Centre No.						Paper Reference							Surname	Initial(s)
Candidate No.						6	6	6	6	/	0	1	Signature	

Paper Reference(s)

6666/01

Edexcel GCE

Core Mathematics C4

Advanced

Thursday 21 June 2012 – Afternoon

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
1	
2	
3	
4	
5	
6	
7	
8	
Total	

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer for each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.
There are 32 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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1. $$\mathrm{f}(x) = \frac{1}{x(3x-1)^2} = \frac{A}{x} + \frac{B}{(3x-1)} + \frac{C}{(3x-1)^2}$$

(a) Find the values of the constants A, B and C.

(4)

(b) (i) Hence find $\int \mathrm{f}(x)\,\mathrm{d}x$.

(ii) Find $\int_1^2 \mathrm{f}(x)\,\mathrm{d}x$, leaving your answer in the form $a + \ln b$, where a and b are constants.

(6)

June 2012
6666 Core Mathematics C4
Mark Scheme

Question Number	Scheme	Marks
1.	(a) $1 = A(3x-1)^2 + Bx(3x-1) + Cx$	B1
	$x \to 0$ $(1 = A)$	M1
	$x \to \frac{1}{3}$ $1 = \frac{1}{3}C \Rightarrow C = 3$ any two constants correct	A1
	Coefficients of x^2	
	$0 = 9A + 3B \Rightarrow B = -3$ all three constants correct	A1 **(4)**
	(b)(i) $\int \left(\frac{1}{x} - \frac{3}{3x-1} + \frac{3}{(3x-1)^2} \right) dx$	
	$= \ln x - \frac{3}{3}\ln(3x-1) + \frac{3}{(-1)3}(3x-1)^{-1} \quad (+C)$	M1 A1ft A1ft
	$\left(= \ln x - \ln(3x-1) - \frac{1}{3x-1} \quad (+C) \right)$	
	(ii) $\int_1^2 f(x)\,dx = \left[\ln x - \ln(3x-1) - \frac{1}{3x-1} \right]_1^2$	
	$= \left(\ln 2 - \ln 5 - \frac{1}{5} \right) - \left(\ln 1 - \ln 2 - \frac{1}{2} \right)$	M1
	$= \ln \frac{2 \times 2}{5} + \ldots$	M1
	$= \frac{3}{10} + \ln\left(\frac{4}{5}\right)$	A1 **(6)**
		[10]

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2.

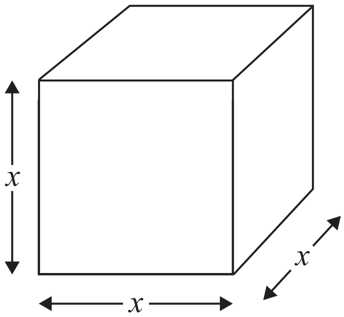

Figure 1

Figure 1 shows a metal cube which is expanding uniformly as it is heated.
At time t seconds, the length of each edge of the cube is x cm,
and the volume of the cube is V cm^3.

(a) Show that $\dfrac{dV}{dx} = 3x^2$ **(1)**

Given that the volume, V cm^3, increases at a constant rate of 0.048 cm^{3}s^{-1},

(b) find $\dfrac{dx}{dt}$, when $x = 8$ **(2)**

(c) find the rate of increase of the total surface area of the cube, in cm^{2}s^{-1}, when $x = 8$ **(3)**

Question Number	Scheme		Marks	
2.	(a) $V = x^3 \Rightarrow \dfrac{\mathrm{d}V}{\mathrm{d}x} = 3x^2$ *	cso	B1	**(1)**
	(b) $\dfrac{\mathrm{d}x}{\mathrm{d}t} = \dfrac{\mathrm{d}x}{\mathrm{d}V} \times \dfrac{\mathrm{d}V}{\mathrm{d}t} = \dfrac{0.048}{3x^2}$		M1	
	At $x = 8$ $\dfrac{\mathrm{d}x}{\mathrm{d}t} = \dfrac{0.048}{3(8^2)} = 0.00025 \; \left(\text{cm s}^{-1}\right)$	2.5×10^{-4}	A1	**(2)**
	(c) $S = 6x^2 \Rightarrow \dfrac{\mathrm{d}S}{\mathrm{d}x} = 12x$		B1	
	$\dfrac{\mathrm{d}S}{\mathrm{d}t} = \dfrac{\mathrm{d}S}{\mathrm{d}x} \times \dfrac{\mathrm{d}x}{\mathrm{d}t} = 12x\left(\dfrac{0.048}{3x^2}\right)$		M1	
	At $x = 8$ $\dfrac{\mathrm{d}S}{\mathrm{d}t} = 0.024 \; \left(\text{cm}^2\,\text{s}^{-1}\right)$		A1	**(3)**
				[6]

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3. $$\mathrm{f}(x) = \frac{6}{\sqrt{(9-4x)}}, \quad |x| < \frac{9}{4}$$

(a) Find the binomial expansion of $\mathrm{f}(x)$ in ascending powers of x, up to and including the term in x^3. Give each coefficient in its simplest form.

(6)

Use your answer to part (a) to find the binomial expansion in ascending powers of x, up to and including the term in x^3, of

(b) $\mathrm{g}(x) = \dfrac{6}{\sqrt{(9+4x)}}, \quad |x| < \dfrac{9}{4}$

(1)

(c) $\mathrm{h}(x) = \dfrac{6}{\sqrt{(9-8x)}}, \quad |x| < \dfrac{9}{8}$

(2)

Question Number	Scheme	Marks
3.	(a) $f(x) = \dots(\dots - \dots x)^{-\frac{1}{2}}$	M1
	$= 6 \times 9^{-\frac{1}{2}}(\dots)$ $\quad \frac{6}{9^{\frac{1}{2}}}, \frac{6}{3}, 2$ or equivalent	B1
	$= \dots\left(1 + \left(-\tfrac{1}{2}\right)(kx); + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2}(kx)^2 + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)}{3!}(kx)^3 + \dots\right)$	M1; A1ft
	$= 2\left(1 + \frac{2}{9}x + \dots\right)$ $\quad$ or $2 + \frac{4}{9}x$	A1
	$= 2 + \frac{4}{9}x + \frac{4}{27}x^2 + \frac{40}{729}x^3 + \dots$	A1 **(6)**
	(b) $g(x) = 2 - \frac{4}{9}x + \frac{4}{27}x^2 - \frac{40}{729}x^3 + \dots$	B1ft **(1)**
	(c) $h(x) = 2 + \frac{4}{9}(2x) + \frac{4}{27}(2x)^2 + \frac{40}{729}(2x)^3 + \dots$	M1 A1 **(2)**
	$\left(= 2 + \frac{8}{9}x + \frac{16}{27}x^2 + \frac{320}{729}x^3 + \dots\right)$	**[9]**

Leave blank

4. Given that $y = 2$ at $x = \frac{\pi}{4}$, solve the differential equation

$$\frac{dy}{dx} = \frac{3}{y\cos^2 x}$$

(5)

Question Number	Scheme		Marks	
4.	$\int y\,\mathrm{d}y = \int \frac{3}{\cos^2 x}\,\mathrm{d}x$ $= \int 3\sec^2 x\,\mathrm{d}x$	Can be implied. Ignore integral signs	B1	
	$\frac{1}{2}y^2 = 3\tan x \quad (+C)$		M1 A1	
	$y = 2,\ x = \frac{\pi}{4}$ $\frac{1}{2}2^2 = 3\tan\frac{\pi}{4} + C$		M1	
	Leading to $C = -1$ $\frac{1}{2}y^2 = 3\tan x - 1$	or equivalent	A1	**(5)**
				[5]

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5. The curve C has equation

$$16y^3 + 9x^2y - 54x = 0$$

(a) Find $\frac{dy}{dx}$ in terms of x and y.

(5)

(b) Find the coordinates of the points on C where $\frac{dy}{dx} = 0$.

(7)

Question Number	Scheme	Marks
5.	(a) Differentiating implicitly to obtain $\pm ay^2\frac{\mathrm{d}y}{\mathrm{d}x}$ and/or $\pm bx^2\frac{\mathrm{d}y}{\mathrm{d}x}$	M1
	$48y^2\frac{\mathrm{d}y}{\mathrm{d}x} + \ \ldots\ -54\ \ldots$	A1
	$9x^2y \rightarrow 9x^2\frac{\mathrm{d}y}{\mathrm{d}x} + 18xy$ or equivalent	B1
	$\left(48y^2+9x^2\right)\frac{\mathrm{d}y}{\mathrm{d}x} + 18xy - 54 = 0$	M1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{54-18xy}{48y^2+9x^2}\quad \left(= \frac{18-6xy}{16y^2+3x^2}\right)$	A1 **(5)**
	(b) $18-6xy=0$	M1
	Using $x = \frac{3}{y}$ or $y = \frac{3}{x}$	
	$16y^3 + 9\left(\frac{3}{y}\right)^2 y - 54\left(\frac{3}{y}\right) = 0$ or $16\left(\frac{3}{x}\right)^3 + 9x^2\left(\frac{3}{x}\right) - 54x = 0$	M1
	Leading to	
	$16y^4 + 81 - 162 = 0$ or $16 + x^4 - 2x^4 = 0$	M1
	$y^4 = \frac{81}{16}$ or $x^4 = 16$	
	$y = \frac{3}{2}, -\frac{3}{2}$ or $x = 2, -2$	A1 A1
	Substituting either of their values into $xy = 3$ to obtain a value of the other variable.	M1
	$\left(2, \frac{3}{2}\right), \left(-2, -\frac{3}{2}\right)$ both	A1 **(7)**
		[12]

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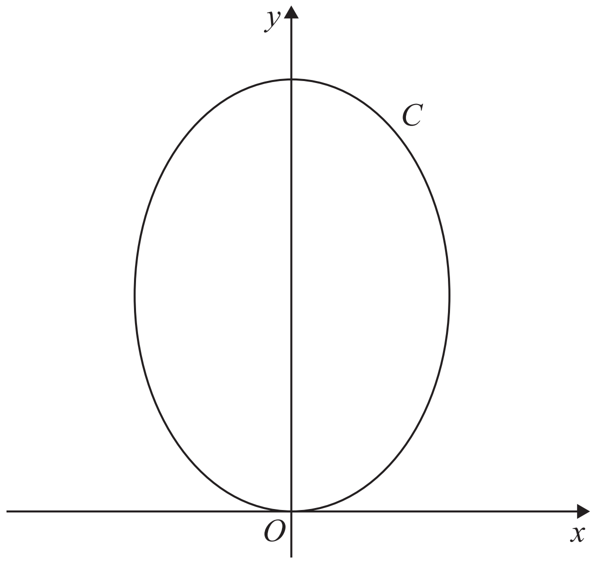

Figure 2

Figure 2 shows a sketch of the curve C with parametric equations

$$x = (\sqrt{3})\sin 2t, \quad y = 4\cos^2 t, \quad 0 \leqslant t \leqslant \pi$$

(a) Show that $\dfrac{dy}{dx} = k(\sqrt{3})\tan 2t$, where k is a constant to be determined.
(5)

(b) Find an equation of the tangent to C at the point where $t = \dfrac{\pi}{3}$.

Give your answer in the form $y = ax + b$, where a and b are constants.
(4)

(c) Find a cartesian equation of C.
(3)

Question Number	Scheme	Marks
6.	(a) $\dfrac{dx}{dt} = 2\sqrt{3}\cos 2t$	B1
	$\dfrac{dy}{dt} = -8\cos t \sin t$	M1 A1
	$\dfrac{dy}{dx} = \dfrac{-8\cos t \sin t}{2\sqrt{3}\cos 2t}$	M1
	$= -\dfrac{4\sin 2t}{2\sqrt{3}\cos 2t}$	
	$\dfrac{dy}{dx} = -\dfrac{2}{3}\sqrt{3}\tan 2t \qquad \left(k = -\dfrac{2}{3}\right)$	A1 **(5)**
	(b) When $t = \dfrac{\pi}{3}$ $\quad x = \dfrac{3}{2},\ y = 1$ can be implied	B1
	$m = -\dfrac{2}{3}\sqrt{3}\tan\left(\dfrac{2\pi}{3}\right) \quad (= 2)$	M1
	$y - 1 = 2\left(x - \dfrac{3}{2}\right)$	M1
	$y = 2x - 2$	A1 **(4)**
	(c) $x = \sqrt{3}\sin 2t = \sqrt{3} \times 2\sin t \cos t$	M1
	$x^2 = 12\sin^2 t\cos^2 t = 12\left(1 - \cos^2 t\right)\cos^2 t$	
	$x^2 = 12\left(1 - \dfrac{y}{4}\right)\dfrac{y}{4}$ or equivalent	M1 A1 **(3)**
		[12]
	Alternative to (c)	
	$y = 2\cos 2t + 2$	M1
	$\sin^2 2t + \cos^2 2t = 1$	
	$\dfrac{x^2}{3} + \dfrac{(y-2)^2}{4} = 1$	M1 A1 **(3)**

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7.

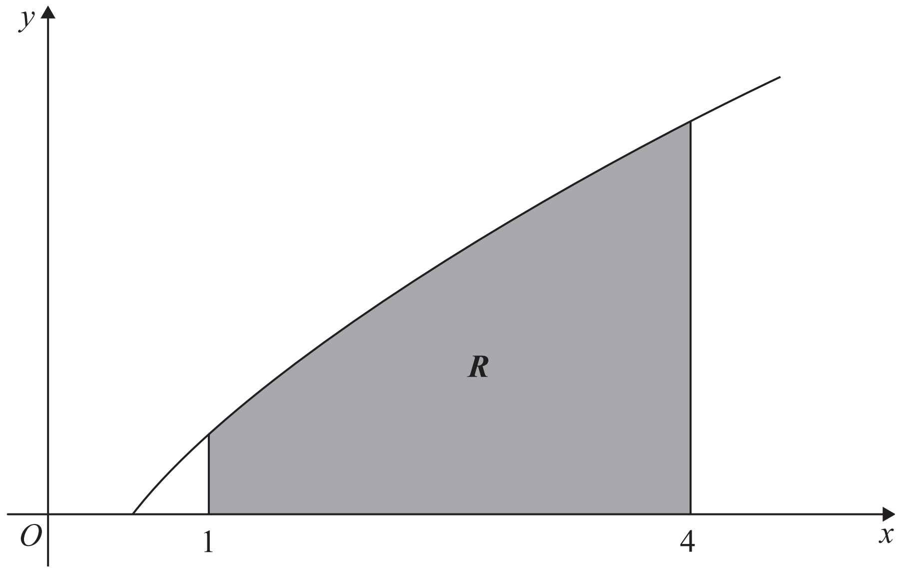

Figure 3

Figure 3 shows a sketch of part of the curve with equation $y = x^{\frac{1}{2}} \ln 2x$.

The finite region R, shown shaded in Figure 3, is bounded by the curve, the x-axis and the lines $x = 1$ and $x = 4$

(a) Use the trapezium rule, with 3 strips of equal width, to find an estimate for the area of R, giving your answer to 2 decimal places.

(4)

(b) Find $\int x^{\frac{1}{2}} \ln 2x \, dx$.

(4)

(c) Hence find the exact area of R, giving your answer in the form $a \ln 2 + b$, where a and b are exact constants.

(3)

Question Number	Scheme	Marks
7.	(a)	
		x	1	2	3	4 | y: ln2, $\sqrt{2}\ln 4$, $\sqrt{3}\ln 6$, 2ln8 0.6931, 1.9605, 3.1034, 4.1589	M1
	Area $=\frac{1}{2}\times 1(\ \ldots)$	B1
	$\approx\ \ldots\ \left(0.6931+2\left(1.9605+3.1034\right)+4.1589\right)$	M1
	$\approx\frac{1}{2}\times 14.97989\ \ldots\ \approx 7.49$ 7.49 cao	A1 **(4)**
	(b) $\int x^{\frac{1}{2}}\ln 2x\,dx=\frac{2}{3}x^{\frac{3}{2}}\ln 2x-\int\frac{2}{3}x^{\frac{3}{2}}\times\frac{1}{x}\,dx$	M1 A1
	$=\frac{2}{3}x^{\frac{3}{2}}\ln 2x-\int\frac{2}{3}x^{\frac{1}{2}}\,dx$	
	$=\frac{2}{3}x^{\frac{3}{2}}\ln 2x-\frac{4}{9}x^{\frac{3}{2}}\quad(+C)$	M1 A1 **(4)**
	(c) $\left[\frac{2}{3}x^{\frac{3}{2}}\ln 2x-\frac{4}{9}x^{\frac{3}{2}}\right]_1^4=\left(\frac{2}{3}4^{\frac{3}{2}}\ln 8-\frac{4}{9}4^{\frac{3}{2}}\right)-\left(\frac{2}{3}\ln 2-\frac{4}{9}\right)$	M1
	$=\left(16\ln 2-\ \ldots\right)-\ \ldots$ Using or implying $\ln 2^n=n\ln 2$	M1
	$=\frac{46}{3}\ln 2-\frac{28}{9}$	A1 **(3)**
		[11]

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8. Relative to a fixed origin O, the point A has position vector $(10\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})$, and the point B has position vector $(8\mathbf{i} + 3\mathbf{j} + 4\mathbf{k})$.

The line l passes through the points A and B.

(a) Find the vector $\overrightarrow{AB}$.

(2)

(b) Find a vector equation for the line l.

(2)

The point C has position vector $(3\mathbf{i} + 12\mathbf{j} + 3\mathbf{k})$.

The point P lies on l. Given that the vector $\overrightarrow{CP}$ is perpendicular to l,

(c) find the position vector of the point P.

(6)

Question Number	Scheme	Marks
8.	(a) $\overrightarrow{AB} = \begin{pmatrix} 8 \\ 3 \\ 4 \end{pmatrix} - \begin{pmatrix} 10 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}$	M1 A1 **(2)**
	(b) $\mathbf{r} = \begin{pmatrix} 10 \\ 2 \\ 3 \end{pmatrix} + t\begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}$ $\quad\quad$ $\mathbf{r} = \begin{pmatrix} 8 \\ 3 \\ 4 \end{pmatrix} + t\begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}$	M1 A1ft **(2)**
	(c) $\overrightarrow{CP} = \begin{pmatrix} 10-2t \\ 2+t \\ 3+t \end{pmatrix} - \begin{pmatrix} 3 \\ 12 \\ 3 \end{pmatrix} = \begin{pmatrix} 7-2t \\ t-10 \\ t \end{pmatrix}$	M1 A1
	$\begin{pmatrix} 7-2t \\ t-10 \\ t \end{pmatrix} . \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix} = -14+4t+t-10+t=0$	M1
	Leading to $\quad t=4$	A1
	Position vector of P is $\begin{pmatrix} 10-8 \\ 2+4 \\ 3+4 \end{pmatrix} = \begin{pmatrix} 2 \\ 6 \\ 7 \end{pmatrix}$	M1 A1 **(6)**
		[10]
	Alternative working for (c)	
	$\overrightarrow{CP} = \begin{pmatrix} 8-2t \\ 3+t \\ 4+t \end{pmatrix} - \begin{pmatrix} 3 \\ 12 \\ 3 \end{pmatrix} = \begin{pmatrix} 5-2t \\ t-9 \\ t+1 \end{pmatrix}$	M1 A1
	$\begin{pmatrix} 5-2t \\ t-9 \\ t+1 \end{pmatrix} . \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix} = -10+4t+t-9+t+1=0$	M1
	Leading to $\quad t=3$	A1
	Position vector of P is $\begin{pmatrix} 8-6 \\ 3+3 \\ 4+3 \end{pmatrix} = \begin{pmatrix} 2 \\ 6 \\ 7 \end{pmatrix}$	M1 A1 **(6)**