)

(b) Use the trapezium rule, with all the values in the completed table, to obtain an estimate for the area of the region R. Give your answer to 4 decimal places.

(4)

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January 2008
6666 Core Mathematics C4
Mark Scheme

Question Number	Scheme		Marks
1. (a)	x: 0, $\frac{\pi}{4}$, $\frac{\pi}{2}$, $\frac{3\pi}{4}$, π; y: 0, 1.844321332…, 4.810477381…, 8.87207, 0		
		awrt 1.84432	B1
		awrt 4.81048 or 4.81047	B1
			[2]
(b) **Way 1**	Area $\approx \frac{1}{2}\times\frac{\pi}{4}\ ;\times\left\{\underline{0+2\left(1.84432+4.81048+8.87207\right)+0}\right\}$ (0 can be implied)	Outside brackets awrt 0.39 or $\frac{1}{2}\times$awrt 0.79; $\frac{1}{2}\times\frac{\pi}{4}$ or $\frac{\pi}{8}$	B1
		<u>For structure of trapezium rule</u> $\underline{\{.............\}}$;	<u>M1</u> $\sqrt{}$
		Correct expression <u>inside brackets</u> which all must be multiplied by their "outside constant".	<u>A1</u> $\sqrt{}$
	$=\frac{\pi}{8}\times 31.05374... = 12.19477518... = \underline{12.1948}$ (4dp)	<u>12.1948</u>	A1 **cao**
			[4]
Aliter (b) **Way 2**	Area $\approx \frac{\pi}{4}\times\left\{\frac{0+1.84432}{2}+\frac{1.84432+4.81048}{2}+\frac{4.81048+8.87207}{2}+\frac{8.87207+0}{2}\right\}$	$\frac{\pi}{4}$ (or awrt 0.79) and a divisor of 2 on all terms inside brackets.	B1
	which is equivalent to:	One of first and last ordinates, two of the middle ordinates inside brackets ignoring the 2.	<u>M1</u> $\sqrt{}$
	Area $\approx \frac{1}{2}\times\frac{\pi}{4}\ ;\times\left\{\underline{0+2\left(1.84432+4.81048+8.87207\right)+0}\right\}$	Correct expression inside brackets if $\frac{1}{2}$ was to be factorised out.	<u>A1</u> $\sqrt{}$
	$=\frac{\pi}{4}\times 15.52687... = 12.19477518... = \underline{12.1948}$ (4dp)	<u>12.1948</u>	A1 **cao**
			[4]
			6 marks

Note an expression like Area $\approx \frac{1}{2}\times\frac{\pi}{4}+2\left(1.84432+4.81048+8.87207\right)$ would score B1M1A0A0

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2. (a) Use the binomial theorem to expand

$$(8-3x)^{\frac{1}{3}}, \qquad |x| < \tfrac{8}{3},$$

in ascending powers of x, up to and including the term in x^3, giving each term as a simplified fraction.

(5)

(b) Use your expansion, with a suitable value of x, to obtain an approximation to $\sqrt[3]{(7.7)}$. Give your answer to 7 decimal places.

(2)

Question Number	Scheme		Marks
	** represents a constant (which must be consistent for first accuracy mark)		
2. (a)	$(8-3x)^{\frac{1}{3}} = \underline{(8)^{\frac{1}{3}}}\left(1-\frac{3x}{8}\right)^{\frac{1}{3}} = \underline{2}\left(1-\frac{3x}{8}\right)^{\frac{1}{3}}$	Takes 8 outside the bracket to give any of $\underline{(8)^{\frac{1}{3}}}$ or $\underline{2}$.	B1
	$= 2\left\{1+(\tfrac{1}{3})(**x);+\frac{(\frac{1}{3})(-\frac{2}{3})}{2!}(**x)^2+\frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})}{3!}(**x)^3+\ldots\right\}$ with $**\neq 1$	Expands $(1+**x)^{\frac{1}{3}}$ to give a simplified or an un-simplified $1+(\frac{1}{3})(**x)$;	M1;
		A correct simplified or an un-simplified $\{\ldots\ldots\ldots\}$ expansion with candidate's followed through $(**x)$	A1 $\sqrt{}$
	$= 2\left\{1+(\tfrac{1}{3})(-\tfrac{3x}{8})+\frac{(\frac{1}{3})(-\frac{2}{3})}{2!}(-\tfrac{3x}{8})^2+\frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})}{3!}(-\tfrac{3x}{8})^3+\ldots\right\}$	Award SC M1 if you see $\frac{(\frac{1}{3})(-\frac{2}{3})}{2!}(**x)^2+\frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})}{3!}(**x)^3$	
	$= 2\left\{1-\tfrac{1}{8}x;-\tfrac{1}{64}x^2-\tfrac{5}{1536}x^3-\ldots\right\}$	Either $2\{1-\frac{1}{8}x\ \ldots\ldots\}$ or anything that cancels to $2-\frac{1}{4}x$;	A1;
	$= 2-\frac{1}{4}x;-\frac{1}{32}x^2-\frac{5}{768}x^3-\ldots$	Simplified $-\frac{1}{32}x^2-\frac{5}{768}x^3$	A1
			[5]
(b)	$(7.7)^{\frac{1}{3}} \approx 2-\frac{1}{4}(0.1)-\frac{1}{32}(0.1)^2-\frac{5}{768}(0.1)^3-\ldots$	***Attempt to substitute*** $x=0.1$ into a candidate's binomial expansion.	M1
	$= 2 - 0.025 - 0.0003125 - 0.0000065104166\ldots$		
	$= 1.97468099\ldots$	awrt 1.9746810	A1
			[2]
			7 marks

You would award B1M1A0 for

$$=2\left\{1+(\tfrac{1}{3})(-\tfrac{3x}{8})+\frac{(\frac{1}{3})(-\frac{2}{3})}{2!}(-\tfrac{3x}{8})^2+\frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})}{3!}(-3x)^3+\ldots\right\}$$

because ** is not consistent.

If you see the constant term "2" in a candidate's final binomial expansion, then you can award B1.

Be wary of calculator value of $(7.7)^{\frac{1}{3}} = 1.974680822\ldots$

Question Number	Scheme		Marks
Aliter **2.** (a) **Way 2**	$(8-3x)^{\frac{1}{3}}$		
		2 or $(8)^{\frac{1}{3}}$ (**See note** ↓)	B1
	$= \left\{ \begin{array}{l} (8)^{\frac{1}{3}} + (\tfrac{1}{3})(8)^{-\frac{2}{3}}(**x); + \frac{(\frac{1}{3})(-\frac{2}{3})}{2!}(8)^{-\frac{5}{3}}(**x)^2 \\ \quad + \frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})}{3!}(8)^{-\frac{8}{3}}(**x)^3 + \ldots \end{array} \right\}$	Expands $(8-3x)^{\frac{1}{3}}$ to give an un-simplified or simplified $(8)^{\frac{1}{3}} + (\tfrac{1}{3})(8)^{-\frac{2}{3}}(**x)$;	M1;
	with $** \neq 1$	A correct un-simplified or simplified $\{\ldots\ldots\ldots\}$ expansion with candidate's followed through $(**x)$	A1 $\sqrt{}$
	$= \left\{ \begin{array}{l} (8)^{\frac{1}{3}} + (\tfrac{1}{3})(8)^{-\frac{2}{3}}(-3x); + \frac{(\frac{1}{3})(-\frac{2}{3})}{2!}(8)^{-\frac{5}{3}}(-3x)^2 \\ \quad + \frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})}{3!}(8)^{-\frac{8}{3}}(-3x)^3 + \ldots \end{array} \right\}$	Award SC M1 if you see $\frac{(\frac{1}{3})(-\frac{2}{3})}{2!}(8)^{-\frac{5}{3}}(**x)^2 + \frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})}{3!}(8)^{-\frac{8}{3}}(**x)^3$	
	$= \left\{2 + (\tfrac{1}{3})(\tfrac{1}{4})(-3x) + (-\tfrac{1}{9})(\tfrac{1}{32})(9x^2) + (\tfrac{5}{81})(\tfrac{1}{256})(-27x^3) + \ldots\right\}$		
	$= 2 - \frac{1}{4}x; - \frac{1}{32}x^2 - \frac{5}{768}x^3 - \ldots$	Anything that cancels to $2 - \frac{1}{4}x$; or $2\{1 - \frac{1}{8}x \ldots\ldots\}$	A1;
		Simplified $-\frac{1}{32}x^2 - \frac{5}{768}x^3$	A1
			[5]

Attempts using Maclaurin expansion should be escalated up to your team leader.

Be wary of calculator value of $(7.7)^{\frac{1}{3}} = 1.974680822\ldots$

If you see the constant term "2" in a candidate's final binomial expansion, then you can award B1.

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3.

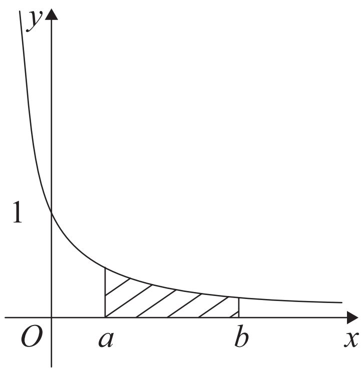

Figure 2

The curve shown in Figure 2 has equation $y = \frac{1}{(2x+1)}$. The finite region bounded by the curve, the x-axis and the lines $x = a$ and $x = b$ is shown shaded in Figure 2. This region is rotated through 360° about the x-axis to generate a solid of revolution.

Find the volume of the solid generated. Express your answer as a single simplified fraction, in terms of a and b.

(5)

Question Number	Scheme		Marks
3.	Volume $= \pi \int_a^b \left(\frac{1}{2x+1}\right)^2 \mathrm{d}x = \pi \int_a^b \frac{1}{(2x+1)^2} \mathrm{d}x$	Use of $V = \pi \int y^2 \mathrm{d}x$. Can be implied. Ignore limits.	B1
	$= \pi \int_a^b (2x+1)^{-2} \mathrm{d}x$		
	$= (\pi)\left[\frac{(2x+1)^{-1}}{(-1)(2)}\right]_a^b$		
	$= (\pi)\left[-\frac{1}{2}(2x+1)^{-1}\right]_a^b$	Integrating to give $\pm p(2x+1)^{-1}$	M1
		$-\frac{1}{2}(2x+1)^{-1}$	A1
	$= (\pi)\left[\left(\frac{-1}{2(2b+1)}\right) - \left(\frac{-1}{2(2a+1)}\right)\right]$	Substitutes limits of b and a and subtracts the correct way round.	dM1
	$= \frac{\pi}{2}\left[\frac{-2a-1+2b+1}{(2a+1)(2b+1)}\right]$		
	$= \frac{\pi}{2}\left[\frac{2(b-a)}{(2a+1)(2b+1)}\right]$		
	$= \frac{\pi(b-a)}{(2a+1)(2b+1)}$	$\frac{\pi(b-a)}{(2a+1)(2b+1)}$	A1 aef
			[5]
			5 marks

Allow other equivalent forms such as

$$\frac{\pi b - \pi a}{(2a+1)(2b+1)} \text{ or } \frac{-\pi(a-b)}{(2a+1)(2b+1)} \text{ or } \frac{\pi(b-a)}{4ab+2a+2b+1} \text{ or } \frac{\pi b - \pi a}{4ab+2a+2b+1}.$$

Note that π is not required for the middle three marks of this question.

Question Number	Scheme		Marks
***Aliter* 3. Way 2**	Volume $= \pi \int_a^b \left(\frac{1}{2x+1}\right)^2 \mathrm{d}x = \pi \int_a^b \frac{1}{(2x+1)^2} \mathrm{d}x$	Use of $V = \pi \int y^2 \, \mathrm{d}x$. Can be implied. Ignore limits.	B1
	$= \pi \int_a^b (2x+1)^{-2} \mathrm{d}x$		
	Applying substitution $u = 2x+1 \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = 2$ and changing limits $x \to u$ so that $a \to 2a+1$ and $b \to 2b+1$, gives		
	$= (\pi) \int_{2a+1}^{2b+1} \frac{u^{-2}}{2} \mathrm{d}u$		
	$= (\pi) \left[\frac{u^{-1}}{(-1)(2)} \right]_{2a+1}^{2b+1}$		
	$= (\pi) \left[-\frac{1}{2} u^{-1} \right]_{2a+1}^{2b+1}$	Integrating to give $\pm p u^{-1}$ $-\frac{1}{2}u^{-1}$	M1 A1
	$= (\pi) \left[\left(\frac{-1}{2(2b+1)}\right) - \left(\frac{-1}{2(2a+1)}\right) \right]$	Substitutes limits of $2b+1$ and $2a+1$ and subtracts the correct way round.	dM1
	$= \frac{\pi}{2} \left[\frac{-2a-1+2b+1}{(2a+1)(2b+1)} \right]$		
	$= \frac{\pi}{2} \left[\frac{2(b-a)}{(2a+1)(2b+1)} \right]$		
	$= \frac{\pi(b-a)}{(2a+1)(2b+1)}$	$\frac{\pi(b-a)}{(2a+1)(2b+1)}$	A1 aef
			[5]
			5 marks

Note that π is not required for the middle three marks of this question.

Allow other equivalent forms such as

$$\frac{\pi b - \pi a}{(2a+1)(2b+1)} \text{ or } \frac{-\pi(a-b)}{(2a+1)(2b+1)} \text{ or } \frac{\pi(b-a)}{4ab+2a+2b+1} \text{ or } \frac{\pi b - \pi a}{4ab+2a+2b+1}.$$

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4. (i) Find $\int \ln(\frac{x}{2})\,dx$.

(4)

(ii) Find the exact value of $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin^2 x\,dx$.

(5)

Question Number	Scheme		Marks
4. (i)	$\int \ln\left(\frac{x}{2}\right) dx = \int 1.\ln\left(\frac{x}{2}\right) dx \Rightarrow \left\{ \begin{array}{ll} u = \ln\left(\frac{x}{2}\right) & \Rightarrow \frac{du}{dx} = \frac{\frac{1}{2}}{\frac{x}{2}} = \frac{1}{x} \\ \frac{dv}{dx} = 1 & \Rightarrow v = x \end{array} \right\}$		
	$\int \ln\left(\frac{x}{2}\right) dx = x\ln\left(\frac{x}{2}\right) - \int x.\frac{1}{x}\, dx$	Use of 'integration by parts' formula in the correct direction. Correct expression.	M1 A1
	$= x\ln\left(\frac{x}{2}\right) - \int \underline{1}\, dx$	An attempt to multiply x by a candidate's $\frac{a}{x}$ or $\frac{1}{bx}$ or $\frac{1}{x}$.	<u>dM1</u>
	$= x\ln\left(\frac{x}{2}\right) - x + c$	Correct integration with + c	A1 aef **[4]**
(ii)	$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin^2 x\, dx$		
	$\left[\text{NB}: \underline{\cos 2x = \pm 1 \pm 2\sin^2 x} \text{ or } \underline{\sin^2 x = \frac{1}{2}(\pm 1 \pm \cos 2x)}\right]$	Consideration of double angle formula for $\cos 2x$	M1
	$= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{1 - \cos 2x}{2} dx = \frac{1}{2}\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (\underline{1 - \cos 2x})\, dx$		
	$= \frac{1}{2}\left[\underline{x - \frac{1}{2}\sin 2x}\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$	<u>Integrating to give</u> <u>$\pm ax \pm b\sin 2x$</u>; $a, b \neq 0$ Correct result of anything equivalent to $\frac{1}{2}x - \frac{1}{4}\sin 2x$	dM1 A1
	$= \frac{1}{2}\left[\left(\frac{\pi}{2} - \frac{\sin(\pi)}{2}\right) - \left(\frac{\pi}{4} - \frac{\sin(\frac{\pi}{2})}{2}\right)\right]$	Substitutes limits of $\frac{\pi}{2}$ and $\frac{\pi}{4}$ and subtracts the correct way round.	ddM1
	$= \frac{1}{2}\left[(\frac{\pi}{2} - 0) - (\frac{\pi}{4} - \frac{1}{2})\right]$		
	$= \frac{1}{2}\left(\frac{\pi}{4} + \frac{1}{2}\right) = \frac{\pi}{8} + \frac{1}{4}$	$\underline{\frac{1}{2}\left(\frac{\pi}{4} + \frac{1}{2}\right)}$ or $\underline{\frac{\pi}{8} + \frac{1}{4}}$ or $\underline{\frac{\pi}{8} + \frac{2}{8}}$ Candidate must collect their π term and constant term together for A1 No fluked answers, hence **cso**.	A1 **aef**, **cso** **[5]**
			9 marks

Note: $\int \ln\left(\frac{x}{2}\right) dx = (\text{their } v)\ln\left(\frac{x}{2}\right) - \int (\text{their } v).(\text{their } \frac{du}{dx})\, dx$ for M1 in part (i).

Note $\frac{\pi}{8} + \frac{1}{4} = 0.64269...$

Question Number	Scheme		Marks
Aliter **4.** (i) **Way 2**	$\int \ln\left(\frac{x}{2}\right)\,dx = \int (\ln x - \ln 2)\,dx = \int \ln x\,dx - \int \ln 2\,dx$		
	$\int \ln x\,dx = \int 1.\ln x\,dx \Rightarrow \left\{\begin{array}{ll} u = \ln x & \Rightarrow \frac{du}{dx} = \frac{1}{x} \\ \frac{dv}{dx} = 1 & \Rightarrow v = x \end{array}\right\}$		
	$\int \ln x\,dx = x\ln x - \int x.\frac{1}{x}\,dx$	Use of 'integration by parts' formula in the correct direction.	M1
	$= x\ln x - x + c$	Correct integration of $\ln x$ with or without $+ c$	A1
	$\int \ln 2\,dx = x\ln 2 + c$	Correct integration of $\ln 2$ with or without $+ c$	M1
	Hence, $\int \ln\left(\frac{x}{2}\right)\,dx = x\ln x - x - x\ln 2 + c$	Correct integration with $+ c$	A1 aef
			[4]

Note: $\int \ln x\,dx = (\text{their } v)\ln x - \int (\text{their } v).\left(\text{their } \frac{du}{dx}\right)\,dx$ for M1 in part (i).

Question Number	Scheme		Marks
Aliter **4.** (i) **Way 3**	$\int \ln\left(\frac{x}{2}\right)\, dx$		
	$u = \frac{x}{2} \Rightarrow \frac{du}{dx} = \frac{1}{2}$		
	$\int \ln\left(\frac{x}{2}\right)\, dx = 2\int \ln u\, du$	Applying substitution correctly to give $\int \ln\left(\frac{x}{2}\right)\, dx = 2\int \ln u\, du$ ***Decide to award 2nd M1 here!***	
	$\int \ln u\, dx = \int 1.\ln u\, du$		
	$\int \ln u\, dx = u\ln u - \int u.\frac{1}{u}\, du$	Use of 'integration by parts' formula in the correct direction.	M1
	$= u\ln u - u + c$	Correct integration of $\ln u$ with or without $+ c$	A1
		Decide to award 2nd M1 here!	M1
	$\int \ln\left(\frac{x}{2}\right) dx = 2(u\ln u - u) + c$		
	Hence, $\int \ln\left(\frac{x}{2}\right)\, dx = x\ln\left(\frac{x}{2}\right) - x + c$	Correct integration with $+ c$	A1 aef
			[4]

Question Number	Scheme		Marks
Aliter **4.** (ii) **Way 2**	$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin^2 x\,dx = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin x.\sin x\,dx$ and $I = \int \sin^2 x\,dx$		
	$\left\{\begin{array}{l} u = \sin x \Rightarrow \frac{du}{dx} = \cos x \\ \frac{dv}{dx} = \sin x \Rightarrow v = -\cos x \end{array}\right\}$		
	$\therefore I = \underline{\left\{-\sin x\cos x + \int \cos^2 x\,dx\right\}}$	An attempt to use the correct by parts formula.	M1
	$\therefore I = \left\{-\sin x\cos x + \int (1-\sin^2 x)\,dx\right\}$		
	$\int \sin^2 x\,dx = \left\{-\sin x\cos x + \int 1\,dx - \int \sin^2 x\,dx\right\}$		
	$2\int \sin^2 x\,dx = \left\{-\sin x\cos x + \int 1\,dx\right\}$	For the LHS becoming $2I$	dM1
	$2\int \sin^2 x\,dx = \{-\sin x\cos x + x\}$		
	$\int \sin^2 x\,dx = \left\{\underline{-\tfrac{1}{2}\sin x\cos x + \tfrac{x}{2}}\right\}$	<u>Correct integration</u>	A1
	$\therefore \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin^2 x\,dx = \left[\left(-\tfrac{1}{2}\sin(\tfrac{\pi}{2})\cos(\tfrac{\pi}{2}) + \tfrac{(\frac{\pi}{2})}{2}\right) - \left(-\tfrac{1}{2}\sin(\tfrac{\pi}{4})\cos(\tfrac{\pi}{4}) + \tfrac{(\frac{\pi}{4})}{2}\right)\right]$ $= \left[(0 + \tfrac{\pi}{4}) - (-\tfrac{1}{4} + \tfrac{\pi}{8})\right]$	Substitutes limits of $\frac{\pi}{2}$ and $\frac{\pi}{4}$ and subtracts the correct way round.	ddM1
	$= \frac{\pi}{8} + \frac{1}{4}$	$\underline{\tfrac{1}{2}\left(\tfrac{\pi}{4}+\tfrac{1}{2}\right)}$ or $\underline{\tfrac{\pi}{8}+\tfrac{1}{4}}$ or $\underline{\tfrac{\pi}{8}+\tfrac{2}{8}}$ Candidate must collect their π term and constant term together for A1 No fluked answers, hence **cso**.	A1 **aef cso** **[5]**

Note $\frac{\pi}{8} + \frac{1}{4} = 0.64269...$

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5. A curve is described by the equation

$$x^3 - 4y^2 = 12xy.$$

(a) Find the coordinates of the two points on the curve where $x = -8$.

(3)

(b) Find the gradient of the curve at each of these points.

(6)

Question Number	Scheme		Marks
5. (a)	$x^3 - 4y^2 = 12xy \quad (\text{ eqn } *)$		
	$x = -8 \Rightarrow -512 - 4y^2 = 12(-8)y$ $-512 - 4y^2 = -96y$	Substitutes $x = -8$ (at least once) into * to obtain a three term quadratic in y. Condone the loss of $= 0$.	M1
	$4y^2 - 96y + 512 = 0$ $y^2 - 24y + 128 = 0$		
	$(y-16)(y-8) = 0$ $y = \dfrac{24 \pm \sqrt{576 - 4(128)}}{2}$	An attempt to solve the quadratic in y by either factorising or by the formula or by ***completing the square***.	dM1
	$y = 16$ or $y = 8$.	Both $\underline{y=16}$ and $\underline{y=8}$. or $\underline{(-8, 8)}$ and $\underline{(-8, 16)}$.	A1
			[3]
(b)	$\left\{\dfrac{\cancel{dy}}{\cancel{dx}} \times \right\}$ $3x^2 - 8y\dfrac{dy}{dx}; = \left(\underline{12y + 12x\dfrac{dy}{dx}}\right)$	Differentiates implicitly to include either $\pm ky\frac{dy}{dx}$ or $12x\frac{dy}{dx}$. Ignore $\frac{dy}{dx} = ...$	M1
		Correct LHS equation;	A1;
		<u>Correct application of product rule</u>	(B1)
	$\left\{\dfrac{dy}{dx} = \dfrac{3x^2 - 12y}{12x + 8y}\right\}$	*not necessarily required.*	
	$@ (-8, 8), \quad \dfrac{dy}{dx} = \dfrac{3(64) - 12(8)}{12(-8) + 8(8)} = \dfrac{96}{-32} = \underline{-3},$	Substitutes $x = -8$ and *at least one* of their y-values to attempt to find any one of $\frac{dy}{dx}$.	dM1
	$@ (-8, 16), \quad \dfrac{dy}{dx} = \dfrac{3(64) - 12(16)}{12(-8) + 8(16)} = \dfrac{0}{32} = \underline{0}.$	One gradient found.	A1
		Both gradients of <u>-3</u> and <u>0</u> ***correctly*** found.	A1 **cso**
			[6]
			9 marks

Question Number	Scheme		Marks
Aliter **5.** (b) **Way 2**	$\left\{\frac{\not{d}x}{\not{d}y} \times\right\}$ $3x^2\frac{dx}{dy} - 8y; = \left(12y\frac{dx}{dy} + 12x\right)$	Differentiates implicitly to include either $\pm kx^2\frac{dx}{dy}$ or $12y\frac{dx}{dy}$. Ignore $\frac{dx}{dy} = \ldots$	M1
		Correct LHS equation	A1;
		<u>Correct application of product rule</u>	(B1)
	$\left\{\frac{dy}{dx} = \frac{3x^2 - 12y}{12x + 8y}\right\}$	*not necessarily required.*	
	@ $(-8, 8)$, $\frac{dy}{dx} = \frac{3(64) - 12(8)}{12(-8) + 8(8)} = \frac{96}{-32} = \underline{-3}$,	Substitutes $x = -8$ and *at least one* of their y-values to attempt to find any one of $\frac{dy}{dx}$ or $\frac{dx}{dy}$.	dM1
	@ $(-8, 16)$, $\frac{dy}{dx} = \frac{3(64) - 12(16)}{12(-8) + 8(16)} = \frac{0}{32} = \underline{0}$.	One gradient found.	A1
		Both gradients of <u>-3</u> and <u>0</u> ***correctly*** found.	A1 **cso**
			[6]

Question Number	Scheme		Marks
Aliter **5.** (b) **Way 3**	$x^3 - 4y^2 = 12xy$ (eqn $*$)		
	$4y^2 + 12xy - x^3 = 0$		
	$y = \dfrac{-12x \pm \sqrt{144x^2 - 4(4)(-x^3)}}{8}$		
	$y = \dfrac{-12x \pm \sqrt{144x^2 + 16x^3}}{8}$		
	$y = \dfrac{-12x \pm 4\sqrt{9x^2 + x^3}}{8}$		
	$y = -\frac{3}{2}x \pm \frac{1}{2}\left(9x^2 + x^3\right)^{\frac{1}{2}}$		
	$\dfrac{dy}{dx} = -\frac{3}{2} \pm \frac{1}{2}\left(\frac{1}{2}\right)\left(9x^2 + x^3\right)^{-\frac{1}{2}};\left(18x + 3x^2\right)$	A credible attempt to make y the subject and an attempt to differentiate either $-\frac{3}{2}x$ or $\frac{1}{2}\left(9x^2 + x^3\right)^{\frac{1}{2}}$.	M1
	$\dfrac{dy}{dx} = -\dfrac{3}{2} \pm \dfrac{18x + 3x^2}{4(9x^2 + x^3)^{\frac{1}{2}}}$	$\dfrac{dy}{dx} = -\frac{3}{2} \pm k\left(9x^2 + x^3\right)^{-\frac{1}{2}}\left(g(x)\right)$	A1
		$\dfrac{dy}{dx} = -\frac{3}{2} \pm \frac{1}{2}\left(\frac{1}{2}\right)\left(9x^2 + x^3\right)^{-\frac{1}{2}};\left(18x + 3x^2\right)$	A1
	@ $x = -8$ $\dfrac{dy}{dx} = -\dfrac{3}{2} \pm \dfrac{18(-8) + 3(64)}{4(9(64) + (-512))^{\frac{1}{2}}}$	Substitutes $x = -8$ find any one of $\frac{dy}{dx}$.	dM1
	$= -\dfrac{3}{2} \pm \dfrac{48}{4\sqrt{(64)}} = -\dfrac{3}{2} \pm \dfrac{48}{32}$		
	$\therefore \dfrac{dy}{dx} = -\dfrac{3}{2} \pm \dfrac{3}{2} = \underline{-3}, \underline{0}.$	One gradient correctly found.	A1
		Both gradients of $\underline{-3}$ and $\underline{0}$ correctly found.	A1
			[6]

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6. The points A and B have position vectors $2\mathbf{i} + 6\mathbf{j} - \mathbf{k}$ and $3\mathbf{i} + 4\mathbf{j} + \mathbf{k}$ respectively.

The line l_1 passes through the points A and B.

(a) Find the vector $\overrightarrow{AB}$.

(2)

(b) Find a vector equation for the line l_1.

(2)

A second line l_2 passes through the origin and is parallel to the vector $\mathbf{i} + \mathbf{k}$. The line l_1 meets the line l_2 at the point C.

(c) Find the acute angle between l_1 and l_2.

(3)

(d) Find the position vector of the point C.

(4)

Question Number	Scheme		Marks
6. (a)	$\overrightarrow{OA} = \begin{pmatrix}2\\6\\-1\end{pmatrix}$ & $\overrightarrow{OB} = \begin{pmatrix}3\\4\\1\end{pmatrix}$		
	$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \begin{pmatrix}3\\4\\1\end{pmatrix} - \begin{pmatrix}2\\6\\-1\end{pmatrix} = \underline{\begin{pmatrix}1\\-2\\2\end{pmatrix}}$	Finding the difference between $\overrightarrow{OB}$ and $\overrightarrow{OA}$.	M1 ±
		Correct answer.	A1
			[2]
(b)	$l_1 : \mathbf{r} = \begin{pmatrix}2\\6\\-1\end{pmatrix} + \lambda\begin{pmatrix}1\\-2\\2\end{pmatrix}$ or $\mathbf{r} = \begin{pmatrix}3\\4\\1\end{pmatrix} + \lambda\begin{pmatrix}1\\-2\\2\end{pmatrix}$	An expression of the form $(\text{vector}) \pm \lambda(\text{vector})$	M1
	$l_1 : \mathbf{r} = \begin{pmatrix}2\\6\\-1\end{pmatrix} + \lambda\begin{pmatrix}-1\\2\\-2\end{pmatrix}$ or $\mathbf{r} = \begin{pmatrix}3\\4\\1\end{pmatrix} + \lambda\begin{pmatrix}-1\\2\\-2\end{pmatrix}$	$\mathbf{r} = \overrightarrow{OA} \pm \lambda(\text{their } \overrightarrow{AB})$ or $\mathbf{r} = \overrightarrow{OB} \pm \lambda(\text{their } \overrightarrow{AB})$ or $\mathbf{r} = \overrightarrow{OA} \pm \lambda(\text{their } \overrightarrow{BA})$ or $\mathbf{r} = \overrightarrow{OB} \pm \lambda(\text{their } \overrightarrow{BA})$ (**r** is needed.)	A1 $\sqrt{}$ aef
			[2]
(c)	$l_2 : \mathbf{r} = \begin{pmatrix}0\\0\\0\end{pmatrix} + \mu\begin{pmatrix}1\\0\\1\end{pmatrix} \Rightarrow \mathbf{r} = \mu\begin{pmatrix}1\\0\\1\end{pmatrix}$		
	$\overrightarrow{AB} = \mathbf{d}_1 = \mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$, $\mathbf{d}_2 = \mathbf{i} + 0\mathbf{j} + \mathbf{k}$ & θ is angle		
	$\cos\theta = \dfrac{\overrightarrow{AB} \bullet \mathbf{d}_2}{\left(\lvert\overrightarrow{AB}\rvert . \lvert\mathbf{d}_2\rvert\right)} = \dfrac{\begin{pmatrix}1\\-2\\2\end{pmatrix} \bullet \begin{pmatrix}1\\0\\1\end{pmatrix}}{\left(\sqrt{(1)^2 + (-2)^2 + (2)^2} . \sqrt{(1)^2 + (0)^2 + (1)^2}\right)}$ ←	Considers dot product between $\mathbf{d}_2$ and their $\overrightarrow{AB}$.	M1 $\sqrt{}$
	$\cos\theta = \dfrac{1 + 0 + 2}{\sqrt{(1)^2 + (-2)^2 + (2)^2} . \sqrt{(1)^2 + (0)^2 + (1)^2}}$	Correct followed through expression or **equation**.	A1 $\sqrt{}$
	$\cos\theta = \dfrac{3}{3.\sqrt{2}} \Rightarrow \underline{\theta = 45^\circ \text{ or } \tfrac{\pi}{4} \text{ or awrt } 0.79}.$	$\underline{\theta = 45^\circ \text{ or } \tfrac{\pi}{4} \text{ or awrt } 0.79}$	A1 **cao**
			[3]

This means that $\cos\theta$ does not necessarily have to be the subject of the equation. It could be of the form $3\sqrt{2}\cos\theta = 3$.

Question Number	Scheme		Marks
6. (d)	If l_1 and l_2 intersect then: $\begin{pmatrix}2\\6\\-1\end{pmatrix}+\lambda\begin{pmatrix}1\\-2\\2\end{pmatrix}=\mu\begin{pmatrix}1\\0\\1\end{pmatrix}$		
	$\mathbf{i}: \ 2+\lambda=\mu$ (1) $\mathbf{j}: \ 6-2\lambda=0$ (2) $\mathbf{k}: -1+2\lambda=\mu$ (3)	**Either** seeing equation (2) written down correctly with or without any other equation **or** seeing equations (1) and (3) written down correctly.	M1 $\sqrt{}$
	(2) yields $\lambda=3$ Any two yields $\lambda=3,\quad \mu=5$	Attempt to solve either equation (2) or simultaneously solve any two of the three equations to find …	dM1
		either one of λ or μ correct.	A1
	$l_1: \mathbf{r}=\begin{pmatrix}2\\6\\-1\end{pmatrix}+3\begin{pmatrix}1\\-2\\2\end{pmatrix}=\underline{\begin{pmatrix}5\\0\\5\end{pmatrix}}$ *or* $\mathbf{r}=5\begin{pmatrix}1\\0\\1\end{pmatrix}=\underline{\begin{pmatrix}5\\0\\5\end{pmatrix}}$	$\underline{\begin{pmatrix}5\\0\\5\end{pmatrix}}$ or $5\mathbf{i}+5\mathbf{k}$ Fully correct solution & no incorrect values of λ or μ seen earlier.	A1 **cso**
			[4]
Aliter 6. (d) **Way 2**	If l_1 and l_2 intersect then: $\begin{pmatrix}3\\4\\1\end{pmatrix}+\lambda\begin{pmatrix}1\\-2\\2\end{pmatrix}=\mu\begin{pmatrix}1\\0\\1\end{pmatrix}$		
	$\mathbf{i}: \ 3+\lambda=\mu$ (1) $\mathbf{j}: \ 4-2\lambda=0$ (2) $\mathbf{k}: \ 1+2\lambda=\mu$ (3)	**Either** seeing equation (2) written down correctly with or without any other equation **or** seeing equations (1) and (3) written down correctly.	M1 $\sqrt{}$
	(2) yields $\lambda=2$ Any two yields $\lambda=2,\quad \mu=5$	Attempt to solve either equation (2) or simultaneously solve any two of the three equations to find …	dM1
		either one of λ or μ correct.	A1
	$l_1: \mathbf{r}=\begin{pmatrix}3\\4\\1\end{pmatrix}+2\begin{pmatrix}1\\-2\\2\end{pmatrix}=\underline{\begin{pmatrix}5\\0\\5\end{pmatrix}}$ *or* $\mathbf{r}=5\begin{pmatrix}1\\0\\1\end{pmatrix}=\underline{\begin{pmatrix}5\\0\\5\end{pmatrix}}$	$\underline{\begin{pmatrix}5\\0\\5\end{pmatrix}}$ or $5\mathbf{i}+5\mathbf{k}$ Fully correct solution & no incorrect values of λ or μ seen earlier.	A1 **cso**
			[4]
			11 marks

Note: Be careful! λ and μ are not defined in the question, so a candidate could interchange these or use different scalar parameters.

Question Number	Scheme		Marks
***Aliter* 6. (d) Way 3**	If l_1 and l_2 intersect then: $\begin{pmatrix}2\\6\\-1\end{pmatrix}+\lambda\begin{pmatrix}-1\\2\\-2\end{pmatrix}=\mu\begin{pmatrix}1\\0\\1\end{pmatrix}$		
	$\mathbf{i}: \; 2-\lambda=\mu \quad (1)$ $\mathbf{j}: \; 6+2\lambda=0 \quad (2)$ $\mathbf{k}: -1-2\lambda=\mu \quad (3)$	**Either** seeing equation (2) written down correctly with or without any other equation **or** seeing equations (1) and (3) written down correctly.	M1 $\sqrt{}$
	(2) yields $\lambda=-3$ Any two yields $\lambda=-3,\ \mu=5$	Attempt to solve either equation (2) or simultaneously solve any two of the three equations to find …	dM1
		either one of λ or μ correct.	A1
	$l_1: \mathbf{r}=\begin{pmatrix}2\\6\\-1\end{pmatrix}-3\begin{pmatrix}-1\\2\\-2\end{pmatrix}=\underline{\begin{pmatrix}5\\0\\5\end{pmatrix}}$ *or* $\mathbf{r}=5\begin{pmatrix}1\\0\\1\end{pmatrix}=\underline{\begin{pmatrix}5\\0\\5\end{pmatrix}}$	$\underline{\begin{pmatrix}5\\0\\5\end{pmatrix}}$ or $5\mathbf{i}+5\mathbf{k}$ Fully correct solution & no incorrect values of λ or μ seen earlier.	A1 **cso**
			[4]
Aliter 6. (d) Way 4	If l_1 and l_2 intersect then: $\begin{pmatrix}3\\4\\1\end{pmatrix}+\lambda\begin{pmatrix}-1\\2\\-2\end{pmatrix}=\mu\begin{pmatrix}1\\0\\1\end{pmatrix}$		
	$\mathbf{i}: \; 3-\lambda=\mu \quad (1)$ $\mathbf{j}: \; 4+2\lambda=0 \quad (2)$ $\mathbf{k}: \; 1-2\lambda=\mu \quad (3)$	**Either** seeing equation (2) written down correctly with or without any other equation **or** seeing equations (1) and (3) written down correctly.	M1 $\sqrt{}$
	(2) yields $\lambda=-2$ Any two yields $\lambda=-2,\ \mu=5$	Attempt to solve either equation (2) or simultaneously solve any two of the three equations to find …	dM1
		either one of λ or μ correct.	A1
	$l_1: \mathbf{r}=\begin{pmatrix}3\\4\\1\end{pmatrix}-2\begin{pmatrix}-1\\2\\-2\end{pmatrix}=\underline{\begin{pmatrix}5\\0\\5\end{pmatrix}}$ *or* $\mathbf{r}=5\begin{pmatrix}1\\0\\1\end{pmatrix}=\underline{\begin{pmatrix}5\\0\\5\end{pmatrix}}$	$\underline{\begin{pmatrix}5\\0\\5\end{pmatrix}}$ or $5\mathbf{i}+5\mathbf{k}$ Fully correct solution & no incorrect values of λ or μ seen earlier.	A1 **cso**
			[4]
			11 marks

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7.

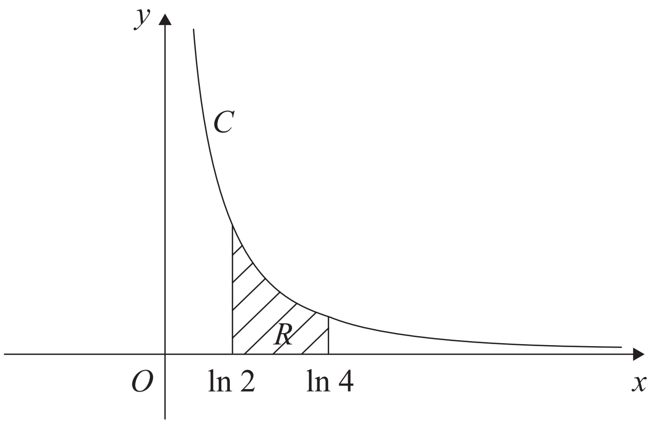

Figure 3

The curve C has parametric equations

$$x = \ln(t+2), \qquad y = \frac{1}{(t+1)}, \qquad t > -1.$$

The finite region R between the curve C and the x-axis, bounded by the lines with equations $x = \ln 2$ and $x = \ln 4$, is shown shaded in Figure 3.

(a) Show that the area of R is given by the integral

$$\int_0^2 \frac{1}{(t+1)(t+2)}\,\mathrm{d}t.$$

(4)

(b) Hence find an exact value for this area.

(6)

(c) Find a cartesian equation of the curve C, in the form $y = \mathrm{f}(x)$.

(4)

(d) State the domain of values for x for this curve.

(1)

Question Number	Scheme		Marks
7. (a)	$\left[x = \ln(t+2),\ y = \dfrac{1}{t+1}\right], \quad \Rightarrow \dfrac{dx}{dt} = \dfrac{1}{t+2}$	Must state $\dfrac{dx}{dt} = \dfrac{1}{t+2}$	B1
	$\text{Area}(R) = \int_{\ln 2}^{\ln 4} \dfrac{1}{t+1}\,dx ;= \int_0^2 \left(\dfrac{1}{t+1}\right)\left(\dfrac{1}{t+2}\right)dt$	Area $= \int \dfrac{1}{t+1}\,dx$. Ignore limits.	M1;
		$\int \left(\dfrac{1}{t+1}\right)\times\left(\dfrac{1}{t+2}\right)dt$. Ignore limits.	A1 **AG**
	Changing limits, when: $x = \ln 2 \Rightarrow \ln 2 = \ln(t+2) \Rightarrow 2 = t+2 \Rightarrow t = 0$ $x = \ln 4 \Rightarrow \ln 4 = \ln(t+2) \Rightarrow 4 = t+2 \Rightarrow t = 2$	changes limits $x \to t$ so that $\ln 2 \to 0$ and $\ln 4 \to 2$	B1
	Hence, $\text{Area}(R) = \int_0^2 \dfrac{1}{(t+1)(t+2)}\,dt$		**[4]**
(b)	$\left(\dfrac{1}{(t+1)(t+2)}\right) = \dfrac{A}{(t+1)} + \dfrac{B}{(t+2)}$	$\dfrac{A}{(t+1)} + \dfrac{B}{(t+2)}$ with A and B found	M1
	$1 = A(t+2) + B(t+1)$		
	Let $t = -1$, $1 = A(1) \Rightarrow \underline{A = 1}$ Let $t = -2$, $1 = B(-1) \Rightarrow \underline{B = -1}$	Finds both A and B correctly. Can be implied. (See note below)	A1
	$\int_0^2 \dfrac{1}{(t+1)(t+2)}\,dt = \int_0^2 \dfrac{1}{(t+1)} - \dfrac{1}{(t+2)}\,dt$		
	$= [\ln(t+1) - \ln(t+2)]_0^2$	Either $\pm a\ln(t+1)$ or $\pm b\ln(t+2)$ Both ln terms correctly ft.	dM1 A1 $\sqrt{}$
	$= (\ln 3 - \ln 4) - (\ln 1 - \ln 2)$	Substitutes ***both*** limits of 2 and 0 and subtracts the correct way round.	ddM1
	$= \ln 3 - \ln 4 + \ln 2 = \ln 3 - \ln 2 = \ln\left(\frac{3}{2}\right)$	$\underline{\ln 3 - \ln 4 + \ln 2}$ or $\underline{\ln\left(\frac{3}{4}\right) - \ln\left(\frac{1}{2}\right)}$ or $\underline{\ln 3 - \ln 2}$ or $\underline{\ln\left(\frac{3}{2}\right)}$ (must deal with ln 1)	A1 aef isw **[6]**

Takes out brackets.

Writing down $\dfrac{1}{(t+1)(t+2)} = \dfrac{1}{(t+1)} + \dfrac{1}{(t+2)}$ means first M1A0 in (b).

Writing down $\dfrac{1}{(t+1)(t+2)} = \dfrac{1}{(t+1)} - \dfrac{1}{(t+2)}$ means first M1A1 in (b).

Question Number	Scheme		Marks
	$x = \ln(t+2), \quad y = \dfrac{1}{t+1}$		
7. (c)	$e^x = t+2 \Rightarrow t = e^x - 2$	Attempt to make $t =$... the subject giving $t = e^x - 2$	M1 A1
	$y = \dfrac{1}{e^x - 2 + 1} \Rightarrow y = \dfrac{1}{e^x - 1}$	Eliminates t by substituting in y giving $y = \dfrac{1}{e^x - 1}$	dM1 A1 **[4]**
Aliter **7.** (c) **Way 2**	$t+1 = \dfrac{1}{y} \Rightarrow t = \dfrac{1}{y} - 1$ or $t = \dfrac{1-y}{y}$ $y(t+1) = 1 \Rightarrow yt + y = 1 \Rightarrow yt = 1 - y \Rightarrow t = \dfrac{1-y}{y}$	Attempt to make $t =$... the subject Giving either $t = \dfrac{1}{y} - 1$ or $t = \dfrac{1-y}{y}$	M1 A1
	$x = \ln\left(\dfrac{1}{y} - 1 + 2\right)$ or $x = \ln\left(\dfrac{1-y}{y} + 2\right)$	Eliminates t by substituting in x	dM1
	$x = \ln\left(\dfrac{1}{y} + 1\right)$		
	$e^x = \dfrac{1}{y} + 1$		
	$e^x - 1 = \dfrac{1}{y}$		
	$y = \dfrac{1}{e^x - 1}$	giving $y = \dfrac{1}{e^x - 1}$	A1 **[4]**
(d)	Domain : $\underline{x > 0}$	$\underline{x > 0}$ or just > 0	B1 **[1]**
			15 marks

Question Number	Scheme		Marks
Aliter **7.** (c) **Way 3**	$e^x = t + 2 \;\Rightarrow\; t + 1 = e^x - 1$	Attempt to make $t+1 =$ ... the subject giving $t + 1 = e^x - 1$	M1 A1
	$y = \dfrac{1}{t+1} \;\Rightarrow\; y = \dfrac{1}{e^x - 1}$	Eliminates t by substituting in y giving $y = \dfrac{1}{e^x - 1}$	dM1 A1
			[4]
Aliter **7.** (c) **Way 4**	$t + 1 = \dfrac{1}{y} \;\Rightarrow\; t + 2 = \dfrac{1}{y} + 1$ or $t + 2 = \dfrac{1+y}{y}$	Attempt to make $t+2 =$... the subject Either $t + 2 = \dfrac{1}{y} + 1$ or $t + 2 = \dfrac{1+y}{y}$	M1 A1
	$x = \ln\left(\dfrac{1}{y} + 1\right)$ or $x = \ln\left(\dfrac{1+y}{y}\right)$	Eliminates t by substituting in x	dM1
	$x = \ln\left(\dfrac{1}{y} + 1\right)$		
	$e^x = \dfrac{1}{y} + 1 \;\Rightarrow\; e^x - 1 = \dfrac{1}{y}$		
	$y = \dfrac{1}{e^x - 1}$	giving $y = \dfrac{1}{e^x - 1}$	A1
			[4]

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8. Liquid is pouring into a large vertical circular cylinder at a constant rate of 1600 $\text{cm}^3\,\text{s}^{-1}$ and is leaking out of a hole in the base, at a rate proportional to the square root of the height of the liquid already in the cylinder. The area of the circular cross section of the cylinder is 4000 cm^2.

(a) Show that at time t seconds, the height h cm of liquid in the cylinder satisfies the differential equation

$$\frac{dh}{dt} = 0.4 - k\sqrt{h}, \text{ where } k \text{ is a positive constant.}$$

(3)

When $h = 25$, water is leaking out of the hole at 400 $\text{cm}^3\,\text{s}^{-1}$.

(b) Show that $k = 0.02$

(1)

(c) Separate the variables of the differential equation

$$\frac{dh}{dt} = 0.4 - 0.02\sqrt{h},$$

to show that the time taken to fill the cylinder from empty to a height of 100 cm is given by

$$\int_0^{100} \frac{50}{20 - \sqrt{h}}\, dh.$$

(2)

Using the substitution $h = (20 - x)^2$, or otherwise,

(d) find the exact value of $\int_0^{100} \frac{50}{20 - \sqrt{h}}\, dh$.

(6)

(e) Hence find the time taken to fill the cylinder from empty to a height of 100 cm, giving your answer in minutes and seconds to the nearest second.

(1)

Question Number	Scheme		Marks
8. (a)	$\frac{dV}{dt} = 1600 - c\sqrt{h}$ or $\frac{dV}{dt} = 1600 - k\sqrt{h}$,	Either of these statements	M1
	$\left(V = 4000h \;\Rightarrow\right) \frac{dV}{dh} = 4000$	$\frac{dV}{dh} = 4000$ or $\frac{dh}{dV} = \frac{1}{4000}$	M1
	$\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt} = \frac{\frac{dV}{dt}}{\frac{dV}{dh}}$		
	Either, $\frac{dh}{dt} = \frac{1600 - c\sqrt{h}}{4000} = \frac{1600}{4000} - \frac{c\sqrt{h}}{4000} = 0.4 - k\sqrt{h}$ or $\frac{dh}{dt} = \frac{1600 - k\sqrt{h}}{4000} = \frac{1600}{4000} - \frac{k\sqrt{h}}{4000} = 0.4 - k\sqrt{h}$	Convincing proof of $\frac{dh}{dt}$	A1 **AG**
			[3]
(b)	When $h = 25$ water ***leaks out such that*** $\frac{dV}{dt} = 400$		
	$400 = c\sqrt{h} \Rightarrow 400 = c\sqrt{25} \Rightarrow 400 = c(5) \Rightarrow c = 80$		
	From above; $k = \frac{c}{4000} = \frac{80}{4000} = 0.02$ as required	Proof that $k = 0.02$	B1 **AG**
			[1]
Aliter (b) **Way 2**	$400 = 4000k\sqrt{h}$		
	$\Rightarrow 400 = 4000k\sqrt{25}$		
	$\Rightarrow 400 = k(20000) \Rightarrow k = \frac{400}{20000} = 0.02$	Using 400, 4000 and $h = 25$ or $\sqrt{h} = 5$. Proof that $k = 0.02$	B1 **AG**
			[1]
(c)	$\frac{dh}{dt} = 0.4 - k\sqrt{h} \Rightarrow \int \frac{dh}{0.4 - k\sqrt{h}} = \int dt$	***Separates the variables*** with $\int \frac{dh}{0.4 - k\sqrt{h}}$ and $\int dt$ on either side with integral signs not necessary.	M1 oe
	$\therefore$ time required $= \int_0^{100} \frac{1}{0.4 - 0.02\sqrt{h}}\, dh \quad \frac{\div 0.02}{\div 0.02}$		
	time required $= \int_0^{100} \frac{50}{20 - \sqrt{h}}\, dh$	Correct proof	A1 **AG**
			[2]

Question Number	Scheme		Marks
8. (d)	$\int_0^{100} \frac{50}{20-\sqrt{h}}\,dh$ with substitution $h=(20-x)^2$		
	$\frac{dh}{dx}=2(20-x)(-1)$ or $\frac{dh}{dx}=-2(20-x)$	Correct $\frac{dh}{dx}$	B1 aef
	$h=(20-x)^2 \Rightarrow \sqrt{h}=20-x \Rightarrow x=20-\sqrt{h}$		
	$\int \frac{50}{20-\sqrt{h}}\,dh = \int \frac{50}{x}.-2(20-x)\,dx$	$\pm\lambda\int\frac{20-x}{x}\,dx$ or $\pm\lambda\int\frac{20-x}{20-(20-x)}\,dx$ where λ is a constant	M1
	$=100\int\frac{x-20}{x}\,dx$		
	$=100\int\left(1-\frac{20}{x}\right)dx$		
	$=100(x-20\ln x)\ (+c)$	$\pm\alpha x \pm \beta\ln x\ ;\ \alpha,\beta\neq 0$ $100x-2000\ln x$	M1 A1
	change limits: when $h=0$ then $x=20$ and when $h=100$ then $x=10$		
	$\int_0^{100}\frac{50}{20-\sqrt{h}}\,dh = \left[100x-2000\ln x\right]_{20}^{10}$		
	or $\int_0^{100}\frac{50}{20-\sqrt{h}}\,dh = \left[100\left(20-\sqrt{h}\right)-2000\ln\left(20-\sqrt{h}\right)\right]_0^{100}$		
	$=(1000-2000\ln 10)-(2000-2000\ln 20)$	Correct use of limits, ie. putting them in the correct way round Either $x=10$ and $x=20$ or $h=100$ and $h=0$	ddM1
	$=2000\ln 20-2000\ln 10-1000$	Combining logs to give...	
	$=2000\ln 2-1000$	$2000\ln 2-1000$ or $-2000\ln\left(\frac{1}{2}\right)-1000$	A1 **aef**
			[6]
(e)	Time required $=2000\ln 2-1000 = 386.2943611...$ sec		
	$=386$ seconds (nearest second)		
	$=6$ minutes and 26 seconds (nearest second)	<u>6 minutes, 26 seconds</u>	B1
			[1]
			13 marks