Centre No.						Paper Reference							Surname	Initial(s)
Candidate No.						6	6	6	6	/	0	1	Signature	

Paper Reference(s)

6666/01

Edexcel GCE

Core Mathematics C4

Advanced

Thursday 12 June 2008 – Morning

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
1	
2	
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4	
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7	
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Total	

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Green)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
You must write your answer for each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.
There are 28 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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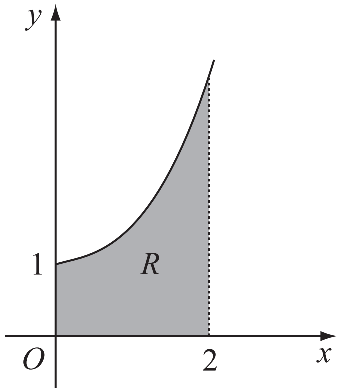

Figure 1

Figure 1 shows part of the curve with equation $y = e^{0.5x^2}$. The finite region R, shown shaded in Figure 1, is bounded by the curve, the x-axis, the y-axis and the line $x = 2$.

(a) Complete the table with the values of y corresponding to $x = 0.8$ and $x = 1.6$.

x	0	0.4	0.8	1.2	1.6	2
y	e^0	$e^{0.08}$		$e^{0.72}$		e^2

(1)

(b) Use the trapezium rule with all the values in the table to find an approximate value for the area of R, giving your answer to 4 significant figures.

(3)

June 2008
6666 Core Mathematics C4
Mark Scheme

Question	Scheme	Marks
1. (a)		

x	0	0.4	0.8	1.2	1.6	2
y	e^0	$e^{0.08}$	$e^{0.32}$	$e^{0.72}$	$e^{1.28}$	e^2
or y	1	1.08329 …	1.37713…	2.05443…	3.59664…	7.38906…

Either $e^{0.32}$ and $e^{1.28}$ or awrt 1.38 and 3.60 (or a mixture of e's and decimals) — B1 **[1]**

(b) **Way 1**

$$\text{Area} \approx \frac{1}{2}\times 0.4\,;\times\left[\ e^0 + 2\left(e^{0.08} + e^{0.32} + e^{0.72} + e^{1.28}\right) + e^2\ \right]$$

Outside brackets $\frac{1}{2}\times 0.4$ or 0.2 — B1;

<u>For structure of trapezium rule</u>$\left[\ \ldots\ldots\ldots\ldots\ \right]$; — <u>M1</u> √

$= 0.2\times 24.61203164\ldots = 4.922406\ldots = \underline{4.922}$ (4sf)

$\underline{4.922}$ — A1 **cao** **[3]**

Aliter
(b)
Way 2

$$\text{Area} \approx 0.4\times\left[\frac{e^0+e^{0.08}}{2} + \frac{e^{0.08}+e^{0.32}}{2} + \frac{e^{0.32}+e^{0.72}}{2} + \frac{e^{0.72}+e^{1.28}}{2} + \frac{e^{1.28}+e^2}{2}\right]$$

0.4 and a divisor of 2 on all terms inside brackets. — B1

which is equivalent to:

One of first and last ordinates, two of the middle ordinates inside brackets ignoring the 2. — <u>M1</u> √

$$\text{Area} \approx \frac{1}{2}\times 0.4\,;\times\left[\ e^0 + 2\left(e^{0.08} + e^{0.32} + e^{0.72} + e^{1.28}\right) + e^2\ \right]$$

$= 0.2\times 24.61203164\ldots = 4.922406\ldots = \underline{4.922}$ (4sf)

$\underline{4.922}$ — A1 **cao** **[3]**

4 marks

Note an expression like $\text{Area} \approx \frac{1}{2}\times 0.4 + e^0 + 2\left(e^{0.08} + e^{0.32} + e^{0.72} + e^{1.28}\right) + e^2$ would score B1M1A0

Allow one term missing (slip!) in the ( ) brackets for

The M1 mark for structure is for the material found in the curly brackets ie
$\left[\ \text{first } y \text{ ordinate} + 2\left(\text{intermediate ft } y \text{ ordinate}\right) + \text{final } y \text{ ordinate}\ \right]$

Leave blank

2. (a) Use integration by parts to find $\int x\,\mathrm{e}^x\,\mathrm{d}x$.

(3)

(b) Hence find $\int x^2\,\mathrm{e}^x\,\mathrm{d}x$.

(3)

Question Number	Scheme		Marks
2. (a)	$\left\{\begin{matrix} u = x & \Rightarrow & \frac{du}{dx} = 1 \\ \frac{dv}{dx} = e^x & \Rightarrow & v = e^x \end{matrix}\right\}$		
	$\int xe^x \, dx = xe^x - \int e^x.1 \, dx$	Use of 'integration by parts' formula in the **correct direction**. (See note.) Correct expression. (Ignore d*x*)	M1 A1
	$= xe^x - \int e^x \, dx$		
	$= xe^x - e^x \; (+c)$	Correct integration with/without + *c*	A1
			[3]
(b)	$\left\{\begin{matrix} u = x^2 & \Rightarrow & \frac{du}{dx} = 2x \\ \frac{dv}{dx} = e^x & \Rightarrow & v = e^x \end{matrix}\right\}$		
	$\int x^2e^x \, dx = x^2 e^x - \int e^x.2x \, dx$	Use of 'integration by parts' formula in the **correct direction**. Correct expression. (Ignore d*x*)	M1 A1
	$= x^2e^x - 2\int xe^x \, dx$		
	$= x^2e^x - 2\left(xe^x - e^x\right) + c$	Correct expression **including + c**. **(seen at any stage! in part (b))** You can ignore subsequent working.	A1 ISW
			[3]
	$\left\{\begin{matrix} = x^2e^x - 2xe^x + 2e^x + c \\ = e^x\left(x^2 - 2x + 2\right) + c \end{matrix}\right\}$	*Ignore subsequent working*	
			6 marks

Note integration by parts in the **correct direction** means that *u* and $\frac{dv}{dx}$ must be assigned/used as $u = x$ and $\frac{dv}{dx} = e^x$ in part (a) for example

+ *c* is not required in part (a).

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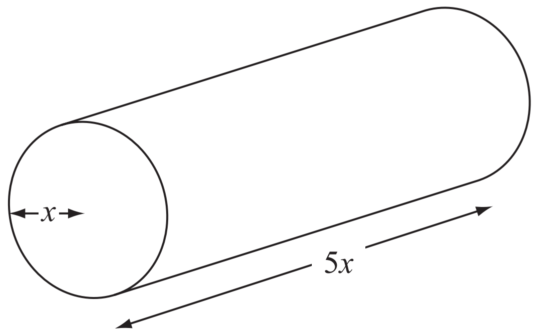

Figure 2

Figure 2 shows a right circular cylindrical metal rod which is expanding as it is heated. After t seconds the radius of the rod is x cm and the length of the rod is $5x$ cm. The cross-sectional area of the rod is increasing at the constant rate of 0.032 $\text{cm}^2\ \text{s}^{-1}$.

(a) Find $\frac{dx}{dt}$ when the radius of the rod is 2 cm, giving your answer to 3 significant figures.

(4)

(b) Find the rate of increase of the volume of the rod when $x = 2$.

(4)

Question Number	Scheme		Marks
3. (a)	From question, $\frac{dA}{dt} = 0.032$	$\frac{dA}{dt} = 0.032$ seen or implied from working.	B1
	$\left\{A = \pi x^2 \Rightarrow \frac{dA}{dx} =\right\} 2\pi x$	$2\pi x$ by itself seen or implied from working	B1
	$\frac{dx}{dt} = \frac{dA}{dt} \div \frac{dA}{dx} = (0.032)\frac{1}{2\pi x}; \left\{= \frac{0.016}{\pi x}\right\}$	$0.032 \div \text{Candidate's } \frac{dA}{dx}$;	M1;
	When $x = 2\text{cm}$, $\frac{dx}{dt} = \frac{0.016}{2\pi}$		
	Hence, $\frac{dx}{dt} = 0.002546479...$ (cm s^{-1})	awrt 0.00255	A1 **cso**
			[4]
(b)	$V = \underline{\pi x^2(5x)} = \underline{5\pi x^3}$	$V = \underline{\pi x^2(5x)}$ or $\underline{5\pi x^3}$	B1
	$\frac{dV}{dx} = 15\pi x^2$	$\frac{dV}{dx} = 15\pi x^2$ or ft from candidate's V **in one variable**	B1 $\sqrt{}$
	$\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dx}{dt} = 15\pi x^2.\left(\frac{0.016}{\pi x}\right); \{= 0.24x\}$	Candidate's $\frac{dV}{dx} \times \frac{dx}{dt}$;	M1 $\sqrt{}$
	When $x = 2\text{cm}$, $\frac{dV}{dt} = 0.24(2) = \underline{0.48}$ ($\text{cm}^3\,\text{s}^{-1}$)	$\underline{0.48}$ or $\underline{\text{awrt } 0.48}$	A1 **cso**
			[4]
			8 marks

Leave blank

4. A curve has equation $3x^2 - y^2 + xy = 4$. The points P and Q lie on the curve. The gradient of the tangent to the curve is $\frac{8}{3}$ at P and at Q.

(a) Use implicit differentiation to show that $y - 2x = 0$ at P and at Q.

(6)

(b) Find the coordinates of P and Q.

(3)

H 3 0 4 2 7 A 0 1 0 2 8

Question Number	Scheme		Marks
4. (a)	$3x^2 - y^2 + xy = 4$ (eqn $*$)		
	$\left\{\frac{\cancel{d}x}{\cancel{d}x} \times\right\}$ $\underline{6x - 2y\frac{dy}{dx}} + \left(\underline{\underline{y + x\frac{dy}{dx}}}\right) = \underline{0}$	Differentiates implicitly to include either $\pm ky\frac{dy}{dx}$ or $x\frac{dy}{dx}$. (Ignore $\left(\frac{dy}{dx} = \right)$)	M1
		Correct application $\underline{\underline{(\ )}}$ of product rule	B1
		$(3x^2 - y^2) \to \left(\underline{6x - 2y\frac{dy}{dx}}\right)$ and $(4 \to \underline{0})$	<u>A1</u>
	$\left\{\frac{dy}{dx} = \frac{-6x - y}{x - 2y}\right\}$ or $\left\{\frac{dy}{dx} = \frac{6x + y}{2y - x}\right\}$	*not necessarily required.*	
	$\frac{dy}{dx} = \frac{8}{3} \Rightarrow \frac{-6x - y}{x - 2y} = \frac{8}{3}$	Substituting $\frac{dy}{dx} = \frac{8}{3}$ into their equation.	M1$*$
	giving $-18x - 3y = 8x - 16y$		
	giving $13y = 26x$	Attempt to combine either terms in *x* or terms in *y* together to give either ax or by.	dM1$*$
	Hence, $y = 2x \Rightarrow \underline{y - 2x = 0}$	simplifying to give $\underline{y - 2x = 0}$ **AG**	A1 cso
			[6]
(b)	At *P* & *Q*, $y = 2x$. Substituting into eqn $*$		
	gives $3x^2 - (2x)^2 + x(2x) = 4$	Attempt replacing y by $2x$ **in at least one** of the *y* terms in eqn$*$	M1
	Simplifying gives, $x^2 = 4 \Rightarrow \underline{x = \pm 2}$	Either $x = 2$ or $x = -2$	<u>A1</u>
	$y = 2x \Rightarrow y = \pm 4$		
	Hence coordinates are $\underline{(2,4)}$ and $\underline{(-2,-4)}$	Both $\underline{(2,4)}$ and $\underline{(-2,-4)}$	<u>A1</u>
			[3]
			9 marks

Leave blank

5. (a) Expand $\dfrac{1}{\sqrt{(4-3x)}}$, where $|x| < \frac{4}{3}$, in ascending powers of x up to and including the term in x^2. Simplify each term.

(5)

(b) Hence, or otherwise, find the first 3 terms in the expansion of $\dfrac{x+8}{\sqrt{(4-3x)}}$ as a series in ascending powers of x.

(4)

Question Number	Scheme		Marks
	** represents a constant (which must be consistent for first accuracy mark)		
5. (a)	$\frac{1}{\sqrt{(4-3x)}} = (4-3x)^{-\frac{1}{2}} = \underline{(4)^{-\frac{1}{2}}}\left(1-\frac{3x}{4}\right)^{-\frac{1}{2}} = \underline{\frac{1}{2}}\left(1-\frac{3x}{4}\right)^{-\frac{1}{2}}$	$\underline{(4)^{-\frac{1}{2}}}$ or $\underline{\frac{1}{2}}$ outside brackets	B1
		Expands $(1+{**}x)^{-\frac{1}{2}}$ to give a simplified or an un-simplified $1+(-\frac{1}{2})({**}x)$;	M1;
	$=\frac{1}{2}\left[\underline{1+(-\frac{1}{2})({**}x);+\frac{(-\frac{1}{2})(-\frac{3}{2})}{2!}({**}x)^2+...}\right]$ with $** \neq 1$	A correct simplified or an un-simplified [.........] expansion with candidate's followed through $({**}x)$	A1 $\sqrt{}$
	$=\frac{1}{2}\left[\underline{1+(-\frac{1}{2})(-\frac{3x}{4})+\frac{(-\frac{1}{2})(-\frac{3}{2})}{2!}(-\frac{3x}{4})^2+...}\right]$	Award SC M1 if you see $(-\frac{1}{2})({**}x)+\frac{(-\frac{1}{2})(-\frac{3}{2})}{2!}({**}x)^2$	
	$=\frac{1}{2}\left[1+\frac{3}{8}x;+\frac{27}{128}x^2+...\right]$	$\frac{1}{2}\left[1+\frac{3}{8}x;...\right]$	A1 isw
		SC: $K\left[1+\frac{3}{8}x+\frac{27}{128}x^2+...\right]$	
		$\frac{1}{2}\left[........;\frac{27}{128}x^2\right]$	A1 isw
	$\left\{=\frac{1}{2}+\frac{3}{16}x;+\frac{27}{256}x^2+...\right\}$	*Ignore subsequent working*	
			[5]
(b)	$(x+8)\left(\frac{1}{2}+\frac{3}{16}x+\frac{27}{256}x^2+...\right)$	Writing $(x+8)$ multiplied by candidate's part (a) expansion.	M1
	$= \frac{1}{2}x+\frac{3}{16}x^2+.....$ $+4+\frac{3}{2}x+\frac{27}{32}x^2+.....$	Multiply out brackets to find a constant term, two x terms and two x^2 terms.	M1
	$= 4+2x;+\frac{33}{32}x^2+...$	Anything that cancels to $4+2x;\ \frac{33}{32}x^2$	A1; A1
			[4]
			9 marks

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6. With respect to a fixed origin O, the lines l_1 and l_2 are given by the equations

$$l_1: \quad \mathbf{r} = (-9\mathbf{i} + 10\mathbf{k}) + \lambda(2\mathbf{i} + \mathbf{j} - \mathbf{k})$$

$$l_2: \quad \mathbf{r} = (3\mathbf{i} + \mathbf{j} + 17\mathbf{k}) + \mu(3\mathbf{i} - \mathbf{j} + 5\mathbf{k})$$

where λ and μ are scalar parameters.

(a) Show that l_1 and l_2 meet and find the position vector of their point of intersection.
(6)

(b) Show that l_1 and l_2 are perpendicular to each other.
(2)

The point A has position vector $5\mathbf{i} + 7\mathbf{j} + 3\mathbf{k}$.

(c) Show that A lies on l_1.
(1)

The point B is the image of A after reflection in the line l_2.

(d) Find the position vector of B.
(3)

Question Number	Scheme		Marks
6. (a)	Lines meet where:		
	$\begin{pmatrix}-9\\0\\10\end{pmatrix}+\lambda\begin{pmatrix}2\\1\\-1\end{pmatrix}=\begin{pmatrix}3\\1\\17\end{pmatrix}+\mu\begin{pmatrix}3\\-1\\5\end{pmatrix}$		
	Any two of $\mathbf{i}: -9+2\lambda=3+3\mu$ (1) $\mathbf{j}: \lambda=1-\mu$ (2) $\mathbf{k}: 10-\lambda=17+5\mu$ (3)	Need any two of these correct equations seen anywhere in part (a).	M1
	(1) - 2**(2)** gives: $-9=1+5\mu \Rightarrow \mu=-2$	Attempts to solve simultaneous equations to find one of either λ or μ	dM1
	(2) gives: $\lambda=1--2=3$	Both $\underline{\lambda=3}$ & $\underline{\mu=-2}$	A1
	$\mathbf{r}=\begin{pmatrix}-9\\0\\10\end{pmatrix}+3\begin{pmatrix}2\\1\\-1\end{pmatrix}$ or $\mathbf{r}=\begin{pmatrix}3\\1\\17\end{pmatrix}-2\begin{pmatrix}3\\-1\\5\end{pmatrix}$	Substitutes their value of **either** λ or μ into the line l_1 or l_2 **respectively.** This mark can be implied by any two correct components of $(-3,3,7)$.	ddM1
	Intersect at $\mathbf{r}=\underline{\begin{pmatrix}-3\\3\\7\end{pmatrix}}$ or $\mathbf{r}=\underline{-3\mathbf{i}+3\mathbf{j}+7\mathbf{k}}$	$\underline{\begin{pmatrix}-3\\3\\7\end{pmatrix}}$ or $\underline{-3\mathbf{i}+3\mathbf{j}+7\mathbf{k}}$ or $(-3,3,7)$	A1
	Either check **k:** $\lambda=3$: LHS $=10-\lambda=10-3=7$ $\mu=-2$: RHS $=17+5\mu=17-10=7$	**Either** check that $\lambda=3$, $\mu=-2$ in a third equation **or** check that $\lambda=3$, $\mu=-2$ give the same coordinates on the other line. Conclusion not needed.	B1
	(As LHS = RHS then the lines intersect.)		**[6]**
(b)	$\mathbf{d}_1=2\mathbf{i}+\mathbf{j}-\mathbf{k}$, $\mathbf{d}_2=3\mathbf{i}-\mathbf{j}+5\mathbf{k}$		
	As $\mathbf{d}_1\bullet\mathbf{d}_2=\begin{pmatrix}2\\1\\-1\end{pmatrix}\bullet\begin{pmatrix}3\\-1\\5\end{pmatrix}=\underline{(2\times3)+(1\times-1)+(-1\times5)}=0$	Dot product calculation between the ***two direction vectors***: $\underline{(2\times3)+(1\times-1)+(-1\times5)}$ or $\underline{6-1-5}$	M1
	Then l_1 is perpendicular to l_2.	Result '=0' and appropriate conclusion	A1
			[2]

Question Number	Scheme		Marks
6. (c)	Equating **i** ; $-9+2\lambda=5 \Rightarrow \lambda=7$		
	$\mathbf{r}=\begin{pmatrix}-9\\0\\10\end{pmatrix}+7\begin{pmatrix}2\\1\\-1\end{pmatrix}=\begin{pmatrix}5\\7\\3\end{pmatrix}$ $(=\overrightarrow{OA}.$ Hence the point A lies on l_1.)	Substitutes candidate's $\lambda=7$ into the line l_1 and finds $5\mathbf{i}+7\mathbf{j}+3\mathbf{k}$. The conclusion on this occasion is not needed.	B1
			[1]
(d)	Let $\overrightarrow{OX}=-3\mathbf{i}+3\mathbf{j}+7\mathbf{k}$ be point of intersection		
	$\overrightarrow{AX}=\overrightarrow{OX}-\overrightarrow{OA}=\underline{\begin{pmatrix}-3\\3\\7\end{pmatrix}-\begin{pmatrix}5\\7\\3\end{pmatrix}}=\begin{pmatrix}-8\\-4\\4\end{pmatrix}$	Finding the difference between their $\overrightarrow{OX}$ (can be implied) and $\overrightarrow{OA}$. $\overrightarrow{AX}=\pm\left(\begin{pmatrix}-3\\3\\7\end{pmatrix}-\begin{pmatrix}5\\7\\3\end{pmatrix}\right)$	M1 $\sqrt{}$ ±
	$\overrightarrow{OB}=\overrightarrow{OA}+\overrightarrow{AB}=\overrightarrow{OA}+2\overrightarrow{AX}$		
	$\overrightarrow{OB}=\begin{pmatrix}5\\7\\3\end{pmatrix}+2\begin{pmatrix}-8\\-4\\4\end{pmatrix}$	$\begin{pmatrix}5\\7\\3\end{pmatrix}+2\begin{pmatrix}\text{their } \overrightarrow{AX}\end{pmatrix}$	dM1 $\sqrt{}$
	Hence, $\overrightarrow{OB}=\underline{\begin{pmatrix}-11\\-1\\11\end{pmatrix}}$ or $\overrightarrow{OB}=\underline{-11\mathbf{i}-\mathbf{j}+11\mathbf{k}}$	$\underline{\begin{pmatrix}-11\\-1\\11\end{pmatrix}}$ or $\underline{-11\mathbf{i}-\mathbf{j}+11\mathbf{k}}$ or $\underline{(-11,-1,11)}$	A1
			[3]
			12 marks

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7. (a) Express $\dfrac{2}{4-y^2}$ in partial fractions. **(3)**

(b) Hence obtain the solution of

$$2\cot x\,\frac{dy}{dx} = (4-y^2)$$

for which $y = 0$ at $x = \frac{\pi}{3}$, giving your answer in the form $\sec^2 x = g(y)$. **(8)**

Question Number	Scheme		Marks
7. (a)	$\dfrac{2}{4-y^2} \equiv \dfrac{2}{(2-y)(2+y)} \equiv \dfrac{A}{(2-y)} + \dfrac{B}{(2+y)}$		
	$2 \equiv A(2+y) + B(2-y)$	Forming this identity. **NB:** A & B are not assigned in this question	M1
	Let $y=-2$, $2 = B(4) \Rightarrow B = \frac{1}{2}$		
	Let $y=2$, $2 = A(4) \Rightarrow A = \frac{1}{2}$	Either one of $A=\frac{1}{2}$ or $B=\frac{1}{2}$	A1
	giving $\dfrac{\frac{1}{2}}{(2-y)} + \dfrac{\frac{1}{2}}{(2+y)}$	$\dfrac{\frac{1}{2}}{(2-y)} + \dfrac{\frac{1}{2}}{(2+y)}$, aef	A1 **cao**
	(If n**o working seen**, but candidate writes down ***correct partial fraction*** then award all three marks. If no working is seen but one of *A* or *B* is incorrect then M0A0A0.)		**[3]**

Question Number	Scheme		Marks
7. (b)	$\int \frac{2}{4-y^2}\,dy = \int \frac{1}{\cot x}\,dx$	Separates variables as shown. Can be implied. Ignore the integral signs, and the '2'.	B1
	$\int \frac{\frac{1}{2}}{(2-y)} + \frac{\frac{1}{2}}{(2+y)}\,dy = \int \tan x\,dx$		
		$\ln(\sec x)$ or $-\ln(\cos x)$	B1
	$\therefore\ -\frac{1}{2}\ln(2-y) + \frac{1}{2}\ln(2+y) = \ln(\sec x)\ \ + (c)$	Either $\pm a\ln(\lambda - y)$ or $\pm b\ln(\lambda + y)$	M1;
		their $\int \frac{1}{\cot x}\,dx =$ LHS correct with ft for their *A* and *B* and no error with the "2" with or without $+c$	A1 $\sqrt{}$
	$y=0,\ x=\frac{\pi}{3}\ \Rightarrow\ -\frac{1}{2}\ln 2 + \frac{1}{2}\ln 2 = \ln\left(\frac{1}{\cos\left(\frac{\pi}{3}\right)}\right) + c$	Use of $y=0$ and $x=\frac{\pi}{3}$ in an integrated equation containing c ;	M1*
	$\{0 = \ln 2 + c\ \Rightarrow\ \underline{c = -\ln 2}\}$		
	$-\frac{1}{2}\ln(2-y) + \frac{1}{2}\ln(2+y) = \ln(\sec x) - \ln 2$		
	$\frac{1}{2}\ln\left(\frac{2+y}{2-y}\right) = \ln\left(\frac{\sec x}{2}\right)$	Using either the quotient (or product) or power laws for logarithms CORRECTLY.	M1
	$\ln\left(\frac{2+y}{2-y}\right) = 2\ln\left(\frac{\sec x}{2}\right)$		
	$\ln\left(\frac{2+y}{2-y}\right) = \ln\left(\frac{\sec x}{2}\right)^2$	Using the log laws correctly to obtain a single log term on both sides of the equation.	dM1*
	$\frac{2+y}{2-y} = \frac{\sec^2 x}{4}$		
	Hence, $\underline{\sec^2 x = \frac{8+4y}{2-y}}$	$\underline{\sec^2 x = \frac{8+4y}{2-y}}$	A1 aef
			[8]
			11 marks

8.

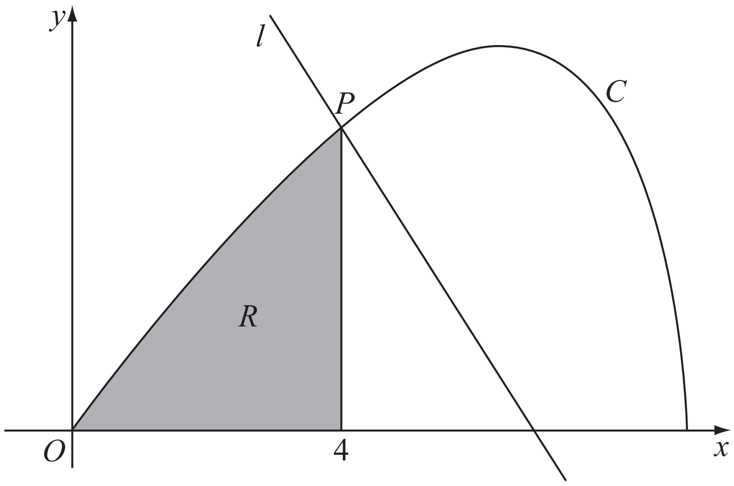

Figure 3

Figure 3 shows the curve C with parametric equations

$$x = 8\cos t, \qquad y = 4\sin 2t, \qquad 0 \leqslant t \leqslant \frac{\pi}{2}.$$

The point P lies on C and has coordinates $(4, 2\sqrt{3})$.

(a) Find the value of t at the point P.

(2)

The line l is a normal to C at P.

(b) Show that an equation for l is $y = -x\sqrt{3} + 6\sqrt{3}$.

(6)

The finite region R is enclosed by the curve C, the x-axis and the line $x = 4$, as shown shaded in Figure 3.

(c) Show that the area of R is given by the integral $\displaystyle\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 64\sin^2 t\cos t\,dt$.

(4)

(d) Use this integral to find the area of R, giving your answer in the form $a + b\sqrt{3}$, where a and b are constants to be determined.

(4)

Question Number	Scheme		Marks
8. (a)	At $P(4,2\sqrt{3})$ either $\underline{4=8\cos t}$ or $\underline{2\sqrt{3}=4\sin 2t}$	$\underline{4=8\cos t}$ or $\underline{2\sqrt{3}=4\sin 2t}$	M1
	$\Rightarrow$ only solution is $\underline{t=\frac{\pi}{3}}$ where $0 \leq t \leq \frac{\pi}{2}$	$\underline{t=\frac{\pi}{3}}$ or awrt 1.05 (radians) only stated in the range $0 \leq t \leq \frac{\pi}{2}$	A1
			[2]
(b)	$x=8\cos t$, $y=4\sin 2t$		
	$\frac{dx}{dt}=-8\sin t$, $\frac{dy}{dt}=8\cos 2t$	Attempt to differentiate both x and y wrt t to give $\pm p\sin t$ and $\pm q\cos 2t$ respectively	M1
		Correct $\frac{dx}{dt}$ and $\frac{dy}{dt}$	A1
	At *P*, $\frac{dy}{dx}=\underline{\frac{8\cos\left(\frac{2\pi}{3}\right)}{-8\sin\left(\frac{\pi}{3}\right)}}$	Divides in correct way round and attempts to substitute their value of t (in degrees or radians) into their $\frac{dy}{dx}$ expression.	M1*
	$\left\{=\frac{8\left(-\frac{1}{2}\right)}{(-8)\left(\frac{\sqrt{3}}{2}\right)}=\frac{1}{\sqrt{3}}=\text{awrt } 0.58\right\}$	You may need to check candidate's substitutions for M1* **Note the next two method marks are dependent on M1***	
	Hence m(**N**) $=-\sqrt{3}$ or $\frac{-1}{\frac{1}{\sqrt{3}}}$	Uses m(**N**) $=-\frac{1}{\text{their m}(\mathbf{T})}$.	dM1*
	N: $y-2\sqrt{3}=-\sqrt{3}(x-4)$	Uses $y-2\sqrt{3}=(\text{their } m_N)(x-4)$ or finds c using $x=4$ and $y=2\sqrt{3}$ and uses $y=(\text{their m}_N)x+\text{"}c\text{"}$.	dM1*
	N: $\underline{y=-\sqrt{3}x+6\sqrt{3}}$ **AG**	$\underline{y=-\sqrt{3}x+6\sqrt{3}}$	A1 **cso AG**
	or $2\sqrt{3}=-\sqrt{3}(4)+c \Rightarrow c=2\sqrt{3}+4\sqrt{3}=6\sqrt{3}$ so N: $\left[\underline{y=-\sqrt{3}x+6\sqrt{3}}\right]$		
			[6]

Question	Scheme		Marks
8. (c)	$A = \int_0^4 y\,\mathrm{d}x = \int_{\frac{\pi}{2}}^{\frac{\pi}{3}} 4\sin 2t.(-8\sin t)\,\mathrm{d}t$	attempt at $A = \int y \frac{\mathrm{d}x}{\mathrm{d}t}\,\mathrm{d}t$	M1
		correct expression (ignore limits and $\mathrm{d}t$)	A1
	$A = \int_{\frac{\pi}{2}}^{\frac{\pi}{3}} -32\sin 2t.\sin t\,\mathrm{d}t = \int_{\frac{\pi}{2}}^{\frac{\pi}{3}} -32(2\sin t\cos t).\sin t\,\mathrm{d}t$	Seeing $\sin 2t = 2\sin t\cos t$ anywhere in PART (c).	M1
	$A = \int_{\frac{\pi}{2}}^{\frac{\pi}{3}} -64.\sin^2 t\cos t\,\mathrm{d}t$ $A = \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 64.\sin^2 t\cos t\,\mathrm{d}t$	Correct proof. Appreciation of how the negative sign affects the limits. **Note that the answer is given in the question.**	A1 **AG**
			[4]
(d)	{Using substitution $u = \sin t \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}t} = \cos t$} {change limits: when $t = \frac{\pi}{3}$, $u = \frac{\sqrt{3}}{2}$ & when $t = \frac{\pi}{2}$, $u = 1$}		
	$A = 64\left[\frac{\sin^3 t}{3}\right]_{\frac{\pi}{3}}^{\frac{\pi}{2}}$ or $A = 64\left[\frac{u^3}{3}\right]_{\frac{\sqrt{3}}{2}}^{1}$	$k\sin^3 t$ or ku^3 with $u = \sin t$	M1
		Correct integration ignoring limits.	A1
	$A = 64\left[\frac{1}{3} - \left(\frac{1}{3}.\frac{\sqrt{3}}{2}.\frac{\sqrt{3}}{2}.\frac{\sqrt{3}}{2}\right)\right]$	Substitutes limits of either $\left(t = \frac{\pi}{2} \text{ and } t = \frac{\pi}{3}\right)$ or $\left(u = 1 \text{ and } u = \frac{\sqrt{3}}{2}\right)$ and subtracts the correct way round.	dM1
	$A = 64\left(\frac{1}{3} - \frac{1}{8}\sqrt{3}\right) = \underline{\frac{64}{3} - 8\sqrt{3}}$	$\underline{\frac{64}{3} - 8\sqrt{3}}$ Aef in the form $a + b\sqrt{3}$, with awrt 21.3 and anything that cancels to $a = \frac{64}{3}$ and $b = -8$.	A1 aef isw
			[4]
	(Note that $a = \frac{64}{3}$, $b = -8$)		
			16 marks