





January 2011  
Core Mathematics C4 6666  
Mark Scheme

Question Number	Scheme	Marks
1.	$\int x \sin 2x \, dx = -\frac{x \cos 2x}{2} + \int \frac{\cos 2x}{2} \, dx$ $= \dots + \frac{\sin 2x}{4}$ $\left[ \dots \right]_0^{\frac{\pi}{2}} = \frac{\pi}{4}$	<p>M1 A1 A1</p> <p>M1</p> <p>M1 A1</p> <p>[6]</p>
2.	<p>At <math>t = 3</math></p> $\frac{dI}{dt} = -16 \ln(0.5) 0.5^t$ $\frac{dI}{dt} = -16 \ln(0.5) 0.5^3$ $= -2 \ln 0.5 = \ln 4$	<p>M1 A1</p> <p>M1</p> <p>M1 A1</p> <p>[5]</p>



January 2011  
Core Mathematics C4 6666  
Mark Scheme

Question Number	Scheme	Marks
1.	$\int x \sin 2x \, dx = -\frac{x \cos 2x}{2} + \int \frac{\cos 2x}{2} \, dx$ $= \dots + \frac{\sin 2x}{4}$ $\left[ \dots \right]_0^{\frac{\pi}{2}} = \frac{\pi}{4}$	<p>M1 A1 A1</p> <p>M1</p> <p>M1 A1</p> <p>[6]</p>
2.	<p>At <math>t = 3</math></p> $\frac{dI}{dt} = -16 \ln(0.5) 0.5^t$ $\frac{dI}{dt} = -16 \ln(0.5) 0.5^3$ $= -2 \ln 0.5 = \ln 4$	<p>M1 A1</p> <p>M1</p> <p>M1 A1</p> <p>[5]</p>



Question Number	Scheme	Marks
3.  (a)	$\frac{5}{(x-1)(3x+2)} = \frac{A}{x-1} + \frac{B}{3x+2}$ $5 = A(3x+2) + B(x-1)$ $x \rightarrow 1 \quad 5 = 5A \Rightarrow A = 1$ $x \rightarrow -\frac{2}{3} \quad 5 = -\frac{5}{3}B \Rightarrow B = -3$	M1 A1 A1 (3)
(b)	$\int \frac{5}{(x-1)(3x+2)} dx = \int \left( \frac{1}{x-1} - \frac{3}{3x+2} \right) dx$ $= \ln(x-1) - \ln(3x+2) \quad (+C) \quad \text{ft constants}$	M1 A1ft A1ft (3)
(c)	$\int \frac{5}{(x-1)(3x+2)} dx = \int \left( \frac{1}{y} \right) dy$ $\ln(x-1) - \ln(3x+2) = \ln y \quad (+C)$ $y = \frac{K(x-1)}{3x+2} \quad \text{depends on first two Ms in (c)}$ <p>Using (2, 8)</p> $8 = \frac{K}{8} \quad \text{depends on first two Ms in (c)}$ $y = \frac{64(x-1)}{3x+2}$	M1 M1 A1 M1 dep M1 dep A1 (6)  [12]





Question Number	Scheme	Marks
4. (a)	$\overrightarrow{AB} = -2\mathbf{i} + 2\mathbf{j} - \mathbf{k} - (\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}) = -3\mathbf{i} + 5\mathbf{j} - 3\mathbf{k}$	M1 A1 (2)
(b)	$\mathbf{r} = \mathbf{i} - 3\mathbf{j} + 2\mathbf{k} + \lambda(-3\mathbf{i} + 5\mathbf{j} - 3\mathbf{k})$ or $\mathbf{r} = -2\mathbf{i} + 2\mathbf{j} - \mathbf{k} + \lambda(-3\mathbf{i} + 5\mathbf{j} - 3\mathbf{k})$	M1 A1ft (2)
(c)	$\overrightarrow{AC} = 2\mathbf{i} + p\mathbf{j} - 4\mathbf{k} - (\mathbf{i} - 3\mathbf{j} + 2\mathbf{k})$ $= \mathbf{i} + (p+3)\mathbf{j} - 6\mathbf{k}$ or $\overrightarrow{CA}$ $\overrightarrow{AC} \cdot \overrightarrow{AB} = \begin{pmatrix} 1 \\ p+3 \\ -6 \end{pmatrix} \cdot \begin{pmatrix} -3 \\ 5 \\ -3 \end{pmatrix} = 0$ $-3 + 5p + 15 + 18 = 0$ Leading to $p = -6$	B1 M1 M1 A1 (4)
(d)	$AC^2 = (2-1)^2 + (-6+3)^2 + (-4-2)^2 (= 46)$ $AC = \sqrt{46}$	M1 A1 accept awrt 6.8 (2) [10]



Question Number	Scheme	Marks
5.  (a)	$(2-3x)^{-2} = 2^{-2} \left(1 - \frac{3}{2}x\right)^{-2}$ $\left(1 - \frac{3}{2}x\right)^{-2} = 1 + (-2)\left(-\frac{3}{2}x\right) + \frac{-2 \cdot -3}{1 \cdot 2} \left(-\frac{3}{2}x\right)^2 + \frac{-2 \cdot -3 \cdot -4}{1 \cdot 2 \cdot 3} \left(-\frac{3}{2}x\right)^3 + \dots$ $= 1 + 3x + \frac{27}{4}x^2 + \frac{27}{2}x^3 + \dots$ $(2-3x)^{-2} = \frac{1}{4} + \frac{3}{4}x + \frac{27}{16}x^2 + \frac{27}{8}x^3 + \dots$	B1  M1 A1  M1 A1 (5)
(b)	$f(x) = (a+bx) \left( \frac{1}{4} + \frac{3}{4}x + \frac{27}{16}x^2 + \frac{27}{8}x^3 + \dots \right)$ <p>Coefficient of <math>x</math>; <math>\frac{3a}{4} + \frac{b}{4} = 0 \quad (3a+b=0)</math></p> <p>Coefficient of <math>x^2</math>; <math>\frac{27a}{16} + \frac{3b}{4} = \frac{9}{16} \quad (9a+4b=3)</math> A1 either correct</p> <p>Leading to <math>a = -1, b = 3</math></p>	M1 M1 A1 M1 A1 (5)
(c)	<p>Coefficient of <math>x^3</math> is <math>\frac{27a}{8} + \frac{27b}{16} = \frac{27}{8} \times (-1) + \frac{27}{16} \times 3</math></p> $= \frac{27}{16}$ <p style="text-align: right;">cao</p>	M1 A1ft A1 (3)  [13]



Question Number	Scheme	Marks
6.  (a)	$\frac{dx}{dt} = \frac{1}{t}, \quad \frac{dy}{dt} = 2t$ $\frac{dy}{dx} = 2t^2$ <p>Using <math>mm' = -1</math>, at <math>t = 3</math></p> $m' = -\frac{1}{18}$ $y - 7 = -\frac{1}{18}(x - \ln 3)$	<p>M1 A1</p> <p>M1 A1</p> <p>M1 A1 (6)</p>
(b)	$x = \ln t \Rightarrow t = e^x$ $y = e^{2x} - 2$	<p>B1</p> <p>M1 A1 (3)</p>
(c)	$V = \pi \int (e^{2x} - 2)^2 dx$ $\int (e^{2x} - 2)^2 dx = \int (e^{4x} - 4e^{2x} + 4) dx$ $= \frac{e^{4x}}{4} - \frac{4e^{2x}}{2} + 4x$ $\pi \left[ \frac{e^{4x}}{4} - \frac{4e^{2x}}{2} + 4x \right]_{\ln 2}^{\ln 4} = \pi [(64 - 32 + 4 \ln 4) - (4 - 8 + 4 \ln 2)]$ $= \pi(36 + 4 \ln 2)$	<p>M1</p> <p>M1</p> <p>M1 A1</p> <p>M1</p> <p>A1</p> <p>(6)</p> <p>[15]</p>
<p><i>Alternative to (c) using parameters</i></p> $V = \pi \int (t^2 - 2)^2 \frac{dx}{dt} dt$ $\int \left( (t^2 - 2)^2 \times \frac{1}{t} \right) dt = \int \left( t^3 - 4t + \frac{4}{t} \right) dt$ $= \frac{t^4}{4} - 2t^2 + 4 \ln t$ <p>The limits are <math>t = 2</math> and <math>t = 4</math></p> $\pi \left[ \frac{t^4}{4} - 2t^2 + 4 \ln t \right]_2^4 = \pi [(64 - 32 + 4 \ln 4) - (4 - 8 + 4 \ln 2)]$ $= \pi(36 + 4 \ln 2)$		<p>M1</p> <p>M1</p> <p>M1 A1</p> <p>M1</p> <p>A1</p> <p>(6)</p>



Question Number	Scheme	Marks
7. (a)	$x = 3 \Rightarrow y = 0.1847$ $x = 5 \Rightarrow y = 0.1667$	awrt B1 awrt or $\frac{1}{6}$ B1 (2)
(b)	$I \approx \frac{1}{2} [0.2 + 0.1667 + 2(0.1847 + 0.1745)]$ $\approx 0.543$	B1 M1 A1ft 0.542 or 0.543 A1 (4)
(c)	$\frac{dx}{du} = 2(u - 4)$ $\int \frac{1}{4 + \sqrt{(x-1)}} dx = \int \frac{1}{u} \times 2(u - 4) du$ $= \int \left( 2 - \frac{8}{u} \right) du$ $= 2u - 8 \ln u$ $x = 2 \Rightarrow u = 5, \quad x = 5 \Rightarrow u = 6$ $[2u - 8 \ln u]_5^6 = (12 - 8 \ln 6) - (10 - 8 \ln 5)$ $= 2 + 8 \ln \left( \frac{5}{6} \right)$	B1 M1 A1 M1 A1 B1 M1 A1 (8) [14]