

Centre No.						Paper Reference						Surname	Initial(s)	
Candidate No.						6	6	6	6	/	0	1	Signature	

Paper Reference(s)

6666/01

Edexcel GCE

Core Mathematics C4

Advanced Level

Tuesday 28 June 2005 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination
Mathematical Formulae (Green)

Items included with question papers
Nil

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

Examiner's use only

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Team Leader's use only

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Question Number	Leave Blank
1	
2	
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6	
7	
8	
Total	

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initial(s) and signature.

Check that you have the correct question paper.

You must write your answer for each question in the space following the question.

When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.

Full marks may be obtained for answers to ALL questions.

The marks for individual questions and the parts of questions are shown in round brackets: e.g. (2).

There are 8 questions in this question paper. The total mark for this paper is 75.

There are 24 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.

You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

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Turn over

June 2005
6666 Core C4
Mark Scheme

Question Number	Scheme	Marks
1.	$(4-9x)^{\frac{1}{2}} = 2\left(1 - \frac{9x}{4}\right)^{\frac{1}{2}}$ $= 2\left(1 + \frac{\frac{1}{2}\left(-\frac{9x}{4}\right)}{1} + \frac{\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{9x}{4}\right)^2}{1.2} + \frac{\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{9x}{4}\right)^3}{1.2.3} + \dots\right)$ $= 2\left(1 - \frac{9}{8}x - \frac{81}{128}x^2 - \frac{729}{1024}x^3 + \dots\right)$ $= 2 - \frac{9}{4}x - \frac{81}{64}x^2 - \frac{729}{512}x^3 + \dots$ <p><i>Note</i> The M1 is gained for $\frac{\frac{1}{2}\left(-\frac{1}{2}\right)}{1.2}\left(\dots\right)^2$ or $\frac{\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{1.2.3}\left(\dots\right)^3$</p> <p><i>Special Case</i> If the candidate reaches $= 2\left(1 - \frac{9}{8}x - \frac{81}{128}x^2 - \frac{729}{1024}x^3 + \dots\right)$ and goes no further allow A1 A0 A0</p>	<p>B1</p> <p>M1</p> <p>A1, A1, A1</p> <p>[5]</p>

Leave blank

2. A curve has equation

$$x^2 + 2xy - 3y^2 + 16 = 0.$$

Find the coordinates of the points on the curve where $\frac{dy}{dx} = 0$.

(7)

Lined area for writing the answer.

(Total 7 marks)

Q2



N 2 0 2 3 2 B 0 3 2 4

Question Number	Scheme	Marks
2.	$2x + \left(2x \frac{dy}{dx} + 2y \right) - 6y \frac{dy}{dx} = 0$ $\frac{dy}{dx} = 0 \Rightarrow x + y = 0 \quad \text{or equivalent}$ <p>Eliminating either variable and solving for at least one value of x or y.</p> $y^2 - 2y^2 - 3y^2 + 16 = 0 \quad \text{or the same equation in } x$ $y = \pm 2 \quad \text{or } x = \pm 2$ $(2, -2), (-2, 2)$ <p>Note: $\frac{dy}{dx} = \frac{x+y}{3y-x}$</p> <p><i>Alternative</i></p> $3y^2 - 2xy - (x^2 + 16) = 0$ $y = \frac{2x \pm \sqrt{(16x^2 + 192)}}{6}$ $\frac{dy}{dx} = \frac{1}{3} \pm \frac{1}{3} \cdot \frac{8x}{\sqrt{(16x^2 + 192)}}$ $\frac{dy}{dx} = 0 \Rightarrow \frac{8x}{\sqrt{(16x^2 + 192)}} = \pm 1$ $64x^2 = 16x^2 + 192$ $x = \pm 2$ $(2, -2), (-2, 2)$	<p>M1 (A1) A1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>[7]</p> <p>M1 A1± A1</p> <p>M1</p> <p>M1 A1</p> <p>A1</p> <p>[7]</p>

Question Number	Scheme	Marks
3.	<p>(a) $\frac{5x+3}{(2x-3)(x+2)} = \frac{A}{2x-3} + \frac{B}{x+2}$</p> $5x+3 = A(x+2) + B(2x-3)$ <p>Substituting $x = -2$ or $x = \frac{3}{2}$ and obtaining A or B; or equating coefficients and solving a pair of simultaneous equations to obtain A or B.</p> $A = 3, B = 1$ <p>If the cover-up rule is used, give M1 A1 for the first of A or B found, A1 for the second.</p> <p>(b) $\int \frac{5x+3}{(2x-3)(x+2)} dx = \frac{3}{2} \ln(2x-3) + \ln(x+2)$</p> $\left[\dots \right]_2^6 = \frac{3}{2} \ln 9 + \ln 2$ $= \ln 54$	<p>M1</p> <p>A1, A1 (3)</p> <p>M1 A1ft</p> <p>M1 A1</p> <p>cao A1 (5) [8]</p>

Question Number	Scheme	Marks
4.	<p style="text-align: center;"> $\int \frac{1}{(1-x^2)^{\frac{1}{2}}} dx = \int \frac{1}{(1-\sin^2 \theta)^{\frac{1}{2}}} \cos \theta d\theta \quad \text{Use of } x = \sin \theta \text{ and } \frac{dx}{d\theta} = \cos \theta$ $= \int \frac{1}{\cos^2 \theta} d\theta$ $= \int \sec^2 \theta d\theta = \tan \theta$ </p> <p>Using the limits 0 and $\frac{\pi}{6}$ to evaluate integral</p> <p style="text-align: center;"> $[\tan \theta]_0^{\frac{\pi}{6}} = \frac{1}{\sqrt{3}} \quad \left(= \frac{\sqrt{3}}{3} \right)$ </p> <p><i>Alternative for final M1 A1</i></p> <p>Returning to the variable x and using the limits 0 and $\frac{1}{2}$ to evaluate integral</p> <p style="text-align: center;"> $\left[\frac{x}{\sqrt{1-x^2}} \right]_0^{\frac{1}{2}} = \frac{1}{\sqrt{3}} \quad \left(= \frac{\sqrt{3}}{3} \right)$ </p>	<p>M1</p> <p>M1 A1</p> <p>M1 A1</p> <p>M1</p> <p>cao A1</p> <p style="text-align: right;">[7]</p> <p>M1</p> <p>cao A1</p>

5.

Figure 1

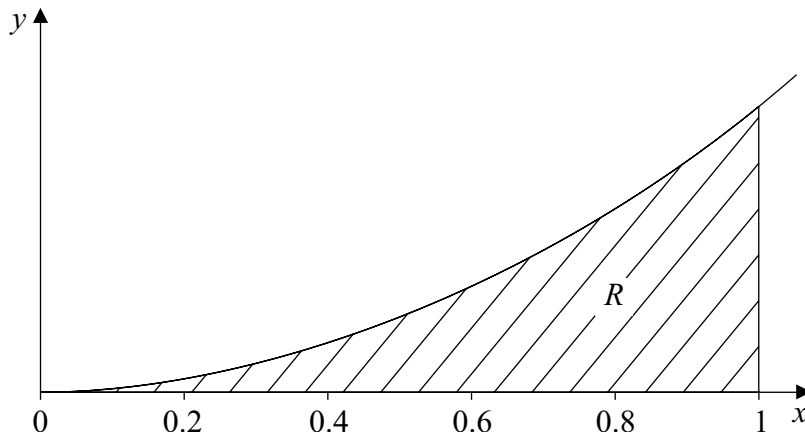


Figure 1 shows the graph of the curve with equation

$$y = xe^{2x}, \quad x \geq 0.$$

The finite region R bounded by the lines $x = 1$, the x -axis and the curve is shown shaded in Figure 1.

(a) Use integration to find the exact value for the area of R . (5)

(b) Complete the table with the values of y corresponding to $x = 0.4$ and 0.8 .

x	0	0.2	0.4	0.6	0.8	1
$y = xe^{2x}$	0	0.29836		1.99207		7.38906

(1)

(c) Use the trapezium rule with all the values in the table to find an approximate value for this area, giving your answer to 4 significant figures.

(4)



Question Number	Scheme	Marks
<p>5.</p>	<p>(a) $\int x e^{2x} dx = \frac{1}{2} x e^{2x} - \frac{1}{2} \int e^{2x} dx$ Attempting parts in the right direction</p> $= \frac{1}{2} x e^{2x} - \frac{1}{4} e^{2x}$ $\left[\frac{1}{2} x e^{2x} - \frac{1}{4} e^{2x} \right]_0^1 = \frac{1}{4} + \frac{1}{4} e^2$	<p>M1 A1</p> <p>A1</p> <p>M1 A1</p> <p>(5)</p>
	<p>(b) $x = 0.4 \Rightarrow y \approx 0.89022$ $x = 0.8 \Rightarrow y \approx 3.96243$ Both are required to 5 d.p</p>	<p>B1</p> <p>(1)</p>
	<p>(c) $I \approx \frac{1}{2} \times 0.2 \times [\dots]$</p> $\approx \dots \times [0 + 7.38906 + 2(0.29836 + .89022 + 1.99207 + 3.96243)]$ <p style="text-align: right;">ft their answers to (b)</p> $\approx 0.1 \times 21.67522$ ≈ 2.168 <p style="text-align: right;">cao</p> <p>Note $\frac{1}{4} + \frac{1}{4} e^2 \approx 2.097 \dots$</p>	<p>B1</p> <p>M1 A1ft</p> <p>A1 (4)</p> <p>[10]</p>

Question Number	Scheme	Marks
<p>6.</p>	<p>(a) $\frac{dx}{dt} = -2 \operatorname{cosec}^2 t, \frac{dy}{dt} = 4 \sin t \cos t$ both</p> <p>$\frac{dy}{dx} = \frac{-2 \sin t \cos t}{\operatorname{cosec}^2 t} (= -2 \sin^3 t \cos t)$</p>	<p>M1 A1</p> <p>M1 A1</p> <p>(4)</p>
	<p>(b) At $t = \frac{\pi}{4}, x = 2, y = 1$ both x and y</p> <p>Substitutes $t = \frac{\pi}{4}$ into an attempt at $\frac{dy}{dx}$ to obtain gradient $\left(-\frac{1}{2}\right)$</p> <p>Equation of tangent is $y - 1 = -\frac{1}{2}(x - 2)$</p> <p>Accept $x + 2y = 4$ or any correct equivalent</p>	<p>B1</p> <p>M1</p> <p>M1 A1</p> <p>(4)</p>
	<p>(c) Uses $1 + \cot^2 t = \operatorname{cosec}^2 t$, or equivalent, to eliminate t</p> <p>$1 + \left(\frac{x}{2}\right)^2 = \frac{2}{y}$ correctly eliminates t</p> <p>$y = \frac{8}{4 + x^2}$ cao</p>	<p>M1</p> <p>A1</p> <p>A1</p>
	<p>The domain is $x \dots 0$</p>	<p>B1</p> <p>(4)</p>
	<p><i>An alternative in (c)</i></p> <p>$\sin t = \left(\frac{y}{2}\right)^{\frac{1}{2}}; \cos t = \frac{x}{2} \sin t = \frac{x}{2} \left(\frac{y}{2}\right)^{\frac{1}{2}}$</p> <p>$\sin^2 t + \cos^2 t = 1 \Rightarrow \frac{y}{2} + \frac{x^2}{4} \times \frac{y}{2} = 1$</p> <p>Leading to $y = \frac{8}{4 + x^2}$</p>	<p>M1 A1</p> <p>A1</p> <p>[12]</p>

7. The line l_1 has vector equation

$$\mathbf{r} = \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix}$$

and the line l_2 has vector equation

$$\mathbf{r} = \begin{pmatrix} 0 \\ 4 \\ -2 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix},$$

where λ and μ are parameters.

The lines l_1 and l_2 intersect at the point B and the acute angle between l_1 and l_2 is θ .

(a) Find the coordinates of B . (4)

(b) Find the value of $\cos \theta$, giving your answer as a simplified fraction. (4)

The point A , which lies on l_1 , has position vector $\mathbf{a} = 3\mathbf{i} + \mathbf{j} + 2\mathbf{k}$.

The point C , which lies on l_2 , has position vector $\mathbf{c} = 5\mathbf{i} - \mathbf{j} - 2\mathbf{k}$.

The point D is such that $ABCD$ is a parallelogram.

(c) Show that $|\vec{AB}| = |\vec{BC}|$. (3)

(d) Find the position vector of the point D . (2)



Question Number	Scheme	Marks
7.	<p>(a) k component $2 + 4\lambda = -2 \Rightarrow \lambda = -1$</p> <p style="text-align: right;"><i>Note $\mu = 2$</i></p> <p>Substituting their λ (or μ) into equation of line and obtaining B</p> <p style="text-align: center;">$B: (2, 2, -2)$ Accept vector forms</p>	<p>M1 A1</p> <p>M1</p> <p>A1</p> <p style="text-align: right;">(4)</p>
	<p>(b) $\left \begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix} \right = \sqrt{18}; \quad \left \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} \right = \sqrt{2}$ both</p> <p style="text-align: center;">$\begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} = 1 + 1 + 0 (= 2)$</p>	<p>B1</p> <p>B1</p>
	<p style="text-align: center;">$\cos \theta = \frac{2}{\sqrt{18}\sqrt{2}} = \frac{1}{3}$ cao</p>	<p>M1 A1</p> <p style="text-align: right;">(4)</p>
	<p>(c) $\overline{AB} = -\mathbf{i} + \mathbf{j} - 4\mathbf{k} \Rightarrow \overline{AB} ^2 = 18$ or $\overline{AB} = \sqrt{18}$ ignore direction of vector</p> <p>$\overline{BC} = 3\mathbf{i} - 3\mathbf{j} \Rightarrow \overline{BC} ^2 = 18$ or $\overline{BC} = \sqrt{18}$ ignore direction of vector</p> <p style="text-align: center;">Hence $\overline{AB} = \overline{BC}$ *</p>	<p>M1</p> <p>M1</p> <p>A1</p> <p style="text-align: right;">(3)</p>
	<p>(d) $\overline{OD} = 6\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$</p> <p style="text-align: right;">Allow first B1 for any two correct Accept column form or coordinates</p>	<p>B1 B1</p> <p style="text-align: right;">(2)</p> <p style="text-align: right;">[13]</p>

8. Liquid is pouring into a container at a constant rate of $20 \text{ cm}^3 \text{ s}^{-1}$ and is leaking out at a rate proportional to the volume of liquid already in the container.

(a) Explain why, at time t seconds, the volume, $V \text{ cm}^3$, of liquid in the container satisfies the differential equation

$$\frac{dV}{dt} = 20 - kV,$$

where k is a positive constant.

(2)

The container is initially empty.

(b) By solving the differential equation, show that

$$V = A + Be^{-kt},$$

giving the values of A and B in terms of k .

(6)

Given also that $\frac{dV}{dt} = 10$ when $t = 5$,

(c) find the volume of liquid in the container at 10 s after the start.

(5)



Question Number	Scheme	Marks
<p>8.</p>	<p>(a) $\frac{dV}{dt}$ is the rate of increase of volume (with respect to time) $-kV$: k is constant of proportionality and the negative shows decrease (or loss) giving $\frac{dV}{dt} = 20 - kV$ * These Bs are to be awarded independently</p>	<p>B1 B1 (2)</p>
	<p>(b) $\int \frac{1}{20 - kV} dV = \int 1 dt$ separating variables $-\frac{1}{k} \ln(20 - kV) = t \quad (+C)$ Using $V = 0, t = 0$ to evaluate the constant of integration $c = -\frac{1}{k} \ln 20$ $t = \frac{1}{k} \ln\left(\frac{20}{20 - kV}\right)$ Obtaining answer in the form $V = A + B e^{-kt}$ $V = \frac{20}{k} - \frac{20}{k} e^{-kt}$ Accept $\frac{20}{k}(1 - e^{-kt})$</p>	<p>M1 M1 A1 M1 M1 A1 (6)</p>
	<p>(c) $\frac{dV}{dt} = 20 e^{-kt}$ Can be implied $\frac{dV}{dt} = 10, t = 5 \Rightarrow 10 = 20 e^{-5k} \Rightarrow k = \frac{1}{5} \ln 2 \approx 0.139$ At $t = 10, V = \frac{75}{\ln 2}$ awrt 108</p>	<p>M1 M1 A1 M1 A1 (5)</p>
	<p><i>Alternative to (b)</i> Using printed answer and differentiating $\frac{dV}{dt} = -k B e^{-kt}$ Substituting into differential equation $-k B e^{-kt} = 20 - k A - k B e^{-kt}$ $A = \frac{20}{k}$ Using $V = 0, t = 0$ in printed answer to obtain $A + B = 0$ $B = -\frac{20}{k}$</p>	<p>[13] M1 M1 M1 A1 M1 A1 (6)</p>