

June 2005
6666 Core C4
Mark Scheme

Question Number	Scheme	Marks
1.	$(4-9x)^{\frac{1}{2}} = 2\left(1 - \frac{9x}{4}\right)^{\frac{1}{2}}$ $= 2\left(1 + \frac{\frac{1}{2}\left(-\frac{9x}{4}\right)}{1} + \frac{\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{9x}{4}\right)^2}{1.2} + \frac{\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{9x}{4}\right)^3}{1.2.3} + \dots\right)$ $= 2\left(1 - \frac{9}{8}x - \frac{81}{128}x^2 - \frac{729}{1024}x^3 + \dots\right)$ $= 2 - \frac{9}{4}x - \frac{81}{64}x^2 - \frac{729}{512}x^3 + \dots$ <p><i>Note</i> The M1 is gained for $\frac{\frac{1}{2}\left(-\frac{1}{2}\right)}{1.2}\left(\dots\right)^2$ or $\frac{\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{1.2.3}\left(\dots\right)^3$</p> <p><i>Special Case</i></p> <p>If the candidate reaches $= 2\left(1 - \frac{9}{8}x - \frac{81}{128}x^2 - \frac{729}{1024}x^3 + \dots\right)$ and goes no further allow A1 A0 A0</p>	<p>B1</p> <p>M1</p> <p>A1, A1, A1</p> <p>[5]</p>

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2.	$2x + \left(2x \frac{dy}{dx} + 2y \right) - 6y \frac{dy}{dx} = 0$ $\frac{dy}{dx} = 0 \Rightarrow x + y = 0 \quad \text{or equivalent}$ <p>Eliminating either variable and solving for at least one value of x or y.</p> $y^2 - 2y^2 - 3y^2 + 16 = 0 \quad \text{or the same equation in } x$ $y = \pm 2 \quad \text{or } x = \pm 2$ $(2, -2), (-2, 2)$ <p>Note: $\frac{dy}{dx} = \frac{x + y}{3y - x}$</p> <p><i>Alternative</i></p> $3y^2 - 2xy - (x^2 + 16) = 0$ $y = \frac{2x \pm \sqrt{(16x^2 + 192)}}{6}$ $\frac{dy}{dx} = \frac{1}{3} \pm \frac{1}{3} \cdot \frac{8x}{\sqrt{(16x^2 + 192)}}$ $\frac{dy}{dx} = 0 \Rightarrow \frac{8x}{\sqrt{(16x^2 + 192)}} = \pm 1$ $64x^2 = 16x^2 + 192$ $x = \pm 2$ $(2, -2), (-2, 2)$	<p>M1 (A1) A1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>[7]</p> <p>M1 A1± A1</p> <p>M1</p> <p>M1 A1</p> <p>A1</p> <p>[7]</p>

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3.	<p>(a)</p> $\frac{5x+3}{(2x-3)(x+2)} = \frac{A}{2x-3} + \frac{B}{x+2}$ $5x+3 = A(x+2) + B(2x-3)$ <p>Substituting $x = -2$ or $x = \frac{3}{2}$ and obtaining A or B; or equating coefficients and solving a pair of simultaneous equations to obtain A or B.</p> $A = 3, B = 1$ <p>If the cover-up rule is used, give M1 A1 for the first of A or B found, A1 for the second.</p> <p>(b)</p> $\int \frac{5x+3}{(2x-3)(x+2)} dx = \frac{3}{2} \ln(2x-3) + \ln(x+2)$ $\left[\dots \right]_2^6 = \frac{3}{2} \ln 9 + \ln 2$ $= \ln 54$	<p>M1</p> <p>A1, A1 (3)</p> <p>M1 A1ft</p> <p>M1 A1</p> <p>cao A1 (5) [8]</p>

Question Number	Scheme	Marks
4.	$\int \frac{1}{(1-x^2)^{\frac{1}{2}}} dx = \int \frac{1}{(1-\sin^2 \theta)^{\frac{1}{2}}} \cos \theta d\theta \quad \text{Use of } x = \sin \theta \text{ and } \frac{dx}{d\theta} = \cos \theta$ $= \int \frac{1}{\cos^2 \theta} d\theta$ $= \int \sec^2 \theta d\theta = \tan \theta$ <p>Using the limits 0 and $\frac{\pi}{6}$ to evaluate integral</p> $[\tan \theta]_0^{\frac{\pi}{6}} = \frac{1}{\sqrt{3}} \quad \left(= \frac{\sqrt{3}}{3} \right)$ <p><i>Alternative for final M1 A1</i></p> <p>Returning to the variable x and using the limits 0 and $\frac{1}{2}$ to evaluate integral</p> $\left[\frac{x}{\sqrt{1-x^2}} \right]_0^{\frac{1}{2}} = \frac{1}{\sqrt{3}} \quad \left(= \frac{\sqrt{3}}{3} \right)$	<p>M1</p> <p>M1 A1</p> <p>M1 A1</p> <p>M1</p> <p>cao A1</p> <p>[7]</p> <p>M1</p> <p>cao A1</p>

5.

Figure 1

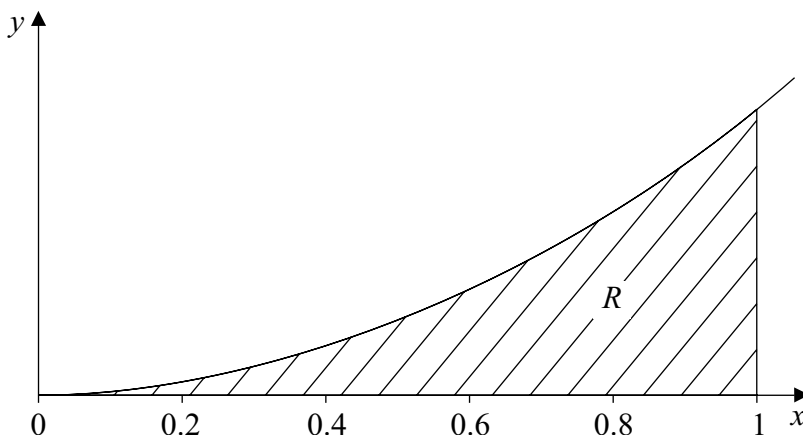


Figure 1 shows the graph of the curve with equation

$$y = xe^{2x}, \quad x \geq 0.$$

The finite region R bounded by the lines $x = 1$, the x -axis and the curve is shown shaded in Figure 1.

(a) Use integration to find the exact value for the area of R . (5)

(b) Complete the table with the values of y corresponding to $x = 0.4$ and 0.8 .

x	0	0.2	0.4	0.6	0.8	1
$y = xe^{2x}$	0	0.29836		1.99207		7.38906

(1)

(c) Use the trapezium rule with all the values in the table to find an approximate value for this area, giving your answer to 4 significant figures.

(4)



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<p>5.</p>	<p>(a) $\int x e^{2x} dx = \frac{1}{2} x e^{2x} - \frac{1}{2} \int e^{2x} dx$ Attempting parts in the right direction</p> $= \frac{1}{2} x e^{2x} - \frac{1}{4} e^{2x}$ $\left[\frac{1}{2} x e^{2x} - \frac{1}{4} e^{2x} \right]_0^1 = \frac{1}{4} + \frac{1}{4} e^2$	<p>M1 A1</p> <p>A1</p> <p>M1 A1</p> <p>(5)</p>
	<p>(b) $x = 0.4 \Rightarrow y \approx 0.89022$ $x = 0.8 \Rightarrow y \approx 3.96243$ Both are required to 5 d.p</p>	<p>B1</p> <p>(1)</p>
	<p>(c) $I \approx \frac{1}{2} \times 0.2 \times [\dots]$ $\approx \dots \times [0 + 7.38906 + 2(0.29836 + .89022 + 1.99207 + 3.96243)]$ $\approx 0.1 \times 21.67522$ ≈ 2.168</p> <p>Note $\frac{1}{4} + \frac{1}{4} e^2 \approx 2.097 \dots$</p>	<p>B1</p> <p>M1 A1ft</p> <p>ft their answers to (b)</p> <p>cao</p> <p>A1</p> <p>(4)</p> <p>[10]</p>

Question Number	Scheme	Marks
<p>6.</p>	<p>(a) $\frac{dx}{dt} = -2 \operatorname{cosec}^2 t, \frac{dy}{dt} = 4 \sin t \cos t$ both</p> <p>$\frac{dy}{dx} = \frac{-2 \sin t \cos t}{\operatorname{cosec}^2 t} (= -2 \sin^3 t \cos t)$</p>	<p>M1 A1</p> <p>M1 A1</p> <p>(4)</p>
	<p>(b) At $t = \frac{\pi}{4}, x = 2, y = 1$ both x and y</p> <p>Substitutes $t = \frac{\pi}{4}$ into an attempt at $\frac{dy}{dx}$ to obtain gradient $\left(-\frac{1}{2}\right)$</p> <p>Equation of tangent is $y - 1 = -\frac{1}{2}(x - 2)$</p> <p>Accept $x + 2y = 4$ or any correct equivalent</p>	<p>B1</p> <p>M1</p> <p>M1 A1</p> <p>(4)</p>
	<p>(c) Uses $1 + \cot^2 t = \operatorname{cosec}^2 t$, or equivalent, to eliminate t</p> <p>$1 + \left(\frac{x}{2}\right)^2 = \frac{2}{y}$ correctly eliminates t</p> <p>$y = \frac{8}{4 + x^2}$ cao</p> <p>The domain is $x \dots 0$</p>	<p>M1</p> <p>A1</p> <p>A1</p> <p>B1</p> <p>(4)</p>
	<p><i>An alternative in (c)</i></p> <p>$\sin t = \left(\frac{y}{2}\right)^{\frac{1}{2}}; \cos t = \frac{x}{2} \sin t = \frac{x}{2} \left(\frac{y}{2}\right)^{\frac{1}{2}}$</p> <p>$\sin^2 t + \cos^2 t = 1 \Rightarrow \frac{y}{2} + \frac{x^2}{4} \times \frac{y}{2} = 1$</p> <p>Leading to $y = \frac{8}{4 + x^2}$</p>	<p>M1 A1</p> <p>A1</p> <p>[12]</p>

Question Number	Scheme	Marks
7.	<p>(a) k component $2 + 4\lambda = -2 \Rightarrow \lambda = -1$</p> <p style="text-align: right;"><i>Note $\mu = 2$</i></p> <p>Substituting their λ (or μ) into equation of line and obtaining B</p> <p style="text-align: center;">$B: (2, 2, -2)$</p> <p style="text-align: right;">Accept vector forms</p>	<p>M1 A1</p> <p>M1</p> <p>A1</p> <p style="text-align: right;">(4)</p>
	<p>(b)</p> $\left \begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix} \right = \sqrt{18}; \quad \left \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} \right = \sqrt{2}$ <p style="text-align: right;">both</p> $\begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} = 1 + 1 + 0 (= 2)$	<p>B1</p> <p>B1</p>
	$\cos \theta = \frac{2}{\sqrt{18}\sqrt{2}} = \frac{1}{3}$ <p style="text-align: right;">cao</p>	<p>M1 A1</p> <p style="text-align: right;">(4)</p>
	<p>(c) $\overline{AB} = -\mathbf{i} + \mathbf{j} - 4\mathbf{k} \Rightarrow \overline{AB} ^2 = 18$ or $\overline{AB} = \sqrt{18}$ ignore direction of vector</p> <p>$\overline{BC} = 3\mathbf{i} - 3\mathbf{j} \Rightarrow \overline{BC} ^2 = 18$ or $\overline{BC} = \sqrt{18}$ ignore direction of vector</p> <p style="text-align: center;">Hence $\overline{AB} = \overline{BC}$ *</p>	<p>M1</p> <p>M1</p> <p>A1</p> <p style="text-align: right;">(3)</p>
	<p>(d) $\overline{OD} = 6\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$</p> <p style="text-align: right;">Allow first B1 for any two correct Accept column form or coordinates</p>	<p>B1 B1</p> <p style="text-align: right;">(2)</p> <p style="text-align: right;">[13]</p>

Question Number	Scheme	Marks
<p>8.</p>	<p>(a) $\frac{dV}{dt}$ is the rate of increase of volume (with respect to time)</p> <p>$-kV$: k is constant of proportionality and the negative shows decrease (or loss)</p> <p>giving $\frac{dV}{dt} = 20 - kV$ * These Bs are to be awarded independently</p>	<p>B1</p> <p>B1</p> <p>(2)</p>
	<p>(b) $\int \frac{1}{20 - kV} dV = \int 1 dt$ separating variables</p> <p>$-\frac{1}{k} \ln(20 - kV) = t \quad (+C)$</p> <p>Using $V = 0, t = 0$ to evaluate the constant of integration</p> <p>$c = -\frac{1}{k} \ln 20$</p> <p>$t = \frac{1}{k} \ln\left(\frac{20}{20 - kV}\right)$</p> <p>Obtaining answer in the form $V = A + B e^{-kt}$</p> <p>$V = \frac{20}{k} - \frac{20}{k} e^{-kt}$ Accept $\frac{20}{k}(1 - e^{-kt})$</p>	<p>M1</p> <p>M1 A1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>(6)</p>
	<p>(c) $\frac{dV}{dt} = 20 e^{-kt}$ Can be implied</p> <p>$\frac{dV}{dt} = 10, t = 5 \Rightarrow 10 = 20 e^{-kt} \Rightarrow k = \frac{1}{5} \ln 2 \approx 0.139$</p> <p>At $t = 10, V = \frac{75}{\ln 2}$ awrt 108</p>	<p>M1</p> <p>M1 A1</p> <p>M1 A1</p> <p>(5)</p>
	<p><i>Alternative to (b)</i></p> <p>Using printed answer and differentiating $\frac{dV}{dt} = -k B e^{-kt}$</p> <p>Substituting into differential equation</p> <p>$-k B e^{-kt} = 20 - k A - k B e^{-kt}$</p> <p>$A = \frac{20}{k}$</p> <p>Using $V = 0, t = 0$ in printed answer to obtain $A + B = 0$</p> <p>$B = -\frac{20}{k}$</p>	<p>M1</p> <p>M1</p> <p>M1 A1</p> <p>M1</p> <p>A1</p> <p>(6)</p>
	<p>[13]</p>	