

June 2009
6666 Core Mathematics C4
Mark Scheme

Question Number	Scheme	Marks
Q1	$f(x) = \frac{1}{\sqrt{4+x}} = (4+x)^{-\frac{1}{2}}$ $= (4)^{-\frac{1}{2}} (1 + \dots)^{-\frac{1}{2}} \qquad \frac{1}{2} (1 + \dots)^{-\frac{1}{2}} \text{ or } \frac{1}{2\sqrt{1+\dots}}$ $= \dots \left(1 + \left(-\frac{1}{2}\right) \left(\frac{x}{4}\right) + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2} \left(\frac{x}{4}\right)^2 + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)}{3!} \left(\frac{x}{4}\right)^3 + \dots \right)$ <p style="text-align: right;">ft their $\left(\frac{x}{4}\right)$</p> $= \frac{1}{2} - \frac{1}{16}x + \frac{3}{256}x^2 - \frac{5}{2048}x^3 + \dots$ <i>Alternative</i> $f(x) = \frac{1}{\sqrt{4+x}} = (4+x)^{-\frac{1}{2}}$ $= 4^{-\frac{1}{2}} + \left(-\frac{1}{2}\right) 4^{-\frac{3}{2}} x + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{1.2} 4^{-\frac{5}{2}} x^2 + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)}{1.2.3} 4^{-\frac{7}{2}} x^3 + \dots$ $= \frac{1}{2} - \frac{1}{16}x + \frac{3}{256}x^2 - \frac{5}{2048}x^3 + \dots$	<p>M1</p> <p>B1</p> <p>M1 A1ft</p> <p>A1, A1 (6)</p> <p style="text-align: right;">[6]</p> <p>M1</p> <p><u>B1</u> M1 A1</p> <p>A1, A1 (6)</p>

2.

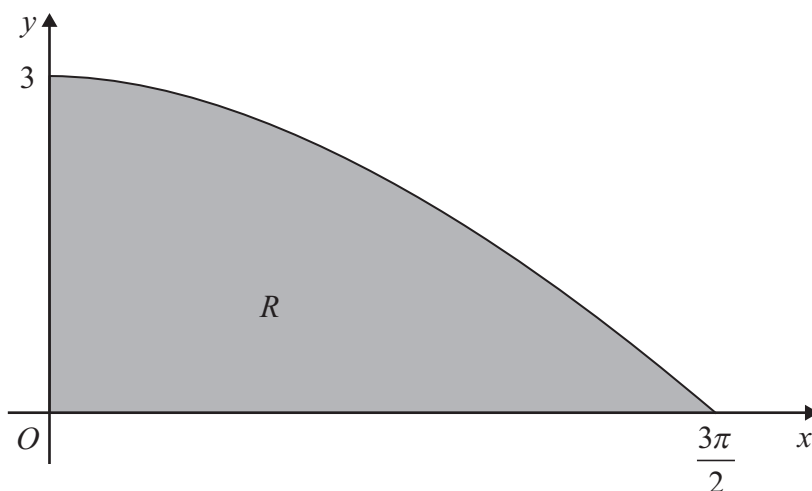


Figure 1

Figure 1 shows the finite region R bounded by the x -axis, the y -axis and the curve with equation $y = 3 \cos\left(\frac{x}{3}\right)$, $0 \leq x \leq \frac{3\pi}{2}$.

The table shows corresponding values of x and y for $y = 3 \cos\left(\frac{x}{3}\right)$.

x	0	$\frac{3\pi}{8}$	$\frac{3\pi}{4}$	$\frac{9\pi}{8}$	$\frac{3\pi}{2}$
y	3	2.77164	2.12132		0

- (a) Complete the table above giving the missing value of y to 5 decimal places. (1)
- (b) Using the trapezium rule, with all the values of y from the completed table, find an approximation for the area of R , giving your answer to 3 decimal places. (4)
- (c) Use integration to find the exact area of R . (3)



Question Number	Scheme	Marks
Q2 (a)	1.14805 awrt 1.14805	B1 (1)
(b)	$A \approx \frac{1}{2} \times \frac{3\pi}{8} (\dots)$ $= \dots (3 + 2(2.77164 + 2.12132 + 1.14805) + 0) \quad 0 \text{ can be implied}$ $= \frac{3\pi}{16} (3 + 2(2.77164 + 2.12132 + 1.14805)) \quad \text{ft their (a)}$ $= \frac{3\pi}{16} \times 15.08202 \dots = 8.884 \quad \text{cao}$	B1 M1 A1ft A1 (4)
(c)	$\int 3 \cos\left(\frac{x}{3}\right) dx = \frac{3 \sin\left(\frac{x}{3}\right)}{\frac{1}{3}}$ $= 9 \sin\left(\frac{x}{3}\right)$ $A = \left[9 \sin\left(\frac{x}{3}\right) \right]_0^{\frac{3\pi}{2}} = 9 - 0 = 9 \quad \text{cao}$	M1 A1 A1 (3) [8]

Question Number	Scheme	Marks
Q3 (a)	$f(x) = \frac{4-2x}{(2x+1)(x+1)(x+3)} = \frac{A}{2x+1} + \frac{B}{x+1} + \frac{C}{x+3}$ $4-2x = A(x+1)(x+3) + B(2x+1)(x+3) + C(2x+1)(x+1)$ <p style="text-align: center;">A method for evaluating one constant</p> $x \rightarrow -\frac{1}{2}, \quad 5 = A\left(\frac{1}{2}\right)\left(\frac{5}{2}\right) \Rightarrow A = 4$ $x \rightarrow -1, \quad 6 = B(-1)(2) \Rightarrow B = -3$ $x \rightarrow -3, \quad 10 = C(-5)(-2) \Rightarrow C = 1$	<p>M1 M1 A1 A1 (4)</p>
	<p>(b)</p> <p>(i) $\int \left(\frac{4}{2x+1} - \frac{3}{x+1} + \frac{1}{x+3} \right) dx$</p> $= \frac{4}{2} \ln(2x+1) - 3 \ln(x+1) + \ln(x+3) + C$ <p style="text-align: center;">All three ln terms correct and "+C"; ft constants</p> <p>(ii) $\left[2 \ln(2x+1) - 3 \ln(x+1) + \ln(x+3) \right]_0^2$</p> $= (2 \ln 5 - 3 \ln 3 + \ln 5) - (2 \ln 1 - 3 \ln 1 + \ln 3)$ $= 3 \ln 5 - 4 \ln 3$ $= \ln \left(\frac{5^3}{3^4} \right)$ $= \ln \left(\frac{125}{81} \right)$	<p>M1 A1ft A1ft (3)</p> <p>M1 M1 A1 (3)</p> <p style="text-align: right;">[10]</p>

Question Number	Scheme	Marks
<p>Q4 (a)</p> <p>(b)</p>	$e^{-2x} \frac{dy}{dx} - 2ye^{-2x} = 2 + 2y \frac{dy}{dx}$ $\frac{d}{dx}(ye^{-2x}) = e^{-2x} \frac{dy}{dx} - 2ye^{-2x}$ $(e^{-2x} - 2y) \frac{dy}{dx} = 2 + 2ye^{-2x}$ $\frac{dy}{dx} = \frac{2 + 2ye^{-2x}}{e^{-2x} - 2y}$ <p>At P , $\frac{dy}{dx} = \frac{2 + 2e^0}{e^0 - 2} = -4$</p> <p>Using $mm' = -1$</p> $m' = \frac{1}{4}$ $y - 1 = \frac{1}{4}(x - 0)$ $x - 4y + 4 = 0$ <p>or any integer multiple</p> <p><i>Alternative for (a) differentiating implicitly with respect to y.</i></p> $e^{-2x} - 2ye^{-2x} \frac{dx}{dy} = 2 \frac{dx}{dy} + 2y$ $\frac{d}{dy}(ye^{-2x}) = e^{-2x} - 2ye^{-2x} \frac{dx}{dy}$ $(2 + 2ye^{-2x}) \frac{dx}{dy} = e^{-2x} - 2y$ $\frac{dx}{dy} = \frac{e^{-2x} - 2y}{2 + 2ye^{-2x}}$ $\frac{dy}{dx} = \frac{2 + 2ye^{-2x}}{e^{-2x} - 2y}$	<p>A1 correct RHS</p> <p>M1 A1</p> <p>B1</p> <p>M1</p> <p>A1 (5)</p> <p>M1</p> <p>M1</p> <p>A1 (4)</p> <p>[9]</p> <p>M1 A1</p> <p>B1</p> <p>M1</p> <p>A1 (5)</p>

Question Number	Scheme	Marks
Q5 (a)	$\frac{dx}{dt} = -4 \sin 2t, \quad \frac{dy}{dt} = 6 \cos t$ $\frac{dy}{dx} = -\frac{6 \cos t}{4 \sin 2t} \quad \left(= -\frac{3}{4 \sin t} \right)$ <p>At $t = \frac{\pi}{3}$, $m = -\frac{3}{4 \times \frac{\sqrt{3}}{2}} = -\frac{\sqrt{3}}{2}$ accept equivalents, awrt -0.87</p>	<p>B1, B1</p> <p>M1</p> <p>A1 (4)</p>
(b)	<p>Use of $\cos 2t = 1 - 2 \sin^2 t$</p> $\cos 2t = \frac{x}{2}, \quad \sin t = \frac{y}{6}$ $\frac{x}{2} = 1 - 2 \left(\frac{y}{6} \right)^2$ <p>Leading to $y = \sqrt{(18 - 9x)} \quad (= 3\sqrt{(2 - x)})$ cao</p> <p>$-2 \leq x \leq 2$ $k = 2$</p>	<p>M1</p> <p>M1</p> <p>A1</p> <p>B1 (4)</p>
(c)	$0 \leq f(x) \leq 6$ either $0 \leq f(x)$ or $f(x) \leq 6$ <p>Fully correct. Accept $0 \leq y \leq 6, [0, 6]$</p>	<p>B1</p> <p>B1 (2)</p>
[10]		
<i>Alternatives to (a) where the parameter is eliminated</i>		
①	$y = (18 - 9x)^{\frac{1}{2}}$ $\frac{dy}{dx} = \frac{1}{2}(18 - 9x)^{-\frac{1}{2}} \times (-9)$ <p>At $t = \frac{\pi}{3}$, $x = \cos \frac{2\pi}{3} = -1$</p> $\frac{dy}{dx} = \frac{1}{2} \times \frac{1}{\sqrt{(27)}} \times -9 = -\frac{\sqrt{3}}{2}$	<p>B1</p> <p>B1</p> <p>M1 A1 (4)</p>
②	$y^2 = 18 - 9x$ $2y \frac{dy}{dx} = -9$ <p>At $t = \frac{\pi}{3}$, $y = 6 \sin \frac{\pi}{3} = 3\sqrt{3}$</p> $\frac{dy}{dx} = -\frac{9}{2 \times 3\sqrt{3}} = -\frac{\sqrt{3}}{2}$	<p>B1</p> <p>B1</p> <p>M1 A1 (4)</p>

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6. (a) Find $\int \sqrt{5-x} dx$. (2)

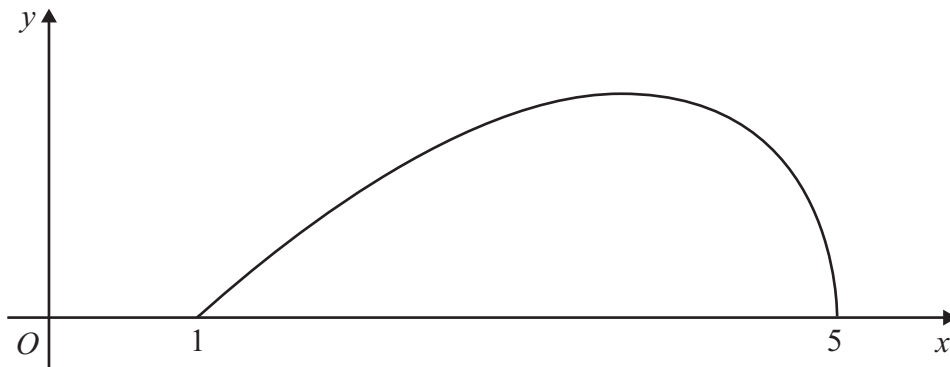


Figure 3

Figure 3 shows a sketch of the curve with equation

$$y = (x - 1)\sqrt{5 - x}, \quad 1 \leq x \leq 5$$

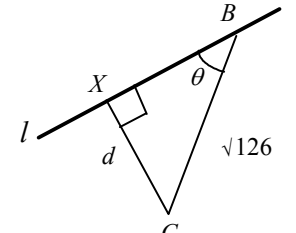
- (b) (i) Using integration by parts, or otherwise, find

$$\int (x-1)\sqrt{5-x} dx \quad (4)$$

- (ii) Hence find $\int_1^5 (x-1)\sqrt{5-x} dx$. (2)



Question Number	Scheme	Marks
Q6 (a)	$\int \sqrt{5-x} dx = \int (5-x)^{\frac{1}{2}} dx = \frac{(5-x)^{\frac{3}{2}}}{-\frac{3}{2}} (+C)$ $\left(= -\frac{2}{3}(5-x)^{\frac{3}{2}} + C \right)$	M1 A1 (2)
(b) (i)	$\int (x-1)\sqrt{5-x} dx = -\frac{2}{3}(x-1)(5-x)^{\frac{3}{2}} + \frac{2}{3} \int (5-x)^{\frac{3}{2}} dx$ $= \dots + \frac{2}{3} \times \frac{(5-x)^{\frac{5}{2}}}{-\frac{5}{2}} (+C)$ $= -\frac{2}{3}(x-1)(5-x)^{\frac{3}{2}} - \frac{4}{15}(5-x)^{\frac{5}{2}} (+C)$	M1 A1ft M1 A1 (4)
(ii)	$\left[-\frac{2}{3}(x-1)(5-x)^{\frac{3}{2}} - \frac{4}{15}(5-x)^{\frac{5}{2}} \right]_1^5 = (0-0) - \left(0 - \frac{4}{15} \times 4^{\frac{5}{2}} \right)$ $= \frac{128}{15} \left(= 8 \frac{8}{15} \approx 8.53 \right) \quad \text{awrt 8.53}$ <p><i>Alternatives for (b) and (c)</i></p> <p>(b) $u^2 = 5-x \Rightarrow 2u \frac{du}{dx} = -1 \left(\Rightarrow \frac{dx}{du} = -2u \right)$</p> $\int (x-1)\sqrt{5-x} dx = \int (4-u^2)u \frac{dx}{du} du = \int (4-u^2)u(-2u) du$ $= \int (2u^4 - 8u^2) du = \frac{2}{5}u^5 - \frac{8}{3}u^3 (+C)$ $= \frac{2}{5}(5-x)^{\frac{5}{2}} - \frac{8}{3}(5-x)^{\frac{3}{2}} (+C)$ <p>(c) $x=1 \Rightarrow u=2, x=5 \Rightarrow u=0$</p> $\left[\frac{2}{5}u^5 - \frac{8}{3}u^3 \right]_2^0 = (0-0) - \left(\frac{64}{5} - \frac{64}{3} \right)$ $= \frac{128}{15} \left(= 8 \frac{8}{15} \approx 8.53 \right) \quad \text{awrt 8.53}$	M1 A1 (2) [8] M1 A1 M1 A1 M1 A1 (2)

Question Number	Scheme	Marks	
Q7 (a)	$\overline{AB} = \overline{OB} - \overline{OA} = \begin{pmatrix} 10 \\ 14 \\ -4 \end{pmatrix} - \begin{pmatrix} 8 \\ 13 \\ -2 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$ <p style="text-align: right;">or $\overline{BA} = \begin{pmatrix} -2 \\ -1 \\ 2 \end{pmatrix}$</p> $\mathbf{r} = \begin{pmatrix} 8 \\ 13 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix} \text{ or } \mathbf{r} = \begin{pmatrix} 10 \\ 14 \\ -4 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$ <p style="text-align: right;">accept equivalents</p>	M1 M1 A1ft (3)	
(b)	$\overline{CB} = \overline{OB} - \overline{OC} = \begin{pmatrix} 10 \\ 14 \\ -4 \end{pmatrix} - \begin{pmatrix} 9 \\ 9 \\ 6 \end{pmatrix} = \begin{pmatrix} 1 \\ 5 \\ -10 \end{pmatrix}$ <p style="text-align: right;">or $\overline{BC} = \begin{pmatrix} -1 \\ -5 \\ 10 \end{pmatrix}$</p> $CB = \sqrt{(1^2 + 5^2 + (-10)^2)} = \sqrt{126} \quad (= 3\sqrt{14} \approx 11.2)$ <p style="text-align: right;">awrt 11.2</p>	M1 A1 (2)	
(c)	$\overline{CB} \cdot \overline{AB} = \overline{CB} \overline{AB} \cos \theta$ $(\pm)(2 + 5 + 20) = \sqrt{126} \sqrt{9} \cos \theta$ $\cos \theta = \frac{3}{\sqrt{14}} \Rightarrow \theta \approx 36.7^\circ$ <p style="text-align: right;">awrt 36.7°</p>	M1 A1 A1 (3)	
(d)	 $\frac{d}{\sqrt{126}} = \sin \theta$ $d = 3\sqrt{5} (\approx 6.7)$ <p style="text-align: right;">awrt 6.7</p>	M1 A1ft A1 (3)	
(e)	$BX^2 = BC^2 - d^2 = 126 - 45 = 81$ $! CBX = \frac{1}{2} \times BX \times d = \frac{1}{2} \times 9 \times 3\sqrt{5} = \frac{27\sqrt{5}}{2} (\approx 30.2)$ <p style="text-align: right;">awrt 30.1 or 30.2</p>	M1 M1 A1 (3)	
[14]			
<p><i>Alternative for (e)</i></p> $! CBX = \frac{1}{2} \times d \times BC \sin \angle XCB$ $= \frac{1}{2} \times 3\sqrt{5} \times \sqrt{126} \sin (90 - 36.7)^\circ$ <p style="text-align: right;">sine of correct angle</p> ≈ 30.2 <p style="text-align: right;">$\frac{27\sqrt{5}}{2}$, awrt 30.1 or 30.2</p>			M1 M1 A1 (3)

Question Number	Scheme	Marks
Q8 (a)	$\int \sin^2 \theta \, d\theta = \frac{1}{2} \int (1 - \cos 2\theta) \, d\theta = \frac{1}{2} \theta - \frac{1}{4} \sin 2\theta \quad (+C)$	M1 A1 (2)
(b)	$x = \tan \theta \Rightarrow \frac{dx}{d\theta} = \sec^2 \theta$ $\pi \int y^2 \, dx = \pi \int y^2 \frac{dx}{d\theta} \, d\theta = \pi \int (2 \sin 2\theta)^2 \sec^2 \theta \, d\theta$ $= \pi \int \frac{(2 \times 2 \sin \theta \cos \theta)^2}{\cos^2 \theta} \, d\theta$ $= 16\pi \int \sin^2 \theta \, d\theta \quad k = 16\pi$ $x = 0 \Rightarrow \tan \theta = 0 \Rightarrow \theta = 0, \quad x = \frac{1}{\sqrt{3}} \Rightarrow \tan \theta = \frac{1}{\sqrt{3}} \Rightarrow \theta = \frac{\pi}{6}$ $\left(V = 16\pi \int_0^{\frac{\pi}{6}} \sin^2 \theta \, d\theta \right)$	M1 A1 M1 A1 B1 (5)
(c)	$V = 16\pi \left[\frac{1}{2} \theta - \frac{\sin 2\theta}{4} \right]_0^{\frac{\pi}{6}}$ $= 16\pi \left[\left(\frac{\pi}{12} - \frac{1}{4} \sin \frac{\pi}{3} \right) - (0 - 0) \right]$ $= 16\pi \left(\frac{\pi}{12} - \frac{\sqrt{3}}{8} \right) = \frac{4}{3} \pi^2 - 2\pi \sqrt{3}$	<div style="border-left: 1px solid black; border-right: 1px solid black; padding: 0 10px;"> M1 M1 A1 (3) </div> <p style="text-align: center;">Use of correct limits</p> $p = \frac{4}{3}, \quad q = -2$ <p style="text-align: right;">[10]</p>