

June 2011
Core Mathematics C4 6666
Mark Scheme

Question Number	Scheme	Marks
1.	$9x^2 = A(x-1)(2x+1) + B(2x+1) + C(x-1)^2$ <p> $x \rightarrow 1 \quad 9 = 3B \Rightarrow B = 3$ </p> <p> $x \rightarrow -\frac{1}{2} \quad \frac{9}{4} = \left(-\frac{3}{2}\right)^2 C \Rightarrow C = 1$ </p> <p> x^2 terms $9 = 2A + C \Rightarrow A = 4$ </p> <p><i>Alternatives for finding A.</i></p> <p> x terms $0 = -A + 2B - 2C \Rightarrow A = 4$ Constant terms $0 = -A + B + C \Rightarrow A = 4$ </p>	<p>B1</p> <p>M1</p> <p>Any two of A, B, C A1</p> <p>All three correct A1</p> <p>(4) [4]</p>

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2.	$f(x) = (\dots + \dots)^{-\frac{1}{2}}$ $= 9^{-\frac{1}{2}} (\dots + \dots)^{\dots}$ $(1+kx^2)^n = 1+nkx^2 + \dots$ $(1+kx^2)^{-\frac{1}{2}} = \dots + \frac{(-\frac{1}{2})(-\frac{3}{2})}{2}(kx^2)^2$ $\left(1 + \frac{4}{9}x^2\right)^{-\frac{1}{2}} = 1 - \frac{2}{9}x^2 + \frac{2}{27}x^4$ $f(x) = \frac{1}{3} - \frac{2}{27}x^2 + \frac{2}{81}x^4$	<p>M1</p> <p>B1 $3^{-1}, \frac{1}{3}$ or $\frac{1}{9^{\frac{1}{2}}}$</p> <p>M1 n not a natural number, $k \neq 1$</p> <p>A1 ft ft their $k \neq 1$</p> <p>A1</p> <p>A1 (6) [6]</p>

Question Number	Scheme	Marks
3.	(a) $\frac{dV}{dh} = \frac{1}{2}\pi h - \pi h^2$	or equivalent M1 A1
	At $h = 0.1$, $\frac{dV}{dh} = \frac{1}{2}\pi(0.1) - \pi(0.1)^2 = 0.04\pi$	$\frac{\pi}{25}$ M1 A1 (4)
	(b) $\frac{dh}{dt} = \frac{dV}{dt} \div \frac{dV}{dh} = \frac{\pi}{800} \times \frac{1}{\frac{1}{2}\pi h - \pi h^2}$	or $\frac{\pi}{800} \div$ their (a) M1
	At $h = 0.1$, $\frac{dh}{dt} = \frac{\pi}{800} \times \frac{25}{\pi} = \frac{1}{32}$	awrt 0.031 A1 (2) [6]

4.

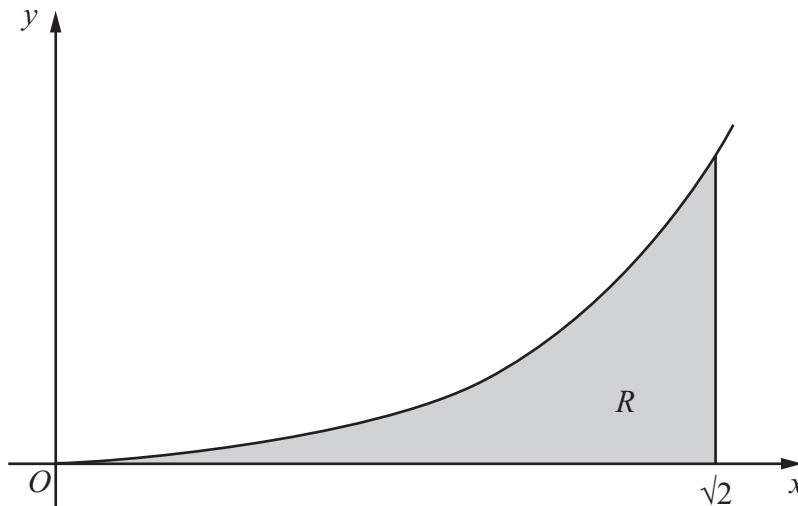


Figure 2

Figure 2 shows a sketch of the curve with equation $y = x^3 \ln(x^2 + 2)$, $x \geq 0$.
The finite region R , shown shaded in Figure 2, is bounded by the curve, the x -axis and the line $x = \sqrt{2}$.

The table below shows corresponding values of x and y for $y = x^3 \ln(x^2 + 2)$.

x	0	$\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{3\sqrt{2}}{4}$	$\sqrt{2}$
y	0		0.3240		3.9210

(a) Complete the table above giving the missing values of y to 4 decimal places. (2)

(b) Use the trapezium rule, with all the values of y in the completed table, to obtain an estimate for the area of R , giving your answer to 2 decimal places. (3)

(c) Use the substitution $u = x^2 + 2$ to show that the area of R is

$$\frac{1}{2} \int_2^4 (u - 2) \ln u \, du \quad (4)$$

(d) Hence, or otherwise, find the exact area of R . (6)



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<p>4.</p>	<p>(a) 0.0333, 1.3596 awrt 0.0333, 1.3596</p>	<p>B1 B1 (2)</p>
	<p>(b) $\text{Area}(R) \approx \frac{1}{2} \times \frac{\sqrt{2}}{4} [\dots]$ $\approx \dots [0 + 2(0.0333 + 0.3240 + 1.3596) + 3.9210]$ ≈ 1.30 Accept</p>	<p>B1 M1 A1 (3)</p>
	<p>(c) $u = x^2 + 2 \Rightarrow \frac{du}{dx} = 2x$ $\text{Area}(R) = \int_0^{\sqrt{2}} x^3 \ln(x^2 + 2) dx$ $\int x^3 \ln(x^2 + 2) dx = \int x^2 \ln(x^2 + 2) x dx = \int (u - 2)(\ln u) \frac{1}{2} du$</p>	<p>B1 B1 M1</p>
	<p>Hence $\text{Area}(R) = \frac{1}{2} \int_2^4 (u - 2) \ln u du$ * cso</p>	<p>A1 (4)</p>
	<p>(d) $\int (u - 2) \ln u du = \left(\frac{u^2}{2} - 2u \right) \ln u - \int \left(\frac{u^2}{2} - 2u \right) \frac{1}{u} du$ $= \left(\frac{u^2}{2} - 2u \right) \ln u - \int \left(\frac{u}{2} - 2 \right) du$ $= \left(\frac{u^2}{2} - 2u \right) \ln u - \left(\frac{u^2}{4} - 2u \right) (+C)$</p>	<p>M1 A1 M1 A1</p>
	<p>$\text{Area}(R) = \frac{1}{2} \left[\left(\frac{u^2}{2} - 2u \right) \ln u - \left(\frac{u^2}{4} - 2u \right) \right]_2^4$ $= \frac{1}{2} [(8 - 8) \ln 4 - 4 + 8 - ((2 - 4) \ln 2 - 1 + 4)]$ $= \frac{1}{2} (2 \ln 2 + 1)$ ln 2 + 1/2</p>	<p>M1 A1 (6) [15]</p>

Question Number	Scheme	Marks	
5.	$\frac{1}{y} \frac{dy}{dx} = \dots$ $\dots = 2 \ln x + 2x \left(\frac{1}{x} \right)$	B1 M1 A1	
	At $x = 2$, leading to	$\ln y = 2(2) \ln 2$ $y = 16$	M1 A1
	At (2, 16)	$\frac{1}{16} \frac{dy}{dx} = 2 \ln 2 + 2$	M1
		$\frac{dy}{dx} = 16(2 + 2 \ln 2)$	A1 (7)
	<i>Alternative</i>		[7]
		$y = e^{2x \ln x}$	B1
		$\frac{d}{dx}(2x \ln x) = 2 \ln x + 2x \left(\frac{1}{x} \right)$	M1 A1
		$\frac{dy}{dx} = \left(2 \ln x + 2x \left(\frac{1}{x} \right) \right) e^{2x \ln x}$	M1 A1
	At $x = 2$,	$\frac{dy}{dx} = (2 \ln 2 + 2) e^{4 \ln 2}$	M1
		$= 16(2 + 2 \ln 2)$	A1 (7)

Question Number	Scheme	Marks
6.	<p>(a) i: $6 - \lambda = -5 + 2\mu$ j: $-3 + 2\lambda = 15 - 3\mu$ leading to $\lambda = 3, \mu = 4$ $\mathbf{r} = \begin{pmatrix} 6 \\ -3 \\ -2 \end{pmatrix} + 3 \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \\ 7 \end{pmatrix}$ or $\mathbf{r} = \begin{pmatrix} -5 \\ 15 \\ 3 \end{pmatrix} + 4 \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \\ 7 \end{pmatrix}$ k: LHS = $-2 + 3(3) = 7$, RHS = $3 + 4(1) = 7$ (As LHS = RHS, lines intersect)</p> <p>Alternatively for B1, showing that $\lambda = 3$ and $\mu = 4$ both give $\begin{pmatrix} 3 \\ 3 \\ 7 \end{pmatrix}$</p> <p>(b) $\begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} = -2 - 6 + 3 = \sqrt{14}\sqrt{14}\cos\theta$ ($\theta \approx 110.92^\circ$) Acute angle is 69.1°</p> <p>(c) $\mathbf{r} = \begin{pmatrix} 6 \\ -3 \\ -2 \end{pmatrix} + 1 \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 5 \\ -1 \\ 1 \end{pmatrix}$ ($\Rightarrow B$ lies on l_1)</p> <p>(d) Let d be shortest distance from B to l_2</p> <div style="display: flex; align-items: center;"> <div style="flex: 1;"> <p>$\overline{AB} = \begin{pmatrix} 5 \\ -1 \\ 1 \end{pmatrix} - \begin{pmatrix} 3 \\ 3 \\ 7 \end{pmatrix} = \begin{pmatrix} 2 \\ -4 \\ -6 \end{pmatrix}$</p> <p>$\overline{AB} = \sqrt{(2)^2 + (-4)^2 + (-6)^2} = \sqrt{56}$</p> <p>$\frac{d}{\sqrt{56}} = \sin\theta$ $d = \sqrt{56}\sin 69.1^\circ \approx 6.99$</p> </div> <div style="flex: 1; text-align: center;"> </div> </div>	<p>Any two equations</p> <p>M1 M1 A1 M1 A1</p> <p>B1 (6)</p> <p>M1 A1</p> <p>A1 (3)</p> <p>B1 (1)</p> <p>M1 A1 M1 A1 (4) [14]</p>

Question Number	Scheme	Marks
7.	(a) $\tan \theta = \sqrt{3}$ or $\sin \theta = \frac{\sqrt{3}}{2}$ $\theta = \frac{\pi}{3}$	M1 awrt 1.05 A1 (2)
	(b) $\frac{dx}{d\theta} = \sec^2 \theta$, $\frac{dy}{d\theta} = \cos \theta$ $\frac{dy}{dx} = \frac{\cos \theta}{\sec^2 \theta} (= \cos^3 \theta)$	M1 A1
	At P, $m = \cos^3 \left(\frac{\pi}{3} \right) = \frac{1}{8}$	Can be implied A1
	Using $mm' = -1$, $m' = -8$ For normal $y - \frac{1}{2}\sqrt{3} = -8(x - \sqrt{3})$	M1 M1
	At Q, $y = 0$ $-\frac{1}{2}\sqrt{3} = -8(x - \sqrt{3})$	
	leading to $x = \frac{17}{16}\sqrt{3}$ ($k = \frac{17}{16}$)	1.0625 A1 (6)
	(c) $\int y^2 dx = \int y^2 \frac{dx}{d\theta} d\theta = \int \sin^2 \theta \sec^2 \theta d\theta$ $= \int \tan^2 \theta d\theta$ $= \int (\sec^2 \theta - 1) d\theta$ $= \tan \theta - \theta (+C)$	M1 A1 A1 M1 A1
	$V = \pi \int_0^{\frac{\pi}{3}} y^2 dx = [\tan \theta - \theta]_0^{\frac{\pi}{3}} = \pi \left[\left(\sqrt{3} - \frac{\pi}{3} \right) - (0 - 0) \right]$	M1
	$= \sqrt{3}\pi - \frac{1}{3}\pi^2$ ($p = 1, q = -\frac{1}{3}$)	A1 (7) [15]

Question Number	Scheme	Marks
8.	<p>(a) $\int (4y+3)^{-\frac{1}{2}} dx = \frac{(4y+3)^{\frac{1}{2}}}{(4)(\frac{1}{2})} (+C)$ $(= \frac{1}{2}(4y+3)^{\frac{1}{2}} + C)$</p> <p>(b) $\int \frac{1}{\sqrt{4y+3}} dy = \int \frac{1}{x^2} dx$ $\int (4y+3)^{-\frac{1}{2}} dy = \int x^{-2} dx$ $\frac{1}{2}(4y+3)^{\frac{1}{2}} = -\frac{1}{x} (+C)$</p> <p>Using $(-2, 1.5)$ $\frac{1}{2}(4 \times 1.5 + 3)^{\frac{1}{2}} = -\frac{1}{-2} + C$ leading to $C = 1$ $\frac{1}{2}(4y+3)^{\frac{1}{2}} = -\frac{1}{x} + 1$ $(4y+3)^{\frac{1}{2}} = 2 - \frac{2}{x}$ $y = \frac{1}{4} \left(2 - \frac{2}{x} \right)^2 - \frac{3}{4}$</p>	<p>M1 A1 (2)</p> <p>B1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1 (6)</p> <p>[8]</p>

or equivalent