Write your name here

Surname	Other names

Pearson Edexcel
International Advanced Level

Centre Number	Candidate Number

Core Mathematics C4
Advanced

Monday 27 January 2014 – Morning
Time: 1 hour 30 minutes

Paper Reference
6666A/01

You must have:
Mathematical Formulae and Statistical Tables (Pink)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions
- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information
- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice
- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

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1. (a) Find the binomial expansion of

$$\frac{1}{(4+3x)^3}, \qquad |x| < \frac{4}{3}$$

in ascending powers of x, up to and including the term in x^3.
Give each coefficient as a simplified fraction.

(6)

In the binomial expansion of

$$\frac{1}{(4-9x)^3}, \qquad |x| < \frac{4}{9}$$

the coefficient of x^2 is A.

(b) Using your answer to part (a), or otherwise, find the value of A.
Give your answer as a simplified fraction.

(2)

Question Number	Scheme		Marks
1. (a)	$\left\{\dfrac{1}{(4+3x)^3} = \right\} (4+3x)^{-3}$	Moving power to the top	M1
	$= \underline{(4)^{-3}}\left(1+\dfrac{3x}{4}\right)^{-3} = \underline{\dfrac{1}{64}}\left(1+\dfrac{3x}{4}\right)^{-3}$	$\underline{4^{-3}}$ or $\underline{\dfrac{1}{64}}$	<u>B1</u>
	$= \left\{\dfrac{1}{64}\right\}\left[1+(-3)(kx)+\dfrac{(-3)(-4)}{2!}(kx)^2+\dfrac{(-3)(-4)(-5)}{3!}(kx)^3+...\right]$	see notes	M1 A1
	$= \left\{\dfrac{1}{64}\right\}\left[1+(-3)\left(\dfrac{3x}{4}\right)+\dfrac{(-3)(-4)}{2!}\left(\dfrac{3x}{4}\right)^2+\dfrac{(-3)(-4)(-5)}{3!}\left(\dfrac{3x}{4}\right)^3+...\right]$		
	$= \dfrac{1}{64}\left[1-\dfrac{9}{4}x+\dfrac{27}{8}x^2-\dfrac{135}{32}x^3+...\right]$		
	$= \dfrac{1}{64}-\dfrac{9}{256}x;+\dfrac{27}{512}x^2-\dfrac{135}{2048}x^3+...$		A1; A1
			[6]
(b)	$\left\{\dfrac{1}{(4-9x)^3}\right\}$, so the coefficient of x^2 is $A=(9)\left(\dfrac{27}{512}\right)=\dfrac{243}{512}$	$9\times$(coeff x^2 in (a))	M1
		$\dfrac{243}{512}$	A1
			[2] **8**

Notes

(a)

M1: Writes down $(4+3x)^{-3}$ or uses power of -3.

This mark can be implied by a constant term of $(4)^{-3}$ or $\dfrac{1}{64}$.

<u>**B1**</u>: $\underline{4^{-3}}$ or $\underline{\dfrac{1}{64}}$ outside brackets or $\dfrac{1}{64}$ as candidate's constant term in their binomial expansion.

M1: Expands $(...+kx)^{-3}$ to give any 2 terms out of 4 terms simplified or un-simplified,

Eg: $1+(-3)(kx)$ or $\dfrac{(-3)(-4)}{2!}(kx)^2+\dfrac{(-3)(-4)(-5)}{3!}(kx)^3$ or $1+\ ...\ +\dfrac{(-3)(-4)}{2!}(kx)^2$

or $\dfrac{(-3)(-4)}{2!}(kx)^2+\dfrac{(-3)(-4)(-5)}{3!}(kx)^3$ **where** $k\neq 1$ are fine for M1.

A1: A correct simplified or un-simplified $1+(-3)(kx)+\dfrac{(-3)(-4)}{2!}(kx)^2+\dfrac{(-3)(-4)(-5)}{3!}(kx)^3$ expansion with consistent (kx). Note that (kx) must be consistent (on the RHS, not necessarily the LHS) in a candidate's expansion. **Note** that $k\neq 1$.

You would award B1M1A0 for $\dfrac{1}{64}\left[1+(-3)\left(\dfrac{3x}{4}\right)+\dfrac{(-3)(-4)}{2!}(3x)^2+\dfrac{(-3)(-4)(-5)}{3!}\left(\dfrac{3x}{4}\right)^3+...\right]$

because (kx) is not consistent.

Notes for Question 1 continued	
1. (a) ctd	***"Incorrect bracketing"*** $=\left\{\frac{1}{64}\right\}\left[1+(-3)\left(\frac{3x}{4}\right)+\frac{(-3)(-4)}{2!}\left(\frac{3x^2}{4}\right)+\frac{(-3)(-4)(-5)}{3!}\left(\frac{3x^3}{4}\right)+\ldots\right]$ is M1A0 unless recovered. **A1:** For $\frac{1}{64}-\frac{9}{256}x$ **(simplified please)** or also allow $0.015625 - 0.03515625x$. **Allow Special Case A1A0 for either SC:** $\frac{1}{64}\left[1-\frac{9}{4}x\,;\ldots\right]$ **or SC**: $\lambda\left[1-\frac{9}{4}x+\frac{27}{8}x^2-\frac{135}{32}x^3+\ldots\right]$ (where λ can be 1 or omitted), with each term in the $[\ldots\ldots\ldots]$ either a simplified fraction or a decimal. **A1:** Accept only $\frac{27}{512}x^2-\frac{135}{2048}x^3$ or $0.052734375x^2 - 0.06591796875x^3$ Candidates who write $=\frac{1}{64}\left[1+(-3)\left(-\frac{3x}{4}\right)+\frac{(-3)(-4)}{2!}\left(-\frac{3x}{4}\right)^2+\frac{(-3)(-4)(-5)}{3!}\left(-\frac{3x}{4}\right)^3+\ldots\right]$ where $k=-\frac{3}{4}$ and not $\frac{3}{4}$ and achieve $=\frac{1}{64}+\frac{9}{256}x;+\frac{27}{512}x^2+\frac{135}{2048}x^3+\ldots$ will get B1M1A1A0A0. **<u>Note for final two marks:</u>** $\frac{1}{64}\left[1-\frac{9}{4}x+\frac{27}{8}x^2-\frac{135}{32}x^3+\ldots\right]=\frac{1}{64}+\frac{9}{256}x+\frac{27}{512}x^2-\frac{135}{2048}x^3+\ldots$ scores final A0A1. $\frac{1}{64}\left[1-\frac{9}{4}x+\frac{27}{8}x^2-\frac{135}{32}x^3+\ldots\right]=\frac{1}{64}-\frac{9}{256}+\frac{27}{512}x^2-\frac{135}{2048}x^3+\ldots$ scores final A0A1 **<u>Special case for the M1 mark</u>** Award Special Case M1 for a correct simplified or un-simplified $1+n(kx)+\frac{n(n-1)}{2!}(kx)^2+\frac{n(n-1)(n-2)}{3!}(kx)^3$ expansion with their $n\neq-3$, $n\neq$ ***positive integer*** and a consistent (kx). Note that (kx) must be consistent (on the RHS, not necessarily the LHS) in a candidate's expansion. **Note** that $k\neq1$**.**
(b)	**M1:** $9\times\left(\text{their } \frac{27}{512}\right)$ or $9\left(\text{their } \frac{27}{512}x^2\right)$ **A1:** For $\frac{243}{512}$. Note that $\frac{243}{512}x^2$ is A0. ***<u>Alternative method for part (b)</u>*** **M1:** for $(4)^{-3}\left(\frac{(-3)(-4)}{2!}\right)\left(-\frac{9}{4}\right)^2$ or $(4)^{-3}\left(\frac{(-3)(-4)}{2!}\right)\left(\frac{9}{4}\right)^2$ or $(4)^{-3}\left(\frac{(-3)(-4)}{2!}\right)\left(\frac{9x}{4}\right)^2$ or $\frac{1}{64}\left(\frac{243x^2}{8}\right)$ Also allow M1 for $\underline{\frac{1}{64}}\left[\ldots\ldots+\underline{\frac{(-3)(-4)}{2!}\left(-\frac{9x}{4}\right)^2}+\ldots\ldots\right]$ or $\underline{\frac{1}{64}}\left[\ldots\ldots+\underline{\frac{(-3)(-4)}{2!}\left(\frac{9x}{4}\right)^2}+\ldots\ldots\right]$ Also allow M1 for $\lambda\left[\ldots\ldots+\underline{\frac{(-3)(-4)}{2!}\left(-\frac{9x}{4}\right)^2}+\ldots\ldots\right]$ or $\lambda\left[\ldots\ldots+\underline{\frac{(-3)(-4)}{2!}\left(\frac{9x}{4}\right)^2}+\ldots\ldots\right]$ where λ is the multiplicative constant used by the candidate in part (a). **Note** that λ can be 1. **A1:** For $\frac{243}{512}$. Note that $\frac{243}{512}x^2$ is A0.

Notes for Question 1 continued

Alternative Methods for part (a)

Alternative method 1: Candidates can apply an alternative form of the binomial expansion.

$$\left\{\frac{1}{(4+3x)^3} = \right\}(4+3x)^{-3} = (4)^{-3} + (-3)(4)^{-4}(3x) + \frac{(-3)(-4)}{2!}(4)^{-5}(3x)^2 + \frac{(-3)(-4)(-5)}{3!}(4)^{-6}(3x)^3$$

M1: Writes down $(4+3x)^{-3}$ or uses power of -3.

B1: 4^{-3} or $\frac{1}{64}$

M1: Any two of four (un-simplified or simplified) terms correct.

A1: All four (un-simplified or simplified) terms correct.

A1: $\frac{1}{64} - \frac{9}{256}x$

A1: $\frac{27}{512}x^2 - \frac{135}{2048}x^3$

Note: The terms in C need to be evaluated,

so ${}^{-3}C_0(4)^{-3} + {}^{-3}C_1(4)^{-4}(3x) + {}^{-3}C_2(4)^{-5}(3x)^2 + {}^{-3}C_3(4)^{-6}(3x)^3$ without further working is B0M0A0.

Alternative Method 2: Maclaurin Expansion

$(4+3x)^{-3}$	Moving power to the top	M1
$f''(x) = 108(4+3x)^{-5}$, $f'''(x) = -1620(4+3x)^{-6}$	Correct $f''(x)$ and $f'''(x)$	B1
$f'(x) = -3(4+3x)^{-4}(3)$	$\pm a(4+3x)^{-4}$; $a \neq \pm 1$	M1
	$-3(4+3x)^{-4}(3)$	A1 oe
$\left\{\therefore f(0) = \frac{1}{64}, f'(0) = -\frac{9}{256}, f''(0) = \frac{27}{256} \text{ and } f'''(0) = -\frac{405}{1024}\right\}$		
$f(x) = \frac{1}{64} - \frac{9}{256}x; + \frac{27}{512}x^2 - \frac{135}{2048}x^3 + \dots$		A1; A1

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2. (i) Find

$$\int x\cos\left(\frac{x}{2}\right)\mathrm{d}x$$

(3)

(ii) (a) Express $\dfrac{1}{x^2(1-3x)}$ in partial fractions.

(4)

(b) Hence find, for $0 < x < \dfrac{1}{3}$

$$\int \frac{1}{x^2(1-3x)}\mathrm{d}x$$

(3)

Question Number	Scheme		Marks
2.	$\int x\cos\left(\frac{x}{2}\right)\mathrm{d}x$, $\left\{\begin{array}{lll} u = x & \Rightarrow & \frac{\mathrm{d}u}{\mathrm{d}x} = 1 \\ \frac{\mathrm{d}v}{\mathrm{d}x} = \cos\left(\frac{x}{2}\right) & \Rightarrow & v = 2\sin\left(\frac{x}{2}\right) \end{array}\right\}$		
(i)	$= 2x\sin\left(\frac{x}{2}\right) - \int 2\sin\left(\frac{x}{2}\right)\{\mathrm{d}x\}$		M1 A1
	$= 2x\sin\left(\frac{x}{2}\right) + 4\cos\left(\frac{x}{2}\right)\{+c\}$		A1
			[3]
(ii)(a)	$\frac{1}{x^2(1-3x)} \equiv \frac{A}{x} + \frac{B}{x^2} + \frac{C}{(1-3x)}$		
		At least one of "B" or "C" correct.	B1
	$B = 1,\ C = 9$	Breaks up their partial fraction correctly into ***three terms* and** both "B" $= 1$ and "C" $= 9$. **See notes below.**	B1 **cso**
	$1 \equiv Ax(1-3x) + B(1-3x) + Cx^2$ $x = 0,\quad 1 = B$ $x = \frac{1}{3},\quad 1 = \frac{1}{9}C \Rightarrow C = 9$ x^2 terms: $0 = -3A + C$ $0 = -3A + 9 \Rightarrow A = 3$	Writes down **a correct identity** and attempts to **find the value** of either one of "A", "B" or "C".	M1
	$x^2: 0 = -3A + C,\quad x: 0 = A - 3B,$ constant: $1 = B$		
	leading to $A = 3$	Correct value for "A" which is found using a correct identity and follows from their partial fraction decomposition.	A1
			[4]
(b)	$\int \frac{1}{x^2(1-3x)}\mathrm{d}x = \int \frac{3}{x} + \frac{1}{x^2} + \frac{9}{(1-3x)}\mathrm{d}x$		
		Either $\pm\frac{P}{x} \to \pm a\ln x$ or $\pm a\ln kx$ or $\pm\frac{Q}{x^2} \to \pm bx^{-1}$ or $\frac{R}{(1-3x)} \to \pm c\ln(1-3x)$	M1
	$= 3\ln x + \frac{x^{-1}}{(-1)} + \frac{9}{(-3)}\ln(1-3x)\ \{+c\}$	***At least*** two terms correctly integrated	A1ft
		All three terms correctly integrated. Ignore absence of '$+c$'	A1ft
			[3] **10**

Notes

2. (i)	**M1:** Integration by parts is applied in the form $\pm\lambda x\sin\left(\frac{x}{2}\right) \pm \int \mu\sin\left(\frac{x}{2}\right)\{\mathrm{d}x\}$ (where $\lambda \neq 0,\ \mu \neq 0$) **A1:** $2x\sin\left(\frac{x}{2}\right) - \int 2\sin\left(\frac{x}{2}\right)\{\mathrm{d}x\}$ or equivalent. Can be un-simplified.
	A1: $2x\sin\left(\frac{x}{2}\right) + 4\cos\left(\frac{x}{2}\right)$ or $2x\sin\left(\frac{x}{2}\right) - -\frac{2}{\left(\frac{1}{2}\right)}\cos\left(\frac{x}{2}\right)$ or equivalent with/without $+c$ Can be un-simplified.

	Notes for Question 2 continued
(ii)(a)	**SPECIAL CASE:** A candidate who uses $u = x$, $\frac{\mathrm{d}v}{\mathrm{d}x} = \cos\left(\frac{x}{2}\right)$, writes down the correct "by parts" formula, but makes only one error when applying it can be awarded Special Case M1. **BE CAREFUL!** Candidates will assign ***their own*** "*A*, *B* and *C*" for this question. **B1:** At least one of "*B*" or "*C*" are correct. **B1:** Breaks up their partial fraction correctly into three terms **and** both $"B" = 1$ and $"C" = 9$. **Note:** If a candidate does not give partial fraction decomposition then: • the 2nd B1 mark can follow from a correct identity. **M1:** Writes down ***a correct identity*** (although this can be implied) and attempts to find the value of either one of "*A*" or "*B*" or "*C*". This can be achieved by ***either*** substituting values into their identity ***or*** comparing coefficients and solving the resulting equations simultaneously. **A1:** Correct value for "*A*" which is found using a correct identity and follows from their partial fraction decomposition. **Note:** If a candidate does not give partial fraction decomposition then: • the final A1 mark can be awarded for a correct "*A*" if a candidate writes out their partial fractions at the end. **Note:** The correct partial fraction from no working scores B1B1M1A1. **Note:** A number of candidates will start this problem by writing out the correct identity and then attempt to find "*A*" or " *B*" or "*C*". Therefore the B1 marks can be awarded from this method. **Note:** $\frac{1}{x^2(1-3x)} \equiv \frac{B}{x^2} + \frac{C}{(1-3x)}$ leading to $"B" = 1$ or $"C" = 9$ will only score a maximum of B1B0M0A0.
(ii)(b)	**M1:** Either $\pm\frac{P}{x} \to \pm a\ln x$ or $\pm\frac{Q}{x^2} \to \pm b x^{-1}$ or $\frac{R}{(1-3x)} \to \pm c\ln(1-3x)$, from their constants *P*, *Q*, *R*. **A1ft:** At least two terms from any of $\pm\frac{P}{x}$ or $\pm\frac{Q}{x^2}$ or $\frac{R}{(1-3x)}$ correctly integrated. Can be un-simplified. **A1ft:** All 3 terms from $\pm\frac{P}{x}$, $\pm\frac{Q}{x^2}$ and $\frac{R}{(1-3x)}$ correctly integrated. Can be un-simplified with/without $+c$. **NOTE:** Ignore subsequent working for applying limits after integration. **NOTE:** Integrating $\frac{9}{(1-3x)}$ to give $-3\ln\lvert 3x-1\rvert$ is correct but $-3\ln(3x-1)$ is incorrect. **NOTE:** The final two marks in (ii)(b) are both follow through accuracy marks. **NOTE:** Some candidates are applying limits of $x = 0$ and $x = \frac{1}{3}$ to their integrated expression. You can award up to all three marks in (ii)(b) for the integrated expression and ignore the application of limits. **NOTE:** A candidate who achieves full marks in (ii)(a), but then mixes up the correct constants when writing their partial fraction can only achieve a maximum of M1A1A0 in (ii)(b).

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3. The number of bacteria, N, present in a liquid culture at time t hours after the start of a scientific study is modelled by the equation

$$N = 5000(1.04)^t, \qquad t \geqslant 0$$

where N is a continuous function of t.

(a) Find the number of bacteria present at the start of the scientific study.
(1)

(b) Find the percentage increase in the number of bacteria present from $t = 0$ to $t = 2$
(2)

Given that $N = 15000$ when $t = T$,

(c) find the value of $\frac{\mathrm{d}N}{\mathrm{d}t}$ when $t = T$, giving your answer to 3 significant figures.
(4)

Question Number	Scheme		Marks
3.	$N = 5000(1.04)^t, \quad t \in \mathbb{R}, t \geqslant 0.$		
(a)	$\{t = 0 \Rightarrow\}\ N = 5000$ (bacteria)	5000	B1 **cao**
			[1]
(b)	$\pm\left(\dfrac{5000(1.04)^2 - 5000}{5000}\right)100$ or $\pm\left(\dfrac{5408 - 5000}{5000}\right)100$		M1
	$= 8.16$ (%)	8 or 8.2 or 8.16	A1
			[2]
(c)	$\dfrac{dN}{dt} = 5000(1.04)^t \ln(1.04)$ or $\dfrac{dN}{dt} = 5000\left(e^{t\ln(1.04)}\right)\ln(1.04)$ or $\dfrac{dN}{dt} = N\ln(1.04)$ or $\dfrac{1}{N}\dfrac{dN}{dt} = \ln(1.04)$		M1 A1
	At $t = T$, $15000 = 5000(1.04)^T \Rightarrow 3 = (1.04)^T \Rightarrow T = \dfrac{\ln 3}{\ln 1.04} = 28.01...$		
	$\{\text{At } t = T,\}\ \dfrac{dN}{dt} = 5000(3)\ln(1.04)$ or $\dfrac{dN}{dt} = 5000(1.04)^{28.01...}\ln(1.04)$	Substitutes their found $(1.04)^T$ or their found T into $\dfrac{dN}{dt}$ or N into $\dfrac{dN}{dt} = N\ln(1.04)$	dM1
	$= 588.3106973...\left(\dfrac{\text{bacteria}}{\text{hour}}\right)$	590 or awrt 588	A1
			[4]
			7

Notes

(a)

B1: 5000 cao.

(b)

M1: A **full method** for finding a percentage increase.

A1: 8 or 8.1 or 8.16

Note: $(1.04)^2$ or 1.0816 or 0.0816 by itself is M0; but followed by either 8 or 8.2 or 8.16 is M1A1.

Note: Applying $\left(\dfrac{5000(1.04)^2 - 5000}{5408}\right)100$ or equivalent (answer of 7.54(%)) is M0A0.

(c)

M1: Award M1 for $\dfrac{dN}{dt} = \pm\lambda(1.04)^t$ or $\dfrac{dN}{dt} = \pm\lambda N$ or $\dfrac{dN}{dt} = \pm\lambda e^{t\ln 1.04}$ or $\dfrac{1}{N}\dfrac{dN}{dt} = \pm\lambda$ where $\lambda \neq 0$ is a constant.

EXCEPTION: Award M0, however, for $\dfrac{dN}{dt} = ...(1.04)^{t-1}$ or $\dfrac{dN}{dt} = ...(1.04)^{t+1}$ or equivalent.

Note: Award M0 for expressions such as $\dfrac{dN}{dt} = 5000(1.04)^{t-1}$ or $\dfrac{dN}{dt} = 5000t(1.04)^{t-1}$

Note: You can award M1 for $\dfrac{dN}{dt} = 5000(1.04)^t$

Be careful: $\dfrac{dN}{dt} = 5000(1.04)^t \ln(1.04)^t$ is M0.

Notes for Question 3 continued	
3. (c) contd.	**A1:** $\frac{\mathrm{d}N}{\mathrm{d}t} = 5000\left(1.04\right)^{t}\ln(1.04)$ or $\frac{\mathrm{d}N}{\mathrm{d}t} = 5000\left(\mathrm{e}^{t\ln(1.04)}\right)\ln(1.04)$ or $\frac{\mathrm{d}N}{\mathrm{d}t} = N\ln(1.04)$ or $\frac{1}{N}\frac{\mathrm{d}N}{\mathrm{d}t} = \ln(1.04)$ or equivalent. **dM1: (dependent on the first M mark)** For substituting their found $(1.04)^{T}$ (or $(1.04)^{t}$) or their found T (or t) into their $\frac{\mathrm{d}N}{\mathrm{d}t} = \mathrm{f}(t)$; or their found N or $N = 15000$ into their $\frac{\mathrm{d}N}{\mathrm{d}t} = \mathrm{f}(N)$. **A1:** 590 or anything that rounds to 588

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4.

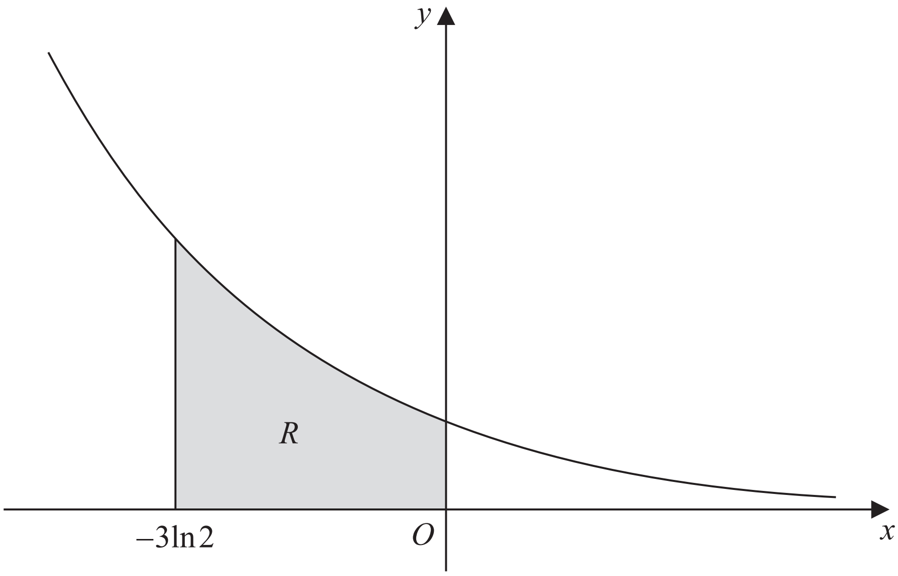

Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = \dfrac{4e^{-x}}{3\sqrt{(1+3e^{-x})}}$

The finite region R, shown shaded in Figure 1, is bounded by the curve, the x-axis, the line $x = -3\ln 2$ and the y-axis.

The table below shows corresponding values of x and y for $y = \dfrac{4e^{-x}}{3\sqrt{(1+3e^{-x})}}$

x	$-3\ln 2$	$-2\ln 2$	$-\ln 2$	0
y	2.1333		1.0079	0.6667

(a) Complete the table above by giving the missing value of y to 4 decimal places.
(1)

(b) Use the trapezium rule, with all the values of y in the completed table, to obtain an estimate for the area of R, giving your answer to 2 decimal places.
(3)

(c) (i) Using the substitution $u = 1 + 3e^{-x}$, or otherwise, find

$$\int \frac{4e^{-x}}{3\sqrt{(1+3e^{-x})}}\,dx$$

(5)

(ii) Hence find the value of the area of R.
(2)

Question Number	Scheme		Marks
4. (a)	1.4792	1.4792	B1 **cao**
			[1]
(b)	Area $\approx \frac{1}{2}\times \ln 2\,; \times \left[\underline{2.1333 + 2(\text{their } 1.4792 + 1.0079) + 0.6667}\right]$		B1 <u>M1</u>
	$= \frac{\ln 2}{2} \times 7.7742... = 2.694332406... = 2.69\ (2\text{ dp})$	awrt 2.69	A1
			[3]
(c)(i)	$\{u = 1 + 3e^{-x}\} \Rightarrow \underline{\frac{du}{dx} = -3e^{-x}}$ or $\underline{\frac{dx}{du} = \frac{-1}{(u-1)}}$		<u>B1</u> oe
	$\left\{\int \frac{4e^{-x}}{3\sqrt{(1+3e^{-x})}}\,dx =\right\} -\frac{4}{9}\int \frac{1}{\sqrt{u}}\,du$	$\pm\lambda \int \frac{1}{\sqrt{u}}\,du$	M1
		$-\frac{4}{9}\int \frac{1}{\sqrt{u}}\,du$	A1 oe
	$= -\frac{4}{9}\left(\frac{u^{\frac{1}{2}}}{\frac{1}{2}}\right) \{+c\}$	giving $\pm\beta u^{\frac{1}{2}}$	dM1
	$= -\frac{8}{9}u^{\frac{1}{2}} \{+c\}$		
	$= -\frac{8}{9}\sqrt{(1+3e^{-x})} \{+c\}$		A1
			[5]
(ii)	$= -\frac{8}{9}\left(\sqrt{(1+3e^{-0})} - \sqrt{(1+3e^{--3\ln 2})}\right)$	Applying limits of $x = -3\ln 2$ and $x = 0$ to an expression of the form $\pm A\sqrt{(1+3e^{-x})}$ and subtracts either way round. **See notes.**	M1
	$= -\frac{8}{9}\left(\sqrt{4} - \sqrt{25}\right)$		
	$= \frac{8}{3}$	$\frac{8}{3}$ or awrt 2.67	A1
			[2]
			11

Notes

(a)	**B1:** 1.4792 correct answer only. Look for this on the table or in the candidate's working.
(b)	**B1**: Outside brackets $\frac{1}{2}\times \ln 2$ or $\frac{\ln 2}{2}$ or awrt 0.35 or $\frac{\text{awrt } 0.69}{2}$. Also allow $-\frac{1}{2}\times \ln 2$ or $-\frac{\ln 2}{2}$ or awrt -0.35 or $-\frac{\text{awrt } 0.69}{2}$. **M1:** <u>For structure of trapezium rule</u>$\left[\underline{\ldots\ldots\ldots\ldots}\right]$ **A1:** anything that rounds to 2.69 **<u>Note:</u>** It can be possible to award : (a) B0 (b) B1M1A1 (awrt 2.69) **<u>Note:</u>** Working must be seen to demonstrate the use of the trapezium rule. **<u>Note</u>**: actual area is 2.66666… **<u>Note:</u>** Award B1M1A1 for $\frac{\ln 2}{2}(2.1333 + 0.6667) + \ln 2(\text{their } 1.4792 + 1.0079) = 2.694332406...$

Notes for Question 4 continued	
4. (b) contd.	<u>Bracketing mistake</u>: **Unless the final answer implies that the calculation has been done correctly,** Award B1M0A0 for $\frac{1}{2}\times\ln 2 + 2.1333 + 2\big(\text{their } 1.4792 + 1.0079\big) + 0.6667$ (nb: answer of 8.12077...). Award B1M0A0 for $\frac{1}{2}\times\ln 2\ \ (2.1333 + 0.6667) + 2\big(\text{their } 1.4792 + 1.0079\big)$ (nb: answer of 5.94461...). *<u>Alternative method for part (b): Adding individual trapezia</u>* Area $\approx\ \ln 2\times\left[\frac{2.1333+1.4792}{2} + \frac{1.4792+1.0079}{2} + \frac{1.0079+0.6667}{2}\right] =\ 2.694332406...$ **B1:** $\ln 2$ and a divisor of 2 on all terms inside brackets. **M1:** First and last ordinates once and the middle ordinates twice inside brackets ignoring the 2. **A1:** anything that rounds to 2.69
(c)(i)	**NOTE: YOU CAN MARK (c)(i) AND (c)(ii) TOGETHER.** **B1:** For $\frac{\mathrm{d}u}{\mathrm{d}x} = -3\mathrm{e}^{-x}$ or $\mathrm{d}u = -3\mathrm{e}^{-x}\,\mathrm{d}x$ or $\frac{\mathrm{d}x}{\mathrm{d}u} = \frac{1}{-3\mathrm{e}^{-x}}$ or $\frac{\mathrm{d}x}{\mathrm{d}u} = -\frac{\mathrm{e}^{x}}{3}$ or equivalent. **Award B1** for $\frac{\mathrm{d}x}{\mathrm{d}u} = \frac{-\frac{1}{3}}{\frac{u-1}{3}}$ **(**which can be obtained from differentiating $x = -\ln\left(\frac{u-1}{3}\right)$). **M1:** Applying the substitution and achieving $\pm\lambda\int\frac{1}{\sqrt{u}}\,(\mathrm{d}u)$ or $\pm\lambda\int u^{-\frac{1}{2}}\,(\mathrm{d}u)$, $\lambda\neq 0$ **Note:** Any $(u-1)$ terms need to be cancelled out for this M1 mark. **A1:** $\int\frac{4}{3u^{\frac{1}{2}}}\frac{(\mathrm{d}u)}{(-3)}$ or $-\frac{4}{9}\int\frac{1}{\sqrt{u}}\,(\mathrm{d}u)$ or $-\frac{4}{9}\int u^{-\frac{1}{2}}\,(\mathrm{d}u)$ or $\int\frac{-\frac{4}{3}}{3\sqrt{u}}\,(\mathrm{d}u)$ or equivalent. **Ignore the presence of limits, but note that** $\int_{-3\ln 2}^{0}\frac{4\mathrm{e}^{-x}}{3\sqrt{(1+3\mathrm{e}^{-x})}}\,\mathrm{d}x = \int_{4}^{25}\frac{4}{9\sqrt{u}}\,\mathrm{d}u$ **dM1: (dependent on the first M mark)** Integrates $\pm\lambda\int\frac{1}{\sqrt{u}}\,\mathrm{d}u$ to give $\pm\beta u^{\frac{1}{2}}$, $\lambda\neq 0$, $\beta\neq 0$ **A1:** $-\frac{8}{9}\sqrt{(1+3\mathrm{e}^{-x})}$, simplified or un-simplified, with/without $+c$ **Note:** $\int\frac{\frac{4(u-1)}{3}}{3\sqrt{u}}\times\frac{-\mathrm{d}u}{(u-1)}$ is 1st M0A0 unless the $(u-1)$ terms have been cancelled out later but $\int\frac{\frac{4\,\cancel{(u-1)}}{3}}{3\sqrt{u}}\times\frac{-\mathrm{d}u}{\cancel{(u-1)}}$ is 1st M1A1.
(c)(ii)	**M1:** Applies limits of $x = -3\ln 2$ or -2.07... and $x = 0$ to an expression in the form $\pm A\sqrt{(1+3\mathrm{e}^{-x})}$ and subtracts either way round. Or attempts to apply limits of $u = 25$ and $u = 4$ to an expression in the form $\pm\beta u^{\frac{1}{2}}$ and subtracts either way round. **A1:** $\frac{8}{3}$ or anything that rounds to 2.67. **Note:** The final A1 mark in (c)(ii) is dependent on (c)(i) B1M1A1M1 and (c)(ii) M1.

Leave blank

5. Given that $y = 2$ at $x = \dfrac{\pi}{8}$, solve the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3y^2}{2\sin^2 2x}$$

giving your answer in the form $y = \mathrm{f}(x)$.

(6)

Question Number	Scheme	Marks
5.	$\frac{dy}{dx} = \frac{3y^2}{2\sin^2 2x}$ $\quad y = 2$ at $x = \frac{\pi}{8}$	
	$\int \frac{1}{y^2}\,dy = \int \frac{3}{2\sin^2 2x}\,dx$ — Separates variables as shown. Can be implied. Ignore the integral signs.	B1
	$\int \frac{1}{y^2}\,dy = \int \frac{3}{2}\text{cosec}^2 2x\,dx$	
	$\frac{1}{y^2} \to -\frac{1}{y}$. **(See notes).**	B1
	$-\frac{1}{y} = \frac{3}{2}\left(-\frac{\cot 2x}{2}\right)\ \{+c\}$ — $\pm\lambda\cot 2x$	M1
	$-\frac{1}{y} = \frac{3}{2}\left(-\frac{\cot 2x}{2}\right)$	A1 oe
	$\left\{y = 2,\ x = \frac{\pi}{8} \Rightarrow\right\}\quad -\frac{1}{2} = -\frac{3}{4}\cot\left(2\left(\frac{\pi}{8}\right)\right) + c$ — Use of $x = \frac{\pi}{8}$ and $y = 2$ in an integrated equation containing c	M1
	$-\frac{1}{2} = -\frac{3}{4} + c \Rightarrow c = \frac{1}{4}$	
	$-\frac{1}{y} = -\frac{3}{4}\cot 2x + \frac{1}{4} = \frac{1 - 3\cot 2x}{4}$	
	So, $\quad y = \frac{-1}{-\frac{3}{4}\cot 2x + \frac{1}{4}}$ or $y = \frac{4}{3\cot 2x - 1}$ or $y = \frac{4\tan 2x}{3 - \tan 2x}$	A1 oe
		[6] 6

Notes

B1: Separates variables as shown. dy and dx should be in the correct positions, though this mark can be implied by later working. Ignore the integral signs. The numbers "3" and "2" may appear on either side.

Eg: $\int \frac{2}{y^2}\,dy = \int \frac{3}{\sin^2 2x}\,dx$, $\quad \int \frac{2}{3y^2}\,dy = \int \frac{1}{\sin^2 2x}\,dx$,

$\int \frac{1}{3y^2}\,dy = \int \frac{1}{2\sin^2 2x}\,dx$ are all fine for B1.

B1: $\frac{1}{y^2} \to -\frac{1}{y}$ or $\frac{2}{y^2} \to -\frac{2}{y}$ or $\frac{2}{3y^2} \to -\frac{2}{3y}$ or $\frac{1}{3y^2} \to -\frac{1}{3y}$

M1: $\frac{1}{\sin^2 2x}$ or $\text{cosec}^2 2x \to \pm\lambda\cot 2x$, $\lambda \neq 0$

A1: $-\frac{1}{y} = \frac{3}{2}\left(-\frac{\cot 2x}{2}\right)$ with/without $+c$ or equivalent. Eg: $\frac{4}{3y} = \cot 2x$

M1: Some evidence of using both $x = \frac{\pi}{8}$ and $y = 2$ in an integrated or changed equation containing c.

Note that is mark can be implied by the correct value of c.

A1: $y = \frac{-1}{-\frac{3}{4}\cot 2x + \frac{1}{4}}$ or $y = \frac{4}{3\cot 2x - 1}$ or $y = \frac{4\tan 2x}{3 - \tan 2x}$ **or any equivalent correct answer.**

Note: You can ignore subsequent working which follows from a correct answer.

Leave blank

6. Oil is leaking from a storage container onto a flat section of concrete at a rate of $0.48\ \text{cm}^3\ \text{s}^{-1}$. The leaking oil spreads to form a pool with an increasing circular cross-section. The pool has a constant uniform thickness of 3 mm.

Find the rate at which the radius r of the pool of oil is increasing at the instant when $r = 5$ cm. Give your answer, in cm s^{-1}, to 3 significant figures.

(5)

Question Number	Scheme		Marks
6.	From question, $\frac{dV}{dt} = 0.48$		
	$V = \pi r^2(0.3)$	$V = 0.3\pi r^2$ (Can be implied.)	B1 oe
	$\frac{dV}{dr} = 0.6\pi r$		B1 ft
	$\left\{\frac{dV}{dr} \times \frac{dr}{dt} = \frac{dV}{dt}\right\} \Rightarrow (0.6\pi r)\frac{dr}{dt} = 0.48$ $\left\{\frac{dr}{dt} = \frac{dV}{dt} \div \frac{dV}{dr}\right\} \Rightarrow \frac{dr}{dt} = (0.48)\frac{1}{0.6\pi r};\ \left\{= \frac{4}{5\pi r}\right\}$	$\left(\text{Candidate's } \frac{dV}{dr}\right) \times \frac{dr}{dt} = 0.48$ or $0.48 \div \text{Candidate's } \frac{dV}{dr}$;	M1; oe
	When $r = 5\text{cm}$, $\frac{dr}{dt} = \frac{0.48}{0.6\pi(5)}$ $\left\{= \frac{4}{5\pi(5)}\right\}$	Substitutes $r = 5$ into an equation containing $\frac{dr}{dt}$.	dM1
	Hence, $\frac{dr}{dt} = 0.05092958179...(\text{cm s}^{-1})$	anything that rounds to 0.0509	A1
			[5] **5**

Notes

B1: $V = \pi r^2(0.3)$ or equivalent.

B1ft: Correct follow through differentiation of their V or their A with respect to r.

M1: $\left(\text{Candidate's } \frac{dV}{dr}\right) \times \frac{dr}{dt} = 0.48$ or $0.48 \div \text{Candidate's } \frac{dV}{dr}$

dM1: (dependent on the previous method mark) Substitutes $r = 5$ into an equation containing $\frac{dr}{dt}$.

A1: anything that rounds to 0.0509

Example 1: Using thickness = 3 (cm) and not 0.3 (cm)

$V = 3\pi r^2 \Rightarrow \frac{dV}{dr} = 6\pi r$ leading to $\left.\frac{dr}{dt}\right|_{t=5} = \frac{0.48}{6\pi(5)} = 0.005092958179...$ gets B0B1ftM1M1A0.

Example 2: Using thickness = 0.03 (cm) and not 0.3 (cm)

$V = 0.03\pi r^2 \Rightarrow \frac{dV}{dr} = 0.06\pi r$ leading to $\left.\frac{dr}{dt}\right|_{t=5} = \frac{0.48}{0.06\pi(5)} = 0.5092958179...$ gets B0B1ftM1M1A0.

Alternative method 1 First 3 marks

$A = \pi r^2$ **and** $\frac{dA}{dt} = \frac{0.48}{0.3}\ \{= 1.6\}$	Can be implied.	B1 oe
$\frac{dA}{dr} = 2\pi r$	ft $\frac{dA}{dr}$	B1 ft
$\left\{\frac{dA}{dr} \times \frac{dr}{dt} = \frac{dA}{dt}\right\} \Rightarrow (2\pi r)\frac{dr}{dt} = 1.6$ or $\frac{dr}{dt} = (1.6)\frac{1}{2\pi r};\ \left\{= \frac{4}{5\pi r}\right\}$		M1; oe

Alternative method 2 First 3 marks

$A = \pi r^2$ **and** $V = 0.3A$	Can be implied.	B1 oe
$\frac{dA}{dr} = 2\pi r$, $\left\{\frac{dV}{dA} = 0.3\right\}$	ft $\frac{dA}{dr}$	B1 ft
$\left\{\frac{dr}{dt} = \frac{dr}{dA} \times \frac{dA}{dV} \times \frac{dV}{dt}\right\} \Rightarrow \frac{dr}{dt} = \frac{1}{2\pi r}\left(\frac{1}{0.3}\right)(0.48);\ \left\{= \frac{4}{5\pi r}\right\}$		M1; oe

Leave blank

7. The curve C has parametric equations

$$x = 2\cos t, \quad y = \sqrt{3}\cos 2t, \qquad 0 \leqslant t \leqslant \pi$$

where t is a parameter.

(a) Find an expression for $\dfrac{dy}{dx}$ in terms of t.

(2)

The point P lies on C where $t = \dfrac{2\pi}{3}$

The line l is a normal to C at P.

(b) Show that an equation for l is

$$2x - 2\sqrt{3}y - 1 = 0$$

(5)

The line l intersects the curve C again at the point Q.

(c) Find the exact coordinates of Q.
You must show clearly how you obtained your answers.

(6)

Question Number	Scheme		Marks
7. (a)	$x = 2\cos t, \quad y = \sqrt{3}\cos 2t, \quad 0 \leqslant t \leqslant \pi$		
	$\frac{dy}{dx} = \frac{-2\sqrt{3}\sin 2t}{-2\sin t}\left\{= \frac{\sqrt{3}\sin 2t}{\sin t} = 2\sqrt{3}\cos t\right\}$	Candidate's $\frac{dy}{dt} \div \frac{dx}{dt}$ Correct simplified or un-simplified result.	M1 A1 oe **cso**
			[2]
(b)	$\left\{\text{When } t = \frac{2\pi}{3},\right\}\quad x = -1,\ y = -\frac{\sqrt{3}}{2}$ (need values)	The point $\left(-1, -\frac{\sqrt{3}}{2} \text{ or awrt } -0.87\right)$. **These coordinates can be implied.**	B1
	$m(\mathbf{T}) = \frac{dy}{dx} = \frac{\sqrt{3}\sin\left(2\left(\frac{2\pi}{3}\right)\right)}{\sin\left(\frac{2\pi}{3}\right)} = \frac{\sqrt{3}\left(-\frac{\sqrt{3}}{2}\right)}{\left(\frac{\sqrt{3}}{2}\right)}\ \left\{= -\sqrt{3}\right\}$	Inserts $t = \frac{2\pi}{3}$ into their $\frac{dy}{dx}$. Can be implied.	M1
	So, $m(\mathbf{N}) = \frac{1}{\sqrt{3}}$	Applies $m(\mathbf{N}) = -\frac{1}{m(\mathbf{T})}$	M1
	N: $y - -\frac{\sqrt{3}}{2} = \frac{1}{\sqrt{3}}(x - -1)$ or $-\frac{\sqrt{3}}{2} = \frac{1}{\sqrt{3}}(-1) + c \Rightarrow c = -\frac{\sqrt{3}}{6}$	See notes.	M1
	N: $y + \frac{\sqrt{3}}{2} = \frac{1}{\sqrt{3}}(x+1)$ **N**: $2\sqrt{3}y + 3 = 2x + 2$ **N**: $2x - 2\sqrt{3}y - 1 = 0$	Proves the result $2x - 2\sqrt{3}y - 1 = 0$ using exact values.	A1 * cso
			[5]
(c)	$2(2\cos t) - 2\sqrt{3}\left(\sqrt{3}\cos 2t\right) - 1 = 0$	Substitutes $x = 2\cos t$ and $y = \sqrt{3}\cos 2t$ into **N** to form an equation in variable t.	M1
	$4\cos t - 6\cos 2t - 1 = 0$ $6\cos 2t - 4\cos t + 1 = 0$		
	$6(2\cos^2 t - 1) - 4\cos t + 1 = 0$	Applies $\cos 2t = 2\cos^2 t - 1$	M1
	$12\cos^2 t - 4\cos t - 5 = 0$	$12\cos^2 t - 4\cos t - 5\ \{= 0\}$. See notes.	A1 oe
	$(6\cos t - 5)(2\cos t + 1) = 0 \Rightarrow \cos t = ...$		ddM1
	$\cos t = \frac{5}{6},\ \left\{\cos t = -\frac{1}{2}\right\}$		
	So $(x, y) = \left(\frac{5}{3}, \frac{7}{18}\sqrt{3}\right)$	At least one of either x or y correct. (See notes) Both x and y correct.	A1 oe A1 oe
			[6] **13**

Notes for Question 7	
7(a)	**Note:** Award M1A0 for a candidate who writes (explicitly) $\frac{\mathrm{d}y}{\mathrm{d}t} = 2\sqrt{3}\sin 2t$, $\frac{\mathrm{d}x}{\mathrm{d}t} = 2\sin t$ followed by $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2\sqrt{3}\sin 2t}{2\sin t}$. **Note:** Award M1A1for $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2\sqrt{3}\sin 2t}{2\sin t}$ with no explicit reference to $\frac{\mathrm{d}y}{\mathrm{d}t}$ and $\frac{\mathrm{d}x}{\mathrm{d}t}$. **Note:** Also award M1A1 for $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y/\mathrm{d}t}{\mathrm{d}x/\mathrm{d}t} = \frac{2\sqrt{3}\sin 2t}{2\sin t}$ with no explicit reference to $\frac{\mathrm{d}y}{\mathrm{d}t}$ and $\frac{\mathrm{d}x}{\mathrm{d}t}$.
(b)	**B1:** $x = -1$, $y = -\frac{\sqrt{3}}{2}$ or $\left(-1, -\frac{\sqrt{3}}{2} \text{ or awrt } -0.87\right)$. You can imply these coordinates from later working. **M1:** Inserts $t = \frac{2\pi}{3}$ into their $\frac{\mathrm{d}y}{\mathrm{d}x}$. This mark can be implied by a correct ft value from their $\left.\frac{\mathrm{d}y}{\mathrm{d}x}\right\|_{t=\frac{2\pi}{3}}$. **M1:** Applies $\mathrm{m}(\mathbf{N}) = -\frac{1}{\mathrm{m}(\mathbf{T})}$. Numerical value for $\mathrm{m}(\mathbf{N})$ is required here. **M1:** Use $y - (\text{their } y_1) = (\text{their } m_N)(x - (\text{their } x_1))$. or ***finds c*** by substituting $\left(\text{their } -1, \text{ their } -\frac{\sqrt{3}}{2}\right)$ into $y = (\text{their } m_N)x + c$ where $m_N = -\frac{1}{\text{their m}(\mathbf{T})}$ or $m_N = \frac{1}{\text{their m}(\mathbf{T})}$ or $m_N = -\text{their m}(\mathbf{T})$. **Note:** Numerical values for their x_1 , y_1 and $\mathrm{m}(\mathbf{N})$ are required here. **A1: (correct solution only from** $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2\sqrt{3}\sin 2t}{2\sin t}$ **)** Convincing proof of $2x - 2\sqrt{3}y - 1 = 0$ **(answer given)** with no errors. Eg 1: $y + \frac{\sqrt{3}}{2} = \frac{1}{\sqrt{3}}(x+1) \Rightarrow 2\sqrt{3}y + 3 = 2x + 2 \Rightarrow 2x - 2\sqrt{3}y - 1 = 0$ Eg 2: $y = \frac{1}{\sqrt{3}}x - \frac{\sqrt{3}}{6} \Rightarrow \sqrt{3}y = x - \frac{1}{2} \Rightarrow 2\sqrt{3}y = 2x - 1 \Rightarrow 2x - 2\sqrt{3}y - 1 = 0$ **Note:** Candidate need to work in exact values to prove $2x - 2\sqrt{3}y - 1 = 0$ for the final A1.
7. (c)	**M1:** Substitutes $x = 2\cos t$ and $y = \sqrt{3}\cos 2t$ into $2x - 2\sqrt{3}y - 1 = 0$ to form an equation in only one variable. **M1:** Applies $\cos 2t = 2\cos^2 t - 1$ **A1:** For obtaining either $12\cos^2 t - 4\cos t - 5\ \{= 0\}$ or $-12\cos^2 t + 4\cos t + 5\ \{= 0\}$ This mark can also awarded for a correct three term equation eg. $12\cos^2 t - 4\cos t = 5$ or $12\cos^2 x = 4\cos x + 5$ etc.

Notes for Question 7 continued

ddM1 (dependent on the previous 2 M marks)

See page 4: Method mark for solving a 3 term quadratic.

- $(6\cos t - 5)(2\cos t + 1) = 0 \Rightarrow \cos t = ...$
- $\cos t = \dfrac{4 \pm \sqrt{16 - 4(12)(-5)}}{2(12)}$
- $\cos^2 t - \dfrac{1}{3}\cos t - \dfrac{5}{12} = 0 \Rightarrow \left(\cos t - \dfrac{1}{6}\right)^2 - \dfrac{1}{36} - \dfrac{5}{12} = 0 \Rightarrow \cos t = ...$
- Or writes down at least one correct root from their quadratic equation.

A1: Any one of either $x = \dfrac{5}{3}$ or 1.66 or awrt 1.67 or $y = \dfrac{7}{18}\sqrt{3}$ or $\dfrac{21}{18\sqrt{3}}$ or awrt 0.67

A1: Both $x = \dfrac{5}{3}$ and $y = \dfrac{7}{18}\sqrt{3}$ or $\dfrac{21}{18\sqrt{3}}$ (both exact values required here for the final A1.)

Note: A candidate cannot obtain any of the final two accuracy marks unless the first three marks (M1M1A1) have already been awarded.

<u>(c): Alternative Method 1</u> Forming a Cartesian equation from $x = 2\cos t,\ y = \sqrt{3}\cos 2t$

$y = \sqrt{3}\cos 2t = \sqrt{3}\left(2\cos^2 t - 1\right)$ — **2nd M1:** For applying $\cos 2t = 2\cos^2 t - 1$

So $y = \sqrt{3}\left(\dfrac{2x^2}{4} - 1\right) = \dfrac{\sqrt{3}}{2}x^2 - \sqrt{3}$

$2x - 2\sqrt{3}\left(\dfrac{\sqrt{3}}{2}x^2 - \sqrt{3}\right) - 1 = 0$ — **1st M1:** For substituting their $y = \dfrac{\sqrt{3}}{2}x^2 - \sqrt{3}$ into **N**.

$3x^2 - 2x - 5 = 0$ — **A1:** For $3x^2 - 2x - 5\ \{= 0\}$ or $3x^2 - 2x = 5$, etc.

$(x+1)(3x-5) = 0 \Rightarrow x = ...$ — **ddM1:** For attempting to solve a quadratic equation.

$x = \dfrac{5}{3},\ y = \dfrac{7}{18}\sqrt{3}$ — **A1A1:** As above.

<u>(c): Alternative Method 2</u> Forming a Cartesian equation from $x = 2\cos t,\ y = \sqrt{3}\cos 2t$

$y = \sqrt{3}\cos 2t = \sqrt{3}\left(2\cos^2 t - 1\right)$ — **2nd M1:** For applying $\cos 2t = 2\cos^2 t - 1$

So $y = \dfrac{\sqrt{3}}{2}x^2 - \sqrt{3} \Rightarrow x^2 = \dfrac{2\sqrt{3}}{3}\left(y + \sqrt{3}\right)$

$2x = 2\sqrt{3}y + 1 \Rightarrow 4x^2 = 12y^2 + 4\sqrt{3}y + 1$

$4\left(\dfrac{2\sqrt{3}}{3}\left(y + \sqrt{3}\right)\right) = 12y^2 + 4\sqrt{3}y + 1$ — **1st M1:** For substituting their $x^2 = \dfrac{2\sqrt{3}}{3}\left(y + \sqrt{3}\right)$ or $x = \sqrt{\dfrac{2\sqrt{3}}{3}\left(y + \sqrt{3}\right)}$ into **N**.

$36y^2 + 4\sqrt{3}y - 21 = 0$ — **A1:** For $36y^2 + 4\sqrt{3}y - 21\ \{= 0\}$ or $12y^2 + \dfrac{4\sqrt{3}}{3}y - 7\ \{= 0\}$ etc.

$\left(18y - 7\sqrt{3}\right)\left(2y + \sqrt{3}\right) = 0 \Rightarrow y = ...$ — **ddM1:** For attempting to solve a quadratic equation.

$y = \dfrac{7}{18}\sqrt{3},\ x = \dfrac{5}{3}$ — **A1A1:** As above.

Leave blank

8. With respect to a fixed origin O, the lines l_1 and l_2 are given by the equations

$$l_1 : \mathbf{r} = \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}, \qquad l_2 : \mathbf{r} = \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix} + \mu \begin{pmatrix} 5 \\ -2 \\ 5 \end{pmatrix}$$

where λ and μ are scalar parameters.

(a) Find, to the nearest 0.1°, the acute angle between l_1 and l_2 **(3)**

The point A has position vector $\begin{pmatrix} 0 \\ 1 \\ 6 \end{pmatrix}$.

(b) Show that A lies on l_1 **(1)**

The lines l_1 and l_2 intersect at the point X.

(c) Write down the coordinates of X. **(1)**

(d) Find the exact value of the distance AX. **(2)**

The distinct points B_1 and B_2 both lie on the line l_2

Given that $AX = XB_1 = XB_2$

(e) find the area of the triangle AB_1B_2 giving your answer to 3 significant figures. **(3)**

Given that the x coordinate of B_1 is positive,

(f) find the exact coordinates of B_1 and the exact coordinates of B_2 **(5)**

Question Number	Scheme		Marks
8.	$l_1 : \mathbf{r} = \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}$, $l_2 : \mathbf{r} = \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix} + \mu \begin{pmatrix} 5 \\ -2 \\ 5 \end{pmatrix}$ So $\mathbf{d}_1 = \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}$, $\mathbf{d}_2 = \begin{pmatrix} 5 \\ -2 \\ 5 \end{pmatrix}$. $\overrightarrow{OA} = \begin{pmatrix} 0 \\ 1 \\ 6 \end{pmatrix}$.		
(a)	So $\mathbf{d}_1 = \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}$, $\mathbf{d}_2 = \begin{pmatrix} 5 \\ -2 \\ 5 \end{pmatrix} \Rightarrow \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix} \bullet \begin{pmatrix} 5 \\ -2 \\ 5 \end{pmatrix}$	Realisation that the dot product is required between $\pm A\mathbf{d_1}$ and $\pm B\mathbf{d}_2$.	M1
	$\cos\theta = \pm\left(\dfrac{-5-4+5}{\sqrt{(-1)^2+(2)^2+(1)^2}\,.\sqrt{(5)^2+(-2)^2+(5)^2}}\right)$	Correct **equation**.	A1
	$\cos\theta = \dfrac{-4}{18} \Rightarrow \theta = 102.8395884...$ So acute angle $= 77.16041159...$	awrt 77.2	A1
			[3]
(b)	$\mathbf{r} = \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix} + 2\begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 6 \end{pmatrix}$. $(= \overrightarrow{OA}$. Hence the point A lies on l_1).	Substitutes candidate's $\lambda = 2$ into l_1 and finds $\mathbf{j} + 6\mathbf{k}$. The conclusion on this occasion is not needed.	B1
			[1]
(c)	$\{l_1 = l_2 \Rightarrow\}$ So $X(2, -3, 4)$.	$X(2, -3, 4)$	B1
			[1]
(d)	$\overrightarrow{AX} = \overrightarrow{OX} - \overrightarrow{OA} = \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \\ 6 \end{pmatrix} = \begin{pmatrix} 2 \\ -4 \\ -2 \end{pmatrix}$		
	$AX = \sqrt{(2)^2 + (-4)^2 + (-2)^2} = \sqrt{24}\ \{= 2\sqrt{6}\}$	Full method for finding AX. $\sqrt{24}$ or $2\sqrt{6}$ or 4.89 or awrt 4.90	M1 A1
			[2]
(e)	Area $AB_1B_2 = \left(\frac{1}{2}(\sqrt{24})^2 \sin 77.1604...^{\circ}\right); \times 2 = 23.3990503...$	awrt 23.4 or $\dfrac{8}{3}\sqrt{77}$	M1; dM1 A1
			[3]

Question Number	Scheme		Marks
8. (f)	$\overrightarrow{XB} = \begin{pmatrix} 5\mu \\ -2\mu \\ 5\mu \end{pmatrix}$ and $XB = \sqrt{24}$		
	$\left\{AX^2 =\right\}\ (5\mu)^2 + (-2\mu)^2 + (5\mu)^2 = 24$		M1
	$\left\{\Rightarrow 54\mu^2 = 24 \Rightarrow \mu^2 = \frac{4}{9} \Rightarrow\right\}\ \mu = \pm\frac{2}{3}$	Either $\mu = \frac{2}{3}$ or $\mu = -\frac{2}{3}$	A1
	$l_2 : \mathbf{r} = \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix} \pm \frac{2}{3}\begin{pmatrix} 5 \\ -2 \\ 5 \end{pmatrix}$	Substitutes at least one of their values of μ into l_2.	dM1
	$\left\{\overrightarrow{OB_1}\right\} = \begin{pmatrix} \frac{16}{3} \\ -\frac{13}{3} \\ \frac{22}{3} \end{pmatrix}$ or $\begin{pmatrix} 5\frac{1}{3} \\ -4\frac{1}{3} \\ 7\frac{1}{3} \end{pmatrix}$, $\left\{\overrightarrow{OB_2}\right\} = \begin{pmatrix} -\frac{4}{3} \\ -\frac{5}{3} \\ \frac{2}{3} \end{pmatrix}$ or $\begin{pmatrix} -1\frac{1}{3} \\ -1\frac{2}{3} \\ \frac{2}{3} \end{pmatrix}$	At least one set of coordinates are correct.	A1
		Both sets of coordinates are correct.	A1
			[5] **15**

Notes

8. (a)

M1: Realisation that the dot product is required between $\pm A\mathbf{d_1}$ and $\pm B\mathbf{d}_2$. Allow one copying slip.

A1: Correct application of the dot product formula $\mathbf{d_1} \bullet \mathbf{d_2} = \pm|\mathbf{d_1}||\mathbf{d_2}|\cos\theta$ or $\cos\theta = \pm\left(\frac{\mathbf{d_1} \bullet \mathbf{d_2}}{|\mathbf{d_1}||\mathbf{d_2}|}\right)$

The dot product must be correctly applied, and the square roots although they can be un-simplified must be correctly applied.

A1: awrt 77.2 $\theta = 1.3467...^c$ or $\theta = 1.7948...^c$ is A0.

Alternative Method: Vector Cross Product

Only apply this scheme if it is clear that a candidate is applying a vector cross product method.

$\mathbf{d}_1 \times \mathbf{d}_2 = \underline{\begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix} \times \begin{pmatrix} 5 \\ -2 \\ 5 \end{pmatrix}} = \left\{ \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & 2 & 1 \\ 5 & -2 & 5 \end{vmatrix} = 12\mathbf{i} + 10\mathbf{j} - 8\mathbf{k} \right\}$ **<u>M1</u>:** Realisation that the vector cross product is required between $\pm A\mathbf{d_1}$ and $\pm B\mathbf{d}_2$. Allow one copying slip.

$$\sin\theta = \frac{\sqrt{(12)^2 + (10)^2 + (-8)^2}}{\sqrt{(-1)^2 + (2)^2 + (1)^2}\,.\sqrt{(5)^2 + (-2)^2 + (5)^2}}$$ **A1:** Correct applied **equation**.

$\sin\theta = \frac{\sqrt{308}}{\sqrt{6}.\sqrt{54}} \Rightarrow \theta = 77.16041159... = 77.2$ (1 dp) **A1:** awrt 77.2

(b) **B1:** Substitutes candidate's $\lambda = 2$ into l_1 and finds $\mathbf{j} + 6\mathbf{k}$. The conclusion on this occasion is not needed.

Note: $\lambda = 2 \Rightarrow r = \mathbf{j} + 6\mathbf{k}$ is not sufficient working for B1.

Note: Writing $2 - \lambda = 0$, $2\lambda - 3 = 1$, $\lambda + 4 = 6$ followed by $\lambda = 2$ is ok for B1.

(c) **B1:** $(2, -3, 4)$ or $2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$ etc.

	Notes for Question 8 continued
8. (d)	**Working must occur in part (d) only.** **M1:** Finds the difference between their $\overrightarrow{OX}$ and $\overrightarrow{OA}$ and applies Pythagoras to the result to find either AX or AX^2. **A1:** $\sqrt{24}$ or $2\sqrt{6}$ or 4.89 or awrt 4.90 . **M1A1** can be awarded for seeing either $\sqrt{24}$ or $2\sqrt{6}$ or 4.89 or awrt 4.90 as their answer.
(e)	**NOTE: Parts (e) and (f) can be marked together.** **M1:** Either $\frac{1}{2}\left(\text{their }"\sqrt{24}"\right)^2\sin\left(\text{their }(77.2^\circ)\text{ from (a)}\right)$ or $\frac{1}{2}\left(\text{their }"\sqrt{24}"\right)^2\sin\left(\text{their }(180-77.2^\circ)\text{ from (a)}\right)$. awrt 11.7 will usually imply this mark. **dM1:** Multiplies one of their areas by 2 for triangle AB_1B_2 or writes down **both areas** for ΔAXB_1 and ΔAXB_2. **A1:** awrt 23.4 __Note:__ **Award M1dM1 for** $\left(\text{their }"\sqrt{24}"\right)^2\sin\left(\text{their }(77.2^\circ)\text{ from (a)}\right)$ __Note:__ **Award M1dM1 for** $(24)\left(\frac{\sqrt{77}}{9}\right)$ __**Alternative Method 1:**__ Some candidates may apply $\frac{1}{2}$(base)(height) "perpendicular" height = $\left(\text{their }"\sqrt{24}"\right)\sin\left(\text{their }(77.2^\circ)\text{ from (a)}\right)$ **Award M1dM1** for $\frac{1}{2}\left(2(\text{their }"\sqrt{24}")\right)\left(\text{their }"\sqrt{24}"\right)\sin\left(\text{their }(77.2^\circ)\text{ from (a)}\right)$, where their $"\sqrt{24}"$'s are consistent, i.e. the same. __**Alternative Method 2:**__ **M1:** $\frac{1}{2}\left(\text{their }"\sqrt{24}"\right)\left(\text{their }AB="6.11"\right)\sin A$, where $A=51.42°$, or $A=\frac{1}{2}\left(180-\text{their }(77.2^\circ)\text{ from (a)}\right)$. **Note:** there must be a full method for finding the length *AB*. (i.e. from either the sine rule or the cosine rule.) **dM1:** Multiplies one of their areas by 2 for triangle AB_1B_2 or writes down **both areas** for ΔAXB_1 and ΔAXB_2. **A1:** awrt 23.4
(f)	**M1:** Writes down an equation relating the $\lvert\overrightarrow{AX}\rvert$ to $\lvert\overrightarrow{XB}\rvert$ or $\lvert\overrightarrow{AX}\rvert^2$ to $\lvert\overrightarrow{XB}\rvert^2$. M1 can also be awarded for either $\frac{\text{their }\lvert\overrightarrow{AX}\rvert}{\text{their }\lvert\mathbf{d}_2\rvert}$ or $\frac{\text{their }\lvert\mathbf{d}_2\rvert}{\text{their }\lvert\overrightarrow{AX}\rvert}$. **A1:** Either $\mu=\frac{2}{3}$ or $\mu=-\frac{2}{3}$ **dM1:** **(Dependent on the previous method mark)** Substitutes at least one of their values of μ into l_2. If no working shown then two out of three of the components must be correctly followed through. **A1:** At least one set of coordinates are correct. Ignore labelling of B_1, B_2 **A1:** Both sets of coordinates are correct. Ignore labelling of B_1, B_2