Centre No.						Paper Reference							Surname	Initial(s)
Candidate No.						6	6	6	6	/	0	1	Signature	

Paper Reference(s)

6666/01

Edexcel GCE

Core Mathematics C4

Advanced Level

Tuesday 28 June 2005 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Green)	Nil

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
1	
2	
3	
4	
5	
6	
7	
8	
Total	

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initial(s) and signature.
Check that you have the correct question paper.
You must write your answer for each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.
There are 24 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

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1. Use the binomial theorem to expand

$$\sqrt{(4-9x)}, \qquad |x| < \frac{4}{9},$$

in ascending powers of x, up to and including the term in x^3, simplifying each term.

(5)

Q1

(Total 5 marks)

June 2005
6666 Core C4
Mark Scheme

Question Number	Scheme	Marks
1.	$(4-9x)^{\frac{1}{2}} = 2\left(1-\frac{9x}{4}\right)^{\frac{1}{2}}$	B1
	$= 2\left(1+\frac{\frac{1}{2}}{1}\left(-\frac{9x}{4}\right)+\frac{\frac{1}{2}\left(-\frac{1}{2}\right)}{1.2}\left(-\frac{9x}{4}\right)^2+\frac{\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{1.2.3}\left(-\frac{9x}{4}\right)^3+\ \ldots\right)$	M1
	$= 2\left(1-\frac{9}{8}x-\frac{81}{128}x^2-\frac{729}{1024}x^3+\ \ldots\right)$	
	$= 2-\frac{9}{4}x, -\frac{81}{64}x^2, -\frac{729}{512}x^3+\ \ldots$	A1, A1, A1
		[5]
	Note The M1 is gained for $\frac{\frac{1}{2}\left(-\frac{1}{2}\right)}{1.2}(\ \ldots\ )^2$ or $\frac{\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{1.2.3}(\ \ldots\ )^3$	
	Special Case	
	If the candidate reaches $= 2\left(1-\frac{9}{8}x-\frac{81}{128}x^2-\frac{729}{1024}x^3+\ \ldots\right)$ and goes no further	
	allow A1 A0 A0	

Leave blank

2. A curve has equation

$$x^2 + 2xy - 3y^2 + 16 = 0.$$

Find the coordinates of the points on the curve where $\frac{dy}{dx} = 0$.

(7)

Q2

(Total 7 marks)

Question Number	Scheme	Marks
2.	$2x + \left(2x\frac{dy}{dx} + 2y\right) - 6y\frac{dy}{dx} = 0$	M1 (A1) A1
	$\frac{dy}{dx} = 0 \quad \Rightarrow \quad x + y = 0$ or equivalent	M1
	Eliminating either variable and solving for at least one value of *x* or *y*.	M1
	$y^2 - 2y^2 - 3y^2 + 16 = 0$ or the same equation in *x*	
	$y = \pm 2$ or $x = \pm 2$	A1
	$(2, -2), (-2, 2)$	A1
		[7]
	Note: $\frac{dy}{dx} = \frac{x+y}{3y-x}$	
	Alternative	
	$3y^2 - 2xy - (x^2 + 16) = 0$	
	$y = \frac{2x \pm \sqrt{(16x^2 + 192)}}{6}$	
	$\frac{dy}{dx} = \frac{1}{3} \pm \frac{1}{3}.\frac{8x}{\sqrt{(16x^2+192)}}$	M1 A1± A1
	$\frac{dy}{dx} = 0 \quad \Rightarrow \quad \frac{8x}{\sqrt{(16x^2+192)}} = \pm 1$	M1
	$64x^2 = 16x^2 + 192$	
	$x = \pm 2$	M1 A1
	$(2, -2), (-2, 2)$	A1
		[7]

Leave blank

3. (a) Express $\frac{5x+3}{(2x-3)(x+2)}$ in partial fractions.

(3)

(b) Hence find the exact value of $\int_2^6 \frac{5x+3}{(2x-3)(x+2)}\,dx$, giving your answer as a single logarithm.

(5)

Question Number	Scheme	Marks
3.	(a) $\dfrac{5x+3}{(2x-3)(x+2)} = \dfrac{A}{2x-3} + \dfrac{B}{x+2}$	
	$5x+3 = A(x+2) + B(2x-3)$	
	Substituting $x=-2$ or $x=\frac{3}{2}$ and obtaining *A* or *B*; or equating coefficients and solving a pair of simultaneous equations to obtain *A* or *B*.	M1
	$A=3,\ B=1$	A1, A1 **(3)**
	If the cover-up rule is used, give M1 A1 for the first of *A* or *B* found, A1 for the second.	
	(b) $\displaystyle\int \frac{5x+3}{(2x-3)(x+2)}\,\mathrm{d}x = \frac{3}{2}\ln(2x-3) + \ln(x+2)$	M1 A1ft
	$\left[\ \dots\ \right]_2^6 = \dfrac{3}{2}\ln 9 + \ln 2$	M1 A1
	$= \ln 54$ cao	A1 **(5)** **[8]**

Leave blank

4. Use the substitution $x = \sin\theta$ to find the exact value of

$$\int_0^{\frac{1}{2}} \frac{1}{(1-x^2)^{\frac{3}{2}}}\,dx.$$

(7)

Question Number	Scheme	Marks
4.	$\int \frac{1}{(1-x^2)^{\frac{1}{2}}}\,dx = \int \frac{1}{(1-\sin^2\theta)^{\frac{3}{2}}}\cos\theta\,d\theta$ Use of $x = \sin\theta$ and $\frac{dx}{d\theta} = \cos\theta$	M1
	$= \int \frac{1}{\cos^2\theta}\,d\theta$	M1 A1
	$= \int \sec^2\theta\,d\theta = \tan\theta$	M1 A1
	Using the limits 0 and $\frac{\pi}{6}$ to evaluate integral	M1
	$[\tan\theta]_0^{\frac{\pi}{6}} = \frac{1}{\sqrt{3}} \quad \left(= \frac{\sqrt{3}}{3}\right)$ cao	A1
		[7]
	Alternative for final M1 A1	
	Returning to the variable x and using the limits 0 and $\frac{1}{2}$ to evaluate integral	M1
	$\left[\frac{x}{\sqrt{(1-x^2)}}\right]_0^{\frac{1}{2}} = \frac{1}{\sqrt{3}} \quad \left(= \frac{\sqrt{3}}{3}\right)$ cao	A1

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5.

Figure 1

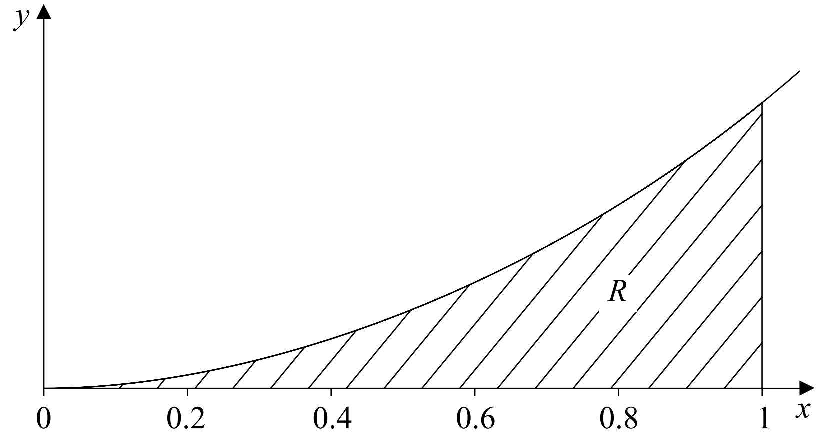

Figure 1 shows the graph of the curve with equation

$$y = xe^{2x}, \qquad x \geqslant 0.$$

The finite region R bounded by the lines $x = 1$, the x-axis and the curve is shown shaded in Figure 1.

(a) Use integration to find the exact value for the area of R.

(5)

(b) Complete the table with the values of y corresponding to $x = 0.4$ and 0.8.

x	0	0.2	0.4	0.6	0.8	1
$y = xe^{2x}$	0	0.29836		1.99207		7.38906

(1)

(c) Use the trapezium rule with all the values in the table to find an approximate value for this area, giving your answer to 4 significant figures.

(4)

Question Number	Scheme	Marks
5.	(a) $\int x\,e^{2x}dx = \frac{1}{2}x\,e^{2x} - \frac{1}{2}\int e^{2x}\,dx$ Attempting parts in the right direction	M1 A1
	$= \frac{1}{2}x\,e^{2x} - \frac{1}{4}e^{2x}$	A1
	$\left[\frac{1}{2}x\,e^{2x} - \frac{1}{4}e^{2x}\right]_0^1 = \frac{1}{4} + \frac{1}{4}e^2$	M1 A1
		(5)
	(b) $x = 0.4 \Rightarrow y \approx 0.890\,22$	
	$x = 0.8 \Rightarrow y \approx 3.962\,43$ Both are required to 5 d.p.	B1
		(1)
	(c) $I \approx \frac{1}{2} \times 0.2 \times [\ \dots\ ]$	B1
	$\approx \dots \times \left[0 + 7.389\,06 + 2\left(0.298\,36 + .890\,22 + 1.992\,07 + 3.962\,43\right)\right]$ ft their answers to (b)	M1 A1ft
	$\approx 0.1 \times 21.675\,22$	
	≈ 2.168 cao	A1 **(4)**
		[10]
	Note $\frac{1}{4} + \frac{1}{4}e^2 \approx 2.097\ \dots$	

Leave blank

6. A curve has parametric equations

$$x = 2\cot t, \quad y = 2\sin^2 t, \quad 0 < t \leqslant \frac{\pi}{2}.$$

(a) Find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of the parameter t.

(4)

(b) Find an equation of the tangent to the curve at the point where $t = \frac{\pi}{4}$.

(4)

(c) Find a cartesian equation of the curve in the form $y = \mathrm{f}(x)$. State the domain on which the curve is defined.

(4)

Question Number	Scheme		Marks	
6.	(a) $\frac{dx}{dt} = -2\operatorname{cosec}^2 t,\ \frac{dy}{dt} = 4\sin t\cos t$	both	M1 A1	
	$\frac{dy}{dx} = \frac{-2\sin t\cos t}{\operatorname{cosec}^2 t}$ $\left(= -2\sin^3 t\cos t\right)$		M1 A1	
				(4)
	(b) At $t = \frac{\pi}{4}$, $x = 2,\ y = 1$	both x and y	B1	
	Substitutes $t = \frac{\pi}{4}$ into an attempt at $\frac{dy}{dx}$ to obtain gradient $\left(-\frac{1}{2}\right)$		M1	
	Equation of tangent is $y - 1 = -\frac{1}{2}(x - 2)$		M1 A1	
	Accept $x + 2y = 4$ or any correct equivalent			**(4)**
	(c) Uses $1 + \cot^2 t = \operatorname{cosec}^2 t$, or equivalent, to eliminate t		M1	
	$1 + \left(\frac{x}{2}\right)^2 = \frac{2}{y}$	correctly eliminates t	A1	
	$y = \frac{8}{4 + x^2}$	cao	A1	
	The domain is $x \ldots 0$		B1	**(4)**
				[12]
	An alternative in (c)			
	$\sin t = \left(\frac{y}{2}\right)^{\frac{1}{2}}$; $\cos t = \frac{x}{2}\sin t = \frac{x}{2}\left(\frac{y}{2}\right)^{\frac{1}{2}}$			
	$\sin^2 t + \cos^2 t = 1 \quad \Rightarrow \quad \frac{y}{2} + \frac{x^2}{4} \times \frac{y}{2} = 1$		M1 A1	
	Leading to $y = \frac{8}{4 + x^2}$		A1	

Leave blank

7. The line l_1 has vector equation

$$\mathbf{r} = \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix}$$

and the line l_2 has vector equation

$$\mathbf{r} = \begin{pmatrix} 0 \\ 4 \\ -2 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix},$$

where λ and μ are parameters.

The lines l_1 and l_2 intersect at the point B and the acute angle between l_1 and l_2 is θ.

(a) Find the coordinates of B.
(4)

(b) Find the value of $\cos\theta$, giving your answer as a simplified fraction.
(4)

The point A, which lies on l_1, has position vector $\mathbf{a} = 3\mathbf{i} + \mathbf{j} + 2\mathbf{k}$.
The point C, which lies on l_2, has position vector $\mathbf{c} = 5\mathbf{i} - \mathbf{j} - 2\mathbf{k}$.
The point D is such that $ABCD$ is a parallelogram.

(c) Show that $|\overrightarrow{AB}| = |\overrightarrow{BC}|$.
(3)

(d) Find the position vector of the point D.
(2)

Question Number	Scheme	Marks
7.	(a) **k** component $2+4\lambda=-2 \;\Rightarrow\; \lambda=-1$	M1 A1
	Note $\mu=2$	
	Substituting their λ (or μ) into equation of line and obtaining B	M1
	B: $(2, 2, -2)$ Accept vector forms	A1
		(4)
	(b) $\left\|\begin{pmatrix}1\\-1\\4\end{pmatrix}\right\|=\sqrt{18}$; $\left\|\begin{pmatrix}1\\-1\\0\end{pmatrix}\right\|=\sqrt{2}$ both	B1
	$\begin{pmatrix}1\\-1\\4\end{pmatrix}\cdot\begin{pmatrix}1\\-1\\0\end{pmatrix}=1+1+0\,(=2)$	B1
	$\cos\theta=\dfrac{2}{\sqrt{18}\sqrt{2}}=\dfrac{1}{3}$ cao	M1 A1
		(4)
	(c) $\overrightarrow{AB}=-\mathbf{i}+\mathbf{j}-4\mathbf{k}\Rightarrow\left\|\overrightarrow{AB}\right\|^2=18$ or $\left\|\overrightarrow{AB}\right\|=\sqrt{18}$ ignore direction of vector	M1
	$\overrightarrow{BC}=3\mathbf{i}-3\mathbf{j}\Rightarrow\left\|\overrightarrow{BC}\right\|^2=18$ or $\left\|\overrightarrow{BC}\right\|=\sqrt{18}$ ignore direction of vector	M1
	Hence $\left\|\overrightarrow{AB}\right\|=\left\|\overrightarrow{BC}\right\|$ ∗	A1 **(3)**
	(d) $\overrightarrow{OD}=6\mathbf{i}-2\mathbf{j}+2\mathbf{k}$ Allow first B1 for any two correct Accept column form or coordinates	B1 B1
		(2)
		[13]

Leave blank

8. Liquid is pouring into a container at a constant rate of $20\ \text{cm}^3\ \text{s}^{-1}$ and is leaking out at a rate proportional to the volume of liquid already in the container.

(a) Explain why, at time t seconds, the volume, $V\ \text{cm}^3$, of liquid in the container satisfies the differential equation

$$\frac{\mathrm{d}V}{\mathrm{d}t} = 20 - kV,$$

where k is a positive constant.

(2)

The container is initially empty.

(b) By solving the differential equation, show that

$$V = A + B\mathrm{e}^{-kt},$$

giving the values of A and B in terms of k.

(6)

Given also that $\frac{\mathrm{d}V}{\mathrm{d}t} = 10$ when $t = 5$,

(c) find the volume of liquid in the container at 10 s after the start.

(5)

Question Number	Scheme	Marks
8.	(a) $\frac{dV}{dt}$ is the rate of increase of volume (with respect to time)	B1
	$-kV$: k is constant of proportionality and the negative shows decrease (or loss)	
	giving $\frac{dV}{dt} = 20 - kV$ ∗ These Bs are to be awarded independently	B1
		(2)
	(b) $\int \frac{1}{20-kV} dV = \int 1 dt$ separating variables	M1
	$-\frac{1}{k}\ln(20-kV) = t \quad (+C)$	M1 A1
	Using $V = 0, t = 0$ to evaluate the constant of integration	M1
	$c = -\frac{1}{k}\ln 20$	
	$t = \frac{1}{k}\ln\left(\frac{20}{20-kV}\right)$	
	Obtaining answer in the form $V = A + Be^{-kt}$	M1
	$V = \frac{20}{k} - \frac{20}{k}e^{-kt}$ Accept $\frac{20}{k}\left(1-e^{-kt}\right)$	A1 **(6)**
	(c) $\frac{dV}{dt} = 20e^{-kt}$ Can be implied	M1
	$\frac{dV}{dt} = 10, t = 5 \Rightarrow 10 = 20e^{-kt} \Rightarrow k = \frac{1}{5}\ln 2 \approx 0.139$	M1 A1
	At $t = 10$, $V = \frac{75}{\ln 2}$ awrt 108	M1 A1 **(5)**
		[13]
	Alternative to (b)	
	Using printed answer and differentiating $\frac{dV}{dt} = -kBe^{-kt}$	M1
	Substituting into differential equation	
	$-kBe^{-kt} = 20 - kA - kBe^{-kt}$	M1
	$A = \frac{20}{k}$	M1 A1
	Using $V = 0, t = 0$ in printed answer to obtain $A + B = 0$	M1
	$B = -\frac{20}{k}$	A1 **(6)**