Centre No.						Paper Reference	Surname	Initial(s)
Candidate No.						6 6 6 6 / 0 1	Signature	

Paper Reference(s)

6666/01

Edexcel GCE

Core Mathematics C4

Advanced Level

Thursday 15 June 2006 – Afternoon

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
1	
2	
3	
4	
5	
6	
7	
Total	

Materials required for examination
Mathematical Formulae (Green)

Items included with question papers
Nil

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initial(s) and signature.
Check that you have the correct question paper.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.
You must write your answer for each question in the space following the question.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 7 questions in this question paper.
The total mark for this paper is 75.
There are 20 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the examiner. Answers without working may gain no credit.

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1. A curve C is described by the equation

$$3x^2 - 2y^2 + 2x - 3y + 5 = 0.$$

Find an equation of the normal to C at the point (0, 1), giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(7)

June 2006
6666 Core Mathematics C4
Mark Scheme

Question Number	Scheme		Marks
1.	$\left\{\frac{\not{d}y}{\not{d}x} \times\right\}$ $6x - 4y\frac{dy}{dx} + 2 - 3\frac{dy}{dx} = 0$	Differentiates implicitly to include either $\pm ky\frac{dy}{dx}$ or $\pm 3\frac{dy}{dx}$. (Ignore $\left(\frac{dy}{dx} = \right)$.) Correct equation.	M1 A1
	$\left\{\frac{dy}{dx} = \frac{6x+2}{4y+3}\right\}$	*not necessarily required.*	
	At (0, 1), $\frac{dy}{dx} = \frac{0+2}{4+3} = \frac{2}{7}$	Substituting x = 0 & y = 1 into an *equation* involving $\frac{dy}{dx}$; to give $\frac{2}{7}$ or $\frac{-2}{-7}$	dM1; A1 **cso**
	Hence m(**N**) = $-\frac{7}{2}$ or $\frac{-1}{\frac{2}{7}}$	Uses m(**T**) to 'correctly' find m(**N**). Can be ft from "their tangent gradient".	A1$\sqrt{}$ oe.
	Either **N**: $y - 1 = -\frac{7}{2}(x - 0)$ or **N**: $y = -\frac{7}{2}x + 1$	$y - 1 = m(x - 0)$ with 'their tangent or normal gradient'; or uses $y = mx + 1$ with 'their tangent or normal gradient';	M1;
	N: 7x + 2y – 2 = 0	Correct equation in the form $'ax + by + c = 0'$, where a, b and c are integers.	A1 oe **cso**
			[7]
			7 marks

Beware: $\frac{dy}{dx} = \frac{2}{7}$ does not necessarily imply the award of all the first four marks in this question.
So please ensure that you check candidates' initial differentiation before awarding the first A1 mark.

Beware: The final accuracy mark is for completely correct solutions. If a candidate flukes the final line then they must be awarded A0.

Beware: A candidate finding an m(**T**) = 0 can obtain A1ft for m(**N**) = ∞, but obtains M0 if they write $y - 1 = \infty(x - 0)$. If they write, however, **N**: x = 0, then can score M1.

Beware: A candidate finding an m(**T**) = ∞ can obtain A1ft for m(**N**) = 0, and also obtains M1 if they write $y - 1 = 0(x - 0)$ or y = 1.

Beware: The final **cso** refers to the whole question.

Question Number	Scheme		Marks
Aliter			
1.	$\left\{ \frac{\not{d}\not{x}}{\not{d}\not{y}} \times \right\}$ $6x\frac{dx}{dy} - 4y + 2\frac{dx}{dy} - 3 = 0$	Differentiates implicitly to include either $\pm kx\frac{dx}{dy}$ or $\pm 2\frac{dx}{dy}$. (Ignore $\left(\frac{dx}{dy} = \right)$.) Correct equation.	M1 A1
Way 2	$\left\{\frac{dx}{dy} = \frac{4y+3}{6x+2}\right\}$	*not necessarily required.*	
	At (0, 1), $\frac{dx}{dy} = \frac{4+3}{0+2} = \frac{7}{2}$	Substituting x = 0 & y = 1 into an *equation* involving $\frac{dx}{dy}$; to give $\frac{7}{2}$	dM1; A1 **cso**
	Hence m(**N**) = $-\frac{7}{2}$ or $\frac{-1}{\frac{2}{7}}$	Uses m(**T**) or $\frac{dx}{dy}$ to 'correctly' find m(**N**). Can be ft using "$-1.\frac{dx}{dy}$".	A1$\sqrt{}$ oe.
	Either **N**: $y - 1 = -\frac{7}{2}(x - 0)$ or **N**: $y = -\frac{7}{2}x + 1$	$y - 1 = m(x - 0)$ with 'their tangent, $\frac{dx}{dy}$ or normal gradient'; or uses $y = mx + 1$ with 'their tangent, $\frac{dx}{dy}$ or normal gradient';	M1;
	N: 7x + 2y – 2 = 0	Correct equation in the form '$ax + by + c = 0$', where a, b and c are integers.	A1 oe **cso**
			7 marks

Question Number	Scheme		Marks
Aliter **1. Way 3**	$2y^2 + 3y - 3x^2 - 2x - 5 = 0$		
	$\left(y + \frac{3}{4}\right)^2 - \frac{9}{16} = \frac{3x^2}{2} + x + \frac{5}{2}$		
	$y = \sqrt{\left(\frac{3x^2}{2} + x + \frac{49}{16}\right)} - \frac{3}{4}$		
	$\frac{dy}{dx} = \frac{1}{2}\left(\frac{3x^2}{2} + x + \frac{49}{16}\right)^{-\frac{1}{2}}(3x+1)$	Differentiates using the chain rule; Correct expression for $\frac{dy}{dx}$.	M1; A1 oe
	At (0, 1), $\frac{dy}{dx} = \frac{1}{2}\left(\frac{49}{16}\right)^{-\frac{1}{2}} = \frac{1}{2}\left(\frac{4}{7}\right) = \frac{2}{7}$	Substituting x = 0 into an *equation* involving $\frac{dy}{dx}$; to give $\frac{2}{7}$ or $\frac{-2}{-7}$	dM1 A1 **cso**
	Hence m(**N**) = $-\frac{7}{2}$	Uses m(**T**) to 'correctly' find m(**N**). Can be ft from "their tangent gradient".	A1$\sqrt{}$
	Either **N**: $y - 1 = -\frac{7}{2}(x - 0)$ or **N**: $y = -\frac{2}{7}x + 1$	$y - 1 = m(x - 0)$ with 'their tangent or normal gradient'; or uses $y = mx + 1$ with 'their tangent or normal gradient'	M1
	N: 7x + 2y – 2 = 0	Correct equation in the form '$ax + by + c = 0$', where a, b and c are integers.	A1 oe
			[7]
			7 marks

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2. $$\mathrm{f}(x) = \frac{3x-1}{(1-2x)^2}, \qquad |x| < \tfrac{1}{2}.$$

Given that, for $x \neq \frac{1}{2}$, $\dfrac{3x-1}{(1-2x)^2} = \dfrac{A}{(1-2x)} + \dfrac{B}{(1-2x)^2}$, where A and B are constants,

(a) find the values of A and B.

(3)

(b) Hence, or otherwise, find the series expansion of f(x), in ascending powers of x, up to and including the term in x^3, simplifying each term.

(6)

Question Number	Scheme		Marks
2. (a)	$3x-1 \equiv A(1-2x)+B$	Considers this identity and either substitutes $x=\frac{1}{2}$, equates coefficients or solves simultaneous equations	*complete* M1
	Let $x=\frac{1}{2}$; $\frac{3}{2}-1=B \Rightarrow B=\frac{1}{2}$		
	Equate x terms; $3=-2A \Rightarrow A=-\frac{3}{2}$	$A=-\frac{3}{2}$; $B=\frac{1}{2}$	A1;A1
	(**No working seen**, but A and B correctly stated $\Rightarrow$ award all three marks. If one of A or B correctly stated give two out of the three marks available for this part.)		**[3]**
(b)	$f(x)=-\frac{3}{2}(1-2x)^{-1}+\frac{1}{2}(1-2x)^{-2}$	Moving powers to top on any one of the two expressions	M1
	$=-\frac{3}{2}\left\{\underline{1+(-1)(-2x);+\frac{(-1)(-2)}{2!}(-2x)^2+\frac{(-1)(-2)(-3)}{3!}(-2x)^3+\dots}\right\}$	Either $1\pm 2x$ or $1\pm 4x$ from either first or second expansions respectively	dM1;
	$+\frac{1}{2}\left\{\underline{1+(-2)(-2x);+\frac{(-2)(-3)}{2!}(-2x)^2+\frac{(-2)(-3)(-4)}{3!}(-2x)^3+\dots}\right\}$	Ignoring $-\frac{3}{2}$ and $\frac{1}{2}$, any one correct $\{\underline{\dots\dots\dots}\}$ expansion. Both $\{\underline{\dots\dots\dots}\}$ correct.	A1 A1
	$=-\frac{3}{2}\{1+2x+4x^2+8x^3+\dots\}+\frac{1}{2}\{1+4x+12x^2+32x^3+\dots\}$		
	$=-1-x\ ;+0x^2+4x^3$	$-1-x$; $(0x^2)+4x^3$	A1; A1 **[6]**
			9 marks

Question Number	Scheme		Marks
Aliter **2.** (b) **Way 2**	$f(x) = (3x-1)(1-2x)^{-2}$	Moving power to top	M1
	$= (3x-1) \times \left(1 + (-2)(-2x) ; + \frac{(-2)(-3)}{2!}(-2x)^2 + \frac{(-2)(-3)(-4)}{3!}(-2x)^3 + \ldots\right)$	$1 \pm 4x$; Ignoring $(3x-1)$, correct $(\ldots\ldots\ldots\ldots)$ expansion	dM1; A1
	$= (3x-1)(1 + 4x + 12x^2 + 32x^3 + \ldots)$		
	$= \underline{3x + 12x^2 + 36x^3 - 1 - 4x - 12x^2 - 32x^3} + \ldots$	Correct expansion	A1
	$= -1 - x ; + 0x^2 + 4x^3$	$-1-x$; $(0x^2) + 4x^3$	A1; A1 **[6]**
Aliter **2.** (b) **Way 3**	Maclaurin expansion		
	$f(x) = -\frac{3}{2}(1-2x)^{-1} + \frac{1}{2}(1-2x)^{-2}$	Bringing both powers to top	M1
	$f'(x) = -3(1-2x)^{-2} + 2(1-2x)^{-3}$	Differentiates to give $a(1-2x)^{-2} \pm b(1-2x)^{-3}$; $-3(1-2x)^{-2} + 2(1-2x)^{-3}$	M1; A1 oe
	$f''(x) = -12(1-2x)^{-3} + 12(1-2x)^{-4}$		
	$f'''(x) = -72(1-2x)^{-4} + 96(1-2x)^{-5}$	Correct $f''(x)$ and $f'''(x)$	A1
	$\therefore f(0) = -1$, $f'(0) = -1$, $f''(0) = 0$ and $f'''(0) = 24$		
	gives $f(x) = -1 - x ; + 0x^2 + 4x^3 + \ldots$	$-1-x$; $(0x^2) + 4x^3$	A1; A1 **[6]**

Question Number	Scheme		Marks
Aliter **2.** (b) **Way 4**	$f(x) = -3(2-4x)^{-1} + \frac{1}{2}(1-2x)^{-2}$	Moving powers to top on any one of the two expressions	M1
	$= -3\left\{\underline{(2)^{-1} + (-1)(2)^{-2}(-4x); + \frac{(-1)(-2)}{2!}(2)^{-3}(-4x)^2 + \frac{(-1)(-2)(-3)}{3!}(2)^{-4}(-4x)^3 + \ldots}\right\}$	Either $\frac{1}{2} \pm x$ or $1 \pm 4x$ from either first or second expansions respectively	dM1;
	$+\frac{1}{2}\left\{\underline{1 + (-2)(-2x); + \frac{(-2)(-3)}{2!}(-2x)^2 + \frac{(-2)(-3)(-4)}{3!}(-2x)^3 + \ldots}\right\}$	Ignoring -3 and $\frac{1}{2}$, any one correct $\{\underline{\ldots\ldots\ldots\ldots}\}$ expansion. Both $\{\underline{\ldots\ldots\ldots\ldots}\}$ correct.	A1 A1
	$= -3\left\{\frac{1}{2} + x + 2x^2 + 4x^3 + \ldots\right\} + \frac{1}{2}\left\{1 + 4x + 12x^2 + 32x^3 + \ldots\right\}$		
	$= -1 - x\ ; + 0x^2 + 4x^3$	$-1 - x\ ;\quad (0x^2) + 4x^3$	A1; A1
			[6]

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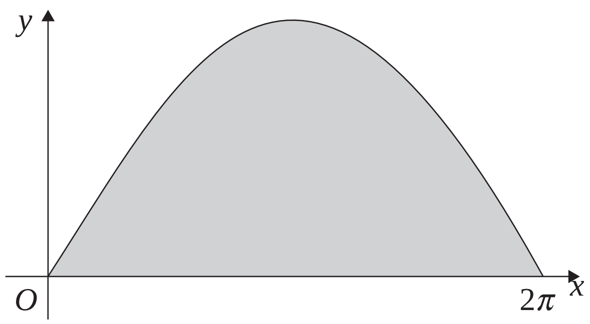
3. **Figure 1**



The curve with equation $y = 3\sin\frac{x}{2}$, $0 \leqslant x \leqslant 2\pi$, is shown in Figure 1. The finite region enclosed by the curve and the x-axis is shaded.

(a) Find, by integration, the area of the shaded region.

(3)

This region is rotated through 2π radians about the x-axis.

(b) Find the volume of the solid generated.

(6)

Question Number	Scheme		Marks
3. (a)	Area Shaded $= \int_0^{2\pi} 3\sin\left(\frac{x}{2}\right)\,dx$		
	$= \left[\dfrac{-3\cos\left(\frac{x}{2}\right)}{\frac{1}{2}}\right]_0^{2\pi}$	Integrating $3\sin\left(\frac{x}{2}\right)$ to give $k\cos\left(\frac{x}{2}\right)$ with $k \neq 1$. Ignore limits.	M1
	$= \left[-6\cos\left(\frac{x}{2}\right)\right]_0^{2\pi}$	$-6\cos\left(\frac{x}{2}\right)$ or $\frac{-3}{\frac{1}{2}}\cos\left(\frac{x}{2}\right)$	A1 oe.
	$= [-6(-1)] - [-6(1)] = 6 + 6 = \underline{12}$	$\underline{12}$	A1 cao **[3]**
	(Answer of 12 with no working scores M0A0A0.)		
(b)	Volume $= \underline{\pi \int_0^{2\pi} \left(3\sin\left(\frac{x}{2}\right)\right)^2}\,dx = 9\pi \int_0^{2\pi} \sin^2\left(\frac{x}{2}\right)\,dx$	**Use** of $\underline{V = \pi\int y^2}\,dx$. Can be implied. Ignore limits.	M1
	$\left[\text{NB}: \underline{\cos 2x = \pm 1 \pm 2\sin^2 x} \text{ gives } \sin^2 x = \frac{1-\cos 2x}{2}\right]$ $\left[\text{NB}: \underline{\cos x = \pm 1 \pm 2\sin^2\left(\frac{x}{2}\right)} \text{ gives } \sin^2\left(\frac{x}{2}\right) = \frac{1-\cos x}{2}\right]$	Consideration of the Half Angle Formula for $\sin^2\left(\frac{x}{2}\right)$ or the Double Angle Formula for $\sin^2 x$	M1∗
	$\therefore$ Volume $= 9(\pi)\int_0^{2\pi}\left(\dfrac{1-\cos x}{2}\right)dx$	Correct expression for Volume Ignore limits and π.	A1
	$= \dfrac{9(\pi)}{2}\int_0^{2\pi} \underline{(1-\cos x)}\,dx$		
	$= \dfrac{9(\pi)}{2}\left[\underline{x - \sin x}\right]_0^{2\pi}$	Integrating to give $\underline{\pm ax \pm b\sin x}$; Correct integration $\underline{k - k\cos x \rightarrow kx - k\sin x}$	depM1∗; A1
	$= \dfrac{9\pi}{2}[(2\pi - 0) - (0 - 0)]$		
	$= \dfrac{9\pi}{2}(2\pi) = \underline{9\pi^2}$ or $\underline{88.8264\ldots}$	Use of limits to give either $9\pi^2$ or awrt 88.8. Solution must be completely correct. No flukes allowed.	A1 **cso** **[6]**
			9 marks

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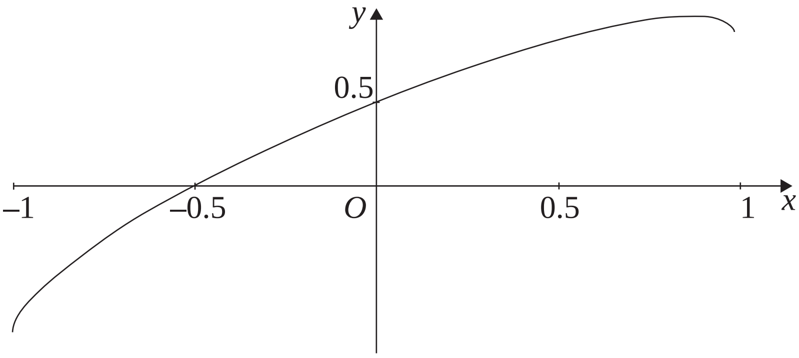
4.

Figure 2



The curve shown in Figure 2 has parametric equations

$$x = \sin t, \quad y = \sin\left(t + \tfrac{\pi}{6}\right), \qquad -\tfrac{\pi}{2} < t < \tfrac{\pi}{2}.$$

(a) Find an equation of the tangent to the curve at the point where $t = \frac{\pi}{6}$.

(6)

(b) Show that a cartesian equation of the curve is

$$y = \frac{\sqrt{3}}{2}x + \frac{1}{2}\sqrt{(1 - x^2)}, \qquad -1 < x < 1.$$

(3)

N 2 3 5 6 3 A 0 8 2 0

Question Number	Scheme		Marks
4. (a)	$x = \sin t, \quad y = \sin\left(t + \frac{\pi}{6}\right)$		
	$\frac{dx}{dt} = \cos t, \quad \frac{dy}{dt} = \cos\left(t + \frac{\pi}{6}\right)$	Attempt to differentiate both x and y wrt t to give two terms in cos Correct $\frac{dx}{dt}$ and $\frac{dy}{dt}$	M1 A1
	When $t = \frac{\pi}{6}$, $\frac{dy}{dx} = \underline{\frac{\cos\left(\frac{\pi}{6}+\frac{\pi}{6}\right)}{\cos\left(\frac{\pi}{6}\right)} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}} = \text{awrt } 0.58}$	Divides in correct way and substitutes for t to give any of the four underlined oe: Ignore the double negative if candidate has differentiated $\sin \to -\cos$	A1
	When $t = \frac{\pi}{6}$, $x = \frac{1}{2}$, $y = \frac{\sqrt{3}}{2}$	The point $\underline{\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)}$ or $\underline{\left(\frac{1}{2}, \text{awrt } 0.87\right)}$	B1
	T: $\underline{y - \frac{\sqrt{3}}{2} = \frac{1}{\sqrt{3}}\left(x - \frac{1}{2}\right)}$	Finding an equation of a tangent with their point and their tangent gradient or finds c and uses $y = (\text{their gradient})x + \text{"c"}$. Correct <u>EXACT</u> equation of <u>tangent</u> oe.	dM1 <u>A1</u> oe
	or $\frac{\sqrt{3}}{2} = \frac{1}{\sqrt{3}}\left(\frac{1}{2}\right) + c \Rightarrow c = \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{6} = \frac{\sqrt{3}}{3}$		
	or **T**: $\left[\underline{y = \frac{\sqrt{3}}{3}x + \frac{\sqrt{3}}{3}}\right]$		
			[6]
(b)	$y = \sin\left(t + \frac{\pi}{6}\right) = \sin t \cos\frac{\pi}{6} + \cos t \sin\frac{\pi}{6}$	Use of compound angle formula for sine.	M1
	Nb: $\sin^2 t + \cos^2 t \equiv 1 \Rightarrow \cos^2 t \equiv 1 - \sin^2 t$		
	$\therefore x = \sin t$ gives $\cos t = \sqrt{(1-x^2)}$	Use of trig identity to find $\cos t$ in terms of x or $\cos^2 t$ in terms of x.	M1
	$\therefore y = \frac{\sqrt{3}}{2}\sin t + \frac{1}{2}\cos t$		
	gives $y = \frac{\sqrt{3}}{2}x + \frac{1}{2}\sqrt{(1-x^2)}$ **AG**	Substitutes for $\sin t$, $\cos\frac{\pi}{6}$, $\cos t$ and $\sin\frac{\pi}{6}$ to give y in terms of x.	A1 cso
			[3]
			9 marks

Question Number	Scheme		Marks
Aliter **4.** (a) **Way 2**	$x = \sin t, \quad y = \sin\left(t + \frac{\pi}{6}\right) = \sin t \cos\frac{\pi}{6} + \cos t \sin\frac{\pi}{6}$	(Do not give this for part (b)) Attempt to differentiate x and y wrt t to give $\frac{dx}{dt}$ in terms of cos and $\frac{dy}{dt}$ in the form $\pm a\cos t \pm b\sin t$	M1
	$\frac{dx}{dt} = \cos t, \quad \frac{dy}{dt} = \cos t \cos\frac{\pi}{6} - \sin t \sin\frac{\pi}{6}$	Correct $\frac{dx}{dt}$ and $\frac{dy}{dt}$	A1
	When $t = \frac{\pi}{6}$, $\frac{dy}{dx} = \underline{\frac{\cos\frac{\pi}{6}\cos\frac{\pi}{6} - \sin\frac{\pi}{6}\sin\frac{\pi}{6}}{\cos\left(\frac{\pi}{6}\right)}}$ $\underline{= \frac{\frac{3}{4} - \frac{1}{4}}{\frac{\sqrt{3}}{2}} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}} = \text{awrt } 0.58}$	Divides in correct way and substitutes for t to give any of the four underlined oe:	A1
	When $t = \frac{\pi}{6}$, $x = \frac{1}{2}, \; y = \frac{\sqrt{3}}{2}$	The point $\underline{\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)}$ or $\underline{\left(\frac{1}{2}, \text{awrt } 0.87\right)}$	B1
	T: $\underline{y - \frac{\sqrt{3}}{2} = \frac{1}{\sqrt{3}}\left(x - \frac{1}{2}\right)}$	Finding an equation of a tangent with their point and their tangent gradient or finds c and uses $y = (\text{their gradient})x + "c"$.	dM1
		Correct EXACT equation of <u>tangent</u> oe.	<u>A1</u> oe
	or $\frac{\sqrt{3}}{2} = \frac{1}{\sqrt{3}}\left(\frac{1}{2}\right) + c \Rightarrow c = \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{6} = \frac{\sqrt{3}}{3}$		
	or **T**: $\left[\underline{y = \frac{\sqrt{3}}{3}x + \frac{\sqrt{3}}{3}}\right]$		
			[6]

Question Number	Scheme		Marks
Aliter **4.** (a) **Way 3**	$y = \frac{\sqrt{3}}{2}x + \frac{1}{2}\sqrt{(1-x^2)}$		
	$\frac{dy}{dx} = \frac{\sqrt{3}}{2} + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)(1-x^2)^{-\frac{1}{2}}(-2x)$	Attempt to differentiate two terms using the chain rule for the second term. Correct $\frac{dy}{dx}$	M1 A1
	$\frac{dy}{dx} = \underline{\frac{\sqrt{3}}{2} + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)(1-(0.5)^2)^{-\frac{1}{2}}(-2(0.5)) = \frac{1}{\sqrt{3}}}$	Correct substitution of $x = \frac{1}{2}$ into a correct $\frac{dy}{dx}$	A1
	When $t = \frac{\pi}{6}$, $x = \frac{1}{2}$, $y = \frac{\sqrt{3}}{2}$	The point $\underline{\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)}$ or $\underline{\left(\frac{1}{2}, \text{awrt } 0.87\right)}$	B1
	T: $\underline{y - \frac{\sqrt{3}}{2} = \frac{1}{\sqrt{3}}\left(x - \frac{1}{2}\right)}$	Finding an equation of a tangent with their point and their tangent gradient or finds c and uses $y = (\text{their gradient})x + "c"$. Correct <u>EXACT</u> equation of <u>tangent</u> oe.	dM1 <u>A1</u> oe
	or $\frac{\sqrt{3}}{2} = \frac{1}{\sqrt{3}}\left(\frac{1}{2}\right) + c \Rightarrow c = \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{6} = \frac{\sqrt{3}}{3}$		
	or **T**: $\left[\underline{y = \frac{\sqrt{3}}{3}x + \frac{\sqrt{3}}{3}}\right]$		
			[6]
Aliter **4.** (b) **Way 2**	$x = \sin t$ gives $y = \frac{\sqrt{3}}{2}\sin t + \frac{1}{2}\sqrt{(1-\sin^2 t)}$	Substitutes $x = \sin t$ into the equation give in y.	M1
	Nb: $\sin^2 t + \cos^2 t \equiv 1 \Rightarrow \cos^2 t \equiv 1 - \sin^2 t$		
	$\cos t = \sqrt{(1-\sin^2 t)}$	Use of trig identity to deduce that $\cos t = \sqrt{(1-\sin^2 t)}$.	M1
	gives $y = \frac{\sqrt{3}}{2}\sin t + \frac{1}{2}\cos t$		
	Hence $y = \sin t \cos\frac{\pi}{6} + \cos t \sin\frac{\pi}{6} = \sin\left(t + \frac{\pi}{6}\right)$	Using the compound angle formula to prove $y = \sin\left(t + \frac{\pi}{6}\right)$	A1 cso
			[3]
			9 marks

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5. The point A, with coordinates $(0, a, b)$ lies on the line l_1, which has equation

$$\mathbf{r} = 6\mathbf{i} + 19\mathbf{j} - \mathbf{k} + \lambda(\mathbf{i} + 4\mathbf{j} - 2\mathbf{k}).$$

(a) Find the values of a and b.

(3)

The point P lies on l_1 and is such that OP is perpendicular to l_1, where O is the origin.

(b) Find the position vector of point P.

(6)

Given that B has coordinates $(5, 15, 1)$,

(c) show that the points A, P and B are collinear and find the ratio $AP : PB$.

(4)

Question Number	Scheme		Marks
5. (a)	Equating **i** ; $0 = 6 + \lambda \Rightarrow \lambda = -6$	$\underline{\lambda = -6}$ Can be implied	B1 $\Rightarrow$ d
	Using $\lambda = -6$ and		
	equating **j** ; $a = 19 + 4(-6) = -5$	For inserting **their stated** λ into either a correct **j** or **k** component Can be implied.	M1 $\Rightarrow$ d
	equating **k** ; $b = -1 - 2(-6) = 11$	$a = -5$ and $b = 11$	A1
			[3]
	With no working… … only one of a or b stated correctly gains the first 2 marks. … both a and b stated correctly gains 3 marks.		
(b)	$\overrightarrow{OP} = (6+\lambda)\mathbf{i} + (19+4\lambda)\mathbf{j} + (-1-2\lambda)\mathbf{k}$		
	direction vector or $l_1 = \mathbf{d} = \mathbf{i} + 4\mathbf{j} - 2\mathbf{k}$		
	$\overrightarrow{OP} \perp l_1 \Rightarrow \underline{\overrightarrow{OP} \bullet \mathbf{d} = 0}$	Allow <u>this statement</u> for M1 if $\overrightarrow{OP}$ and **d** are defined as above.	
	ie. $\underline{\begin{pmatrix} 6+\lambda \\ 19+4\lambda \\ -1-2\lambda \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 4 \\ -2 \end{pmatrix} = 0}$ (or $\underline{x + 4y - 2z = 0}$)	Allow either of these two <u>underlined statements</u>	M1
	$\therefore\ 6 + \lambda + 4(19 + 4\lambda) - 2(-1 - 2\lambda) = 0$	Correct equation	A1 oe
	$6 + \lambda + 76 + 16\lambda + 2 + 4\lambda = 0$	Attempt to solve the equation in λ	dM1
	$21\lambda + 84 = 0 \Rightarrow \lambda = -4$	$\lambda = -4$	A1
	$\overrightarrow{OP} = (6-4)\mathbf{i} + (19+4(-4))\mathbf{j} + (-1-2(-4))\mathbf{k}$	Substitutes their λ into an expression for $\overrightarrow{OP}$	M1
	$\overrightarrow{OP} = 2\mathbf{i} + 3\mathbf{j} + 7\mathbf{k}$	$2\mathbf{i} + 3\mathbf{j} + 7\mathbf{k}$ or P(2, 3, 7)	A1
			[6]

Question Number	Scheme		Marks
Aliter (b) **Way 2**	$\overrightarrow{OP} = (6+\lambda)\mathbf{i} + (19+4\lambda)\mathbf{j} + (-1-2\lambda)\mathbf{k}$		
	$\overrightarrow{AP} = (6+\lambda-0)\mathbf{i} + (19+4\lambda+5)\mathbf{j} + (-1-2\lambda-11)\mathbf{k}$		
	direction vector or $l_1 = \mathbf{d} = \mathbf{i} + 4\mathbf{j} - 2\mathbf{k}$		
	$\overrightarrow{AP} \perp \overrightarrow{OP} \Rightarrow \underline{\overrightarrow{AP} \bullet \overrightarrow{OP} = 0}$	Allow <u>this statement</u> for M1 if $\overrightarrow{AP}$ and $\overrightarrow{OP}$ are defined as above.	
	ie. $\underline{\begin{pmatrix} 6+\lambda \\ 24+4\lambda \\ -12-2\lambda \end{pmatrix} \bullet \begin{pmatrix} 6+\lambda \\ 19+4\lambda \\ -1-2\lambda \end{pmatrix} = 0}$	<u>underlined statement</u>	M1
	$\therefore (6+\lambda)(6+\lambda) + (24+4\lambda)(19+4\lambda) + (-12-2\lambda)(-1-2\lambda) = 0$	Correct equation	A1 oe
	$36 + 12\lambda + \lambda^2 + 456 + 96\lambda + 76\lambda + 16\lambda^2 + 12 + 24\lambda + 2\lambda + 4\lambda^2 = 0$	Attempt to solve the equation in λ	dM1
	$21\lambda^2 + 210\lambda + 504 = 0$		
	$\lambda^2 + 10\lambda + 24 = 0 \Rightarrow (\lambda = -6)\ \ \underline{\lambda = -4}$	$\lambda = -4$	A1
	$\overrightarrow{OP} = (6-4)\mathbf{i} + (19+4(-4))\mathbf{j} + (-1-2(-4))\mathbf{k}$	Substitutes their λ into an expression for $\overrightarrow{OP}$	M1
	$\overrightarrow{OP} = 2\mathbf{i} + 3\mathbf{j} + 7\mathbf{k}$	$2\mathbf{i} + 3\mathbf{j} + 7\mathbf{k}$ or P(2, 3, 7)	A1
			[6]

Question Number	Scheme		Marks
5. (c)	$\overrightarrow{OP} = 2\mathbf{i} + 3\mathbf{j} + 7\mathbf{k}$		
	$\overrightarrow{OA} = 0\mathbf{i} - 5\mathbf{j} + 11\mathbf{k}$ and $\overrightarrow{OB} = 5\mathbf{i} + 15\mathbf{j} + \mathbf{k}$		
	$\overrightarrow{AP} = \pm(2\mathbf{i} + 8\mathbf{j} - 4\mathbf{k})$, $\overrightarrow{PB} = \pm(3\mathbf{i} + 12\mathbf{j} - 6\mathbf{k})$ $\overrightarrow{AB} = \pm(5\mathbf{i} + 20\mathbf{j} - 10\mathbf{k})$	Subtracting vectors to find any two of $\overrightarrow{AP}$, $\overrightarrow{PB}$ or $\overrightarrow{AB}$; and both are correctly ft using candidate's $\overrightarrow{OA}$ and $\overrightarrow{OP}$ found in parts (a) and (b) respectively.	M1; A1 $\sqrt{}$ $\pm$
	As $\overrightarrow{AP} = \frac{2}{3}(3\mathbf{i} + 12\mathbf{j} - 6\mathbf{k}) = \frac{2}{3}\overrightarrow{PB}$ or $\overrightarrow{AB} = \frac{5}{2}(2\mathbf{i} + 8\mathbf{j} - 4\mathbf{k}) = \frac{5}{2}\overrightarrow{AP}$ or $\overrightarrow{AB} = \frac{5}{3}(3\mathbf{i} + 12\mathbf{j} - 6\mathbf{k}) = \frac{5}{3}\overrightarrow{PB}$ or $\overrightarrow{PB} = \frac{3}{2}(2\mathbf{i} + 8\mathbf{j} - 4\mathbf{k}) = \frac{3}{2}\overrightarrow{AP}$ or $\overrightarrow{AP} = \frac{2}{5}(5\mathbf{i} + 20\mathbf{j} - 10\mathbf{k}) = \frac{2}{5}\overrightarrow{AB}$ or $\overrightarrow{PB} = \frac{3}{5}(5\mathbf{i} + 20\mathbf{j} - 10\mathbf{k}) = \frac{3}{5}\overrightarrow{AB}$ etc…	$\overrightarrow{AP} = \frac{2}{3}\overrightarrow{PB}$ or $\overrightarrow{AB} = \frac{5}{2}\overrightarrow{AP}$ or $\overrightarrow{AB} = \frac{5}{3}\overrightarrow{PB}$ or $\overrightarrow{PB} = \frac{3}{2}\overrightarrow{AP}$ or $\overrightarrow{AP} = \frac{2}{5}\overrightarrow{AB}$ or $\overrightarrow{PB} = \frac{3}{5}\overrightarrow{AB}$	
	alternatively candidates could say for example that $\overrightarrow{AP} = 2(\mathbf{i} + 4\mathbf{j} - 2\mathbf{k})$ $\overrightarrow{PB} = 3(\mathbf{i} + 4\mathbf{j} - 2\mathbf{k})$		
	then <u>the points A, P and B are collinear</u>.	<u>A, P and B are collinear</u> Completely correct proof.	A1
	$\therefore \overrightarrow{AP} : \overrightarrow{PB} = 2:3$	2:3 or 1: $\frac{3}{2}$ or $\sqrt{84} : \sqrt{189}$ aef allow SC $\frac{2}{3}$	B1 oe **[4]**
Aliter **5.** (c) Way 2	At B; <u>$5 = 6 + \lambda$</u>, <u>$15 = 19 + 4\lambda$</u> or <u>$1 = -1 - 2\lambda$</u> or at B; $\lambda = -1$	Writing down any of the three <u>underlined equations</u>.	M1
	gives $\lambda = -1$ for all three equations. or when $\lambda = -1$, this gives $\mathbf{r} = 5\mathbf{i} + 15\mathbf{j} + \mathbf{k}$	$\lambda = -1$ for all three equations or $\lambda = -1$ gives $\mathbf{r} = 5\mathbf{i} + 15\mathbf{j} + \mathbf{k}$	A1
	<u>Hence B lies on l_1</u>. As stated in the question both A and P lie on l_1. $\therefore$ <u>A, P and B are collinear</u>.	<u>Must state B lies on l_1</u> $\Rightarrow$ A, P and B are collinear	A1
	$\therefore \overrightarrow{AP} : \overrightarrow{PB} = 2:3$	2:3 or aef	B1 oe
			[4]
			13 marks

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Figure 3

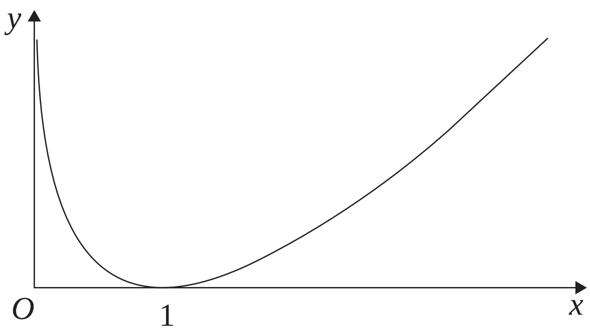

Figure 3 shows a sketch of the curve with equation $y = (x - 1)\ln x, \quad x > 0$.

(a) Complete the table with the values of y corresponding to $x = 1.5$ and $x = 2.5$.

x	1	1.5	2	2.5	3
y	0		$\ln 2$		$2\ln 3$

(1)

Given that $I = \int_1^3 (x-1)\ln x \, dx$,

(b) use the trapezium rule

(i) with values of y at $x = 1$, 2 and 3 to find an approximate value for I to 4 significant figures,

(ii) with values of y at $x = 1$, 1.5, 2, 2.5 and 3 to find another approximate value for I to 4 significant figures.

(5)

(c) Explain, with reference to Figure 3, why an increase in the number of values improves the accuracy of the approximation.

(1)

(d) Show, by integration, that the exact value of $\int_1^3 (x-1)\ln x \, dx$ is $\frac{3}{2}\ln 3$.

(6)

Question Number	Scheme		Marks
6. (a)			

x	1	1.5	2	2.5	3
y	0	0.5 ln 1.5	ln 2	1.5 ln 2.5	2 ln 3
or y	0	0.2027325541 …	ln2	1.374436098 …	2 ln 3

Question Number	Scheme		Marks
		Either 0.5 ln 1.5 and 1.5 ln 2.5 or awrt 0.20 and 1.37 (or mixture of decimals and ln's)	B1 **[1]**
(b)(i)	$I_1 \approx \frac{1}{2} \times 1 \times \underline{\{0 + 2(\ln 2) + 2\ln 3\}}$	For structure of trapezium rule $\{.............\}$;	M1;
	$= \frac{1}{2} \times 3.583518938... = 1.791759... = 1.792$ (4sf)	1.792	A1 cao
(ii)	$I_2 \approx \frac{1}{2} \times 0.5\ ; \times \underline{\{0 + 2(0.5\ln 1.5 + \ln 2 + 1.5\ln 2.5) + 2\ln 3\}}$	Outside brackets $\frac{1}{2} \times 0.5$	B1;
		For structure of trapezium rule $\{.............\}$;	M1 $\sqrt{}$
	$= \frac{1}{4} \times 6.737856242... = 1.684464...$	awrt 1.684	A1 **[5]**
(c)	With increasing ordinates, <u>the line segments at the top of the trapezia are closer to the curve.</u>	<u>Reason</u> or an appropriate diagram elaborating the correct reason.	B1 **[1]**

Question Number	Scheme		Marks
6. (d)	$\begin{cases} u = \ln x & \Rightarrow \frac{du}{dx} = \frac{1}{x} \\ \frac{dv}{dx} = x - 1 & \Rightarrow v = \frac{x^2}{2} - x \end{cases}$	Use of 'integration by parts' formula in the correct direction	M1
	$I = \left(\frac{x^2}{2} - x\right)\ln x - \int \frac{1}{x}\left(\frac{x^2}{2} - x\right) dx$	Correct expression	A1
	$= \left(\frac{x^2}{2} - x\right)\ln x - \underline{\int\left(\frac{x}{2} - 1\right) dx}$	An attempt to multiply at least one term through by $\frac{1}{x}$ and an attempt to ...	
	$= \left(\frac{x^2}{2} - x\right)\ln x - \underline{\left(\frac{x^2}{4} - x\right)} \quad (+c)$	… integrate; correct integration	M1; A1
	$\therefore I = \left[\left(\frac{x^2}{2} - x\right)\ln x - \frac{x^2}{4} + x\right]_1^3$		
	$= \left(\frac{3}{2}\ln 3 - \frac{9}{4} + 3\right) - \left(-\frac{1}{2}\ln 1 - \frac{1}{4} + 1\right)$	Substitutes limits of 3 and 1 and subtracts.	ddM1
	$= \frac{3}{2}\ln 3 + \frac{3}{4} + 0 - \frac{3}{4} \quad = \underline{\frac{3}{2}\ln 3}$ **AG**	$\frac{3}{2}\ln 3$	A1 cso
			[6]
Aliter **6.** (d) **Way 2**	$\int (x-1)\ln x\, dx = \int x\ln x\, dx - \int \ln x\, dx$		
	$\int x\ln x\, dx = \frac{x^2}{2}\ln x - \int \frac{x^2}{2}.\left(\frac{1}{x}\right) dx$	Correct application of 'by parts'	M1
	$= \frac{x^2}{2}\ln x - \frac{x^2}{4} \quad (+ c)$	Correct integration	A1
	$\int \ln x\, dx = x\ln x - \int x.\left(\frac{1}{x}\right) dx$	Correct application of 'by parts'	M1
	$= x\ln x - x \quad (+ c)$	Correct integration	A1
	$\therefore \int_1^3 (x-1)\ln x\, dx = \left(\frac{9}{2}\ln 3 - 2\right) - \left(3\ln 3 - 2\right) = \frac{3}{2}\ln 3$ **AG**	Substitutes limits of 3 and 1 into both integrands and subtracts. $\frac{3}{2}\ln 3$	ddM1 A1 cso
			[6]

Question Number	Scheme		Marks
Aliter 6. (d) **Way 3**	$\left\{\begin{array}{ll} u = \ln x & \Rightarrow \frac{du}{dx} = \frac{1}{x} \\ \frac{dv}{dx} = (x-1) & \Rightarrow v = \frac{(x-1)^2}{2} \end{array}\right\}$	Use of 'integration by parts' formula in the correct direction	M1
	$I = \frac{(x-1)^2}{2}\ln x - \int \frac{(x-1)^2}{2x}\,dx$	Correct expression	A1
	$= \frac{(x-1)^2}{2}\ln x - \int \frac{x^2 - 2x + 1}{2x}\,dx$	Candidate multiplies out numerator to obtain three terms…	
	$= \frac{(x-1)^2}{2}\ln x - \int \underline{\left(\frac{1}{2}x - 1 + \frac{1}{2x}\right)}\,dx$	… multiplies at least one term through by $\frac{1}{x}$ and then attempts to ...	
	$= \frac{(x-1)^2}{2}\ln x - \underline{\left(\frac{x^2}{4} - x + \frac{1}{2}\ln x\right)}$ (+c)	… integrate the result; <u>correct integration</u>	M1; A1
	$\therefore I = \left[\frac{(x-1)^2}{2}\ln x - \frac{x^2}{4} + x - \frac{1}{2}\ln x\right]_1^3$		
	$= \left(2\ln 3 - \frac{9}{4} + 3 - \frac{1}{2}\ln 3\right) - \left(0 - \frac{1}{4} + 1 - 0\right)$	Substitutes limits of 3 and 1 and subtracts.	ddM1
	$= 2\ln 3 - \frac{1}{2}\ln 3 + \frac{3}{4} + \frac{1}{4} - 1 \quad = \underline{\frac{3}{2}\ln 3}$ **AG**	$\frac{3}{2}\ln 3$	A1 cso
			[6]

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Question Number	Scheme		Marks
Aliter **6.** (d) **Way 4**	By substitution $u = \ln x \Rightarrow \frac{du}{dx} = \frac{1}{x}$		
	$I = \int (e^u - 1).ue^u\, du$	Correct expression	
	$= \int u(e^{2u} - e^u)\, du$	Use of 'integration by parts' formula in the correct direction	M1
	$= u\left(\frac{1}{2}e^{2u} - e^u\right) - \int \underline{\left(\frac{1}{2}e^{2u} - e^u\right)}\, dx$	Correct expression	A1
	$= u\left(\frac{1}{2}e^{2u} - e^u\right) - \underline{\left(\frac{1}{4}e^{2u} - e^u\right)}$ (+c)	Attempt <u>to integrate</u>; <u>correct integration</u>	M1; A1
	$\therefore I = \left[\frac{1}{2}ue^{2u} - ue^u - \frac{1}{4}e^{2u} + e^u\right]_{\ln 1}^{\ln 3}$		
	$= \left(\frac{9}{2}\ln 3 - 3\ln 3 - \frac{9}{4} + 3\right) - \left(0 - 0 - \frac{1}{4} + 1\right)$	Substitutes limits of ln3 and ln1 and subtracts.	ddM1
	$= \frac{3}{2}\ln 3 + \frac{3}{4} + \frac{1}{4} - 1 = \underline{\frac{3}{2}\ln 3}$ **AG**	$\frac{3}{2}\ln 3$	A1 cso
			[6]
			13 marks

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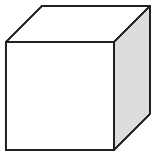
7.



At time t seconds the length of the side of a cube is x cm, the surface area of the cube is S cm^2, and the volume of the cube is V cm^3.

The surface area of the cube is increasing at a constant rate of 8 cm^2 s^{-1}.

Show that

(a) $\frac{dx}{dt} = \frac{k}{x}$, where k is a constant to be found,

(4)

(b) $\frac{dV}{dt} = 2V^{\frac{1}{3}}$.

(4)

Given that $V = 8$ when $t = 0$,

(c) solve the differential equation in part (b), and find the value of t when $V = 16\sqrt{2}$.

(7)

N 2 3 5 6 3 A 0 1 8 2 0

Question Number	Scheme		Marks
7. (a)	From question, $\frac{dS}{dt} = 8$	$\frac{dS}{dt} = 8$	B1
	$S = 6x^2 \Rightarrow \frac{dS}{dx} = 12x$	$\frac{dS}{dx} = 12x$	B1
	$\frac{dx}{dt} = \frac{dS}{dt} \div \frac{dS}{dx} = \underline{\frac{8}{12x}}\ ; = \frac{\frac{2}{3}}{x} \Rightarrow \left(k = \frac{2}{3}\right)$	Candidate's $\frac{dS}{dt} \div \frac{dS}{dx}$; $\underline{\frac{8}{12x}}$	M1; A1oe
			[4]
(b)	$V = x^3 \Rightarrow \frac{dV}{dx} = 3x^2$	$\frac{dV}{dx} = 3x^2$	B1
	$\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dx}{dt} = 3x^2.\left(\frac{2}{3x}\right)\ ; = 2x$	Candidate's $\frac{dV}{dx} \times \frac{dx}{dt}$; λx	M1; A1 $\sqrt{\ }$
	As $x = V^{\frac{1}{3}}$, then $\frac{dV}{dt} = 2V^{\frac{1}{3}}$ **AG**	Use of $x = V^{\frac{1}{3}}$, to give $\frac{dV}{dt} = 2V^{\frac{1}{3}}$	A1
			[4]
(c)	$\int \frac{dV}{V^{\frac{1}{3}}} = \int 2\ dt$	Separates the variables with $\int \frac{dV}{V^{\frac{1}{3}}}$ or $\int V^{-\frac{1}{3}} dV$ on one side and $\int 2\ dt$ on the other side. integral signs not necessary.	B1
	$\int V^{-\frac{1}{3}}\ dV = \int 2\ dt$		
	$\frac{3}{2}V^{\frac{2}{3}} = 2t\ \ (+c)$	Attempts to integrate and … … must see $V^{\frac{2}{3}}$ and 2t; Correct equation with/without + c.	M1; A1
	$\frac{3}{2}(8)^{\frac{2}{3}} = 2(0) + c \Rightarrow c = 6$	Use of V = 8 and t = 0 in a changed equation containing c ; $c = 6$	M1∗; A1
	Hence: $\frac{3}{2}V^{\frac{2}{3}} = 2t + 6$		
	$\frac{3}{2}\left(16\sqrt{2}\right)^{\frac{2}{3}} = 2t + 6 \Rightarrow 12 = 2t + 6$	Having found their "c" candidate … … substitutes $V = 16\sqrt{2}$ into an equation involving V, t and "c".	depM1 ∗
	giving t = 3.	t = 3	A1 cao **[7]**
			15 marks

Question Number	Scheme		Marks
Aliter **7.** (b) **Way 2**	$x = V^{\frac{1}{3}}$ & $S = 6x^2 \Rightarrow S = 6V^{\frac{2}{3}}$	$S = 6V^{\frac{2}{3}}$	B1 $\sqrt{\ }$
	$\frac{dS}{dV} = 4V^{-\frac{1}{3}}$ or $\frac{dV}{dS} = \frac{1}{4}V^{\frac{1}{3}}$	$\frac{dS}{dV} = 4V^{-\frac{1}{3}}$ or $\frac{dV}{dS} = \frac{1}{4}V^{\frac{1}{3}}$	B1
	$\frac{dV}{dt} = \frac{dS}{dt} \times \frac{dV}{dS} = 8.\left(\frac{1}{4V^{-\frac{1}{3}}}\right); = \frac{2}{V^{-\frac{1}{3}}} = 2V^{\frac{1}{3}}$ **AG**	Candidate's $\frac{dS}{dt} \times \frac{dV}{dS}$; $2V^{\frac{1}{3}}$	M1; A1
		In ePEN, award Marks for Way 2 in the order they appear on this mark scheme.	
			[4]
Aliter **7.** (c) **Way 2**	$\int \frac{dV}{2V^{\frac{1}{3}}} = \int 1\, dt$	Separates the variables with $\int \frac{dV}{2V^{\frac{1}{3}}}$ or $\int \frac{1}{2}V^{-\frac{1}{3}}dV$ oe on one side and $\int 1\, dt$ on the other side. integral signs not necessary.	B1
	$\frac{1}{2}\int V^{-\frac{1}{3}}\, dV = \int 1\, dt$		
	$\left(\frac{1}{2}\right)\left(\frac{3}{2}\right)V^{\frac{2}{3}} = t \quad (+c)$	Attempts to integrate and … … must see $V^{\frac{2}{3}}$ and t; Correct equation with/without + c.	M1; A1
	$\frac{3}{4}(8)^{\frac{2}{3}} = (0) + c \Rightarrow c = 3$	Use of V = 8 and t = 0 in a changed equation containing c ; $c = 3$	M1∗; A1
	Hence: $\frac{3}{4}V^{\frac{2}{3}} = t + 3$		
	$\frac{3}{4}\left(16\sqrt{2}\right)^{\frac{2}{3}} = t + 3 \Rightarrow 6 = t + 3$	Having found their "c" candidate … … substitutes $V = 16\sqrt{2}$ into an equation involving V, t and "c".	depM1 ∗
	giving t = 3.	t = 3	A1 cao **[7]**

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Question Number	Scheme		Marks
Aliter	*similar to way 1.*		
(b)	$V = x^3 \Rightarrow \frac{dV}{dx} = 3x^2$	$\frac{dV}{dx} = 3x^2$	B1
Way 3			
	$\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dS}{dt} \times \frac{dx}{dS} = 3x^2.8.\left(\frac{1}{12x}\right); = 2x$	Candidate's $\frac{dV}{dx} \times \frac{dS}{dt} \times \frac{dx}{dS}$; λx	M1; A1 $\sqrt{}$
	As $x = V^{\frac{1}{3}}$, then $\frac{dV}{dt} = 2V^{\frac{1}{3}}$ **AG**	Use of $x = V^{\frac{1}{3}}$, to give $\frac{dV}{dt} = 2V^{\frac{1}{3}}$	A1
			[4]
Aliter			
(c)	$\int \frac{dV}{V^{\frac{1}{3}}} = \int 2\ dt$	Separates the variables with $\int \frac{dV}{V^{\frac{1}{3}}}$ or $\int V^{-\frac{1}{3}} dV$ on one side and $\int 2\ dt$ on the other side. integral signs not necessary.	B1
Way 3			
	$\int V^{-\frac{1}{3}}\ dV = \int 2\ dt$		
	$V^{\frac{2}{3}} = \frac{4}{3}t$ (+c)	Attempts to integrate and … … must see $V^{\frac{2}{3}}$ and $\frac{4}{3}t$; Correct equation with/without + c.	M1; A1
	$(8)^{\frac{2}{3}} = \frac{4}{3}(0) + c \Rightarrow c = 4$	Use of V = 8 and t = 0 in a changed equation containing c ; $c = 4$	M1∗; A1
	Hence: $V^{\frac{2}{3}} = \frac{4}{3}t + 4$		
	$\left(16\sqrt{2}\right)^{\frac{2}{3}} = \frac{4}{3}t + 6 \Rightarrow 8 = \frac{4}{3}t + 4$	Having found their "c" candidate … … substitutes $V = 16\sqrt{2}$ into an equation involving V, t and "c".	depM1 ∗
	giving t = 3.	t = 3	A1 cao
			[7]