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| Centre No. | | | | | | Paper Reference | | | | | | Surname | Initial(s) | |
| Candidate No. | | | | | | 6 | 6 | 6 | 6 | / | 0 | 1 | Signature | |

Paper Reference(s)

6666/01

**Edexcel GCE
Core Mathematics C4
Advanced**

Friday 18 June 2010 – Afternoon
Time: 1 hour 30 minutes

Examiner's use only

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Team Leader's use only

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Materials required for examination **Items included with question papers**
Mathematical Formulae (Pink) Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

| Question Number | Leave Blank |
|-----------------|-------------|
| 1 | |
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Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature. Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer to each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. (2).
There are 8 questions in this question paper. The total mark for this paper is 75.
There are 32 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.



Turn over

1.

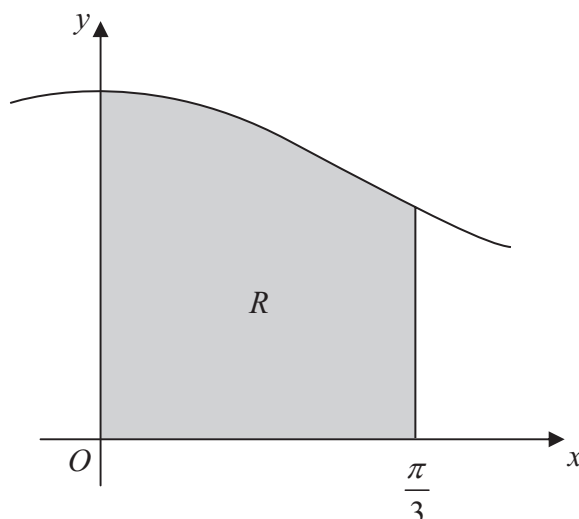


Figure 1

Figure 1 shows part of the curve with equation $y = \sqrt{0.75 + \cos^2 x}$. The finite region R , shown shaded in Figure 1, is bounded by the curve, the y -axis, the x -axis and the line with equation $x = \frac{\pi}{3}$.

(a) Complete the table with values of y corresponding to $x = \frac{\pi}{6}$ and $x = \frac{\pi}{4}$.

| | | | | | |
|-----|--------|------------------|-----------------|-----------------|-----------------|
| x | 0 | $\frac{\pi}{12}$ | $\frac{\pi}{6}$ | $\frac{\pi}{4}$ | $\frac{\pi}{3}$ |
| y | 1.3229 | 1.2973 | | | 1 |

(2)

(b) Use the trapezium rule

(i) with the values of y at $x = 0$, $x = \frac{\pi}{6}$ and $x = \frac{\pi}{3}$ to find an estimate of the area of R .

Give your answer to 3 decimal places.

(ii) with the values of y at $x = 0$, $x = \frac{\pi}{12}$, $x = \frac{\pi}{6}$, $x = \frac{\pi}{4}$ and $x = \frac{\pi}{3}$ to find a

further estimate of the area of R . Give your answer to 3 decimal places.

(6)



June 2010
6666 Core Mathematics C4
Mark Scheme

| Question Number | Scheme | Marks |
|-----------------|---|---|
| 1. | (a) $y\left(\frac{\pi}{6}\right) \approx 1.2247, y\left(\frac{\pi}{4}\right) = 1.1180$ accept awrt 4 d.p. | B1 B1 (2) |
| | (b)(i) $I \approx \left(\frac{\pi}{12}\right)(1.3229 + 2 \times 1.2247 + 1)$ ≈ 1.249 | B1 for $\frac{\pi}{12}$ B1 M1 A1 |
| | (ii) $I \approx \left(\frac{\pi}{24}\right)(1.3229 + 2 \times (1.2973 + 1.2247 + 1.1180) + 1)$ ≈ 1.257 | B1 for $\frac{\pi}{24}$ B1 M1 A1 (6) [8] |

| Question Number | Scheme | Marks |
|-----------------|---|---|
| 2. | $\frac{du}{dx} = -\sin x$ $\int \sin x e^{\cos x+1} dx = -\int e^u du$ $= -e^u$ $= -e^{\cos x+1}$ $\left[-e^{\cos x+1}\right]_0^{\frac{\pi}{2}} = -e^1 - (-e^2)$ $= e(e-1) *$ | <p>B1</p> <p>M1 A1</p> <p>A1ft</p> <p>ft sign error</p> <p>or equivalent with u</p> <p>M1</p> <p>A1</p> <p>cso</p> <p>(6)</p> <p>[6]</p> |

| Question Number | Scheme | Marks |
|-----------------|---|--|
| 3. | $\frac{d}{dx}(2^x) = \ln 2 \cdot 2^x$ $\ln 2 \cdot 2^x + 2y \frac{dy}{dx} = 2y + 2x \frac{dy}{dx}$ <p>Substituting (3, 2)</p> $8 \ln 2 + 4 \frac{dy}{dx} = 4 + 6 \frac{dy}{dx}$ $\frac{dy}{dx} = 4 \ln 2 - 2$ <p>Accept exact equivalents</p> | <p>B1</p> <p>M1 A1= A1</p> <p>M1</p> <p>M1 A1 (7)</p> <p>[7]</p> |

| Question Number | Scheme | Marks |
|-----------------|--|---|
| 4. | <p>(a) $\frac{dx}{dt} = 2 \sin t \cos t, \frac{dy}{dt} = 2 \sec^2 t$</p> <p>$\frac{dy}{dx} = \frac{\sec^2 t}{\sin t \cos t} \left(= \frac{1}{\sin t \cos^3 t} \right)$ or equivalent</p> <p>(b) At $t = \frac{\pi}{3}, x = \frac{3}{4}, y = 2\sqrt{3}$</p> <p>$\frac{dy}{dx} = \frac{\sec^2 \frac{\pi}{3}}{\sin \frac{\pi}{3} \cos \frac{\pi}{3}} = \frac{16}{\sqrt{3}}$</p> <p>$y - 2\sqrt{3} = \frac{16}{\sqrt{3}} \left(x - \frac{3}{4} \right)$</p> <p>$y = 0 \Rightarrow x = \frac{3}{8}$</p> | <p>B1 B1</p> <p>M1 A1 (4)</p> <p>B1</p> <p>M1 A1</p> <p>M1</p> <p>M1 A1 (6)</p> <p>[10]</p> |

| Question Number | Scheme | Marks |
|-----------------|--|--|
| 5. | <p>(a) $A = 2$ $2x^2 + 5x - 10 = A(x-1)(x+2) + B(x+2) + C(x-1)$ $x \rightarrow 1 \quad -3 = 3B \Rightarrow B = -1$ $x \rightarrow -2 \quad -12 = -3C \Rightarrow C = 4$</p> <p>(b) $\frac{2x^2 + 5x - 10}{(x-1)(x+2)} = 2 + (1-x)^{-1} + 2\left(1 + \frac{x}{2}\right)^{-1}$ $(1-x)^{-1} = 1 + x + x^2 + \dots$ $\left(1 + \frac{x}{2}\right)^{-1} = 1 - \frac{x}{2} + \frac{x^2}{4} + \dots$ $\frac{2x^2 + 5x - 10}{(x-1)(x+2)} = (2+1+2) + (1-1)x + \left(1 + \frac{1}{2}\right)x^2 + \dots$ $= 5 + \dots$ ft their $A - B + \frac{1}{2}C$ $= \dots + \frac{3}{2}x^2 + \dots$ $0x$ stated or implied</p> | <p>B1 M1 A1 A1 (4)</p> <p>M1 B1 B1 M1 A1 ft A1 A1 (7) [11]</p> |

| Question Number | Scheme | Marks |
|------------------|--|---|
| <p>6.</p> | <p>(a)</p> $f(\theta) = 4\cos^2\theta - 3\sin^2\theta$ $= 4\left(\frac{1}{2} + \frac{1}{2}\cos 2\theta\right) - 3\left(\frac{1}{2} - \frac{1}{2}\cos 2\theta\right)$ $= \frac{1}{2} + \frac{7}{2}\cos 2\theta \quad *$ | <p>M1 M1</p> <p>cso A1 (3)</p> |
| | <p>(b)</p> $\int \theta \cos 2\theta \, d\theta = \frac{1}{2}\theta \sin 2\theta - \frac{1}{2} \int \sin 2\theta \, d\theta$ $= \frac{1}{2}\theta \sin 2\theta + \frac{1}{4}\cos 2\theta$ $\int \theta f(\theta) \, d\theta = \frac{1}{4}\theta^2 + \frac{7}{4}\theta \sin 2\theta + \frac{7}{8}\cos 2\theta$ $\left[\dots \right]_0^{\frac{\pi}{2}} = \left[\frac{\pi^2}{16} + 0 - \frac{7}{8} \right] - \left[0 + 0 + \frac{7}{8} \right]$ $= \frac{\pi^2}{16} - \frac{7}{4}$ | <p>M1 A1</p> <p>A1</p> <p>M1 A1</p> <p>M1</p> <p>A1 (7)</p> <p>[10]</p> |

Leave blank

7. The line l_1 has equation $\mathbf{r} = \begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$, where λ is a scalar parameter.

The line l_2 has equation $\mathbf{r} = \begin{pmatrix} 0 \\ 9 \\ -3 \end{pmatrix} + \mu \begin{pmatrix} 5 \\ 0 \\ 2 \end{pmatrix}$, where μ is a scalar parameter.

Given that l_1 and l_2 meet at the point C , find

(a) the coordinates of C . (3)

The point A is the point on l_1 where $\lambda = 0$ and the point B is the point on l_2 where $\mu = -1$.

(b) Find the size of the angle ACB . Give your answer in degrees to 2 decimal places. (4)

(c) Hence, or otherwise, find the area of the triangle ABC . (5)



| Question Number | Scheme | Marks |
|-----------------|--|---|
| 7. | <p>(a) j components $3 + 2\lambda = 9 \Rightarrow \lambda = 3$ $(\mu = 1)$ Leading to $C : (5, 9, -1)$ accept vector forms</p> <p>(b) Choosing correct directions or finding \overrightarrow{AC} and \overrightarrow{BC} $\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 5 \\ 0 \\ 2 \end{pmatrix} = 5 + 2 = \sqrt{6}\sqrt{29} \cos \angle ACB$ use of scalar product $\angle ACB = 57.95^\circ$ awrt 57.95°</p> <p>(c) $A : (2, 3, -4)$ $B : (-5, 9, -5)$ $\overrightarrow{AC} = \begin{pmatrix} 3 \\ 6 \\ 3 \end{pmatrix}, \overrightarrow{BC} = \begin{pmatrix} 10 \\ 0 \\ 4 \end{pmatrix}$ $AC^2 = 3^2 + 6^2 + 3^2 \Rightarrow AC = 3\sqrt{6}$ $BC^2 = 10^2 + 4^2 \Rightarrow BC = 2\sqrt{29}$ $\Delta ABC = \frac{1}{2} AC \times BC \sin \angle ACB$ $= \frac{1}{2} 3\sqrt{6} \times 2\sqrt{29} \sin \angle ACB \approx 33.5$ $15\sqrt{5}$, awrt 34</p> | <p>M1 A1 A1 (3)</p> <p>M1 M1 A1 A1 (4)</p> <p>M1 A1 A1 M1 A1 (5) [12]</p> |
| | <p><i>Alternative method for (b) and (c)</i></p> <p>(b) $A : (2, 3, -4)$ $B : (-5, 9, -5)$ $C : (5, 9, -1)$ $AB^2 = 7^2 + 6^2 + 1^2 = 86$ $AC^2 = 3^2 + 6^2 + 3^2 = 54$ $BC^2 = 10^2 + 0^2 + 4^2 = 116$ Finding all three sides $\cos \angle ACB = \frac{116 + 54 - 86}{2\sqrt{116}\sqrt{54}} (= 0.53066 \dots)$ $\angle ACB = 57.95^\circ$ awrt 57.95°</p> <p>If this method is used some of the working may gain credit in part (c) and appropriate marks may be awarded if there is an attempt at part (c).</p> | <p>M1 M1 A1 A1 (4)</p> |

8.

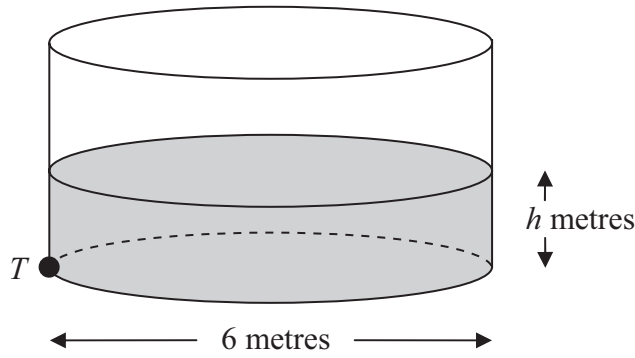


Figure 2

Figure 2 shows a cylindrical water tank. The diameter of a circular cross-section of the tank is 6 m. Water is flowing into the tank at a constant rate of $0.48\pi \text{ m}^3 \text{ min}^{-1}$. At time t minutes, the depth of the water in the tank is h metres. There is a tap at a point T at the bottom of the tank. When the tap is open, water leaves the tank at a rate of $0.6\pi h \text{ m}^3 \text{ min}^{-1}$.

(a) Show that t minutes after the tap has been opened

$$75 \frac{dh}{dt} = (4 - 5h) \tag{5}$$

When $t = 0$, $h = 0.2$

(b) Find the value of t when $h = 0.5$ (6)



| Question Number | Scheme | Marks |
|-----------------|---|--|
| 8. | <p>(a)</p> $\frac{dV}{dt} = 0.48\pi - 0.6\pi h$ $V = 9\pi h \Rightarrow \frac{dV}{dt} = 9\pi \frac{dh}{dt}$ $9\pi \frac{dh}{dt} = 0.48\pi - 0.6\pi h$ <p>Leading to $75 \frac{dh}{dt} = 4 - 5h$ *</p> | <p>M1 A1</p> <p>B1</p> <p>M1</p> <p>cs0 A1 (5)</p> |
| | <p>(b)</p> $\int \frac{75}{4-5h} dh = \int 1 dt$ <p>separating variables</p> $-15 \ln(4-5h) = t (+C)$ $-15 \ln(4-5h) = t + C$ <p>When $t = 0, h = 0.2$</p> $-15 \ln 3 = C$ $t = 15 \ln 3 - 15 \ln(4-5h)$ <p>When $h = 0.5$</p> $t = 15 \ln 3 - 15 \ln 1.5 = 15 \ln \left(\frac{3}{1.5} \right) = 15 \ln 2$ <p>awrt 10.4</p> <p><i>Alternative for last 3 marks</i></p> $t = \left[-15 \ln(4-5h) \right]_{0.2}^{0.5}$ $= -15 \ln 1.5 + 15 \ln 3$ $= 15 \ln \left(\frac{3}{1.5} \right) = 15 \ln 2$ <p>awrt 10.4</p> | <p>M1</p> <p>M1 A1</p> <p>M1</p> <p>M1 A1</p> <p>M1 M1</p> <p>A1 (6)</p> |