Centre No.						Paper Reference	Surname	Initial(s)
Candidate No.						6 6 6 6 / 0 1	Signature	

Paper Reference(s)

6666/01

Edexcel GCE

Core Mathematics C4

Advanced

Wednesday 25 January 2012 – Afternoon

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
1	
2	
3	
4	
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7	
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Total	

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer for each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.
There are 28 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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1. The curve C has the equation $2x + 3y^2 + 3x^2 y = 4x^2$.
The point P on the curve has coordinates $(-1, 1)$.

(a) Find the gradient of the curve at P.

(5)

(b) Hence find the equation of the normal to C at P, giving your answer in the form $ax+by+c = 0$, where a, b and c are integers.

(3)

January 2012
6666 Core Mathematics C4
Mark Scheme

Question Number	Scheme	Marks
1. (a)	$\left\{\cancel{\frac{dy}{dx}} \times\right\}$ $\quad \underline{2+6y\frac{dy}{dx}} + \left(\underline{\underline{6xy+3x^2\frac{dy}{dx}}}\right) = \underline{8x}$	M1 A1 B1
	$\left\{\frac{dy}{dx} = \frac{8x-2-6xy}{6y+3x^2}\right\}$ *not necessarily required.*	
	At $P(-1,1)$, $\mathrm{m}(\mathbf{T}) = \frac{dy}{dx} = \frac{8(-1)-2-6(-1)(1)}{6(1)+3(-1)^2} = -\frac{4}{9}$	dM1 A1 **cso**
		[5]
(b)	So, $\mathrm{m}(\mathbf{N}) = \frac{-1}{-\frac{4}{9}}\ \left\{= \frac{9}{4}\right\}$	M1
	N: $y - 1 = \frac{9}{4}(x+1)$	M1
	N: $9x - 4y + 13 = 0$	A1
		[3]
		8
(a)	**M1**: Differentiates implicitly to include either $\pm ky\frac{dy}{dx}$ or $3x^2\frac{dy}{dx}$. (Ignore $\left(\frac{dy}{dx} = \right)$).	
	A1: $(2x+3y^2) \to \left(2+6y\frac{dy}{dx}\right)$ and $(4x^2 \to \underline{8x})$. **Note:** If an extra "sixth" term appears then award A0.	
	B1: $6xy + 3x^2\frac{dy}{dx}$.	
	dM1: Substituting $x=-1$ and $y=1$ into an equation involving $\frac{dy}{dx}$. Allow this mark if either the numerator or denominator of $\frac{dy}{dx} = \frac{8x-2-6xy}{6y+3x^2}$ is substituted into or evaluated correctly.	
	If it is clear, however, that the candidate is intending to substitute $x=1$ and $y=-1$, then award M0.	
	Candidates who substitute $x=1$ and $y=-1$, will usually achieve $\mathrm{m}(\mathbf{T}) = -4$	
	Note that this mark is dependent on the previous method mark being awarded.	
	A1: For $-\frac{4}{9}$ or $-\frac{8}{18}$ or $-0.\dot{4}$ or awrt -0.44	
	If the candidate's solution is not completely correct, then do not give this mark.	
(b)	**M1:** Applies $\mathrm{m}(\mathbf{N}) = -\frac{1}{\text{their } \mathrm{m}(\mathbf{T})}$.	
	M1: Uses $y-1=(m_N)(x--1)$ or finds c using $x=-1$ and $y=1$ and uses $y=(m_N)x+"c"$, Where $m_N = -\frac{1}{\text{their } \mathrm{m}(\mathbf{T})}$ or $m_N = \frac{1}{\text{their } \mathrm{m}(\mathbf{T})}$ or $m_N = -\text{their } \mathrm{m}(\mathbf{T})$.	
	A1: $9x-4y+13=0$ or $-9x+4y-13=0$ or $4y-9x-13=0$ or $18x-8y+26=0$ etc. Must be " = 0". So do not allow $9x+13=4y$ etc.	
	Note: $m_N = -\left(\frac{6y+3x^2}{8x-2-6xy}\right)$ is M0M0 unless a numerical value is then found for m_N.	

Alternative method for part (a): Differentiating with respect to y

$$\left\{\cancel{\frac{\mathrm{d}x}{\mathrm{d}y}} \times\right\} \quad \underline{2\frac{\mathrm{d}x}{\mathrm{d}y} + 6y} + \left(\underline{\underline{6xy\frac{\mathrm{d}x}{\mathrm{d}y} + 3x^2}}\right) = \underline{8x\frac{\mathrm{d}x}{\mathrm{d}y}}$$

M1: Differentiates implicitly to include either $2\frac{\mathrm{d}x}{\mathrm{d}y}$ or $6xy\frac{\mathrm{d}x}{\mathrm{d}y}$ or $\pm kx\frac{\mathrm{d}x}{\mathrm{d}y}$. (Ignore $\left(\frac{\mathrm{d}x}{\mathrm{d}y} =\right)$).

A1: $\left(2x+3y^2\right) \rightarrow \left(\underline{2\frac{\mathrm{d}x}{\mathrm{d}y} + 6y}\right)$ and $\left(4x^2 \rightarrow \underline{8x\frac{\mathrm{d}x}{\mathrm{d}y}}\right)$. **Note:** If an extra "sixth" term appears then award A0.

B1: $6xy + 3x^2\frac{\mathrm{d}y}{\mathrm{d}x}$.

dM1: Substituting $x = -1$ and $y = 1$ into an equation involving $\frac{\mathrm{d}x}{\mathrm{d}y}$ or $\frac{\mathrm{d}y}{\mathrm{d}x}$. Allow this mark if either the numerator or denominator of $\frac{\mathrm{d}x}{\mathrm{d}y} = \frac{6y+3x^2}{8x-2-6xy}$ is substituted into or evaluated correctly.

If it is clear, however, that the candidate is intending to substitute $x = 1$ and $y = -1$, then award M0.

Candidates who substitute $x = 1$ and $y = -1$, will usually achieve $\mathrm{m}(\mathbf{T}) = -4$

Note that this mark is dependent on the previous method mark being awarded.

A1: For $-\frac{4}{9}$ or $-\frac{8}{18}$ or $-0.\dot{4}$ or awrt -0.44

If the candidate's solution is not completely correct, then do not give this mark.

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2. (a) Use integration by parts to find $\int x \sin 3x \, dx$.

(3)

(b) Using your answer to part (a), find $\int x^2 \cos 3x \, dx$.

(3)

Question Number	Scheme	Marks
2. (a)	$\int x\sin 3x \, \mathrm{d}x = -\frac{1}{3}x\cos 3x - \int -\frac{1}{3}\cos 3x \, \{\mathrm{d}x\}$	M1 A1
	$= -\frac{1}{3}x\cos 3x + \frac{1}{9}\sin 3x \, \{+c\}$	A1
		[3]
(b)	$\int x^2\cos 3x \, \mathrm{d}x = \frac{1}{3}x^2\sin 3x - \int \frac{2}{3}x\sin 3x \, \{\mathrm{d}x\}$	M1 A1
	$= \frac{1}{3}x^2\sin 3x - \frac{2}{3}\left(-\frac{1}{3}x\cos 3x + \frac{1}{9}\sin 3x\right) \, \{+c\}$	A1 **isw**
	$\left\{= \frac{1}{3}x^2\sin 3x + \frac{2}{9}x\cos 3x - \frac{2}{27}\sin 3x \quad \{+c\}\right\}$ *Ignore subsequent working*	**[3]**
		6

(a) **M1:** Use of 'integration by parts' formula $uv - \int vu'$ (whether stated or not stated) in the **correct direction**, where $u = x \to u' = 1$ and $v' = \sin 3x \to v = k\cos 3x$ (seen or implied), where k is a positive or negative constant. (Allow $k = 1$).

This means that the candidate must achieve $x(k\cos 3x) - \int (k\cos 3x)$, where k is a consistent constant.

If x^2 appears after the integral, this would imply that the candidate is applying integration by parts in the wrong direction, so M0.

A1: $-\frac{1}{3}x\cos 3x - \int -\frac{1}{3}\cos 3x \, \{\mathrm{d}x\}$. Can be un-simplified. Ignore the $\{\mathrm{d}x\}$.

A1: $-\frac{1}{3}x\cos 3x + \frac{1}{9}\sin 3x$ with/without + c. Can be un-simplified.

(b) **M1:** Use of 'integration by parts' formula $uv - \int vu'$ (whether stated or not stated) in the **correct direction**, where $u = x^2 \to u' = 2x$ or x and $v' = \cos 3x \to v = \lambda\sin 3x$ (seen or implied), where λ is a positive or negative constant. (Allow $\lambda = 1$).

This means that the candidate must achieve $x^2(\lambda\sin 3x) - \int 2x(\lambda\sin 3x)$, where $u' = 2x$

or $x^2(\lambda\sin 3x) - \int x(\lambda\sin 3x)$, where $u' = x$.

If x^3 appears after the integral, this would imply that the candidate is applying integration by parts in the wrong direction, so M0.

A1: $\frac{1}{3}x^2\sin 3x - \int \frac{2}{3}x\sin 3x \, \{\mathrm{d}x\}$. Can be un-simplified. Ignore the $\{\mathrm{d}x\}$.

A1: $\frac{1}{3}x^2\sin 3x - \frac{2}{3}\left(-\frac{1}{3}x\cos 3x + \frac{1}{9}\sin 3x\right)$ with/without + c, can be un-simplified.

You can ignore subsequent working here.

Special Case: If the candidate scores the first two marks of M1A1 in part (b), then you can award the final A1 as a follow through for $\frac{1}{3}x^2\sin 3x - \frac{2}{3}(\text{their follow through part(a) answer})$.

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3. (a) Expand

$$\frac{1}{(2-5x)^2}, \quad |x| < \frac{2}{5}$$

in ascending powers of x, up to and including the term in x^2, giving each term as a simplified fraction.

(5)

Given that the binomial expansion of $\dfrac{2+kx}{(2-5x)^2}$, $|x| < \dfrac{2}{5}$, is

$$\frac{1}{2} + \frac{7}{4}x + Ax^2 + \ldots$$

(b) find the value of the constant k,

(2)

(c) find the value of the constant A.

(2)

Question Number	Scheme		Marks
3. (a)	$\dfrac{1}{(2-5x)^2} = (2-5x)^{-2} = \underline{(2)^{-2}}\left(1-\dfrac{5x}{2}\right)^{-2} = \underline{\dfrac{1}{4}}\left(1-\dfrac{5x}{2}\right)^{-2}$	$\underline{(2)^{-2}}$ or $\underline{\dfrac{1}{4}}$	<u>B1</u>
	$=\left\{\dfrac{1}{4}\right\}\left[1+(-2)(**x)+\dfrac{(-2)(-3)}{2!}(**x)^2+\dots\right]$	see notes	M1 A1ft
	$=\left\{\dfrac{1}{4}\right\}\left[\underline{1+(-2)\left(-\dfrac{5x}{2}\right)+\dfrac{(-2)(-3)}{2!}\left(-\dfrac{5x}{2}\right)^2+\dots}\right]$		
	$=\dfrac{1}{4}\left[1+5x;+\dfrac{75}{4}x^2+\dots\right]$	See notes below!	
	$=\dfrac{1}{4}+\dfrac{5}{4}x;+\dfrac{75}{16}x^2+\dots$		A1; A1
			[5]
(b)	$\left\{\dfrac{2+kx}{(2-5x)^2}\right\} = (2+kx)\left(\dfrac{1}{4}+\dfrac{5}{4}x+\left\{\dfrac{75}{16}x^2+\dots\right\}\right)$	*Can be implied by later work even in part (c).*	M1
	x terms: $\dfrac{2(5x)}{4}+\dfrac{kx}{4}=\dfrac{7x}{4}$		
	giving, $10+k=7 \Rightarrow \underline{k=-3}$	$\underline{k=-3}$	A1
			[2]
(c)	x^2 terms: $\dfrac{150x^2}{16}+\dfrac{5kx^2}{4}$		M1
	So, $A=\dfrac{75}{8}+\dfrac{5(-3)}{4} = \dfrac{75}{8}-\dfrac{15}{4} = \underline{\dfrac{45}{8}}$	$\underline{\dfrac{45}{8}}$ or $\underline{5\dfrac{5}{8}}$ or $\underline{5.625}$	A1
			[2] **9**

(a)

<u>**B1**</u>: $\underline{(2)^{-2}}$ or $\underline{\dfrac{1}{4}}$ outside brackets or $\underline{\dfrac{1}{4}}$ as candidate's constant term in their binomial expansion.

M1: Expands to give a simplified or an un-simplified,

$1+(-2)(**x)$ or $(-2)(**x)+\dfrac{(-2)(-3)}{2!}(**x)^2$ or $1+\dots\dots+\dfrac{(-2)(-3)}{2!}(**x)^2$, **where** $**\neq 1$**.**

A1: A correct simplified or an un-simplified $1+(-2)(**x)+\dfrac{(-2)(-3)}{2!}(**x)^2$ expansion with candidate's follow through $(**x)$. Note that $(**x)$ must be consistent.

You would award B1M1A0 for $=\dfrac{1}{4}\left[\underline{1+(-2)\left(-\dfrac{5x}{2}\right)+\dfrac{(-2)(-3)}{2!}(-5x)^2+\dots}\right]$ because ** is not consistent.

Invisible brackets $\left\{\dfrac{1}{4}\right\}\left[\underline{1+(-2)\left(-\dfrac{5x}{2}\right)+\dfrac{(-2)(-3)}{2!}\left(-\dfrac{5x^2}{2}\right)+\dots}\right]$ is M1A0 unless recovered.

A1: For $\dfrac{1}{4}+\dfrac{5}{4}x$ **(simplified fractions)** or Also allow $0.25+1.25x$ or $\dfrac{1}{4}+1\dfrac{1}{4}x$**.**

Allow Special Case A1 for either SC: $\dfrac{1}{4}[1+5x;\dots]$ or **SC**: $K\left[1+5x+\dfrac{75}{4}x^2+\dots\right]$.

A1: Accept only $\dfrac{75}{16}x^2$ or $4\dfrac{11}{16}x^2$ or $4.6875x^2$

<u>**Alternative method:**</u> Candidates can apply an alternative form of the binomial expansion. (See next page).

3. (b)	**M1:** Candidate writes down $(2 + kx)\big($their part (a) answer, at least up to the term in x.$\big)$ $(2+kx)\left(\frac{1}{4}+\frac{5}{4}x+\ ...\right)$ or $(2+kx)\left(\frac{1}{4}+\frac{5}{4}x+\frac{75}{16}x^2+...\right)$ are fine. This mark can also be implied by candidate multiplying out to find two terms (or coefficients) in x. **A1:** $k = -3$
(c)	**M1:** Multiplies out their $(2+kx)\left(\frac{1}{4}+\frac{5}{4}x+\frac{75}{16}x^2+...\right)$ to give **exactly** two terms (or coefficients) in x^2 and attempts to find A using a numerical value of k. **A1:** Either $\frac{45}{8}$ or $5\frac{5}{8}$ or 5.625 **Note:** $\frac{45}{8}x^2$ is A0. *Alternative method for part (a)* $(2-5x)^{-2} = (2)^{-2}+(-2)(2)^{-3}(-5x);+\frac{(-2)(-3)}{2!}(2)^{-4}(-5x)^2$ **B1:** $\frac{1}{4}$ or $(2)^{-2}$, **M1:** Any two of three (un-simplified) terms correct. **A1:** All three (un-simplified) terms correct. **A1:** $\frac{1}{4}+\frac{5}{4}x$ **A1:** $\frac{75}{16}x^2$ **Note:** The terms in C need to be evaluated, so ${}^{-2}C_0(2)^{-2}+{}^{-2}C_1(2)^{-3}(-5x);+{}^{-2}C_2(2)^{-4}(-5x)^2$ without further working is B0M0A0. *Alternative method for parts (b) and (c)* $(2+kx) = (2-5x)^2\left(\frac{1}{2}+\frac{7}{4}x+Ax^2+...\right)$ $(2+kx) = (4-20x+25x^2)\left(\frac{1}{2}+\frac{7}{4}x+Ax^2+...\right)$ $(2+kx) = 2+(7x-10x)+\left(4Ax^2-35x^2+\frac{25}{2}x^2\right)$ Equate x terms: $\underline{k=-3}$ Equate x^2 terms: $0 = 4A-35+\frac{25}{2} \Rightarrow 4A = \frac{45}{2} \Rightarrow \underline{A=\frac{45}{8}}$
(b)	**M1:** For $(2+kx) = (4\pm\lambda x+25x^2)\left(\frac{1}{2}+\frac{7}{4}x+Ax^2+...\right)$, where $\lambda \neq 0$ **A1:** $k = -3$
(c)	**M1:** Multiplies out to obtain three x^2 terms/coefficients, equates to 0 and attempts to find A. **A1:** Either $\frac{45}{8}$ or $5\frac{5}{8}$ or 5.625 **Note:** $\frac{45}{8}x^2$ is A0.

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4.

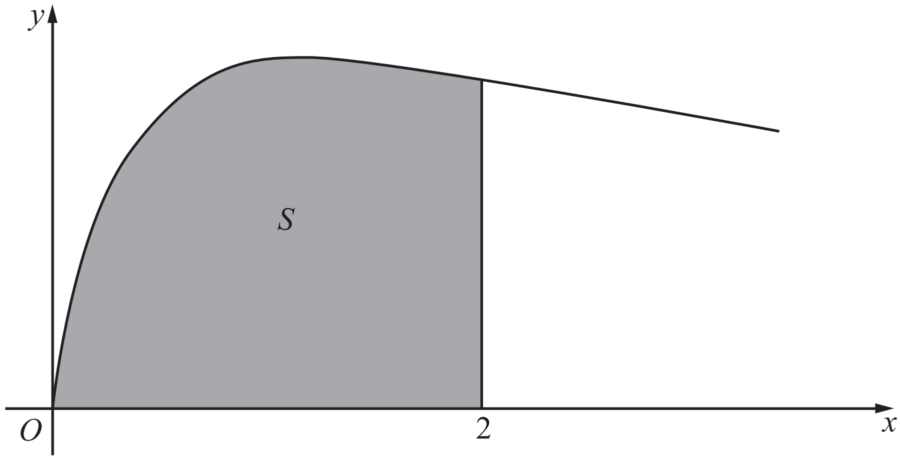

Figure 1

Figure 1 shows the curve with equation

$$y = \sqrt{\left(\frac{2x}{3x^2 + 4}\right)}, \quad x \geqslant 0$$

The finite region S, shown shaded in Figure 1, is bounded by the curve, the x-axis and the line $x = 2$

The region S is rotated 360° about the x-axis.

Use integration to find the exact value of the volume of the solid generated, giving your answer in the form $k \ln a$, where k and a are constants.

(5)

Question Number	Scheme		Marks
4.	Volume $= \underline{\pi \int_0^2 \left(\sqrt{\left(\frac{2x}{3x^2+4}\right)}\right)^2 \, dx}$	Use of $V = \underline{\pi \int y^2} \, dx$.	<u>B1</u>
	$= (\pi)\left[\frac{1}{3}\ln\left(3x^2+4\right)\right]_0^2$	$\pm k\ln\left(3x^2+4\right)$	M1
		$\frac{1}{3}\ln\left(3x^2+4\right)$	A1
	$= (\pi)\left[\left(\frac{1}{3}\ln 16\right) - \left(\frac{1}{3}\ln 4\right)\right]$	Substitutes limits of 2 and 0 and subtracts the correct way round.	dM1
	So Volume $= \underline{\frac{1}{3}\pi\ln 4}$	$\underline{\frac{1}{3}\pi\ln 4}$ or $\underline{\frac{2}{3}\pi\ln 2}$	A1 oe isw
			[5] 5

NOTE: π is required for the B1 mark and the final A1 mark. It is not required for the 3 intermediate marks.

B1: For applying $\pi\int y^2$. Ignore limits and dx. This can be implied by later working, but the pi and $\int \frac{2x}{3x^2+4}$ must appear on one line somewhere in the candidate's working.

B1 can also be implied by a correct final answer. **Note**: $\pi\left(\int y\right)^2$ would be B0.

<u>Working in x</u>

M1: For $\pm k\ln\left(3x^2+4\right)$ or $\pm k\ln\left(x^2+\frac{4}{3}\right)$ where k is a constant and k can be 1.

Note: M0 for $\pm k\,x\ln\left(3x^2+4\right)$.

Note: M1 can also be given for $\pm k\ln\left(p\left(3x^2+4\right)\right)$, where k and p are constants and k can be 1.

A1: For $\frac{1}{3}\ln\left(3x^2+4\right)$ or $\frac{1}{3}\ln\left(\frac{1}{3}\left(3x^2+4\right)\right)$ or $\frac{1}{3}\ln\left(x^2+\frac{4}{3}\right)$ or $\frac{1}{3}\ln\left(p(3x^2+4)\right)$.

You may allow M1 A1 for $\frac{1}{3}\left(\frac{x}{x}\right)\ln\left(3x^2+4\right)$ or $\frac{1}{3}\left(\frac{2x}{6x}\right)\ln\left(3x^2+4\right)$

dM1: Substitutes limits of 2 and 0 and subtracts the correct way round. Working in decimals is fine for dM1.

A1: For either $\frac{1}{3}\pi\ln 4$, $\frac{1}{3}\ln 4^{\pi}$, $\frac{2}{3}\pi\ln 2$, $\pi\ln 4^{\frac{1}{3}}$, $\pi\ln 2^{\frac{2}{3}}$, $\frac{1}{3}\pi\ln\left(\frac{16}{4}\right)$, $2\pi\ln\left(\frac{16^{\frac{1}{6}}}{4^{\frac{1}{6}}}\right)$, etc.

Note: $\frac{1}{3}\pi(\ln 16 - \ln 4)$ would be A0.

<u>Working in u:</u> where $u = 3x^2+4$,

M1: For $\pm k\ln u$ where k is a constant and k can be 1.

Note: M1 can also be given for $\pm k\ln\left(pu\right)$, where k and p are constants and k can be 1.

A1: For $\frac{1}{3}\ln u$ or $\frac{1}{3}\ln 3u$ or $\frac{1}{3}\ln pu$.

dM1: Substitutes limits of 16 and 4 in u or limits of 2 and 0 in x and subtracts the correct way round.

A1: As above!

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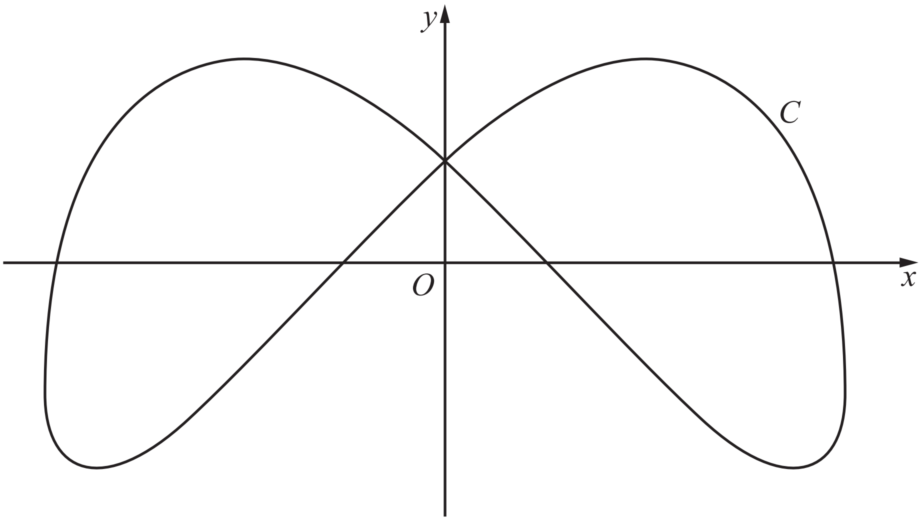

Figure 2

Figure 2 shows a sketch of the curve C with parametric equations

$$x = 4\sin\left(t + \frac{\pi}{6}\right), \qquad y = 3\cos 2t, \qquad 0 \leqslant t < 2\pi$$

(a) Find an expression for $\frac{dy}{dx}$ in terms of t.

(3)

(b) Find the coordinates of all the points on C where $\frac{dy}{dx} = 0$

(5)

Question Number	Scheme	Marks
5.	$x = 4\sin\left(t + \frac{\pi}{6}\right), \quad y = 3\cos 2t, \quad 0 \leqslant t < 2\pi$	
(a)	$\frac{dx}{dt} = 4\cos\left(t + \frac{\pi}{6}\right), \quad \frac{dy}{dt} = -6\sin 2t$	B1 B1
	So, $\frac{dy}{dx} = \underline{\frac{-6\sin 2t}{4\cos\left(t + \frac{\pi}{6}\right)}}$	B1$\sqrt{}$ oe
		[3]
(b)	$\left\{\frac{dy}{dx} = 0 \Rightarrow\right\} -6\sin 2t = 0$	M1 oe
	@ $t = 0$, $x = 4\sin\left(\frac{\pi}{6}\right) = 2$, $y = 3\cos 0 = 3 \quad \rightarrow (2, 3)$	M1
	@ $t = \frac{\pi}{2}$, $x = 4\sin\left(\frac{2\pi}{3}\right) = \frac{4\sqrt{3}}{2}$, $y = 3\cos\pi = -3 \rightarrow (2\sqrt{3}, -3)$	
	@ $t = \pi$, $x = 4\sin\left(\frac{7\pi}{6}\right) = -2$, $y = 3\cos 2\pi = 3 \rightarrow (-2, 3)$	
	@ $t = \frac{3\pi}{2}$, $x = 4\sin\left(\frac{5\pi}{3}\right) = \frac{4(-\sqrt{3})}{2}$, $y = 3\cos 3\pi = -3 \rightarrow (-2\sqrt{3}, -3)$	A1A1A1
		[5] **8**
(a)	**B1:** Either one of $\frac{dx}{dt} = 4\cos\left(t + \frac{\pi}{6}\right)$ or $\frac{dy}{dt} = -6\sin 2t$. They do not have to be simplified. **B1:** Both $\frac{dx}{dt}$ and $\frac{dy}{dt}$ correct. They do not have to be simplified. Any or both of the first two marks can be implied. Don't worry too much about their notation for the first two B1 marks. **B1:** Their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$ or their $\frac{dy}{dt} \times \frac{1}{\text{their}\left(\frac{dx}{dt}\right)}$. **Note:** This is a follow through mark. <u>*Alternative differentiation in part (a)*</u> $x = 2\sqrt{3}\sin t + 2\cos t \Rightarrow \frac{dx}{dt} = 2\sqrt{3}\cos t - 2\sin t$ $y = 3(2\cos^2 t - 1) \Rightarrow \frac{dy}{dt} = 3(-4\cos t\sin t)$ or $y = 3\cos^2 t - 3\sin^2 t \Rightarrow \frac{dy}{dt} = -6\cos t\sin t - 6\sin t\cos t$ or $y = 3(1 - 2\sin^2 t) \Rightarrow \frac{dy}{dt} = 3(-4\cos t\sin t)$	

5. (b)	**M1:** Candidate sets their numerator from part (a) or their $\frac{\mathrm{d}y}{\mathrm{d}t}$ equal to 0. Note that their numerator must be a trig function. Ignore $\frac{\mathrm{d}x}{\mathrm{d}t}$ equal to 0 at this stage. **M1:** Candidate substitutes a found value of t, to attempt to find either one of x or y. The first two method marks can be implied by ONE correct set of coordinates for $(x,\ y)$ or $(y,\ x)$ interchanged. A correct point coming from NO WORKING can be awarded M1M1. **A1:** At least TWO sets of coordinates. **A1:** At least THREE sets of coordinates. **A1:** ONLY FOUR correct sets of coordinates. If there are more than 4 sets of coordinates then award A0. **Note:** Candidate can use the diagram's symmetry to write down some of their coordinates. **Note:** When $x = 4\sin\left(\frac{\pi}{6}\right) = 2$, $y = 3\cos 0 = 3$ is acceptable for a pair of coordinates. Also it is fine for candidates to display their coordinates on a table of values. **Note:** The coordinates must be exact for the accuracy marks. Ie (3.46..., -3) or (-3.46..., -3) is A0. **Note:** $\frac{\mathrm{d}y}{\mathrm{d}x} = 0 \Rightarrow \sin t = 0$ ONLY is fine for the first M1, and potentially the following M1A1A0A0. **Note:** $\frac{\mathrm{d}y}{\mathrm{d}x} = 0 \Rightarrow \cos t = 0$ ONLY is fine for the first M1 and potentially the following M1A1A0A0. **Note:** $\frac{\mathrm{d}y}{\mathrm{d}x} = 0 \Rightarrow \sin t = 0\ \&\ \cos t = 0$ has the potential to achieve all five marks. **Note:** It is possible for a candidate to gain full marks in part (b) if they make sign errors in part (a). *(b) An alternative method for finding the coordinates of the two maximum points.* Some candidates may use $y = 3\cos 2t$ to write down that the y-coordinate of a maximum point is 3. They will then deduce that $t = 0$ or π and proceed to find the x-coordinate of their maximum point. These candidates will receive no credit until they attempt to find one of the x-coordinates for the maximum point. **M1M1:** Candidate states $y = 3$ and attempts to substitute $t = 0$ or π into $x = 4\sin\left(t + \frac{\pi}{6}\right)$. M1M1 can be implied by candidate stating either $(2, 3)$ or $(2, -3)$. **Note:** these marks can only be awarded together for a candidate using this method. **A1:** For both $(2, 3)$ or $(-2, 3)$. **A0A0**: Candidate cannot achieve the final two marks by using this method. They can, however, achieve these marks by subsequently solving their numerator equal to 0.

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6.

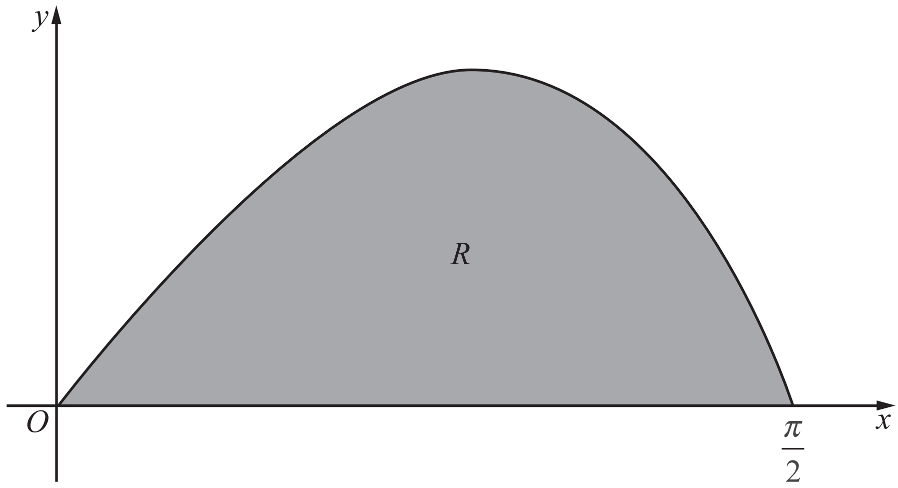

Figure 3

Figure 3 shows a sketch of the curve with equation $y = \frac{2\sin 2x}{(1+\cos x)}$, $0 \leqslant x \leqslant \frac{\pi}{2}$.

The finite region R, shown shaded in Figure 3, is bounded by the curve and the x-axis.

The table below shows corresponding values of x and y for $y = \frac{2\sin 2x}{(1+\cos x)}$.

x	0	$\frac{\pi}{8}$	$\frac{\pi}{4}$	$\frac{3\pi}{8}$	$\frac{\pi}{2}$
y	0		1.17157	1.02280	0

(a) Complete the table above giving the missing value of y to 5 decimal places.
(1)

(b) Use the trapezium rule, with all the values of y in the completed table, to obtain an estimate for the area of R, giving your answer to 4 decimal places.
(3)

(c) Using the substitution $u = 1 + \cos x$, or otherwise, show that

$$\int \frac{2\sin 2x}{(1+\cos x)}\,dx = 4\ln(1+\cos x) - 4\cos x + k$$

where k is a constant.
(5)

(d) Hence calculate the error of the estimate in part (b), giving your answer to 2 significant figures.
(3)

Question Number	Scheme		Marks
6. (a)	0.73508		B1 **cao** **[1]**
(b)	Area $\approx \frac{1}{2}\times\frac{\pi}{8}$; $\times\underline{\left[0+2\left(\text{their } 0.73508+1.17157+1.02280\right)+0\right]}$		B1 <u>M1</u>
	$=\frac{\pi}{16}\times 5.8589... = 1.150392325... = 1.1504$ (4 dp)	awrt 1.1504	A1 **[3]**
(c)	$\underline{\{u=1+\cos x\} \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x}=-\sin x}$		<u>B1</u>
	$\left\{\int\frac{2\sin 2x}{(1+\cos x)}\,\mathrm{d}x=\right\}\int\frac{2(2\sin x\cos x)}{(1+\cos x)}\,\mathrm{d}x$	$\sin 2x = 2\sin x\cos x$	B1
	$=\int\frac{4(u-1)}{u}.(-1)\,\mathrm{d}u \quad \left\{= 4\int\frac{(1-u)}{u}\,\mathrm{d}u\right\}$		M1
	$= 4\int\left(\frac{1}{u}-1\right)\mathrm{d}u = 4(\ln u - u)+c$		dM1
	$= 4\ln(1+\cos x)-4(1+\cos x)+c = 4\ln(1+\cos x)-4\cos x+k$	**AG**	A1 **cso** **[5]**
(d)	$=\left[4\ln\left(1+\cos\frac{\pi}{2}\right)-4\cos\frac{\pi}{2}\right]-\left[4\ln(1+\cos 0)-4\cos 0\right]$	Applying limits $x=\frac{\pi}{2}$ and $x=0$ either way round.	M1
	$=[4\ln 1-0]-[4\ln 2-4]$		
	$= 4-4\ln 2 \ \{= 1.227411278...\}$	$\pm 4(1-\ln 2)$ or $\pm(4-4\ln 2)$ or awrt ± 1.2, ***however*** found.	A1
	Error $= \left\|(4-4\ln 2)-1.1504...\right\|$ $= 0.0770112776... = 0.077$ (2sf)	awrt ± 0.077 or awrt $\pm 6.3(\%)$	A1 **cso** **[3]**
			12

(a) **B1:** 0.73508 correct answer only. Look for this on the table or in the candidate's working.

(b) **B1**: Outside brackets $\frac{1}{2}\times\frac{\pi}{8}$ or $\frac{\pi}{16}$ or awrt 0.196

M1: <u>For structure of trapezium rule</u>$\left[\underline{\quad\ldots\ldots\ldots\ldots\quad}\right]$; (0 can be implied).

A1: anything that rounds to 1.1504

<u>Bracketing mistake</u>: **Unless the final answer implies that the calculation has been done correctly**

Award B1M0A0 for $\frac{1}{2}\times\frac{\pi}{8} + 2\left(\text{their } 0.73508+1.17157+1.02280\right)$ (nb: answer of 6.0552).

Award B1M0A0 for $\frac{1}{2}\times\frac{\pi}{8}$ $(0+0)+2\left(\text{their } 0.73508+1.17157+1.02280\right)$ (nb: answer of 5.8589).

<u>Alternative method for part (b): Adding individual trapezia</u>

$$\text{Area} \approx \frac{\pi}{8}\times\left[\frac{0+0.73508}{2}+\frac{0.73508+1.17157}{2}+\frac{1.17157+1.02280}{2}+\frac{1.02280+0}{2}\right] = 1.150392325...$$

B1: $\frac{\pi}{8}$ and a divisor of 2 on all terms inside brackets.

M1: One of first and last ordinates, two of the middle ordinates inside brackets ignoring the 2.

A1: anything that rounds to 1.1504

6. (c)	**B1:** $\frac{\mathrm{d}u}{\mathrm{d}x} = -\sin x$ or $\mathrm{d}u = -\sin x\,\mathrm{d}x$ or $\frac{\mathrm{d}x}{\mathrm{d}u} = \frac{1}{-\sin x}$ oe. **B1:** For seeing, applying or implying $\sin 2x = 2\sin x\cos x$. **M1:** After applying substitution candidate achieves $\pm k\int \frac{(u-1)}{u}(\mathrm{d}u)$ or $\pm k\int \frac{(1-u)}{u}(\mathrm{d}u)$. Allow M1 for "invisible" brackets here, eg: $\pm\int \frac{(\lambda u-1)}{u}(\mathrm{d}u)$ or $\pm\int \frac{(-\lambda+u)}{u}(\mathrm{d}u)$, where λ is a positive constant. **dM1:** An attempt to divide through each term by u and $\pm k\int\left(\frac{1}{u}-1\right)\mathrm{d}u \to \pm k(\ln u - u)$ with/without $+c$. Note that this mark is dependent on the previous M1 mark being awarded. **<u>Alternative method:</u>** Candidate can also gain this mark for applying integration by parts followed by a correct method for integrating $\ln u$. (See below). **A1:** Correctly combines their $+c$ and $"-4"$ together to give $\underline{4\ln(1+\cos x) - 4\cos x + k}$ As a minimum candidate must write either $4\ln(1+\cos x) - 4(1+\cos x) + c \to 4\ln(1+\cos x) - 4\cos x + k$ or $4\ln(1+\cos x) - 4(1+\cos x) + k \to 4\ln(1+\cos x) - 4\cos x + k$ **Note**: that this mark is also for a correct solution only. **Note**: those candidates who attempt to find the value of k will usually achieve A0.
(d)	**M1:** Substitutes limits of $x = \frac{\pi}{2}$ and $x = 0$ into $\{4\ln(1+\cos x) - 4\cos x\}$ or their answer from part (c) and subtracts the either way round. **Note** that: $\left[4\ln\left(1+\cos\frac{\pi}{2}\right) - 4\cos\frac{\pi}{2}\right] - [0]$ is M0. **A1:** $4(1-\ln 2)$ or $4 - 4\ln 2$ or awrt 1.2, however found. This mark can be implied by the final answer of either awrt ± 0.077 or awrt ± 6.3 **A1:** For either awrt ± 0.077 or awrt ± 6.3 (for percentage error). Note this mark is for a **correct solution only**. Therefore if there if a candidate substitutes limits the incorrect way round and final achieves (usually fudges) the final correct answer then this mark can be withheld. Note that awrt 6.7 (for percentage error) is A0. *<u>Alternative method for dM1 in part (c)</u>* $\int \frac{(1-u)}{u}\,\mathrm{d}u = \left((1-u)\ln u - \int -\ln u\,du\right) = \left((1-u)\ln u + u\ln u - \int \frac{u}{u}\,\mathrm{d}u\right) = \left((1-u)\ln u + u\ln u - u\right)$ or $\int \frac{(u-1)}{u}\,\mathrm{d}u = \left((u-1)\ln u - \int \ln u\,\mathrm{d}u\right) = \left((u-1)\ln u - \left(u\ln u - \int \frac{u}{u}\,\mathrm{d}u\right)\right) = \left((u-1)\ln u - u\ln u + u\right)$ So **dM1** is for $\int \frac{(1-u)}{u}\,\mathrm{d}u$ going to $\left((1-u)\ln u + u\ln u - u\right)$ or $\left((u-1)\ln u - u\ln u + u\right)$ oe. *<u>Alternative method for part (d)</u>* **M1A1** for $\left\{4\int_2^1\left(\frac{1}{u}-1\right)\mathrm{d}u = \right\} 4\left[\ln u - u\right]_2^1 = 4\left[(\ln 1 - 1) - (\ln 2 - 2)\right] = 4(1-\ln 2)$ *<u>Alternative method for part (d): Using an extra constant λ from their integration.</u>* $\left[4\ln\left(1+\cos\frac{\pi}{2}\right) - 4\cos\frac{\pi}{2} + \lambda\right] - \left[4\ln(1+\cos 0) - 4\cos 0 + \lambda\right]$ λ is usually -4, but can be a value of k that the candidate has found in part (d). **Note:** The extra constant λ should cancel out and so the candidate can gain all three marks using this method, even the final A1 cso.

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7. Relative to a fixed origin O, the point A has position vector $(2\mathbf{i} - \mathbf{j} + 5\mathbf{k})$, the point B has position vector $(5\mathbf{i} + 2\mathbf{j} + 10\mathbf{k})$, and the point D has position vector $(-\mathbf{i} + \mathbf{j} + 4\mathbf{k})$.

The line l passes through the points A and B.

(a) Find the vector $\overrightarrow{AB}$.
(2)

(b) Find a vector equation for the line l.
(2)

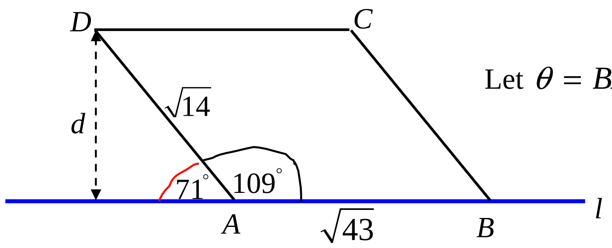
(c) Show that the size of the angle BAD is 109°, to the nearest degree.
(4)

The points A, B and D, together with a point C, are the vertices of the parallelogram $ABCD$, where $\overrightarrow{AB} = \overrightarrow{DC}$.

(d) Find the position vector of C.
(2)

(e) Find the area of the parallelogram $ABCD$, giving your answer to 3 significant figures.
(3)

(f) Find the shortest distance from the point D to the line l, giving your answer to 3 significant figures.
(2)

Question Number	Scheme		Marks
7.	$\overrightarrow{OA} = 2\mathbf{i} - \mathbf{j} + 5\mathbf{k}$, $\overrightarrow{OB} = 5\mathbf{i} + 2\mathbf{j} + 10\mathbf{k}$, $\{\overrightarrow{OC} = 2\mathbf{i} + 4\mathbf{j} + 9\mathbf{k}\}$ & $\overrightarrow{OD} = -\mathbf{i} + \mathbf{j} + 4\mathbf{k}$		
(a)	$\overrightarrow{AB} = = \pm((5\mathbf{i} + 2\mathbf{j} + 10\mathbf{k}) - (2\mathbf{i} - \mathbf{j} + 5\mathbf{k})); = 3\mathbf{i} + 3\mathbf{j} + 5\mathbf{k}$		M1; A1
			[2]
(b)	$l: \mathbf{r} = \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 3 \\ 5 \end{pmatrix}$ or $\mathbf{r} = \begin{pmatrix} 5 \\ 2 \\ 10 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 3 \\ 5 \end{pmatrix}$	See notes	M1 A1ft
			[2]
	 Let $\theta = B\hat{A}D$	Let d be the shortest distance from C to l.	
(c)	$\overrightarrow{AD} = \overrightarrow{OD} - \overrightarrow{OA} = \begin{pmatrix} -1 \\ 1 \\ 4 \end{pmatrix} - \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix} = \begin{pmatrix} -3 \\ 2 \\ -1 \end{pmatrix}$ or $\overrightarrow{DA} = \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix}$		M1
	$\cos\theta = \dfrac{\overrightarrow{AB} \bullet \overrightarrow{AD}}{\lvert\overrightarrow{AB}\rvert . \lvert\overrightarrow{AD}\rvert} = \dfrac{\begin{pmatrix} 3 \\ 3 \\ 5 \end{pmatrix} \bullet \begin{pmatrix} -3 \\ 2 \\ -1 \end{pmatrix}}{\sqrt{(3)^2 + (3)^2 + (5)^2} . \sqrt{(-3)^2 + (2)^2 + (-1)^2}}$	Applies dot product formula between their $(\overrightarrow{AB}$ or $\overrightarrow{BA})$ and their $(\overrightarrow{AD}$ or $\overrightarrow{DA})$.	M1
	$\cos\theta = \pm\left(\dfrac{-9+6-5}{\sqrt{(3)^2 + (3)^2 + (5)^2} . \sqrt{(-3)^2 + (2)^2 + (-1)^2}}\right)$	Correct followed through expression or **equation**.	A1$\sqrt{\ }$
	$\cos\theta = \dfrac{-8}{\sqrt{43}.\sqrt{14}} \Rightarrow \theta = 109.029544... = 109$ (nearest °)	awrt 109	A1 **cso AG**
			[4]
(d)	$\overrightarrow{OC} = \overrightarrow{OD} + \overrightarrow{DC} = \overrightarrow{OD} + \overrightarrow{AB} = (-\mathbf{i} + \mathbf{j} + 4\mathbf{k}) + (3\mathbf{i} + 3\mathbf{j} + 5\mathbf{k})$ $\overrightarrow{OC} = \overrightarrow{OB} + \overrightarrow{BC} = \overrightarrow{OB} + \overrightarrow{AD} = (5\mathbf{i} + 2\mathbf{j} + 10\mathbf{k}) + (-3\mathbf{i} + 2\mathbf{j} - \mathbf{k})$		M1
	So, $\overrightarrow{OC} = 2\mathbf{i} + 4\mathbf{j} + 9\mathbf{k}$		A1
			[2]
(e)	Area $ABCD = \left(\frac{1}{2}(\sqrt{43})(\sqrt{14})\sin 109^\circ\right); \times 2 = 23.19894905$	awrt 23.2	M1; dM1 A1
			[3]
(f)	$\dfrac{d}{\sqrt{14}} = \sin 71$ or $\sqrt{43}\,d = 23.19894905...$		M1
	$\therefore\ d = \sqrt{14}\sin 71^\circ = 3.537806563...$	awrt 3.54	A1
			[2]
			15

7. (a)	**M1:** Finding the difference between $\overrightarrow{OB}$ and $\overrightarrow{OA}$. Can be implied by two out of three components correct in $3\mathbf{i}+3\mathbf{j}+5\mathbf{k}$ or $-3\mathbf{i}-3\mathbf{j}-5\mathbf{k}$ **A1:** $3\mathbf{i}+3\mathbf{j}+5\mathbf{k}$
(b)	**M1:** An expression of the form $(3 \text{ component vector}) \pm \lambda(3 \text{ component vector})$ **A1ft:** $\mathbf{r}=\overrightarrow{OA}+\lambda(\text{their } \pm\overrightarrow{AB})$ or $\mathbf{r}=\overrightarrow{OB}+\lambda(\text{their } \pm\overrightarrow{AB})$. **Note:** Candidate must begin writing their line as $\mathbf{r}=$ or $l=...$ or $\begin{pmatrix}x\\y\\z\end{pmatrix}=...$ So, Line = ... would be A0.
(c)	**M1:** An attempt to find either the vector $\overrightarrow{AD}$ or $\overrightarrow{DA}$. Can be implied by two out of three components correct in $-3\mathbf{i}+2\mathbf{j}-\mathbf{k}$ or $3\mathbf{i}-2\mathbf{j}+\mathbf{k}$, respectively. **M1:** Applies dot product formula between their $(\overrightarrow{AB}$ or $\overrightarrow{BA})$ and their $(\overrightarrow{AD}$ or $\overrightarrow{DA})$. **A1ft:** Correct followed through expression or **equation**. The dot product must be correctly followed through correctly and the square roots although they can be un-simplified must be followed through correctly. **A1:** Obtains an angle of awrt 109 ***by correct solution only***. Award the final A1 mark if candidate achieves awrt 109 by either taking the dot product between: (i) $\begin{pmatrix}3\\3\\5\end{pmatrix}$ and $\begin{pmatrix}-3\\2\\-1\end{pmatrix}$ or (ii) $\begin{pmatrix}-3\\-3\\-5\end{pmatrix}$ and $\begin{pmatrix}3\\-2\\1\end{pmatrix}$. Ignore if any of these vectors are labelled incorrectly. Award A0, cso for those candidates who take the dot product between: (iii) $\begin{pmatrix}-3\\-3\\-5\end{pmatrix}$ and $\begin{pmatrix}-3\\2\\-1\end{pmatrix}$ or (iv) $\begin{pmatrix}3\\3\\5\end{pmatrix}$ and $\begin{pmatrix}3\\-2\\1\end{pmatrix}$. They will usually find awrt 71 and apply 180 – awrt 71 to give awrt 109. If these candidates give a convincing detailed explanation which must include reference to the direction of their vectors then this can be given A1 cso. If still in doubt, here, send to review.
(d)	**M1:** Applies either $\overrightarrow{OD}$ + their $\overrightarrow{AB}$ or $\overrightarrow{OB}$ + their $\overrightarrow{AD}$. This mark can be implied by two out of three correctly followed through components in their $\overrightarrow{OD}$. **A1:** For $2\mathbf{i}+4\mathbf{j}+9\mathbf{k}$.
(e)	**M1:** $\frac{1}{2}(\text{their } AB)(\text{their } CB)\sin(\text{their } 109^\circ \text{ or } 71^\circ \text{ from (b)})$. Awrt 11.6 will usually imply this mark. **dM1:** Multiplies this by 2 for the parallelogram. Can be implied. **Note:** $\frac{1}{2}((\text{their } AB+\text{their } AB))(\text{their } CB)\sin(\text{their } 109^\circ \text{ or } 71^\circ \text{ from (b)})$ **A1:** awrt 23.2
(f)	**M1:** $\dfrac{d}{\text{their } AD}=\sin(\text{their } 109^\circ \text{ or } 71^\circ \text{ from (b)})$ or $(\text{their } AB)\,d=(\text{their Area } ABCD)$ Award M0 for $(\text{their } AB)$ in part (f), if the area of their parallelogram in part (e) is $(\text{their } AB)(\text{their } CB)$. Award M0 for $\dfrac{d}{\text{their } \sqrt{43}}=\sin 71$ or $(\text{their } \sqrt{14})\,d=23.19894905...$ **A1:** awrt 3.54 **Note: Some candidates will use their answer to part (f) in order to answer part (e).**

7.	*Alternative method for part (c): Applying the cosine rule:*	
	$\overrightarrow{AD} = \overrightarrow{OD} - \overrightarrow{OA} = \begin{pmatrix}-1\\1\\4\end{pmatrix} - \begin{pmatrix}2\\-1\\5\end{pmatrix} = \begin{pmatrix}-3\\2\\-1\end{pmatrix}$ or $\overrightarrow{DA} = \begin{pmatrix}3\\-2\\1\end{pmatrix}$	**M1:** as above.
	$\overrightarrow{DB} = \overrightarrow{OD} - \overrightarrow{OA} = \begin{pmatrix}5\\2\\10\end{pmatrix} - \begin{pmatrix}-1\\1\\4\end{pmatrix} = \begin{pmatrix}6\\1\\6\end{pmatrix}$ or $\overrightarrow{BD} = \begin{pmatrix}-6\\-1\\-6\end{pmatrix}$	
	So $\lvert\overrightarrow{AB}\rvert = \sqrt{43}$, $\lvert\overrightarrow{AD}\rvert = \sqrt{14}$ and $\lvert\overrightarrow{DB}\rvert = \sqrt{73}$	
	$\cos\theta = \dfrac{(\sqrt{43})^2 + (\sqrt{14})^2 - (\sqrt{73})^2}{2\sqrt{43}.\sqrt{14}}$	**M1:** Cosine rule structure of $\cos\theta = \dfrac{a^2+b^2-c^2}{2ab}$ assigned each of $\lvert\overrightarrow{AB}\rvert$, $\lvert\overrightarrow{AD}\rvert$ and $\lvert\overrightarrow{DB}\rvert$ in any order as their *a*, *b* and *c*. **A1:** Correct application of cosine rule.
	$\left\{\cos\theta = \dfrac{-16}{2\sqrt{43}.\sqrt{14}} \Rightarrow \theta = 109.029544...\right\} = 109$ (nearest °)	A1: awrt 109 (no errors seen). **AG**
	Alternative method for part (d):	
	$\overrightarrow{OE} = \begin{pmatrix}2\\-1\\5\end{pmatrix} + \lambda\begin{pmatrix}3\\3\\5\end{pmatrix}$	
	$\overrightarrow{DE} = \begin{pmatrix}2+3\lambda\\-1+3\lambda\\5+5\lambda\end{pmatrix} - \begin{pmatrix}-1\\1\\4\end{pmatrix} = \begin{pmatrix}3+3\lambda\\-2+3\lambda\\1+5\lambda\end{pmatrix}$	
	$\overrightarrow{DE} \bullet \overrightarrow{AB} = 0 \Rightarrow \begin{pmatrix}3+3\lambda\\-2+3\lambda\\1+5\lambda\end{pmatrix} \bullet \begin{pmatrix}3\\3\\5\end{pmatrix} = 0$	**M1:** Takes the dot product between $\overrightarrow{DE}$ and $\overrightarrow{AB}$ and progresses to find a value of λ
	$9 + 9\lambda - 6 + 9\lambda + 5 + 3\lambda = 0 \Rightarrow \lambda = -\dfrac{8}{43}$	
	$\overrightarrow{DE} = \begin{pmatrix}2+3\lambda\\-1+3\lambda\\5+5\lambda\end{pmatrix} - \begin{pmatrix}-1\\1\\4\end{pmatrix} = \begin{pmatrix}\frac{103}{43}\\-\frac{110}{43}\\\frac{3}{43}\end{pmatrix}$	**dM1:** Uses their value of λ to find $\overrightarrow{DE}$
	Length DE = 3.537806563...	**A1:** awrt 3.54

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8. (a) Express $\frac{1}{P(5-P)}$ in partial fractions.

(3)

A team of conservationists is studying the population of meerkats on a nature reserve. The population is modelled by the differential equation

$$\frac{dP}{dt} = \frac{1}{15}P(5 - P), \quad t \geqslant 0$$

where P, in thousands, is the population of meerkats and t is the time measured in years since the study began.

Given that when $t = 0$, $P = 1$,

(b) solve the differential equation, giving your answer in the form,

$$P = \frac{a}{b + c\mathrm{e}^{-\frac{1}{3}t}}$$

where a, b and c are integers.

(8)

(c) Hence show that the population cannot exceed 5000

(1)

Question Number	Scheme		Marks
8. (a)	$1 = A(5-P) + BP$	Can be implied.	M1
	$A = \frac{1}{5},\ B = \frac{1}{5}$	Either one.	A1
	giving $\dfrac{\frac{1}{5}}{P} + \dfrac{\frac{1}{5}}{(5-P)}$	See notes.	A1 **cao, aef**
			[3]
(b)	$\int \frac{1}{P(5-P)}\,dP = \int \frac{1}{15}\,dt$		B1
	$\frac{1}{5}\ln P - \frac{1}{5}\ln(5-P) = \frac{1}{15}t\ \ (+c)$		M1* A1ft
	$\{t=0,\ P=1 \Rightarrow\}\quad \frac{1}{5}\ln 1 - \frac{1}{5}\ln(4) = 0 + c \quad \left\{\Rightarrow\ c = -\frac{1}{5}\ln 4\right\}$		dM1*
	eg: $\frac{1}{5}\ln\left(\frac{P}{5-P}\right) = \frac{1}{15}t - \frac{1}{5}\ln 4$	Using any of the subtraction (or addition) laws for logarithms CORRECTLY	dM1*
	$\ln\left(\frac{4P}{5-P}\right) = \frac{1}{3}t$		
	eg: $\frac{4P}{5-P} = e^{\frac{1}{3}t}$ or eg: $\frac{5-P}{4P} = e^{-\frac{1}{3}t}$	Eliminate ln's correctly.	dM1*
	gives $4P = 5e^{\frac{1}{3}t} - Pe^{\frac{1}{3}t} \Rightarrow P(4 + e^{\frac{1}{3}t}) = 5e^{\frac{1}{3}t}$		
	$P = \frac{5e^{\frac{1}{3}t}}{(4+e^{\frac{1}{3}t})} \quad \left\{\frac{(\div e^{\frac{1}{3}t})}{(\div e^{\frac{1}{3}t})}\right\}$	Make P the subject.	dM1*
	$P = \frac{5}{(1+4e^{-\frac{1}{3}t})}$ or $P = \frac{25}{(5+20e^{-\frac{1}{3}t})}$ etc.		A1
			[8]
(c)	$1 + 4e^{-\frac{1}{3}t} > 1 \Rightarrow P < 5$. So population cannot exceed 5000.		B1
			[1] **12**
(a)	**M1:** Forming a correct identity. For example, $1 = A(5-P) + BP$. **Note** A and B not referred to in question. **A1:** Either one of $A = \frac{1}{5}$ or $B = \frac{1}{5}$. **A1:** $\dfrac{\frac{1}{5}}{P} + \dfrac{\frac{1}{5}}{(5-P)}$ or any equivalent form, eg: $\frac{1}{5P} + \frac{1}{25-5P}$, etc. Ignore subsequent working. This answer must be stated in part (a) only. A1 can also be given for a candidate who finds both $A = \frac{1}{5}$ and $B = \frac{1}{5}$ and $\frac{A}{P} + \frac{B}{5-P}$ is seen in their working. Candidate can use 'cover-up' rule to write down $\dfrac{\frac{1}{5}}{P} + \dfrac{\frac{1}{5}}{(5-P)}$, as so gain all three marks. Candidate cannot gain the marks for part (a) in part (b).		

8. (b)	**B1:** Separates variables as shown. $\mathrm{d}P$ and $\mathrm{d}t$ should be in the correct positions, though this mark can be implied by later working. Ignore the integral signs. **M1*:** **Both** $\pm\lambda\ln P$ and $\pm\mu\ln(\pm 5 \pm P)$, where λ and μ are constants. **Or** $\pm\lambda\ln mP$ and $\pm\mu\ln(n(\pm 5 \pm P))$, where λ, μ, m and n are constants. **A1ft:** Correct follow through integration of both sides from their $\int \frac{\lambda}{P} + \frac{\mu}{(5-P)}\,\mathrm{d}P = \int K\,\mathrm{d}t$ with or without $+c$ **dM1*:** Use of $t=0$ and $P=1$ in an integrated equation containing c **dM1*:** Using ANY of the subtraction (or addition) laws for logarithms CORRECTLY. **dM1*:** Apply logarithms (or take exponentials) to eliminate ln's CORRECTLY from their equation. **dM1*:** A full ACCEPTABLE method of rearranging to make P the subject. **(See below for examples!)** **A1:** $P = \frac{5}{(1+4\mathrm{e}^{-\frac{1}{3}t})}$ $\{$where $a=5, b=1, c=4\}$. Also allow any "integer" multiples of this expression. For example: $P = \frac{25}{(5+20\mathrm{e}^{-\frac{1}{3}t})}$ **Note: If the first method mark (M1*) is not awarded then the candidate cannot gain any of the six remaining marks for this part of the question.** **Note:** $\int \frac{1}{P(5-P)}\,\mathrm{d}P = \int 15\,\mathrm{d}t \Rightarrow \int \frac{\frac{1}{5}}{P} + \frac{\frac{1}{5}}{(5-P)}\,\mathrm{d}P = \int 15\,\mathrm{d}t \Rightarrow \ln P - \ln(5-P) = 15t$ is B0M1A1ft. ***<u>dM1* for making P the subject</u>*** **Note there are three type of manipulations here which are considered acceptable to make *P* the subject.** **(1)** M1 for $\frac{P}{5-P} = \mathrm{e}^{\frac{1}{3}t} \Rightarrow P = 5\mathrm{e}^{\frac{1}{3}t} - P\mathrm{e}^{\frac{1}{3}t} \Rightarrow P(1+\mathrm{e}^{\frac{1}{3}t}) = 5\mathrm{e}^{\frac{1}{3}t} \Rightarrow P = \frac{5}{(1+\mathrm{e}^{-\frac{1}{3}t})}$ **(2)** M1 for $\frac{P}{5-P} = \mathrm{e}^{\frac{1}{3}t} \Rightarrow \frac{5-P}{P} = \mathrm{e}^{\frac{1}{3}t} \Rightarrow \frac{5}{P} - 1 = \mathrm{e}^{\frac{1}{3}t} \Rightarrow \frac{5}{P} = \mathrm{e}^{\frac{1}{3}t} + 1 \Rightarrow P = \frac{5}{(1+\mathrm{e}^{\frac{1}{3}t})}$ **(3)** M1 for $P(5-P) = 4\mathrm{e}^{\frac{1}{3}t} \Rightarrow P^2 - 5P = -4\mathrm{e}^{\frac{1}{3}t} \Rightarrow \left(P - \frac{5}{2}\right)^2 - \frac{25}{4} = -4\mathrm{e}^{\frac{1}{3}t}$ leading to $P = \ldots$ **Note**: The incorrect manipulation of $\frac{P}{5-P} = \frac{P}{5} - 1$ or equivalent is awarded this dM0*. **Note:** $(P) - (5-P) = \mathrm{e}^{\frac{1}{3}t} \Rightarrow 2P - 5 = \frac{1}{3}t$ leading to $P = \ldots$ or equivalent is awarded this dM0*
(c)	**B1:** $1 + 4\mathrm{e}^{-\frac{1}{3}t} > 1$ **and** $P < 5$ **and** a conclusion relating population (or even P) or meerkats to 5000. **For** $P = \frac{25}{(5+20\mathrm{e}^{-\frac{1}{3}t})}$, B1 can be awarded for $5 + 20\mathrm{e}^{-\frac{1}{3}t} > 5$ **and** $P < 5$ **and** a conclusion relating population (or even P) or meerkats to 5000. B1 can only be obtained if candidates have correct values of a and b in their $P = \frac{a}{(b+c\mathrm{e}^{-\frac{1}{3}t})}$. **Award B0 for:** As $t \to \infty$, $\mathrm{e}^{-\frac{1}{3}t} \to 0$. So $P \to \frac{5}{(1+0)} = 5$, so population cannot exceed 5000, **unless** the candidate also proves that $P = \frac{5}{(1+4\mathrm{e}^{-\frac{1}{3}t})}$ oe. is an increasing function. **If unsure here, then send to review!**

8.	<u>*Alternative method for part (b)*</u> **B1M1*A1:** as before for $\frac{1}{5}\ln P - \frac{1}{5}\ln(5-P) = \frac{1}{15}t \ \ (+c)$ Award 3rd M1for $\ln\left(\frac{P}{5-P}\right) = \frac{1}{3}t + c$ Award 4th M1 for $\frac{P}{5-P} = Ae^{\frac{1}{3}t}$ Award 2nd M1 for $t=0,\ P=1 \Rightarrow \frac{1}{5-1} = Ae^0 \ \left\{\Rightarrow\ A = \frac{1}{4}\right\}$ $\frac{P}{5-P} = \frac{1}{4}e^{\frac{1}{3}t}$ then award the final M1A1 in the same way.