Centre No.						Paper Reference	Surname	Initial(s)
Candidate No.						6 6 6 6 / 0 1	Signature	

Paper Reference(s)

6666/01

Edexcel GCE

Core Mathematics C4

Advanced

Monday 18 June 2007 – Morning

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
1	
2	
3	
4	
5	
6	
7	
8	
Total	

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Green)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
You must write your answer for each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.
There are 24 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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1. $$f(x) = (3 + 2x)^{-3}, \quad |x| < \tfrac{3}{2}.$$

Find the binomial expansion of f(x), in ascending powers of x, as far as the term in x^3.

Give each coefficient as a simplified fraction.

(5)

June 2007
6666 Core Mathematics C4
Mark Scheme

Question Number	Scheme		Marks
	** represents a constant		
1. (a)	$f(x) = (3+2x)^{-3} = \underline{(3)^{-3}}\left(1+\frac{2x}{3}\right)^{-3} = \underline{\frac{1}{27}}\left(1+\frac{2x}{3}\right)^{-3}$	Takes 3 outside the bracket to give any of $(3)^{-3}$ or $\frac{1}{27}$. **See note below.**	B1
	$= \frac{1}{27}\left\{\underline{1+(-3)(**x);+\frac{(-3)(-4)}{2!}(**x)^2+\frac{(-3)(-4)(-5)}{3!}(**x)^3+...}\right\}$	Expands $(1+**x)^{-3}$ to give a simplified or an un-simplified $1+(-3)(**x)$;	M1;
	with $** \neq 1$	A correct simplified or an un-simplified $\{\underline{..........}\}$ expansion with candidate's followed thro' $(**x)$	A1 $\sqrt{\ }$
	$= \frac{1}{27}\left\{\underline{1+(-3)(\tfrac{2x}{3})+\frac{(-3)(-4)}{2!}(\tfrac{2x}{3})^2+\frac{(-3)(-4)(-5)}{3!}(\tfrac{2x}{3})^3+...}\right\}$		
	$= \frac{1}{27}\left\{1-2x+\frac{8x^2}{3}-\frac{80}{27}x^3+...\right\}$		
	$= \frac{1}{27}-\frac{2x}{27};+\frac{8x^2}{81}-\frac{80x^3}{729}+...$	Anything that cancels to $\frac{1}{27}-\frac{2x}{27}$;	A1;
		Simplified $\frac{8x^2}{81}-\frac{80x^3}{729}$	A1
			[5]
			5 marks

Note: You would award: B1M1A0 for

$$= \frac{1}{27}\left\{\underline{1+(-3)(\tfrac{2x}{3})+\frac{(-3)(-4)}{2!}(2x)^2+\frac{(-3)(-4)(-5)}{3!}(2x)^3+...}\right\}$$

because $**$ is not consistent.

Special Case: If you see the constant $\frac{1}{27}$ in a candidate's final binomial expression, then you can award B1

Question Number	Scheme		Marks
Aliter **1. Way 2**	$f(x) = (3+2x)^{-3}$		
		$\frac{1}{27}$ or $(3)^{-3}$ (**See note** ↓)	B1
	$= \left\{ (3)^{-3} + (-3)(3)^{-4}(**x); + \frac{(-3)(-4)}{2!}(3)^{-5}(**x)^2 + \underline{\frac{(-3)(-4)(-5)}{3!}(3)^{-6}(**x)^3 + ...} \right\}$	Expands $(3+2x)^{-3}$ to give an un-simplified or simplified $(3)^{-3} + (-3)(3)^{-4}(**x)$;	M1
	with $** \neq 1$	A correct un-simplified or simplified $\{\underline{..........}\}$ expansion with candidate's followed thro' $(**x)$	A1 $\sqrt{}$
	$= \left\{ (3)^{-3} + (-3)(3)^{-4}(2x); + \frac{(-3)(-4)}{2!}(3)^{-5}(2x)^2 + \underline{\frac{(-3)(-4)(-5)}{3!}(3)^{-6}(2x)^3 + ...} \right\}$		
	$= \left\{ \frac{1}{27} + (-3)(\frac{1}{81})(2x); + (6)(\frac{1}{243})(4x^2) + (-10)(\frac{1}{729})(8x^3) + ... \right\}$		
	$= \frac{1}{27} - \frac{2x}{27}; + \frac{8x^2}{81} - \frac{80x^3}{729} + ...$	Anything that cancels to $\frac{1}{27} - \frac{2x}{27}$;	A1;
		Simplified $\frac{8x^2}{81} - \frac{80x^3}{729}$	A1
			[5]
			5 marks

Attempts using Maclaurin expansions need to be escalated up to your team leader.

If you feel the mark scheme does not apply fairly to a candidate please escalate the response up to your team leader.

> **Special Case**: If you see the constant $\frac{1}{27}$ in a candidate's final binomial expression, then you can award B1

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2. Use the substitution $u = 2^x$ to find the exact value of

$$\int_0^1 \frac{2^x}{(2^x + 1)^2}\, dx.$$

(6)

Question Number	Scheme		Marks
2.	$\int_0^1 \frac{2^x}{(2^x+1)^2}\,dx$, with substitution $u = 2^x$		
	$\frac{du}{dx} = 2^x.\ln 2 \Rightarrow \frac{dx}{du} = \frac{1}{2^x.\ln 2}$	$\frac{du}{dx} = 2^x.\ln 2$ or $\frac{du}{dx} = u.\ln 2$ or $\left(\frac{1}{u}\right)\frac{du}{dx} = \ln 2$	B1
	$\int \frac{2^x}{(2^x+1)^2}\,dx = \left(\frac{1}{\ln 2}\right)\int \frac{1}{(u+1)^2}\,du$	$k\int \frac{1}{(u+1)^2}\,du$ where k is constant	M1*
	$= \left(\frac{1}{\ln 2}\right)\left(\frac{-1}{(u+1)}\right) + c$	$(u+1)^{-2} \to a(u+1)^{-1}$ $(u+1)^{-2} \to -1.(u+1)^{-1}$	M1 A1
	change limits: when $x = 0$ & $x = 1$ then $u = 1$ & $u = 2$		
	$\int_0^1 \frac{2^x}{(2^x+1)^2}\,dx = \frac{1}{\ln 2}\left[\frac{-1}{(u+1)}\right]_1^2$		
	$= \frac{1}{\ln 2}\left[\left(-\frac{1}{3}\right) - \left(-\frac{1}{2}\right)\right]$	Correct use of limits $u = 1$ and $u = 2$	depM1*
	$= \frac{1}{6\ln 2}$	$\frac{1}{6\ln 2}$ or $\frac{1}{\ln 4} - \frac{1}{\ln 8}$ or $\frac{1}{2\ln 2} - \frac{1}{3\ln 2}$ **Exact value only!**	A1 aef **[6]**
	Alternatively candidate can revert back to x …		
	$\int_0^1 \frac{2^x}{(2^x+1)^2}\,dx = \frac{1}{\ln 2}\left[\frac{-1}{(2^x+1)}\right]_0^1$		
	$= \frac{1}{\ln 2}\left[\left(-\frac{1}{3}\right) - \left(-\frac{1}{2}\right)\right]$	Correct use of limits $x = 0$ and $x = 1$	depM1*
	$= \frac{1}{6\ln 2}$	$\frac{1}{6\ln 2}$ or $\frac{1}{\ln 4} - \frac{1}{\ln 8}$ or $\frac{1}{2\ln 2} - \frac{1}{3\ln 2}$ **Exact value only!**	A1 aef
			6 marks

If you see this **integration** applied anywhere in a candidate's working then you can award M1, A1

There are other acceptable answers for A1, eg: $\frac{1}{2\ln 8}$ or $\frac{1}{\ln 64}$

NB: Use your calculator to check eg. 0.240449…

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3. (a) Find $\int x \cos 2x \, dx$.

(4)

(b) Hence, using the identity $\cos 2x = 2\cos^2 x - 1$, deduce $\int x \cos^2 x \, dx$.

(3)

Question Number	Scheme		Marks
3. (a)	$\left\{\begin{array}{ll} u = x & \Rightarrow \frac{du}{dx} = 1 \\ \frac{dv}{dx} = \cos 2x & \Rightarrow v = \frac{1}{2}\sin 2x \end{array}\right\}$		
		(see note below)	
	Int $= \int x\cos 2x \, dx = \frac{1}{2}x\sin 2x - \int \frac{1}{2}\sin 2x.1 \, dx$	Use of 'integration by parts' formula in the correct direction.	M1
		Correct expression.	A1
	$= \frac{1}{2}x\sin 2x - \frac{1}{2}\left(-\frac{1}{2}\cos 2x\right) + c$	$\sin 2x \to -\frac{1}{2}\cos 2x$ or $\sin kx \to -\frac{1}{k}\cos kx$ with $k \neq 1, k > 0$	dM1
	$= \frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x + c$	Correct expression with $+c$	A1
			[4]
(b)	$\int x\cos^2 x \, dx = \int x\left(\frac{\cos 2x + 1}{2}\right) dx$	Substitutes **correctly** for $\cos^2 x$ in the given integral	M1
	$= \frac{1}{2}\int x\cos 2x \, dx + \frac{1}{2}\int x \, dx$		
	$= \underline{\frac{1}{2}\left(\frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x\right)}; + \frac{1}{2}\int x \, dx$	$\frac{1}{2}$(their answer to (a)); or underlined expression	A1; $\sqrt{}$
	$= \frac{1}{4}x\sin 2x + \frac{1}{8}\cos 2x + \frac{1}{4}x^2 \;(+c)$	Completely correct expression with/without $+c$	A1
			[3]
			7 marks

Notes:

(b)	Int $= \int x\cos 2x \, dx = \frac{1}{2}x\sin 2x \pm \int \frac{1}{2}\sin 2x.1 \, dx$	This is acceptable for M1	M1
	$\left\{\begin{array}{ll} u = x & \Rightarrow \frac{du}{dx} = 1 \\ \frac{dv}{dx} = \cos 2x & \Rightarrow v = \lambda\sin 2x \end{array}\right\}$		
	Int $= \int x\cos 2x \, dx = \lambda x\sin 2x \pm \int \lambda\sin 2x.1 \, dx$	This is also acceptable for M1	M1

Question Number	Scheme		Marks
***Aliter* 3. (b) Way 2**	$\int x\cos^2 x\, dx = \int x\left(\frac{\cos 2x+1}{2}\right) dx$	Substitutes **correctly** for $\cos^2 x$ in the given integral …	M1
	$\left\{\begin{array}{ll} u = x & \Rightarrow \frac{du}{dx} = 1 \\ \frac{dv}{dx} = \frac{1}{2}\cos 2x + \frac{1}{2} & \Rightarrow v = \frac{1}{4}\sin 2x + \frac{1}{2}x \end{array}\right\}$	… or $u = x$ and $\frac{dv}{dx} = \frac{1}{2}\cos 2x + \frac{1}{2}$	
	$= \frac{1}{4}x\sin 2x + \frac{1}{2}x^2 - \int\left(\frac{1}{4}\sin 2x + \frac{1}{2}x\right) dx$		
	$= \underline{\frac{1}{4}x\sin 2x} + \frac{1}{2}x^2 + \underline{\frac{1}{8}\cos 2x} - \frac{1}{4}x^2 + c$	$\frac{1}{2}$(their answer to (a)); or <u>underlined expression</u>	A1 $\sqrt{}$
	$= \frac{1}{4}x\sin 2x + \frac{1}{8}\cos 2x + \frac{1}{4}x^2\ (+c)$	Completely correct expression with/without $+c$	A1
			[3]
***Aliter* (b) Way 3**	$\int x\cos 2x\, dx = \int x\left(2\cos^2 x - 1\right) dx$	Substitutes **correctly** for $\cos 2x$ in $\int x\cos 2x\, dx$	M1
	$\Rightarrow 2\int x\cos^2 x\, dx - \int x\, dx = \frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x + c$		
	$\Rightarrow \int x\cos^2 x\, dx = \underline{\frac{1}{2}\left(\frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x\right)}; + \frac{1}{2}\int x\, dx$	$\frac{1}{2}$(their answer to (a)); or <u>underlined expression</u>	A1; $\sqrt{}$
	$= \frac{1}{4}x\sin 2x + \frac{1}{8}\cos 2x + \frac{1}{4}x^2\ (+c)$	Completely correct expression with/without $+c$	A1
			[3]
			7 marks

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4.

$$\frac{2(4x^2+1)}{(2x+1)(2x-1)} \equiv A + \frac{B}{(2x+1)} + \frac{C}{(2x-1)}.$$

(a) Find the values of the constants *A*, *B* and *C*.

(4)

(b) Hence show that the exact value of $\int_1^2 \frac{2(4x^2+1)}{(2x+1)(2x-1)}\,dx$ is $2 + \ln k$, giving the value of the constant *k*.

(6)

Question Number	Scheme		Marks
4. (a) **Way 1**	A method of long division gives,		
	$\dfrac{2(4x^2+1)}{(2x+1)(2x-1)} \equiv 2 + \dfrac{4}{(2x+1)(2x-1)}$	$A = 2$	B1
	$\dfrac{4}{(2x+1)(2x-1)} \equiv \dfrac{B}{(2x+1)} + \dfrac{C}{(2x-1)}$		
	$4 \equiv B(2x-1) + C(2x+1)$ or their remainder, $Dx + E \equiv B(2x-1) + C(2x+1)$	Forming any one of these two identities. Can be implied.	M1
	Let $x = -\frac{1}{2}$, $4 = -2B \Rightarrow B = -2$		
	Let $x = \frac{1}{2}$, $4 = 2C \Rightarrow C = 2$	**See note below** either one of $B = -2$ or $C = 2$ both B and C correct	A1 A1 **[4]**
Aliter **4.** (a) **Way 2**	$\dfrac{2(4x^2+1)}{(2x+1)(2x-1)} \equiv A + \dfrac{B}{(2x+1)} + \dfrac{C}{(2x-1)}$		
	See below for the award of B1	***decide to award B1 here!! …*** ***… for*** $A = 2$	B1
	$2(4x^2+1) \equiv A(2x+1)(2x-1) + B(2x-1) + C(2x+1)$	Forming this identity. Can be implied.	M1
	Equate x^2, $8 = 4A \Rightarrow A = 2$		
	Let $x = -\frac{1}{2}$, $4 = -2B \Rightarrow B = -2$		
	Let $x = \frac{1}{2}$, $4 = 2C \Rightarrow C = 2$	**See note below** either one of $B = -2$ or $C = 2$ both B and C correct	A1 A1 **[4]**

If a candidate states one of either B or C correctly then the method mark M1 can be implied.

Question Number	Scheme		Marks
4. (b)	$\int \frac{2(4x^2+1)}{(2x+1)(2x-1)}\,dx = \int 2 - \frac{2}{(2x+1)} + \frac{2}{(2x-1)}\,dx$		
	$= 2x - \frac{2}{2}\ln(2x+1) + \frac{2}{2}\ln(2x-1)\ (+c)$	Either $p\ln(2x+1)$ or $q\ln(2x-1)$ or either $p\ln 2x+1$ or $q\ln 2x-1$	M1∗
		$A \to Ax$	B1√
		$-\frac{2}{2}\ln(2x+1) + \frac{2}{2}\ln(2x-1)$ or $-\ln(2x+1)+\ln(2x-1)$ **See note below.**	A1 cso & aef
	$\int_1^2 \frac{2(4x^2+1)}{(2x+1)(2x-1)}\,dx = \left[2x - \ln(2x+1) + \ln(2x-1)\right]_1^2$		
	$= (4 - \ln 5 + \ln 3) - (2 - \ln 3 + \ln 1)$	Substitutes limits of 2 and 1 and subtracts the correct way round. (Invisible brackets okay.)	depM1∗
	$= 2 + \ln 3 + \ln 3 - \ln 5$		
	$= 2 + \ln\left(\frac{3(3)}{5}\right)$	Use of correct product (or power) and/or quotient laws for logarithms to obtain a single logarithmic term for ***their numerical*** expression.	M1
	$= 2 + \ln\left(\frac{9}{5}\right)$	$2 + \ln\left(\frac{9}{5}\right)$ Or $2 - \ln\left(\frac{5}{9}\right)$ **and** k stated as $\frac{9}{5}$.	A1 **[6]**
			10 marks

Some candidates may find rational values for *B* and *C*. They may combine the denominator of their *B* or C with (2x +1) or (2x – 1). Hence: Either $\frac{a}{b(2x-1)} \to k\ln(b(2x-1))$ or $\frac{a}{b(2x+1)} \to k\ln(b(2x+1))$ is okay for M1.

Candidates are not allowed to fluke $-\ln(2x+1)+\ln(2x-1)$ for A1. Hence **cso**. If they do fluke this, however, they can gain the final A1 mark for this part of the question.

To award this M1 mark, the candidate must use the appropriate law(s) of logarithms for their ln terms to give a **one single** logarithmic term. Any error ***in applying the laws of logarithms*** would then earn M0.

Note: This is not a dependent method mark.

Leave blank

5. The line l_1 has equation $\mathbf{r} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$.

The line l_2 has equation $\mathbf{r} = \begin{pmatrix} 1 \\ 3 \\ 6 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$.

(a) Show that l_1 and l_2 do not meet.

(4)

The point A is on l_1 where $\lambda = 1$, and the point B is on l_2 where $\mu = 2$.

(b) Find the cosine of the acute angle between AB and l_1.

(6)

Question Number	Scheme		Marks
5. (a)	If l_1 and l_2 intersect then:		
	$\begin{pmatrix}1\\0\\-1\end{pmatrix}+\lambda\begin{pmatrix}1\\1\\0\end{pmatrix}=\begin{pmatrix}1\\3\\6\end{pmatrix}+\mu\begin{pmatrix}2\\1\\-1\end{pmatrix}$		
	Any two of **i**: $1+\lambda=1+2\mu$ (1) **j**: $\lambda=3+\mu$ (2) **k**: $-1=6-\mu$ (3)	Writes down any two of these equations correctly.	M1
	(1) & (2) yields $\lambda=6,\ \mu=3$ (1) & (3) yields $\lambda=14,\ \mu=7$ (2) & (3) yields $\lambda=10,\ \mu=7$	Solves two of the above equations to find … either one of λ or μ correct both λ and μ correct	A1 A1
	Either checking eqn (3), $-1\neq 3$ checking eqn (2), $14\neq 10$ checking eqn (1), $11\neq 15$	Complete method of putting their values of λ and μ into a third equation to show a contradiction.	B1 $\sqrt{}$
	or for example: checking eqn (3), LHS = -1, RHS = 3 $\Rightarrow$ Lines l_1 and l_2 do not intersect	this type of explanation is also allowed for B1 $\sqrt{}$.	
			[4]
Aliter **5. (a) Way 2**	**k**: $-1=6-\mu \Rightarrow \mu=7$ **i**: $1+\lambda=1+2\mu \Rightarrow 1+\lambda=1+2(7)$ **j**: $\lambda=3+\mu \Rightarrow \lambda=3+(7)$	Uses the **k** component to find μ and substitutes their value of μ into either one of the **i** or **j** component.	M1
	i: $\lambda=14$ **j**: $\lambda=10$	either one of the λ's correct both of the λ's correct	A1 A1
	Either: These equations are then inconsistent Or: $14\neq 10$ Or: Lines l_1 and l_2 do not intersect	Complete method giving rise to any one of these three explanations.	B1 $\sqrt{}$
			[4]

Question Number	Scheme		Marks
Aliter **5.** (a) **Way 3**	If l_1 and l_2 intersect then:		
	$\begin{pmatrix}1\\0\\-1\end{pmatrix} + \lambda\begin{pmatrix}1\\1\\0\end{pmatrix} = \begin{pmatrix}1\\3\\6\end{pmatrix} + \mu\begin{pmatrix}2\\1\\-1\end{pmatrix}$		
	Any two of $\mathbf{i}: \; 1+\lambda = 1+2\mu \quad (1)$ $\mathbf{j}: \; \lambda = 3+\mu \quad (2)$ $\mathbf{k}: \; -1 = 6-\mu \quad (3)$	Writes down any two of these equations	M1
	(1) & (2) yields $\mu = 3$	either one of the μ's correct	A1
	(3) yields $\mu = 7$	both of the μ's correct	A1
	Either: These equations are then inconsistent Or: $3 \neq 7$ Or: Lines l_1 and l_2 do not intersect	Complete method giving rise to any one of these three explanations.	B1 $\sqrt{}$
			[4]
Aliter **5.** (a) **Way 4**	Any two of $\mathbf{i}: \; 1+\lambda = 1+2\mu \quad (1)$ $\mathbf{j}: \; \lambda = 3+\mu \quad (2)$ $\mathbf{k}: \; -1 = 6-\mu \quad (3)$	Writes down any two of these equations	M1
	(1) & (2) yields $\mu = 3$	$\mu = 3$	A1
	(3) RHS $= 6 - 3 = 3$	RHS of (3) = 3	A1
	(3) yields $-1 \neq 3$	Complete method giving rise to this explanation.	B1 $\sqrt{}$
			[4]

Question Number	Scheme		Marks
5. (b)	$\lambda = 1 \Rightarrow \overrightarrow{OA} = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$ & $\mu = 2 \Rightarrow \overrightarrow{OB} = \begin{pmatrix} 5 \\ 5 \\ 4 \end{pmatrix}$	**Only one of either** $\overrightarrow{OA} = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$ or $\overrightarrow{OB} = \begin{pmatrix} 5 \\ 5 \\ 4 \end{pmatrix}$ or $A(2,1,-1)$ or $B(5,5,4)$. (can be implied)	B1
	$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \begin{pmatrix} 5 \\ 5 \\ 4 \end{pmatrix} - \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ 5 \end{pmatrix}$ or $\overrightarrow{BA} = \begin{pmatrix} -3 \\ -4 \\ -5 \end{pmatrix}$	Finding the difference between their $\overrightarrow{OB}$ and $\overrightarrow{OA}$. (can be implied)	M1 $\sqrt{}$
		Applying the dot product formula between "allowable" vectors. **See notes below.**	M1
	$\overrightarrow{AB} = 3\mathbf{i} + 4\mathbf{j} + 5\mathbf{k}$, $\mathbf{d}_1 = \mathbf{i} + \mathbf{j} + 0\mathbf{k}$ & θ is angle		
	$\cos\theta = \dfrac{\overrightarrow{AB} \bullet \mathbf{d}_1}{\lvert\overrightarrow{AB}\rvert . \lvert\mathbf{d}_1\rvert} = \pm\left(\dfrac{3+4+0}{\sqrt{50}.\sqrt{2}}\right)$	Applies dot product formula between $\mathbf{d}_1$ and their $\pm\overrightarrow{AB}$.	M1 $\sqrt{}$
		Correct expression.	A1
	$\cos\theta = \frac{7}{10}$	$\frac{7}{10}$ or 0.7 or $\frac{7}{\sqrt{100}}$ but not $\frac{7}{\sqrt{50}\sqrt{2}}$	A1 **cao** **[6]**
			10 marks

Candidates can score this mark if there is a complete method for finding the dot product between their vectors in the following cases:

Case 1: their ft $\pm\overrightarrow{AB} = \pm(3\mathbf{i} + 4\mathbf{j} + 5\mathbf{k})$ and $\mathbf{d}_1 = \mathbf{i} + \mathbf{j} + 0\mathbf{k}$
$\Rightarrow \cos\theta = \pm\left(\dfrac{3+4+0}{\sqrt{50}.\sqrt{2}}\right)$

Case 2: $\mathbf{d}_1 = \mathbf{i} + \mathbf{j} + 0\mathbf{k}$ and $\mathbf{d}_2 = 2\mathbf{i} + \mathbf{j} - 1\mathbf{k}$
$\Rightarrow \cos\theta = \dfrac{2+1+0}{\sqrt{2}.\sqrt{6}}$

Case 3: $\mathbf{d}_1 = \mathbf{i} + \mathbf{j} + 0\mathbf{k}$ and $\mathbf{d}_2 = 2(2\mathbf{i} + \mathbf{j} - 1\mathbf{k})$
$\Rightarrow \cos\theta = \dfrac{4+2+0}{\sqrt{2}.\sqrt{24}}$

Case 4: their ft $\pm\overrightarrow{AB} = \pm(3\mathbf{i} + 4\mathbf{j} + 5\mathbf{k})$ and $\mathbf{d}_2 = 2\mathbf{i} + \mathbf{j} - \mathbf{k}$
$\Rightarrow \cos\theta = \pm\left(\dfrac{6+4-5}{\sqrt{50}.\sqrt{6}}\right)$

Case 5: their ft $\overrightarrow{OA} = 2\mathbf{i} + 1\mathbf{j} - 1\mathbf{k}$ and their ft $\overrightarrow{OB} = 5\mathbf{i} + 5\mathbf{j} + 4\mathbf{k}$
$\Rightarrow \cos\theta = \pm\left(\dfrac{10+5-4}{\sqrt{6}.\sqrt{66}}\right)$

Note: If candidate use cases 2, 3, 4 and 5 they cannot gain the final three marks for this part.
Note: Candidate can only gain some/all of the final three marks if they use case 1.

Examples of awarding of marks M1M1A1 in 5.(b)

Example	Marks
$\sqrt{50}.\sqrt{2}\ \cos\theta = \pm(3+4+0)$	M1M1A1 (Case 1)
$\sqrt{2}.\sqrt{6}\ \cos\theta = 3$	M1M0A0 (Case 2)
$\sqrt{2}.\sqrt{24}\ \cos\theta = 4+2$	M1M0A0 (Case 3)

Leave blank

6. A curve has parametric equations

$$x = \tan^2 t, \qquad y = \sin t, \qquad 0 < t < \frac{\pi}{2}.$$

(a) Find an expression for $\frac{dy}{dx}$ in terms of t. You need not simplify your answer.
(3)

(b) Find an equation of the tangent to the curve at the point where $t = \frac{\pi}{4}$.

Give your answer in the form $y = ax + b$, where a and b are constants to be determined.
(5)

(c) Find a cartesian equation of the curve in the form $y^2 = f(x)$.
(4)

Question Number	Scheme		Marks
6. (a)	$x = \tan^2 t$, $y = \sin t$		
	$\frac{dx}{dt} = 2(\tan t)\sec^2 t$, $\frac{dy}{dt} = \cos t$	Correct $\frac{dx}{dt}$ and $\frac{dy}{dt}$	B1
	$\therefore \frac{dy}{dx} = \frac{\cos t}{2\tan t \sec^2 t}$ $\left(= \frac{\cos^4 t}{2\sin t}\right)$	$\frac{\pm \cos t}{\text{their } \frac{dx}{dt}}$	M1
		$\frac{+\cos t}{\text{their } \frac{dx}{dt}}$	A1 $\sqrt{}$
			[3]
(b)	When $t = \frac{\pi}{4}$, $x = 1$, $y = \frac{1}{\sqrt{2}}$ (need values)	The point $\underline{\left(1, \frac{1}{\sqrt{2}}\right)}$ or $\underline{(1, \text{awrt } 0.71)}$ **These coordinates can be implied.** ($y = \sin\left(\frac{\pi}{4}\right)$ is not sufficient for B1)	B1, B1
	When $t = \frac{\pi}{4}$, m(**T**) $= \frac{dy}{dx} = \frac{\cos\frac{\pi}{4}}{2\tan\frac{\pi}{4}\sec^2\frac{\pi}{4}}$		
	$= \underline{\frac{\frac{1}{\sqrt{2}}}{2.(1)\left(\frac{1}{\frac{1}{\sqrt{2}}}\right)^2}} = \underline{\frac{\frac{1}{\sqrt{2}}}{2.(1)\left(\frac{1}{\frac{1}{2}}\right)}} = \underline{\frac{\frac{1}{\sqrt{2}}}{2.(1)(2)}} = \underline{\frac{1}{4\sqrt{2}}} = \underline{\frac{\sqrt{2}}{8}}$	any of the five underlined expressions or awrt 0.18	B1 aef
	T: $y - \frac{1}{\sqrt{2}} = \frac{1}{4\sqrt{2}}(x - 1)$	Finding an equation of a tangent with ***their point*** and ***their tangent gradient*** or finds c by using $y = \underline{(\text{their gradient})}x + "\underline{c}"$.	M1 $\sqrt{}$ aef
	T: $\underline{y = \frac{1}{4\sqrt{2}}x + \frac{3}{4\sqrt{2}}}$ or $\underline{y = \frac{\sqrt{2}}{8}x + \frac{3\sqrt{2}}{8}}$	Correct simplified EXACT equation of <u>tangent</u>	<u>A1</u> aef **cso**
	or $\frac{1}{\sqrt{2}} = \frac{1}{4\sqrt{2}}(1) + c \Rightarrow c = \frac{1}{\sqrt{2}} - \frac{1}{4\sqrt{2}} = \frac{3}{4\sqrt{2}}$		
	Hence **T:** $\underline{y = \frac{1}{4\sqrt{2}}x + \frac{3}{4\sqrt{2}}}$ or $\underline{y = \frac{\sqrt{2}}{8}x + \frac{3\sqrt{2}}{8}}$		
			[5]

Note: The x and y coordinates must be the right way round.

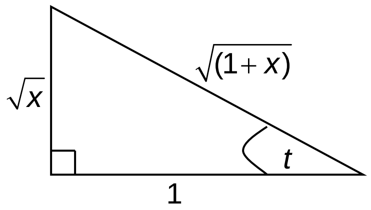
A candidate who incorrectly differentiates $\tan^2 t$ to give $\frac{dx}{dt} = 2\sec^2 t$ or $\frac{dx}{dt} = \sec^4 t$ is then able to fluke the correct answer in part (b). Such candidates can potentially get: (a) B0M1A1 $\sqrt{}$ (b) B1B1B1M1A0 **cso**.
Note: cso means "correct solution only".
Note: part (a) not fully correct implies candidate can achieve a maximum of 4 out of 5 marks in part (b).

Question Number	Scheme		Marks
6. (c) **Way 1**	$x = \tan^2 t = \dfrac{\sin^2 t}{\cos^2 t}$ $y = \sin t$		
	$x = \dfrac{\sin^2 t}{1-\sin^2 t}$	Uses $\cos^2 t = 1-\sin^2 t$	M1
	$x = \dfrac{y^2}{1-y^2}$	Eliminates '*t*' *to* write an equation involving *x* and *y*.	M1
	$x(1-y^2) = y^2 \Rightarrow x - xy^2 = y^2$		
	$x = y^2 + xy^2 \Rightarrow x = y^2(1+x)$	Rearranging and factorising with an attempt to make y^2 the subject.	ddM1
	$y^2 = \dfrac{x}{1+x}$	$\dfrac{x}{1+x}$	A1
			[4]
Aliter **6.** (c) **Way 2**	$1+\cot^2 t = cosec^2 t$	Uses $1+\cot^2 t = cosec^2 t$	M1
	$= \dfrac{1}{\sin^2 t}$	Uses $cosec^2 t = \dfrac{1}{\sin^2 t}$	M1 implied
	Hence, $1+\dfrac{1}{x} = \dfrac{1}{y^2}$	Eliminates '*t*' *to* write an equation involving *x* and *y*.	ddM1
	Hence, $y^2 = 1 - \dfrac{1}{(1+x)}$ or $\dfrac{x}{1+x}$	$1 - \dfrac{1}{(1+x)}$ or $\dfrac{x}{1+x}$	A1
			[4]

$\dfrac{1}{1+\frac{1}{x}}$ is an acceptable response for the final accuracy A1 mark.

Question Number	Scheme		Marks
Aliter **6.** (c) **Way 3**	$x = \tan^2 t \qquad y = \sin t$		
	$1 + \tan^2 t = \sec^2 t$	Uses $1 + \tan^2 t = \sec^2 t$	M1
	$= \dfrac{1}{\cos^2 t}$	Uses $\sec^2 t = \dfrac{1}{\cos^2 t}$	M1
	$= \dfrac{1}{1 - \sin^2 t}$		
	Hence, $1 + x = \dfrac{1}{1 - y^2}$	Eliminates '*t*' *to* write an equation involving *x* and *y*.	ddM1
	Hence, $y^2 = 1 - \dfrac{1}{(1+x)}$ or $\dfrac{x}{1+x}$	$1 - \dfrac{1}{(1+x)}$ or $\dfrac{x}{1+x}$	A1
			[4]
Aliter **6.** (c) **Way 4**	$y^2 = \sin^2 t = 1 - \cos^2 t$	Uses $\sin^2 t = 1 - \cos^2 t$	M1
	$= 1 - \dfrac{1}{\sec^2 t}$	Uses $\cos^2 t = \dfrac{1}{\sec^2 t}$	M1
	$= 1 - \dfrac{1}{(1 + \tan^2 t)}$	then uses $\sec^2 t = 1 + \tan^2 t$	ddM1
	Hence, $y^2 = 1 - \dfrac{1}{(1+x)}$ or $\dfrac{x}{1+x}$	$1 - \dfrac{1}{(1+x)}$ or $\dfrac{x}{1+x}$	A1
			[4]

$\dfrac{1}{1+\frac{1}{x}}$ is an acceptable response for the final accuracy A1 mark.

Question Number	Scheme		Marks
Aliter **6.** (c) **Way 5**	$x = \tan^2 t \qquad y = \sin t$		
	$x = \tan^2 t \;\Rightarrow\; \tan t = \sqrt{x}$		
		Draws a right-angled triangle and places both $\sqrt{x}$ and 1 on the triangle	M1
		Uses Pythagoras to deduce the hypotenuse	M1
	Hence, $y = \sin t = \dfrac{\sqrt{x}}{\sqrt{1+x}}$	Eliminates '*t*' *to* write an equation involving *x* and *y*.	ddM1
	Hence, $y^2 = \dfrac{x}{1+x}$	$\dfrac{x}{1+x}$	A1
			[4]
			12 marks

$\dfrac{1}{1+\frac{1}{x}}$ is an acceptable response for the final accuracy A1 mark.

There are so many ways that a candidate can proceed with part (c). If a candidate produces a correct solution then please award all four marks. If they use a method commensurate with the five ways as detailed on the mark scheme then award the marks appropriately. If you are unsure of how to apply the scheme please escalate your response up to your team leader.

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7.

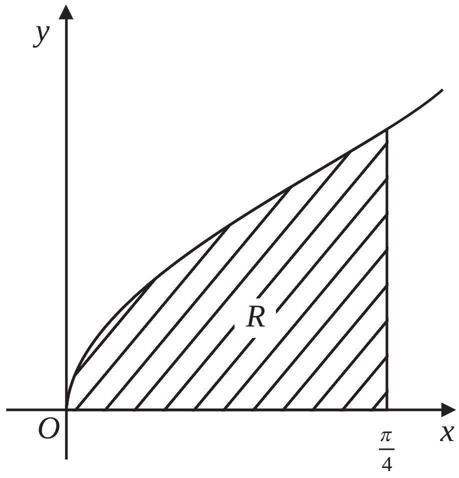

Figure 1

Figure 1 shows part of the curve with equation $y = \sqrt{(\tan x)}$. The finite region R, which is bounded by the curve, the x-axis and the line $x = \frac{\pi}{4}$, is shown shaded in Figure 1.

(a) Given that $y = \sqrt{(\tan x)}$, complete the table with the values of y corresponding to $x = \frac{\pi}{16}, \frac{\pi}{8}$ and $\frac{3\pi}{16}$, giving your answers to 5 decimal places.

x	0	$\frac{\pi}{16}$	$\frac{\pi}{8}$	$\frac{3\pi}{16}$	$\frac{\pi}{4}$
y	0				1

(3)

(b) Use the trapezium rule with all the values of y in the completed table to obtain an estimate for the area of the shaded region R, giving your answer to 4 decimal places.

(4)

The region R is rotated through 2π radians around the x-axis to generate a solid of revolution.

(c) Use integration to find an exact value for the volume of the solid generated.

(4)

Question Number	Scheme	Marks
7. (a)	x: 0, $\frac{\pi}{16}$, $\frac{\pi}{8}$, $\frac{3\pi}{16}$, $\frac{\pi}{4}$; y: 0, 0.445995927…, 0.643594252…, 0.817421946…, 1	
	Enter marks into ePEN in the correct order. 0.446 or awrt 0.44600	B1
	awrt 0.64359	B1
	awrt 0.81742	B1
		[3]
(b) **Way 1**	Outside brackets $\frac{1}{2}\times\frac{\pi}{16}$ or $\frac{\pi}{32}$	B1
	Area $\approx \frac{1}{2}\times\frac{\pi}{16}$;$\times\underline{\left\{0+2(0.44600+0.64359+0.81742)+1\right\}}$ (0 can be implied) For structure of trapezium rule $\{............\}$;	M1 √
	Correct expression inside brackets which all must be multiplied by $\frac{h}{2}$.	A1 √
	$=\frac{\pi}{32}\times 4.81402... = 0.472615308... = \underline{0.4726}$ (4dp) for seeing $\underline{0.4726}$	A1 **cao**
		[4]
Aliter (b) **Way 2**	Area $\approx \frac{\pi}{16}\times\left\{\frac{0+0.44600}{2}+\frac{0.44600+0.64359}{2}+\frac{0.64359+0.81742}{2}+\frac{0.81742+1}{2}\right\}$ — $\frac{\pi}{16}$ and a divisor of 2 on all terms inside brackets.	B1
	which is equivalent to: One of first and last ordinates, two of the middle ordinates inside brackets ignoring the 2.	M1 √
	Area $\approx \frac{1}{2}\times\frac{\pi}{16}$;$\times\underline{\left\{0+2(0.44600+0.64359+0.81742)+1\right\}}$ Correct expression inside brackets if $\frac{1}{2}$ was to be factorised out.	A1 √
	$=\frac{\pi}{16}\times 2.40701... = 0.472615308... = \underline{0.4726}$ $\underline{0.4726}$	A1 **cao**
		[4]

Area $=\frac{1}{2}\times\frac{\pi}{20}\times\{0+2(0.44600+0.64359+0.81742)+1\}=0.3781$, gains B0M1A1A0

In (a) for $x=\frac{\pi}{16}$ writing 0.4459959… then 0.45600 gains B1 for awrt 0.44600 even though 0.45600 is incorrect.

In (b) you can follow though a candidate's values from part (a) to award M1 ft, A1 ft

Question Number	Scheme		Marks
7. (c)	Volume $= (\pi)\int_0^{\frac{\pi}{4}}\left(\sqrt{\tan x}\right)^2 dx = (\pi)\int_0^{\frac{\pi}{4}} \tan x \, dx$	$\int\left(\sqrt{\tan x}\right)^2 dx$ or $\int \tan x \, dx$ Can be implied. Ignore limits and (π)	M1
	$= (\pi)\left[\ln \sec x\right]_0^{\frac{\pi}{4}}$ or $= (\pi)\left[-\ln \cos x\right]_0^{\frac{\pi}{4}}$	$\tan x \to \ln \sec x$ or $\tan x \to -\ln \cos x$	A1
	$= (\pi)\left[\left(\ln \sec \frac{\pi}{4}\right) - \left(\ln \sec 0\right)\right]$ or $= (\pi)\left[\left(-\ln \cos \frac{\pi}{4}\right) - \left(\ln \cos 0\right)\right]$	The correct use of limits on a function other than tan x; ie $x = \frac{\pi}{4}$ 'minus' $x = 0$. $\ln(\sec 0) = 0$ may be implied. Ignore (π)	dM1
	$= \pi\left[\ln\left(\frac{1}{\frac{1}{\sqrt{2}}}\right) - \ln\left(\frac{1}{1}\right)\right] = \pi\left[\ln\sqrt{2} - \ln 1\right]$ or $= \pi\left[-\ln\left(\frac{1}{\sqrt{2}}\right) - \ln(1)\right]$		
	$= \pi \ln\sqrt{2}$ or $\pi \ln \frac{2}{\sqrt{2}}$ or $\frac{1}{2}\pi \ln 2$ or $-\pi \ln\left(\frac{1}{\sqrt{2}}\right)$ or $\frac{\pi}{2}\ln\left(\frac{1}{2}\right)$	$\pi \ln\sqrt{2}$ or $\pi \ln \frac{2}{\sqrt{2}}$ or $\frac{1}{2}\pi \ln 2$ or $-\pi \ln\left(\frac{1}{\sqrt{2}}\right)$ or $\frac{\pi}{2}\ln\left(\frac{1}{2}\right)$ **must be exact.**	A1 aef **[4]**
			11 marks

If a candidate gives the correct exact answer and then writes 1.088779…, then such a candidate can be awarded A1 (aef). The subsequent working would then be ignored. (isw)

Beware: In part (c) the factor of π is not needed for the first three marks.

Beware: In part (b) a candidate can also add up individual trapezia in this way:

$$\text{Area} \approx \tfrac{1}{2}.\tfrac{\pi}{16}(0 + 0.44600) + \tfrac{1}{2}.\tfrac{\pi}{16}(0.44600 + 0.64359) + \tfrac{1}{2}.\tfrac{\pi}{16}(0.64359 + 0.81742) + \tfrac{1}{2}.\tfrac{\pi}{16}(0.81742 + 1)$$

Leave blank

8. A population growth is modelled by the differential equation

$$\frac{dP}{dt} = kP \,,$$

where P is the population, t is the time measured in days and k is a positive constant.

Given that the initial population is P_0,

(a) solve the differential equation, giving P in terms of P_0, k and t.
(4)

Given also that $k = 2.5$,

(b) find the time taken, to the nearest minute, for the population to reach $2P_0$.
(3)

In an improved model the differential equation is given as

$$\frac{dP}{dt} = \lambda P \cos \lambda t \,,$$

where P is the population, t is the time measured in days and λ is a positive constant.

Given, again, that the initial population is P_0 and that time is measured in days,

(c) solve the second differential equation, giving P in terms of P_0, λ and t.
(4)

Given also that $\lambda = 2.5$,

(d) find the time taken, to the nearest minute, for the population to reach $2P_0$ for the first time, using the improved model.
(3)

Question Number	Scheme		Marks
8. (a)	$\frac{dP}{dt} = kP$ and $t = 0,\ P = P_0$ (1)		
	$\int \frac{dP}{P} = \int k\ dt$	Separates the variables with $\int \frac{dP}{P}$ and $\int k\,dt$ on either side with integral signs not necessary.	M1
	$\ln P = kt; (+c)$	Must see $\ln P$ and kt; Correct equation with/without + c.	A1
	When $t = 0,\ P = P_0 \Rightarrow \ln P_0 = c$ $\left(\text{or } P = Ae^{kt} \Rightarrow P_0 = A\right)$	Use of boundary condition (1) to attempt to find the constant of integration.	M1
	$\ln P = kt + \ln P_0 \Rightarrow e^{\ln P} = e^{kt + \ln P_0} = e^{kt}.e^{\ln P_0}$		
	Hence, $\underline{P = P_0 e^{kt}}$	$\underline{P = P_0 e^{kt}}$	A1
			[4]
(b)	$P = 2P_0$ & $k = 2.5 \Rightarrow \underline{2P_0 = P_0 e^{2.5t}}$	Substitutes $P = 2P_0$ into an expression involving P	M1
	$e^{2.5t} = 2 \Rightarrow \underline{\ln e^{2.5t} = \ln 2}$ or $\underline{2.5t = \ln 2}$ ...or $e^{kt} = 2 \Rightarrow \underline{\ln e^{kt} = \ln 2}$ or $\underline{kt = \ln 2}$	Eliminates P_0 and takes ln of both sides	M1
	$\Rightarrow t = \frac{1}{2.5}\ln 2 = 0.277258872...$ days		
	$t = 0.277258872... \times 24 \times 60 = 399.252776...$ minutes		
	$t = \underline{399\,\text{min}}$ or $t = \underline{6\text{ hr } 39\text{ mins}}$ (to nearest minute)	awrt $t = \underline{399}$ or $\underline{6\text{ hr } 39\text{ mins}}$	A1
			[3]

$\underline{P = P_0 e^{kt}}$ written down without the first M1 mark given scores all four marks in part (a).

Question Number	Scheme		Marks
8. (c)	$\dfrac{dP}{dt} = \lambda P\cos\lambda t$ and $t = 0,\ P = P_0$ (1)		
	$\displaystyle\int \frac{dP}{P} = \int \lambda\cos\lambda t\ dt$	Separates the variables with $\int \frac{dP}{P}$ and $\int \lambda\cos\lambda t\,dt$ on either side with integral signs not necessary.	M1
	$\ln P = \sin\lambda t;(+c)$	Must see $\ln P$ and $\sin\lambda t$; Correct equation with/without + c.	A1
	When $t = 0,\ P = P_0 \Rightarrow \ln P_0 = c$ $\left(\text{or } P = Ae^{\sin\lambda t} \Rightarrow P_0 = A\right)$	Use of boundary condition (1) to attempt to find the constant of integration.	M1
	$\ln P = \sin\lambda t + \ln P_0 \Rightarrow e^{\ln P} = e^{\sin\lambda t + \ln P_0} = e^{\sin\lambda t}.e^{\ln P_0}$		
	Hence, $\underline{P = P_0 e^{\sin\lambda t}}$	$\underline{P = P_0 e^{\sin\lambda t}}$	A1
			[4]
(d)	$P = 2P_0$ & $\lambda = 2.5 \Rightarrow 2P_0 = P_0 e^{\sin 2.5t}$		
	$e^{\sin 2.5t} = 2 \Rightarrow \underline{\sin 2.5t = \ln 2}$ …or … $e^{\lambda t} = 2 \Rightarrow \underline{\sin\lambda t = \ln 2}$	Eliminates P_0 and makes $\sin\lambda t$ or $\sin 2.5t$ the subject by taking ln's	M1
	$\underline{t = \frac{1}{2.5}\sin^{-1}(\ln 2)}$	Then rearranges to make t the subject. (must use $\sin^{-1}$)	dM1
	$t = 0.306338477...$		
	$t = 0.306338477... \times 24 \times 60 = 441.1274082...$ minutes		
	$t = \underline{441\text{min}}$ or $t = \underline{7\text{ hr } 21\text{ mins}}$ (to nearest minute)	awrt $t = \underline{441}$ or $\underline{7\text{ hr } 21\text{ mins}}$	A1
			[3]
			14 marks

$\underline{P = P_0 e^{\sin\lambda t}}$ written down without the first M1 mark given scores all four marks in part (c).

Question Number	Scheme		Marks
	$\frac{\mathrm{d}P}{\mathrm{d}t} = kP$ and $t = 0,\ P = P_0$ (1)		
***Aliter* 8. (a) Way 2**	$\int \frac{\mathrm{d}P}{kP} = \int 1\,\mathrm{d}t$	Separates the variables with $\int \frac{\mathrm{d}P}{kP}$ and $\int \mathrm{d}t$ on either side with integral signs not necessary.	M1
	$\frac{1}{k}\ln P = t;(+c)$	Must see $\frac{1}{k}\ln P$ and t; Correct equation with/without + c.	A1
	When $t = 0,\ P = P_0 \Rightarrow \frac{1}{k}\ln P_0 = c$ $\left(\text{or } P = Ae^{kt} \Rightarrow P_0 = A\right)$	Use of boundary condition (1) to attempt to find the constant of integration.	M1
	$\frac{1}{k}\ln P = t + \frac{1}{k}\ln P_0 \Rightarrow \ln P = kt + \ln P_0$ $\Rightarrow e^{\ln P} = e^{kt + \ln P_0} = e^{kt}.e^{\ln P_0}$		
	Hence, $\underline{P = P_0 e^{kt}}$	$\underline{P = P_0 e^{kt}}$	A1
			[4]
***Aliter* 8. (a) Way 3**	$\int \frac{\mathrm{d}P}{kP} = \int 1\,\mathrm{d}t$	Separates the variables with $\int \frac{\mathrm{d}P}{kP}$ and $\int \mathrm{d}t$ on either side with integral signs not necessary.	M1
	$\frac{1}{k}\ln(kP) = t;(+c)$	Must see $\frac{1}{k}\ln(kP)$ and t; Correct equation with/without + c.	A1
	When $t = 0,\ P = P_0 \Rightarrow \frac{1}{k}\ln(kP_0) = c$ $\left(\text{or } kP = Ae^{kt} \Rightarrow kP_0 = A\right)$	Use of boundary condition (1) to attempt to find the constant of integration.	M1
	$\frac{1}{k}\ln(kP) = t + \frac{1}{k}\ln(kP_0) \Rightarrow \ln(kP) = kt + \ln(kP_0)$ $\Rightarrow e^{\ln(kP)} = e^{kt + \ln(kP_0)} = e^{kt}.e^{\ln(kP_0)}$ $\Rightarrow kP = e^{kt}.(kP_0) \Rightarrow kP = kP_0 e^{kt}$ $\left(\text{or } kP = kP_0 e^{kt}\right)$		
	Hence, $\underline{P = P_0 e^{kt}}$	$\underline{P = P_0 e^{kt}}$	A1
			[4]

Question Number	Scheme		Marks
***Aliter* 8. (c) Way 2**	$\frac{dP}{dt} = \lambda P \cos \lambda t$ and $t = 0,\ P = P_0$ (1)		
	$\int \frac{dP}{\lambda P} = \int \cos \lambda t \ dt$	Separates the variables with $\int \frac{dP}{\lambda P}$ and $\int \cos \lambda t \, dt$ on either side with integral signs not necessary.	M1
	$\frac{1}{\lambda} \ln P = \frac{1}{\lambda} \sin \lambda t ; (+c)$	Must see $\frac{1}{\lambda} \ln P$ and $\frac{1}{\lambda} \sin \lambda t$; Correct equation with/without + c.	A1
	When $t = 0,\ P = P_0 \Rightarrow \frac{1}{\lambda} \ln P_0 = c$ $\left(\text{or } P = Ae^{\sin \lambda t} \Rightarrow P_0 = A\right)$	Use of boundary condition (1) to attempt to find the constant of integration.	M1
	$\frac{1}{\lambda} \ln P = \frac{1}{\lambda} \sin \lambda t + \frac{1}{\lambda} \ln P_0 \Rightarrow \ln P = \sin \lambda t + \ln P_0$		
	$\Rightarrow e^{\ln P} = e^{\sin \lambda t + \ln P_0} = e^{\sin \lambda t} . e^{\ln P_0}$		
	Hence, $\underline{P = P_0 e^{\sin \lambda t}}$	$\underline{P = P_0 e^{\sin \lambda t}}$	A1 **[4]**

$\underline{P = P_0 e^{kt}}$ written down without the first M1 mark given scores all four marks in part (a).

$\underline{P = P_0 e^{\sin \lambda t}}$ written down without the first M1 mark given scores all four marks in part (c).

Question Number	Scheme		Marks
***Aliter* 8. (c) Way 3**	$\frac{dP}{dt} = \lambda P\cos\lambda t$ and $t = 0,\ P = P_0$ (1)		
	$\int \frac{dP}{\lambda P} = \int \cos\lambda t\ dt$	Separates the variables with $\int \frac{dP}{\lambda P}$ and $\int \cos\lambda t\, dt$ on either side with integral signs not necessary.	M1
	$\frac{1}{\lambda}\ln(\lambda P) = \frac{1}{\lambda}\sin\lambda t;(+c)$	Must see $\frac{1}{\lambda}\ln(\lambda P)$ and $\frac{1}{\lambda}\sin\lambda t$; Correct equation with/without + c.	A1
	When $t = 0,\ P = P_0 \Rightarrow \frac{1}{\lambda}\ln(\lambda P_0) = c$ $\left(\text{or } \lambda P = Ae^{\sin\lambda t} \Rightarrow \lambda P_0 = A\right)$	Use of boundary condition (1) to attempt to find the constant of integration.	M1
	$\frac{1}{\lambda}\ln(\lambda P) = \frac{1}{\lambda}\sin\lambda t + \frac{1}{\lambda}\ln(\lambda P_0)$		
	$\Rightarrow \ln(\lambda P) = \sin\lambda t + \ln(\lambda P_0)$		
	$\Rightarrow e^{\ln(\lambda P)} = e^{\sin\lambda t + \ln(\lambda P_0)} = e^{\sin\lambda t}.e^{\ln(\lambda P_0)}$		
	$\Rightarrow \lambda P = e^{\sin\lambda t}.(\lambda P_0)$ $\left(\text{or } \lambda P = \lambda P_0 e^{\sin\lambda t}\right)$		
	Hence, $\underline{P = P_0\, e^{\sin\lambda t}}$	$\underline{P = P_0\, e^{\sin\lambda t}}$	A1
			[4]

- Note: dM1 denotes a method mark which is dependent upon the award of the previous method mark.
  ddM1 denotes a method mark which is dependent upon the award of the previous two method marks.
  depM1$*$ denotes a method mark which is dependent upon the award of M1$*$.
  ft denotes "follow through"
  cao denotes "correct answer only"
  aef denotes "any equivalent form"