

January 2010
6666 Core Mathematics C4
Mark Scheme

Question Number	Scheme	Marks
Q1	<p>(a) $(1-8x)^{\frac{1}{2}} = 1 + \left(\frac{1}{2}\right)(-8x) + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2}(-8x)^2 + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{3!}(-8x)^3 + \dots$ $= 1 - 4x - 8x^2; -32x^3 - \dots$</p> <p>(b) $\sqrt{(1-8x)} = \sqrt{\left(1 - \frac{8}{100}\right)}$ $= \sqrt{\frac{92}{100}} = \sqrt{\frac{23}{25}} = \frac{\sqrt{23}}{5} \quad *$</p> <p>(c) $1 - 4x - 8x^2 - 32x^3 = 1 - 4(0.01) - 8(0.01)^2 - 32(0.01)^3$ $= 1 - 0.04 - 0.0008 - 0.00032 = 0.959168$ $\sqrt{23} = 5 \times 0.959168$ $= 4.79584$</p>	<p>M1 A1 A1; A1 (4)</p> <p>M1 cs0 A1 (2)</p> <p>M1 M1 cao A1 (3) [9]</p>

2.

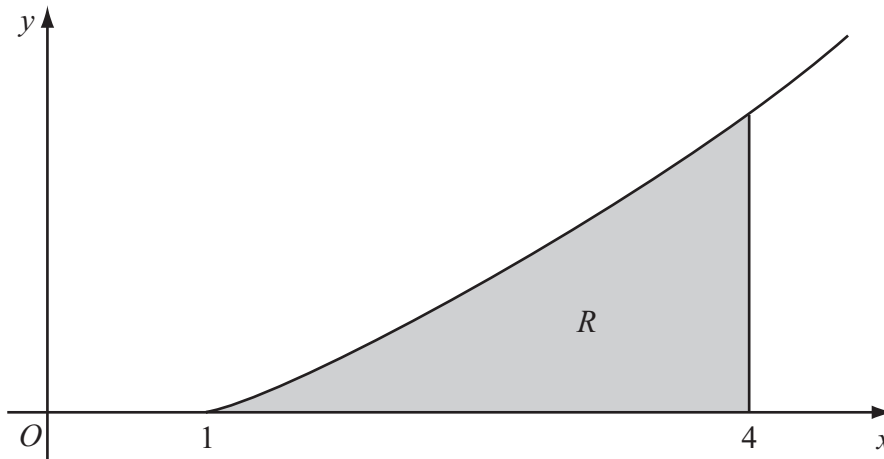


Figure 1

Figure 1 shows a sketch of the curve with equation $y = x \ln x$, $x \geq 1$. The finite region R , shown shaded in Figure 1, is bounded by the curve, the x -axis and the line $x = 4$.

The table shows corresponding values of x and y for $y = x \ln x$.

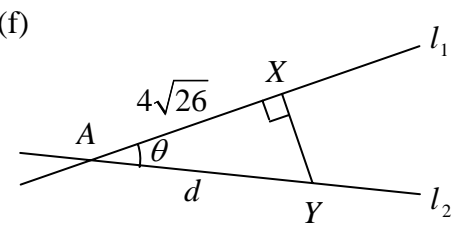
x	1	1.5	2	2.5	3	3.5	4
y	0	0.608			3.296	4.385	5.545

- (a) Complete the table with the values of y corresponding to $x = 2$ and $x = 2.5$, giving your answers to 3 decimal places. (2)
- (b) Use the trapezium rule, with all the values of y in the completed table, to obtain an estimate for the area of R , giving your answer to 2 decimal places. (4)
- (c) (i) Use integration by parts to find $\int x \ln x \, dx$.
- (ii) Hence find the exact area of R , giving your answer in the form $\frac{1}{4}(a \ln 2 + b)$, where a and b are integers. (7)



Question Number	Scheme	Marks
Q2	(a) 1.386, 2.291 awrt 1.386, 2.291	B1 B1 (2)
	(b) $A \approx \frac{1}{2} \times 0.5(\dots)$ $= \dots (0 + 2(0.608 + 1.386 + 2.291 + 3.296 + 4.385) + 5.545)$ $= 0.25(0 + 2(0.608 + 1.386 + 2.291 + 3.296 + 4.385) + 5.545)$ ft their (a) $= 0.25 \times 29.477 \dots \approx 7.37$ cao	B1 M1 A1ft A1 (4)
	(c)(i) $\int x \ln x \, dx = \frac{x^2}{2} \ln x - \int \frac{x^2}{2} \times \frac{1}{x} \, dx$ $= \frac{x^2}{2} \ln x - \int \frac{x}{2} \, dx$ $= \frac{x^2}{2} \ln x - \frac{x^2}{4} (+C)$	M1 A1 M1 A1
	(ii) $\left[\frac{x^2}{2} \ln x - \frac{x^2}{4} \right]_1^4 = (8 \ln 4 - 4) - \left(-\frac{1}{4} \right)$ $= 8 \ln 4 - \frac{15}{4}$	M1
	$= 8(2 \ln 2) - \frac{15}{4}$ ln 4 = 2 ln 2 seen or implied	M1
	$= \frac{1}{4}(64 \ln 2 - 15)$ $a = 64, b = -15$	A1 (7) [13]

Question Number	Scheme	Marks
Q3	(a) $-2\sin 2x - 3\sin 3y \frac{dy}{dx} = 0$ $\frac{dy}{dx} = -\frac{2\sin 2x}{3\sin 3y}$ Accept $\frac{2\sin 2x}{-3\sin 3y}, \frac{-2\sin 2x}{3\sin 3y}$	M1 A1 A1 (3)
	(b) At $x = \frac{\pi}{6}$, $\cos\left(\frac{2\pi}{6}\right) + \cos 3y = 1$ $\cos 3y = \frac{1}{2}$ $3y = \frac{\pi}{3} \Rightarrow y = \frac{\pi}{9}$ awrt 0.349	M1 A1 A1 (3)
	(c) At $\left(\frac{\pi}{6}, \frac{\pi}{9}\right)$, $\frac{dy}{dx} = -\frac{2\sin 2\left(\frac{\pi}{6}\right)}{3\sin 3\left(\frac{\pi}{9}\right)} = -\frac{2\sin \frac{\pi}{3}}{3\sin \frac{\pi}{3}} = -\frac{2}{3}$ $y - \frac{\pi}{9} = -\frac{2}{3}\left(x - \frac{\pi}{6}\right)$ Leading to $6x + 9y - 2\pi = 0$	M1 M1 A1 (3) [9]

Question Number	Scheme	Marks
Q4	(a) $A: (-6, 4, -1)$ Accept vector forms	B1 (1)
	(b) $\begin{pmatrix} 4 \\ -1 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -4 \\ 1 \end{pmatrix} = 12 + 4 + 3 = \sqrt{4^2 + (-1)^2 + 3^2} \sqrt{3^2 + (-4)^2 + 1^2} \cos \theta$	M1 A1
	$\cos \theta = \frac{19}{26}$ awrt 0.73	A1 (3)
	(c) $X: (10, 0, 11)$ Accept vector forms	B1 (1)
	(d) $\vec{AX} = \begin{pmatrix} 10 \\ 0 \\ 11 \end{pmatrix} - \begin{pmatrix} -6 \\ 4 \\ -1 \end{pmatrix}$ Either order	M1
	$= \begin{pmatrix} 16 \\ -4 \\ 12 \end{pmatrix}$ cao	A1 (2)
(e) $ \vec{AX} = \sqrt{16^2 + (-4)^2 + 12^2}$ $= \sqrt{416} = \sqrt{16 \times 26} = 4\sqrt{26} *$ Do not penalise if consistent incorrect signs in (d)	M1 A1 (2)	
(f)  Use of correct right angled triangle	M1 M1 A1 (3)	
$\frac{ \vec{AX} }{d} = \cos \theta$ $d = \frac{4\sqrt{26}}{\frac{19}{26}} \approx 27.9$ awrt 27.9		A1 (3)
		[12]

Question Number	Scheme	Marks
Q5	(a) $\int \frac{9x+6}{x} dx = \int \left(9 + \frac{6}{x}\right) dx$ $= 9x + 6 \ln x (+C)$	M1 A1 (2)
	(b) $\int \frac{1}{y^{\frac{1}{3}}} dy = \int \frac{9x+6}{x} dx$ $\int y^{-\frac{1}{3}} dy = \int \frac{9x+6}{x} dx$	Integral signs not necessary B1
	$\frac{y^{\frac{2}{3}}}{\frac{2}{3}} = 9x + 6 \ln x (+C)$	$\pm ky^{\frac{2}{3}} = \text{their (a)}$ M1
	$\frac{3}{2} y^{\frac{2}{3}} = 9x + 6 \ln x (+C)$	ft their (a) A1ft
	$y = 8, x = 1$	
	$\frac{3}{2} 8^{\frac{2}{3}} = 9 + 6 \ln 1 + C$	M1
	$C = -3$	A1
	$y^{\frac{2}{3}} = \frac{2}{3}(9x + 6 \ln x - 3)$	
	$y^2 = (6x + 4 \ln x - 2)^3 \quad (= 8(3x + 2 \ln x - 1)^3)$	A1 (6) [8]

Question Number	Scheme	Marks
Q6	$\frac{dA}{dt} = 1.5$ $A = \pi r^2 \Rightarrow \frac{dA}{dr} = 2\pi r$ <p>When $A = 2$</p> $2 = \pi r^2 \Rightarrow r = \sqrt{\frac{2}{\pi}} (= 0.797\ 884 \dots)$ $\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt}$ $1.5 = 2\pi r \frac{dr}{dt}$ $\frac{dr}{dt} = \frac{1.5}{2\pi\sqrt{\frac{2}{\pi}}} \approx 0.299$	<p>B1</p> <p>B1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p style="text-align: right;">[5]</p>

7.

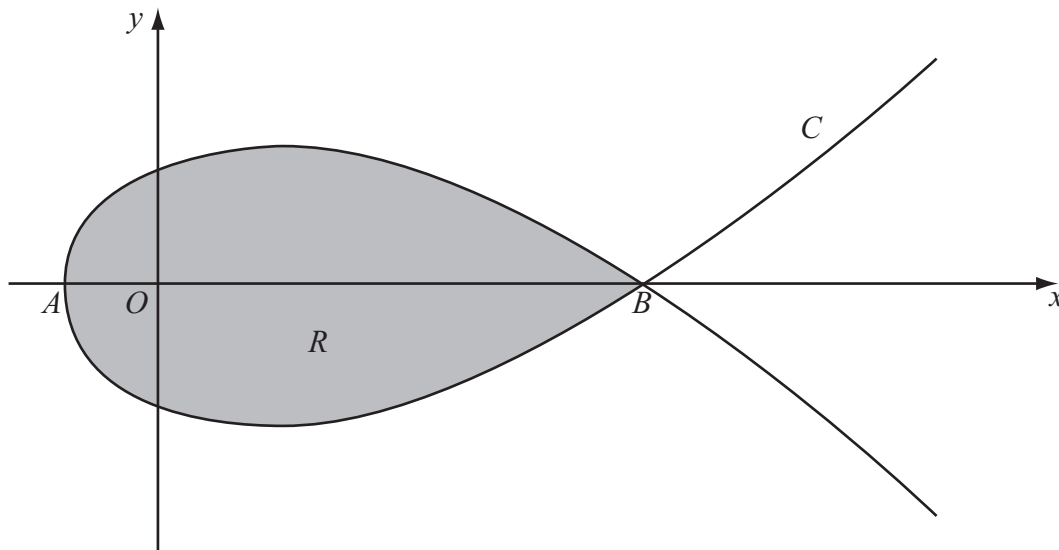


Figure 2

Figure 2 shows a sketch of the curve C with parametric equations

$$x = 5t^2 - 4, \quad y = t(9 - t^2)$$

The curve C cuts the x -axis at the points A and B .

- (a) Find the x -coordinate at the point A and the x -coordinate at the point B . (3)

The region R , as shown shaded in Figure 2, is enclosed by the loop of the curve.

- (b) Use integration to find the area of R . (6)



Question Number	Scheme	Marks
Q7	<p>(a) $y = 0 \Rightarrow t(9 - t^2) = t(3 - t)(3 + t) = 0$ $t = 0, 3, -3$ Any one correct value</p> <p>At $t = 0$, $x = 5(0)^2 - 4 = -4$ Method for finding one value of x</p> <p>At $t = 3$, $x = 5(3)^2 - 4 = 41$</p> <p>(At $t = -3$, $x = 5(-3)^2 - 4 = 41$)</p> <p>At A, $x = -4$; at B, $x = 41$ Both</p>	<p>B1</p> <p>M1</p> <p>A1 (3)</p>
	<p>(b) $\frac{dx}{dt} = 10t$ Seen or implied</p> <p>$\int y dx = \int y \frac{dx}{dt} dt = \int t(9 - t^2)10t dt$</p> <p>$= \int (90t^2 - 10t^4) dt$</p> <p>$= \frac{90t^3}{3} - \frac{10t^5}{5} (+C) \quad (= 30t^3 - 2t^5 (+C))$</p> <p>$\left[\frac{90t^3}{3} - \frac{10t^5}{5} \right]_0^3 = 30 \times 3^3 - 2 \times 3^5 \quad (= 324)$</p> <p>$A = 2 \int y dx = 648 \quad (\text{units}^2)$</p>	<p>B1</p> <p>M1 A1</p> <p>A1</p> <p>M1</p> <p>A1 (6)</p> <p>[9]</p>

Question Number	Scheme	Marks
Q8	<p>(a) $\frac{dx}{du} = -2 \sin u$</p> $\int \frac{1}{x^2 \sqrt{4-x^2}} dx = \int \frac{1}{(2 \cos u)^2 \sqrt{4-(2 \cos u)^2}} \times -2 \sin u du$ $= \int \frac{-2 \sin u}{4 \cos^2 u \sqrt{4 \sin^2 u}} du \quad \text{Use of } 1 - \cos^2 u = \sin^2 u$ $= -\frac{1}{4} \int \frac{1}{\cos^2 u} du \quad \pm k \int \frac{1}{\cos^2 u} du$ $= -\frac{1}{4} \tan u (+C) \quad \pm k \tan u$ <p>$x = \sqrt{2} \Rightarrow \sqrt{2} = 2 \cos u \Rightarrow u = \frac{\pi}{4}$</p> <p>$x = 1 \Rightarrow 1 = 2 \cos u \Rightarrow u = \frac{\pi}{3}$</p> $\left[-\frac{1}{4} \tan u \right]_{\frac{\pi}{3}}^{\frac{\pi}{4}} = -\frac{1}{4} \left(\tan \frac{\pi}{4} - \tan \frac{\pi}{3} \right)$ $= -\frac{1}{4} (1 - \sqrt{3}) \quad \left(= \frac{\sqrt{3}-1}{4} \right)$	<p>B1</p> <p>M1</p> <p>M1</p> <p>M1</p> <p>M1</p> <p>M1</p> <p>A1 (7)</p>
	<p>(b) $V = \pi \int_1^{\sqrt{2}} \left(\frac{4}{x(4-x^2)^{\frac{1}{4}}} \right)^2 dx$</p> $= 16\pi \int_1^{\sqrt{2}} \frac{1}{x^2 \sqrt{4-x^2}} dx \quad 16\pi \times \text{integral in (a)}$ $= 16\pi \left(\frac{\sqrt{3}-1}{4} \right) \quad 16\pi \times \text{their answer to part (a)}$	<p>M1</p> <p>M1</p> <p>A1ft (3)</p> <p>[10]</p>