





Question Number	Scheme		Marks
<b>1.</b>	$x^3 + 2xy - x - y^3 - 20 = 0$		
(a)	$\left\{ \begin{array}{l} \cancel{dx} \\ \cancel{dx} \end{array} \right\} \times \left\{ \begin{array}{l} 3x^2 \\ \cancel{dx} \end{array} \right\} + \left( \underline{2y + 2x \frac{dy}{dx}} \right) - 1 - 3y^2 \frac{dy}{dx} = 0$ $3x^2 + 2y - 1 + (2x - 3y^2) \frac{dy}{dx} = 0$ $\frac{dy}{dx} = \frac{3x^2 + 2y - 1}{3y^2 - 2x} \quad \text{or} \quad \frac{1 - 3x^2 - 2y}{2x - 3y^2}$	<p>M1 <u>A1</u> <u>B1</u></p> <p>dM1</p> <p>A1 <b>cso</b></p>	[5]
(b)	<p>At <math>P(3, -2)</math>, <math>m(\mathbf{T}) = \frac{dy}{dx} = \frac{3(3)^2 + 2(-2) - 1}{3(-2)^2 - 2(3)} = \frac{22}{6}</math> or <math>\frac{11}{3}</math></p> <p><b>and either</b> <math>\mathbf{T}: y - -2 = \frac{11}{3}(x - 3)</math></p> <p><b>or</b> <math>(-2) = \left(\frac{11}{3}\right)(3) + c \Rightarrow c = \dots</math>,</p>	see notes	M1
	$\mathbf{T}: 11x - 3y - 39 = 0$ or $K(11x - 3y - 39) = 0$	A1 <b>cso</b>	
			[2] 7
	<b>Alternative method for part (a)</b>		
(a)	$\left\{ \begin{array}{l} \cancel{dx} \\ \cancel{dx} \end{array} \right\} \times \left\{ \begin{array}{l} 3x^2 \frac{dx}{dy} \\ \cancel{dx} \end{array} \right\} + \left( \underline{2y \frac{dx}{dy} + 2x} \right) - \frac{dx}{dy} - 3y^2 = 0$ $2x - 3y^2 + (3x^2 + 2y - 1) \frac{dx}{dy} = 0$ $\frac{dy}{dx} = \frac{3x^2 + 2y - 1}{3y^2 - 2x} \quad \text{or} \quad \frac{1 - 3x^2 - 2y}{2x - 3y^2}$	<p>M1 <u>A1</u> <u>B1</u></p> <p>dM1</p> <p>A1 <b>cso</b></p>	[5]
<b>Question 1 Notes</b>			
(a) <b>General</b>	<b>Note</b>	Writing down $\frac{dy}{dx} = \frac{3x^2 + 2y - 1}{3y^2 - 2x}$ or $\frac{1 - 3x^2 - 2y}{2x - 3y^2}$ from <b>no working</b> is full marks.	
	<b>Note</b>	Writing down $\frac{dy}{dx} = \frac{3x^2 + 2y - 1}{2x - 3y^2}$ or $\frac{1 - 3x^2 - 2y}{3y^2 - 2x}$ from <b>no working</b> is M1A0B0M1A0	
	<b>Note</b>	Few candidates will write $3x^2 + 2y + 2x dy - 1 - 3y^2 dy = 0$ leading to $\frac{dy}{dx} = \frac{3x^2 + 2y - 1}{3y^2 - 2x}$ , o.e. This should get full marks.	
<b>1. (a)</b>	<b>M1</b>	Differentiates implicitly to include either $2x \frac{dy}{dx}$ or $-y^3 \rightarrow \pm k y^2 \frac{dy}{dx}$ . (Ignore $\left(\frac{dy}{dx} = \right)$ ).	
	<b>A1</b>	$x^3 \rightarrow 3x^2$ <b>and</b> $-x - y^3 - 20 = 0 \rightarrow -1 - 3y^2 \frac{dy}{dx} = 0$	
	<b>B1</b>	$2xy \rightarrow 2y + 2x \frac{dy}{dx}$	
	<b>Note</b>	If an extra term appears then award 1 <sup>st</sup> A0.	

<p>1. (a) ctd</p>	<p><b>Note</b>  <b>dM1</b>          <b>Note</b>  <b>A1</b></p>	<p><math>3x^2 + 2y + 2x \frac{dy}{dx} - 1 - 3y^2 \frac{dy}{dx} \rightarrow 3x^2 + 2y - 1 = 3y^2 \frac{dy}{dx} - 2x \frac{dy}{dx}</math>                  will get 1<sup>st</sup> A1 (implied) as the "= 0" can be implied by rearrangement of their equation.  <b>dependent on the first method mark being awarded.</b>                  An attempt to factorise out <b>all the terms in</b> <math>\frac{dy}{dx}</math> as long as there are <b>at least two terms</b> in <math>\frac{dy}{dx}</math>.                  ie. <math>\dots + (2x - 3y^2) \frac{dy}{dx} = \dots</math>                  Placing an extra <math>\frac{dy}{dx}</math> at the beginning and then including it in their factorisation is fine for dM1.                  For <math>\frac{1 - 2y - 3x^2}{2x - 3y^2}</math> or equivalent. Eg: <math>\frac{3x^2 + 2y - 1}{3y^2 - 2x}</math>  <b>cs0:</b> If the candidate's solution is not completely correct, then do not give this mark.  <b>isw:</b> You can, however, ignore subsequent working following on from correct solution.</p>
<p>1. (b)</p>	<p><b>M1</b>          <b>Note</b>  <b>A1</b>  <b>cs0</b>  <b>isw</b></p>	<p><b>Some</b> attempt to substitute <b>both</b> <math>x = 3</math> <b>and</b> <math>y = -2</math> into their <math>\frac{dy}{dx}</math> which contains both <math>x</math> and <math>y</math> to find <math>m_T</math> <b>and</b></p> <ul style="list-style-type: none"> <li><b>either</b> applies <math>y - -2 = (\text{their } m_T)(x - 3)</math>, where <math>m_T</math> is a numerical value.</li> <li><b>or</b> finds <math>c</math> by solving <math>(-2) = (\text{their } m_T)(3) + c</math>, where <math>m_T</math> is a numerical value.</li> </ul> <p>Using a changed gradient (i.e. applying <math>\frac{-1}{\text{their } \frac{dy}{dx}}</math> or <math>\frac{1}{\text{their } \frac{dy}{dx}}</math> is M0).                  Accept any integer multiple of <math>11x - 3y - 39 = 0</math> or <math>11x - 39 - 3y = 0</math> or <math>-11x + 3y + 39 = 0</math>, where their tangent equation is equal to 0.                  A correct solution is required from a correct <math>\frac{dy}{dx}</math>.                  You can ignore subsequent working following a correct solution.</p>
<p>1. (a)</p>	<p><b>M1</b>          <b>A1</b>  <b>B1</b>  <b>dM1</b>          <b>A1</b></p>	<p><b>Alternative method for part (a): Differentiating with respect to y</b></p> <p>Differentiates implicitly to include either <math>2y \frac{dx}{dy}</math> or <math>x^3 \rightarrow \pm kx^2 \frac{dx}{dy}</math> or <math>-x \rightarrow -\frac{dx}{dy}</math> (Ignore <math>\left(\frac{dx}{dy} = \right)</math>).</p> <p><math>x^3 \rightarrow 3x^2 \frac{dx}{dy}</math> <b>and</b> <math>-x - y^3 - 20 = 0 \rightarrow -\frac{dx}{dy} - 3y^2 = 0</math></p> <p><math>2xy \rightarrow 2y \frac{dx}{dy} + 2x</math></p> <p><b>dependent on the first method mark being awarded.</b>                  An attempt to factorise out <b>all the terms in</b> <math>\frac{dx}{dy}</math> as long as there are <b>at least two terms</b> in <math>\frac{dx}{dy}</math>.                  For <math>\frac{1 - 2y - 3x^2}{2x - 3y^2}</math> or equivalent. Eg: <math>\frac{3x^2 + 2y - 1}{3y^2 - 2x}</math>  <b>cs0:</b> If the candidate's solution is not completely correct, then do not give this mark.</p>

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2. Given that the binomial expansion of  $(1 + kx)^{-4}$ ,  $|kx| < 1$ , is

$$1 - 6x + Ax^2 + \dots$$

(a) find the value of the constant  $k$ ,

(2)

(b) find the value of the constant  $A$ , giving your answer in its simplest form.

(3)

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Question Number	Scheme	Marks
2.	$\left\{ (1 + kx)^{-4} = 1 + (-4)(kx) + \frac{(-4)(-4-1)}{2!}(kx)^2 + \dots \right\}$	
(a)	<p><b>Either</b> <math>(-4)k = -6</math> <b>or</b> <math>(1 + kx)^{-4} = 1 + (-4)(kx)</math> <b>see notes</b></p> <p>leading to <math>k = \frac{3}{2}</math> <span style="margin-left: 200px;"><math>k = \frac{3}{2}</math> or 1.5 or <math>\frac{6}{4}</math></span></p>	M1 A1 <b>[2]</b>
(b)	<p><math>\frac{(-4)(-5)}{2}(k)^2</math> <b>Either</b> <math>\frac{(-4)(-5)}{2!}</math> <b>or</b> <math>(k)^2</math> <b>or</b> <math>(kx)^2</math> M1</p> <p><b>Either</b> <math>\frac{(-4)(-5)}{2!}(k)^2</math> <b>or</b> <math>\frac{(-4)(-5)}{2!}(kx)^2</math> M1</p> <p><math>\left\{ A = \frac{(-4)(-5)}{2!} \left( \frac{3}{2} \right)^2 \right\} \Rightarrow A = \frac{45}{2}</math> <span style="margin-left: 200px;"><math>\frac{45}{2}</math> or 22.5</span> A1</p>	M1 M1 A1 <b>[3]</b> <b>5</b>

**Question 2 Notes**

**Note** In this question ignore part labelling and mark part (a) and part (b) together.

**Note** Writing down  $\left\{ (1 + kx)^{-4} \right\} = 1 + (-4)(kx) + \frac{(-4)(-4-1)}{2!}(kx)^2 + \dots$  gets all the method marks in Q2. i.e. (a) M1 and (b) M1M1

(a) **M1** Award M1 for

- **either** writing down  $(-4)k = -6$  or  $4k = 6$
- **or** expanding  $(1 + kx)^{-4}$  to give  $1 + (-4)(kx)$
- **or** writing down  $(-4)kx = -6$  or  $(-4k) = -6x$  or  $-4kx = -6x$

**A1**  $k = \frac{3}{2}$  or 1.5 or  $\frac{6}{4}$  **from no incorrect sign errors.**

**Note** The M1 mark can be implied by a candidate writing down the correct value of  $k$ .

**Note** Award M1 for writing down  $4k = 6$  and then A1 for  $k = 1.5$  (or equivalent).

**Note** Award M0 for  $4k = -6$  (if there is no evidence that  $(1 + kx)^{-4}$  expands to give  $1 + (-4)(kx) + \dots$ )

**Note**  $1 + (-4)(kx)$  leading to  $(-4)k = 6$  leading to  $k = \frac{3}{2}$  is M1A0.

(b) **M1** For **either**  $\frac{(-4)(-4-1)}{2!}$  **or**  $\frac{(-4)(-5)}{2!}$  **or** 10 **or**  $(k)^2$  **or**  $(kx)^2$

**M1** **Either**  $\frac{(-4)(-4-1)}{2!}(k)^2$  **or**  $\frac{(-4)(-5)}{2!}(k)^2$  **or**  $\frac{(-4)(-5)}{2!}(kx)^2$  **or**  $\frac{(-4)(-5)}{2!}(\text{their } k)^2$  **or**  $10k^2$

**Note** Candidates are allowed to use 2 instead of 2!

**A1** Uses  $k = 1.5$  to give  $A = \frac{45}{2}$  or 22.5

**Note**  $A = \frac{90}{4}$  which has not been simplified is A0.

**Note** Award A0 for  $A = \frac{45}{2}x^2$ .

**Note** Allow A1 for  $A = \frac{45}{2}x^2$  followed by  $A = \frac{45}{2}$

**Note**  $k = -1.5$  leading to  $A = \frac{45}{2}$  or 22.5 is A0.

3.

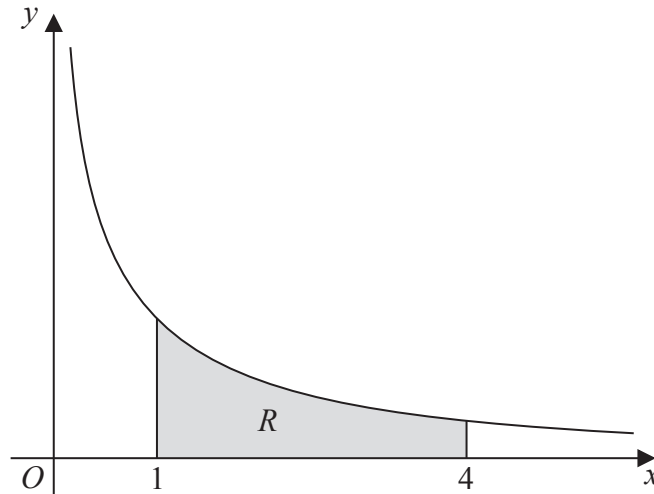


Figure 1

Figure 1 shows a sketch of part of the curve with equation  $y = \frac{10}{2x + 5\sqrt{x}}$ ,  $x > 0$

The finite region  $R$ , shown shaded in Figure 1, is bounded by the curve, the  $x$ -axis, and the lines with equations  $x = 1$  and  $x = 4$

The table below shows corresponding values of  $x$  and  $y$  for  $y = \frac{10}{2x + 5\sqrt{x}}$

$x$	1	2	3	4
$y$	1.42857	0.90326		0.55556

- (a) Complete the table above by giving the missing value of  $y$  to 5 decimal places. (1)
- (b) Use the trapezium rule, with all the values of  $y$  in the completed table, to find an estimate for the area of  $R$ , giving your answer to 4 decimal places. (3)
- (c) By reference to the curve in Figure 1, state, giving a reason, whether your estimate in part (b) is an overestimate or an underestimate for the area of  $R$ . (1)
- (d) Use the substitution  $u = \sqrt{x}$ , or otherwise, to find the exact value of

$$\int_1^4 \frac{10}{2x + 5\sqrt{x}} dx$$
(6)

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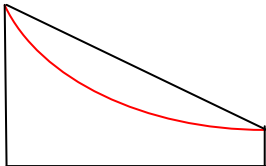


Question Number	Scheme					Marks	
3.	$\frac{x}{y}$	1	2	3	4	$y = \frac{10}{2x + 5\sqrt{x}}$	
(a)	{At $x = 3,$ } $y = 0.68212$ (5 dp)					0.68212	B1 cao [1]
(b)	$\frac{1}{2} \times 1 \times [1.42857 + 0.55556 + 2(0.90326 + \text{their } 0.68212)]$					Outside brackets $\frac{1}{2} \times 1$ or $\frac{1}{2}$ For structure of [.....]	B1 aef M1
(c)	<ul style="list-style-type: none"> <li>Overestimate</li> </ul> and a reason such as <ul style="list-style-type: none"> <li>{top of} <u>trapezia lie above the curve</u></li> <li>a diagram which gives reference to the extra area</li> <li>concave or convex</li> <li><math>\frac{d^2y}{dx^2} &gt; 0</math> (can be implied)</li> <li>bends inwards</li> <li>curves downwards</li> </ul>					anything that rounds to 2.5774	A1 [3]
(d)	$\{u = \sqrt{x} \Rightarrow \frac{du}{dx} = \frac{1}{2}x^{-\frac{1}{2}} \text{ or } \frac{dx}{du} = 2u$						B1
	$\int \frac{10}{2u^2 + 5u} \cdot 2u \, du$					Either $\left\{ \int \frac{\pm k u}{\alpha u^2 \pm \beta u} \{du\} \right\}$ or $\left\{ \int \frac{\pm k}{u(\alpha u^2 \pm \beta u)} \{du\} \right\}$	M1
	$\left\{ = \int \frac{20}{2u + 5} \, du \right\} = \frac{20}{2} \ln(2u + 5)$					$\pm \lambda \ln(2u + 5)$ or $\pm \lambda \ln\left(u + \frac{5}{2}\right), \lambda \neq 0$ <b>with no other terms.</b>	M1
	$\left\{ \left[ \frac{20}{2} \ln(2u + 5) \right]_1^2 \right\} = 10 \ln(2(2) + 5) - 10 \ln(2(1) + 5)$					$\frac{20}{2u + 5} \rightarrow \frac{20}{2} \ln(2u + 5)$ or $10 \ln\left(u + \frac{5}{2}\right)$	A1 cso
	$10 \ln 9 - 10 \ln 7$ or $10 \ln\left(\frac{9}{7}\right)$ or $20 \ln 3 - 10 \ln 7$					Substitutes limits of 2 and 1 in $u$ (or 4 and 1 in $x$ ) and subtracts the correct way round.	M1
							A1 oe cso
							[6] 11

Question 3 Notes

3. (a)	<b>B1</b>	0.68212 correct answer only. Look for this on the table or in the candidate's working.
(b)	<b>B1</b>	Outside brackets $\frac{1}{2} \times 1$ or $\frac{1}{2}$ or equivalent.
	<b>M1</b>	For structure of trapezium rule [ ..... ]
	<b>Note</b>	No errors are allowed [eg. an omission of a y-ordinate or an extra y-ordinate or a repeated y ordinate].
	<b>A1</b>	anything that rounds to 2.5774
	<b>Note</b>	Working must be seen to demonstrate the use of the trapezium rule. (Actual area is 2.51314428...)



<p>3. (b) contd</p>	<p><b>Note</b> Award B1M1A1 for <math>\frac{1}{2}(1.42857 + 0.55556) + (0.90326 + \text{their } 0.68212) = 2.577445</math></p> <p><b>Bracketing mistake:</b> Unless the final answer implies that the calculation has been done correctly award B1M0A0 for <math>\frac{1}{2} \times 1 + 1.42857 + 2(0.90326 + \text{their } 0.68212) + 0.55556</math> (nb: answer of 5.65489).</p> <p>award B1M0A0 for <math>\frac{1}{2} \times 1 (1.42857 + 0.55556) + 2(0.90326 + \text{their } 0.68212)</math> (nb: answer of 4.162825).</p> <p><b>Alternative method: Adding individual trapezia</b></p> $\text{Area} \approx 1 \times \left[ \frac{1.42857 + 0.90326}{2} + \frac{0.90326 + "0.68212"}{2} + \frac{"0.68212" + 0.55556}{2} \right] = 2.577445$ <p><b>B1</b> B1: 1 and a divisor of 2 on all terms inside brackets.</p> <p><b>M1</b> M1: First and last ordinates once and two of the middle ordinates twice inside brackets ignoring the 2.</p> <p><b>A1</b> A1: anything that rounds to 2.5774</p>
<p>(c)</p>	<p><b>B1</b> Overestimate <b>and</b> either trapezia lie above curve <b>or</b> a diagram that gives reference to the extra area</p> <p><b>eg.</b> This diagram is sufficient. It must show the top of a trapezium lying above the curve.</p>  <p><b>or</b> concave or convex <b>or</b> <math>\frac{d^2y}{dx^2} &gt; 0</math> (can be implied) <b>or</b> bends inwards <b>or</b> curves downwards.</p> <p><b>Note</b> Reason of "gradient is negative" by itself is B0.</p>
<p>(d)</p>	<p><b>B1</b> <math>\frac{du}{dx} = \frac{1}{2}x^{-\frac{1}{2}}</math> or <math>du = \frac{1}{2\sqrt{x}} dx</math> or <math>2\sqrt{x} du = dx</math> or <math>dx = 2u du</math> or <math>\frac{dx}{du} = 2u</math> o.e.</p> <p><b>M1</b> Applying the substitution and achieving <math>\left\{ \int \right\} \frac{\pm ku}{\alpha u^2 \pm \beta u} \{du\}</math> or <math>\left\{ \int \right\} \frac{\pm k}{u(\alpha u^2 \pm \beta u)} \{du\}</math>, <math>k, \alpha, \beta \neq 0</math>. Integral sign and <math>du</math> not required for this mark.</p> <p><b>M1</b> Cancelling <math>u</math> and integrates to achieve <math>\pm \lambda \ln(2u + 5)</math> or <math>\pm \lambda \ln\left(u + \frac{5}{2}\right)</math>, <math>\lambda \neq 0</math> <b>with no other terms</b>.</p> <p><b>A1</b> <b>cso.</b> Integrates <math>\frac{20}{2u + 5}</math> to give <math>\frac{20}{2} \ln(2u + 5)</math> or <math>10 \ln\left(u + \frac{5}{2}\right)</math>, un-simplified or simplified.</p> <p><b>Note</b> BE CAREFUL! Candidates must be integrating <math>\frac{20}{2u + 5}</math> or equivalent.</p> <p>So <math>\int \frac{10}{2u + 5} du = 10 \ln(2u + 5)</math> WOULD BE A0 and final A0.</p> <p><b>M1</b> Applies limits of 2 and 1 in <math>u</math> or 4 and 1 in <math>x</math> in their (i.e. any) changed function and subtracts the correct way round.</p> <p><b>A1</b> Exact answers of either <math>10 \ln 9 - 10 \ln 7</math> or <math>10 \ln\left(\frac{9}{7}\right)</math> or <math>20 \ln 3 - 10 \ln 7</math> or <math>20 \ln\left(\frac{3}{\sqrt{7}}\right)</math> or <math>\ln\left(\frac{9^{10}}{7^{10}}\right)</math> or equivalent. <b>Correct solution only.</b></p> <p><b>Note</b> You can ignore subsequent working which follows from a correct answer.</p> <p><b>Note</b> A decimal answer of 2.513144283... (without a correct <b>exact</b> answer) is A0.</p>



Question Number	Scheme	Marks
4.	$\frac{dV}{dt} = 80\pi$ , $V = 4\pi h(h + 4) = 4\pi h^2 + 16\pi h$ , $\frac{dV}{dh} = 8\pi h + 16\pi$	$\pm \alpha h \pm \beta$ , $\alpha \neq 0$ , $\beta \neq 0$ $8\pi h + 16\pi$ M1 A1
	$\left\{ \frac{dV}{dh} \times \frac{dh}{dt} = \frac{dV}{dt} \Rightarrow \right\} (8\pi h + 16\pi) \frac{dh}{dt} = 80\pi$ $\left\{ \frac{dh}{dt} = \frac{dV}{dt} \div \frac{dV}{dh} \Rightarrow \right\} \frac{dh}{dt} = 80\pi \times \frac{1}{8\pi h + 16\pi}$	$\left( \text{Candidate's } \frac{dV}{dh} \right) \times \frac{dh}{dt} = 80\pi$ or $80\pi \div \text{Candidate's } \frac{dV}{dh}$ M1 oe
	When $h = 6$ , $\left\{ \frac{dh}{dt} = \right\} \frac{1}{8\pi(6) + 16\pi} \times 80\pi \left\{ = \frac{80\pi}{64\pi} \right\}$ $\frac{dh}{dt} = \underline{1.25}$ (cms <sup>-1</sup> )	<b>dependent on the previous M1 see notes</b> $1.25$ or $\frac{5}{4}$ or $\frac{10}{8}$ or $\frac{80}{64}$ dM1 A1 oe
<b>[5]</b>		
<b>Alternative Method for the first M1A1</b> Product rule: $\begin{cases} u = 4\pi h & v = h + 4 \\ \frac{du}{dh} = 4\pi & \frac{dv}{dh} = 1 \end{cases}$ $\frac{dV}{dh} = 4\pi(h + 4) + 4\pi h$		
<b>Question 4 Notes</b>		
<b>M1</b>	An expression of the form $\pm \alpha h \pm \beta$ , $\alpha \neq 0$ , $\beta \neq 0$ . <b>Can be simplified or un-simplified.</b>	
<b>A1</b>	Correct simplified or un-simplified differentiation of $V$ . eg. $8\pi h + 16\pi$ or $4\pi(h + 4) + 4\pi h$ or $8\pi(h + 2)$ or equivalent.	
<b>Note</b>	Some candidates will use the product rule to differentiate $V$ with respect to $h$ . <b>(See Alt Method 1).</b>	
<b>Note</b>	$\frac{dV}{dh}$ does not have to be explicitly stated, but it should be clear that they are differentiating their $V$ .	
<b>M1</b>	$\left( \text{Candidate's } \frac{dV}{dh} \right) \times \frac{dh}{dt} = 80\pi$ or $80\pi \div \text{Candidate's } \frac{dV}{dh}$	
<b>Note</b>	Also allow 2 <sup>nd</sup> M1 for $\left( \text{Candidate's } \frac{dV}{dh} \right) \times \frac{dh}{dt} = \mathbf{80}$ or $\mathbf{80} \div \text{Candidate's } \frac{dV}{dh}$	
<b>Note</b>	Give 2 <sup>nd</sup> M0 for $\left( \text{Candidate's } \frac{dV}{dh} \right) \times \frac{dh}{dt} = \mathbf{80\pi t}$ or $\mathbf{80k}$ or $\mathbf{80\pi t}$ or $\mathbf{80k} \div \text{Candidate's } \frac{dV}{dh}$	
<b>dM1</b>	<b>which is dependent on the previous M1 mark.</b> Substitutes $h = 6$ into an expression which is a result of a quotient of their $\frac{dV}{dh}$ and $80\pi$ (or 80)	
<b>A1</b>	$1.25$ or $\frac{5}{4}$ or $\frac{10}{8}$ or $\frac{80}{64}$ (units are not required).	
<b>Note</b>	$\frac{80\pi}{64\pi}$ as a final answer is A0.	
<b>Note</b>	Substituting $h = 6$ into a correct $\frac{dV}{dh}$ gives $64\pi$ but the final M1 mark can only be awarded if this is used as a quotient with $80\pi$ (or 80)	

5.

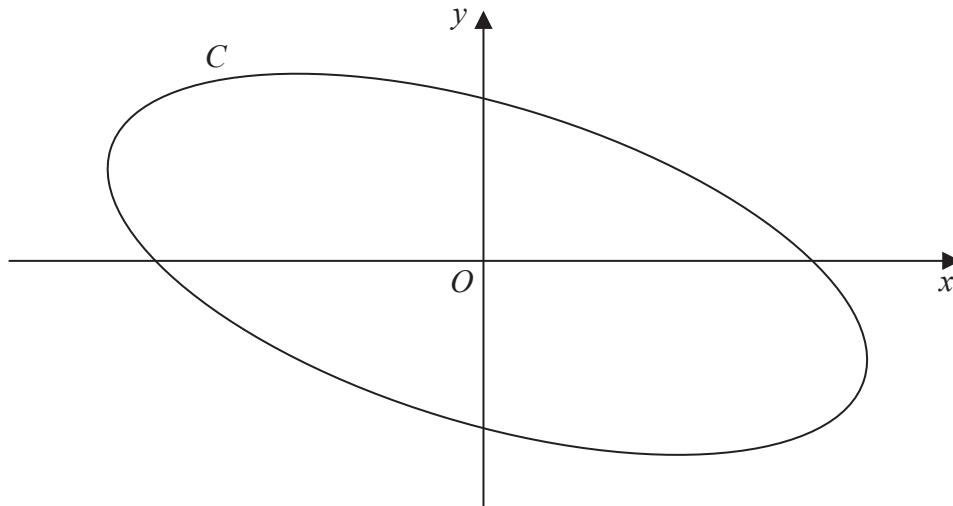


Figure 3

Figure 3 shows a sketch of the curve  $C$  with parametric equations

$$x = 4\cos\left(t + \frac{\pi}{6}\right), \quad y = 2\sin t, \quad 0 \leq t < 2\pi$$

(a) Show that

$$x + y = 2\sqrt{3} \cos t \tag{3}$$

(b) Show that a cartesian equation of  $C$  is

$$(x + y)^2 + ay^2 = b$$

where  $a$  and  $b$  are integers to be determined.

(2)

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Question Number	Scheme	Marks
5.	$x = 4 \cos\left(t + \frac{\pi}{6}\right), \quad y = 2 \sin t$	
(a)	<p><b>Main Scheme</b></p> $x = 4 \left( \cos t \cos\left(\frac{\pi}{6}\right) - \sin t \sin\left(\frac{\pi}{6}\right) \right)$ $\cos\left(t + \frac{\pi}{6}\right) \rightarrow \cos t \cos\left(\frac{\pi}{6}\right) \pm \sin t \sin\left(\frac{\pi}{6}\right)$ <p>So, <math>\{x + y\} = 4 \left( \cos t \cos\left(\frac{\pi}{6}\right) - \sin t \sin\left(\frac{\pi}{6}\right) \right) + 2 \sin t</math></p> <p style="text-align: right;">Adds their expanded <math>x</math> (which is in terms of <math>t</math>) to <math>2 \sin t</math></p> $= 4 \left( \left(\frac{\sqrt{3}}{2}\right) \cos t - \left(\frac{1}{2}\right) \sin t \right) + 2 \sin t$ $= 2\sqrt{3} \cos t \quad *$ <p style="text-align: right;">Correct proof</p>	<p>M1 oe</p> <p>dM1</p> <p>A1 *</p> <p>[3]</p>
(a)	<p><b>Alternative Method 1</b></p> $x = 4 \left( \cos t \cos\left(\frac{\pi}{6}\right) - \sin t \sin\left(\frac{\pi}{6}\right) \right)$ $= 4 \left( \left(\frac{\sqrt{3}}{2}\right) \cos t - \left(\frac{1}{2}\right) \sin t \right) = 2\sqrt{3} \cos t - 2 \sin t$ $\cos\left(t + \frac{\pi}{6}\right) \rightarrow \cos t \cos\left(\frac{\pi}{6}\right) \pm \sin t \sin\left(\frac{\pi}{6}\right)$ <p>So, <math>x = 2\sqrt{3} \cos t - y</math></p> <p style="text-align: right;">Forms an equation in <math>x, y</math> and <math>t</math>.</p> $x + y = 2\sqrt{3} \cos t \quad *$ <p style="text-align: right;">Correct proof</p>	<p>M1 oe</p> <p>dM1</p> <p>A1 *</p> <p>[3]</p>
(b)	<p><b>Main Scheme</b></p> $\left(\frac{x+y}{2\sqrt{3}}\right)^2 + \left(\frac{y}{2}\right)^2 = 1$ $\Rightarrow \frac{(x+y)^2}{12} + \frac{y^2}{4} = 1$ $\Rightarrow (x+y)^2 + 3y^2 = 12$ <p style="text-align: right;">Applies <math>\cos^2 t + \sin^2 t = 1</math> to achieve an equation containing <b>only</b> <math>x</math>'s and <math>y</math>'s.</p> $(x+y)^2 + 3y^2 = 12$ <p style="text-align: right;"><math>\{a = 3, b = 12\}</math></p>	<p>M1</p> <p>A1</p> <p>[2]</p>
(b)	<p><b>Alternative Method 1</b></p> $(x+y)^2 = 12 \cos^2 t = 12(1 - \sin^2 t) = 12 - 12 \sin^2 t$ <p>So, <math>(x+y)^2 = 12 - 3y^2</math></p> $\Rightarrow (x+y)^2 + 3y^2 = 12$ <p style="text-align: right;">Applies <math>\cos^2 t + \sin^2 t = 1</math> to achieve an equation containing <b>only</b> <math>x</math>'s and <math>y</math>'s.</p> $(x+y)^2 + 3y^2 = 12$	<p>M1</p> <p>A1</p> <p>[2]</p>
(b)	<p><b>Alternative Method 2</b></p> $(x+y)^2 = 12 \cos^2 t$ <p>As <math>12 \cos^2 t + 12 \sin^2 t = 12</math></p> <p>then <math>(x+y)^2 + 3y^2 = 12</math></p>	<p>M1, A1</p> <p>[2]</p>
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Question 5 Notes		
5. (a)	M1	$\cos\left(t + \frac{\pi}{6}\right) \rightarrow \cos t \cos\left(\frac{\pi}{6}\right) \pm \sin t \sin\left(\frac{\pi}{6}\right) \quad \text{or} \quad \cos\left(t + \frac{\pi}{6}\right) \rightarrow \left(\frac{\sqrt{3}}{2}\right)\cos t \pm \left(\frac{1}{2}\right)\sin t$
	Note	<p>If a candidate states <math>\cos(A + B) = \cos A \cos B \pm \sin A \sin B</math>, but there is an error <i>in its application</i> then give M1.</p> <p><b><u>Awarding the dM1 mark which is dependent on the first method mark</u></b></p>
Main	dM1	Adds their expanded $x$ (which is in terms of $t$ ) to $2 \sin t$
	Note	Writing $x + y = \dots$ is not needed in the <b>Main Scheme</b> method.
Alt 1	dM1	Forms an equation in $x$ , $y$ and $t$ .
	A1*	Evidence of $\cos\left(\frac{\pi}{6}\right)$ and $\sin\left(\frac{\pi}{6}\right)$ evaluated and the proof is correct with no errors.
	Note	$\{x + y\} = 4 \cos\left(t + \frac{\pi}{6}\right) + 2 \sin t$ , by itself is M0M0A0.
(b)	M1	Applies $\cos^2 t + \sin^2 t = 1$ to achieve an equation containing <b>only</b> $x$ 's and $y$ 's.
	A1	leading $(x + y)^2 + 3y^2 = 12$
	SC	<p>Award <b>Special Case B1B0</b> for a candidate who writes down <b>either</b></p> <ul style="list-style-type: none"> <li>• <math>(x + y)^2 + 3y^2 = 12</math> from no working</li> <li>• <math>a = 3, b = 12</math>, but <b><u>does not provide a correct proof.</u></b></li> </ul>
	Note	Alternative method 2 is fine for M1 A1
	Note	Writing $(x + y)^2 = 12 \cos^2 t$ followed by $12 \cos^2 t + a(4 \sin^2 t) = b \Rightarrow a = 3, b = 12$ is SC: B1B0
	Note	<p>Writing <math>(x + y)^2 = 12 \cos^2 t</math> followed by <math>12 \cos^2 t + a(4 \sin^2 t) = b</math></p> <ul style="list-style-type: none"> <li>• states <math>a = 3, b = 12</math></li> <li>• and refers to either <math>\cos^2 t + \sin^2 t = 1</math> or <math>12 \cos^2 t + 12 \sin^2 t = 12</math></li> <li>• and there is no incorrect working</li> </ul> <p>would get M1A1</p>

6. (i) Find

$$\int x e^{4x} dx$$

**(3)**

(ii) Find

$$\int \frac{8}{(2x - 1)^3} dx, \quad x > \frac{1}{2}$$

**(2)**

(iii) Given that  $y = \frac{\pi}{6}$  at  $x = 0$ , solve the differential equation

$$\frac{dy}{dx} = e^x \operatorname{cosec} 2y \operatorname{cosec} y$$

**(7)**

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Question Number	Scheme	Marks
6. (i)	$\int xe^{4x} dx = \frac{1}{4}xe^{4x} - \int \frac{1}{4}e^{4x} \{dx\}$ $= \frac{1}{4}xe^{4x} - \frac{1}{16}e^{4x} \{+ c\}$	<p><math>\pm \alpha xe^{4x} - \int \beta e^{4x} \{dx\}, \alpha \neq 0, \beta &gt; 0</math> M1</p> <p><math>\frac{1}{4}xe^{4x} - \int \frac{1}{4}e^{4x} \{dx\}</math> A1</p> <p><math>\frac{1}{4}xe^{4x} - \frac{1}{16}e^{4x}</math> A1</p> <p>[3]</p>
(ii)	$\int \frac{8}{(2x-1)^3} dx = \frac{8(2x-1)^{-2}}{(2)(-2)} \{+ c\}$ $\{= -2(2x-1)^{-2} \{+ c\}\}$	<p><math>\pm \lambda(2x-1)^{-2}</math> M1</p> <p><math>\frac{8(2x-1)^{-2}}{(2)(-2)}</math> or equivalent. A1</p> <p>{Ignore subsequent working}. [2]</p>
(iii)	$\frac{dy}{dx} = e^x \operatorname{cosec} 2y \operatorname{cosec} y \quad y = \frac{\pi}{6} \text{ at } x = 0$	
	<p><b>Main Scheme</b></p> $\int \frac{1}{\operatorname{cosec} 2y \operatorname{cosec} y} dy = \int e^x dx \quad \text{or} \quad \int \sin 2y \sin y dy = \int e^x dx$ $\int 2 \sin y \cos y \sin y dy = \int e^x dx$ $\frac{2}{3} \sin^3 y = e^x \{+ c\}$ $\frac{2}{3} \sin^3 \left(\frac{\pi}{6}\right) = e^0 + c \quad \text{or} \quad \frac{2}{3} \left(\frac{1}{8}\right) - 1 = c$ $\left\{ \Rightarrow c = -\frac{11}{12} \right\} \quad \text{giving} \quad \frac{2}{3} \sin^3 y = e^x - \frac{11}{12}$	<p>B1 oe</p> <p>Applying <math>\frac{1}{\operatorname{cosec} 2y}</math> or <math>\sin 2y \rightarrow 2 \sin y \cos y</math> M1</p> <p>Integrates to give <math>\pm \mu \sin^3 y</math> M1</p> <p><math>2 \sin^2 y \cos y \rightarrow \frac{2}{3} \sin^3 y</math> A1</p> <p><math>e^x \rightarrow e^x</math> B1</p> <p>Use of <math>y = \frac{\pi}{6}</math> and <math>x = 0</math> M1</p> <p>in an integrated equation containing <math>c</math></p> <p><math>\frac{2}{3} \sin^3 y = e^x - \frac{11}{12}</math> A1</p> <p>[7]</p>
	<p><b>Alternative Method 1</b></p> $\int \frac{1}{\operatorname{cosec} 2y \operatorname{cosec} y} dy = \int e^x dx \quad \text{or} \quad \int \sin 2y \sin y dy = \int e^x dx$ $\int -\frac{1}{2}(\cos 3y - \cos y) dy = \int e^x dx$ $-\frac{1}{2} \left( \frac{1}{3} \sin 3y - \sin y \right) = e^x \{+ c\}$ $-\frac{1}{2} \left( \frac{1}{3} \sin \left(\frac{3\pi}{6}\right) - \sin \left(\frac{\pi}{6}\right) \right) = e^0 + c \quad \text{or} \quad -\frac{1}{2} \left( \frac{1}{3} - \frac{1}{2} \right) - 1 = c$ $\left\{ \Rightarrow c = -\frac{11}{12} \right\} \quad \text{giving} \quad -\frac{1}{6} \sin 3y + \frac{1}{2} \sin y = e^x - \frac{11}{12}$	<p>B1 oe</p> <p><math>\sin 2y \sin y \rightarrow \pm \lambda \cos 3y \pm \lambda \cos y</math> M1</p> <p>Integrates to give <math>\pm \alpha \sin 3y \pm \beta \sin y</math> M1</p> <p><math>-\frac{1}{2} \left( \frac{1}{3} \sin 3y - \sin y \right)</math> A1</p> <p><math>e^x \rightarrow e^x</math> as part of solving their DE. B1</p> <p>Use of <math>y = \frac{\pi}{6}</math> and <math>x = 0</math> in an integrated equation containing <math>c</math> M1</p> <p><math>-\frac{1}{6} \sin 3y + \frac{1}{2} \sin y = e^x - \frac{11}{12}</math> A1</p> <p>[7]</p>
		12



Question 6 Notes														
6. (i)	<p><b>M1</b> Integration by parts is applied in the form <math>\pm \alpha x e^{4x} - \int \beta e^{4x} \{dx\}</math>, where <math>\alpha \neq 0, \beta &gt; 0</math>. (must be in this form).</p> <p><b>A1</b> <math>\frac{1}{4} x e^{4x} - \int \frac{1}{4} e^{4x} \{dx\}</math> or equivalent.</p> <p><b>A1</b> <math>\frac{1}{4} x e^{4x} - \frac{1}{16} e^{4x}</math> with/without + c. Can be un-simplified.</p> <p><b>isw</b> You can ignore subsequent working following on from a correct solution.</p> <p><b>SC</b> <b>SPECIAL CASE:</b> A candidate who uses <math>u = x, \frac{dv}{dx} = e^{4x}</math>, writes down the correct “by parts” formula, but makes only one error when applying it can be awarded Special Case M1.</p>													
(ii)	<p><b>M1</b> <math>\pm \lambda (2x - 1)^{-2}, \lambda \neq 0</math>. <b>Note</b> that <math>\lambda</math> can be 1.</p> <p><b>A1</b> <math>\frac{8(2x - 1)^{-2}}{(2)(-2)}</math> or <math>-2(2x - 1)^{-2}</math> or <math>\frac{-2}{(2x - 1)^2}</math> with/without + c. Can be un-simplified.</p> <p><b>Note</b> You can ignore subsequent working which follows from a correct answer.</p>													
(iii)	<p><b>B1</b> Separates variables as shown. dy and dx should be in the correct positions, though this mark can be implied by later working. Ignore the integral signs.</p> <p><b>Note</b> <b>Allow B1 for</b> <math>\int \frac{1}{\operatorname{cosec} 2y \operatorname{cosec} y} = \int e^x</math> or <math>\int \sin 2y \sin y = \int e^x</math></p> <p><b>M1</b> <math>\frac{1}{\operatorname{cosec} 2y} \rightarrow 2 \sin y \cos y</math> or <math>\sin 2y \rightarrow 2 \sin y \cos y</math> or <math>\sin 2y \sin y \rightarrow \pm \lambda \cos 3y \pm \lambda \cos y</math> seen anywhere in the candidate’s working to (iii).</p> <p><b>M1</b> Integrates to give <math>\pm \mu \sin^3 y, \mu \neq 0</math> or <math>\pm \alpha \sin 3y \pm \beta \sin y, \alpha \neq 0, \beta \neq 0</math></p> <p><b>A1</b> <math>2 \sin^2 y \cos y \rightarrow \frac{2}{3} \sin^3 y</math> (with no extra terms) or integrates to give <math>-\frac{1}{2} \left( \frac{1}{3} \sin 3y - \sin y \right)</math></p> <p><b>B1</b> Evidence that <math>e^x</math> has been integrated to give <math>e^x</math> <b>as part of solving their DE.</b></p> <p><b>M1</b> Some evidence of using both <math>y = \frac{\pi}{6}</math> and <math>x = 0</math> in an integrated or changed equation containing c.</p> <p><b>Note</b> that is mark can be implied by the correct value of c.</p> <p><b>A1</b> <math>\frac{2}{3} \sin^3 y = e^x - \frac{11}{12}</math> or <math>-\frac{1}{6} \sin 3y + \frac{1}{2} \sin y = e^x - \frac{11}{12}</math> or any equivalent correct answer.</p> <p><b>Note</b> You can ignore subsequent working which follows from a correct answer.</p>													
<p><b>Alternative Method 2 (Using integration by parts twice)</b></p> <table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 60%;"><math>\int \sin 2y \sin y \, dy = \int e^x \, dx</math></td> <td style="width: 40%; text-align: right;">B1 oe</td> </tr> <tr> <td></td> <td style="text-align: right;"><b>M2</b></td> </tr> <tr> <td style="text-align: center;"><math>\frac{1}{3} \cos y \sin 2y - \frac{2}{3} \sin y \cos 2y = e^x \{ + c \}</math></td> <td style="text-align: right;">A1</td> </tr> <tr> <td></td> <td style="text-align: right;"><b>B1</b></td> </tr> <tr> <td></td> <td style="text-align: right;">M1</td> </tr> <tr> <td style="text-align: center;"><math>\frac{1}{3} \cos y \sin 2y - \frac{2}{3} \sin y \cos 2y = e^x - \frac{11}{12}</math></td> <td style="text-align: right;">A1</td> </tr> </table>			$\int \sin 2y \sin y \, dy = \int e^x \, dx$	B1 oe		<b>M2</b>	$\frac{1}{3} \cos y \sin 2y - \frac{2}{3} \sin y \cos 2y = e^x \{ + c \}$	A1		<b>B1</b>		M1	$\frac{1}{3} \cos y \sin 2y - \frac{2}{3} \sin y \cos 2y = e^x - \frac{11}{12}$	A1
$\int \sin 2y \sin y \, dy = \int e^x \, dx$	B1 oe													
	<b>M2</b>													
$\frac{1}{3} \cos y \sin 2y - \frac{2}{3} \sin y \cos 2y = e^x \{ + c \}$	A1													
	<b>B1</b>													
	M1													
$\frac{1}{3} \cos y \sin 2y - \frac{2}{3} \sin y \cos 2y = e^x - \frac{11}{12}$	A1													

7.

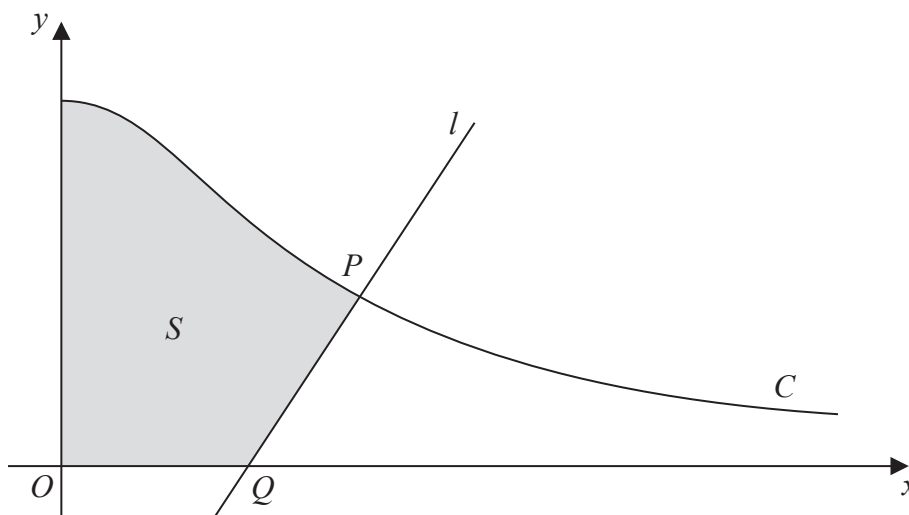


Figure 4

Figure 4 shows a sketch of part of the curve  $C$  with parametric equations

$$x = 3 \tan \theta, \quad y = 4 \cos^2 \theta, \quad 0 \leq \theta < \frac{\pi}{2}$$

The point  $P$  lies on  $C$  and has coordinates  $(3, 2)$ .

The line  $l$  is the normal to  $C$  at  $P$ . The normal cuts the  $x$ -axis at the point  $Q$ .

- (a) Find the  $x$  coordinate of the point  $Q$ . (6)

The finite region  $S$ , shown shaded in Figure 4, is bounded by the curve  $C$ , the  $x$ -axis, the  $y$ -axis and the line  $l$ . This shaded region is rotated  $2\pi$  radians about the  $x$ -axis to form a solid of revolution.

- (b) Find the exact value of the volume of the solid of revolution, giving your answer in the form  $p\pi + q\pi^2$ , where  $p$  and  $q$  are rational numbers to be determined.

[You may use the formula  $V = \frac{1}{3}\pi r^2 h$  for the volume of a cone.] (9)

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Question Number	Scheme	Marks
7. (a)	$x = 3 \tan \theta, \quad y = 4 \cos^2 \theta \quad \text{or} \quad y = 2 + 2 \cos 2\theta, \quad 0 \leq \theta < \frac{\pi}{2}.$ $\frac{dx}{d\theta} = 3 \sec^2 \theta, \quad \frac{dy}{d\theta} = -8 \cos \theta \sin \theta \quad \text{or} \quad \frac{dy}{d\theta} = -4 \sin 2\theta$	
	$\frac{dy}{dx} = \frac{-8 \cos \theta \sin \theta}{3 \sec^2 \theta} \left\{ = -\frac{8}{3} \cos^3 \theta \sin \theta = -\frac{4}{3} \sin 2\theta \cos^2 \theta \right\}$	their $\frac{dy}{d\theta}$ divided by their $\frac{dx}{d\theta}$ Correct $\frac{dy}{dx}$ M1 A1 oe
	At $P(3, 2), \theta = \frac{\pi}{4}, \frac{dy}{dx} = -\frac{8}{3} \cos^3 \left(\frac{\pi}{4}\right) \sin \left(\frac{\pi}{4}\right) \left\{ = -\frac{2}{3} \right\}$ So, $m(N) = \frac{3}{2}$	Some evidence of substituting $\theta = \frac{\pi}{4}$ into their $\frac{dy}{dx}$ applies $m(N) = \frac{-1}{m(T)}$ M1 M1
	Either N: $y - 2 = \frac{3}{2} (x - 3)$ or $2 = \left(\frac{3}{2}\right)(3) + c$	see notes M1
	{At Q, $y = 0$ , so, $-2 = \frac{3}{2}(x - 3)$ } giving $x = \frac{5}{3}$	$x = \frac{5}{3}$ or $1\frac{2}{3}$ or awrt 1.67 A1 cso
(b)	$\left\{ \int y^2 dx = \int y^2 \frac{dx}{d\theta} d\theta \right\} = \left\{ \int \right\} (4 \cos^2 \theta)^2 3 \sec^2 \theta \{d\theta\}$ So, $\pi \int y^2 dx = \pi \int (4 \cos^2 \theta)^2 3 \sec^2 \theta \{d\theta\}$ $\int y^2 dx = \int 48 \cos^2 \theta d\theta$ $= \{48\} \int \left(\frac{1 + \cos 2\theta}{2}\right) d\theta \left\{ = \int (24 + 24 \cos 2\theta) d\theta \right\}$	see notes see notes $\int 48 \cos^2 \theta \{d\theta\}$ Applies $\cos 2\theta = 2 \cos^2 \theta - 1$ M1 M1 M1
	$= \{48\} \left(\frac{1}{2}\theta + \frac{1}{4} \sin 2\theta\right) \left\{ = 24\theta + 12 \sin 2\theta \right\}$	Dependent on the first method mark. For $\pm \alpha \theta \pm \beta \sin 2\theta$ $\cos^2 \theta \rightarrow \left(\frac{1}{2}\theta + \frac{1}{4} \sin 2\theta\right)$ dM1 A1
	$\int_0^{\frac{\pi}{4}} y^2 dx \left\{ = 48 \left[\frac{1}{2}\theta + \frac{1}{4} \sin 2\theta\right]_0^{\frac{\pi}{4}} \right\} = \{48\} \left(\left(\frac{\pi}{8} + \frac{1}{4}\right) - (0 + 0)\right) \left\{ = 6\pi + 12 \right\}$ {So $V = \pi \int_0^{\frac{\pi}{4}} y^2 dx = 6\pi^2 + 12\pi$ } $V_{\text{cone}} = \frac{1}{3} \pi (2)^2 \left(3 - \frac{5}{3}\right) \left\{ = \frac{16\pi}{9} \right\}$ $\left\{ \text{Vol}(S) = 6\pi^2 + 12\pi - \frac{16\pi}{9} \right\} \Rightarrow \text{Vol}(S) = \frac{92}{9} \pi + 6\pi^2$	Dependent on the third method mark. $V_{\text{cone}} = \frac{1}{3} \pi (2)^2 (3 - \text{their } (a))$ $\frac{92}{9} \pi + 6\pi^2$ $\left\{ p = \frac{92}{9}, q = 6 \right\}$ dM1 M1 A1

[6]

[9]

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Question 7 Notes		
7. (a)	<b>1<sup>st</sup> M1</b>	Applies their $\frac{dy}{d\theta}$ divided by their $\frac{dx}{d\theta}$ <b>or</b> applies $\frac{dy}{d\theta}$ multiplied by their $\frac{d\theta}{dx}$
	<b>SC</b>	Award <b>Special Case 1<sup>st</sup> M1</b> if <b>both</b> $\frac{dx}{d\theta}$ <b>and</b> $\frac{dy}{d\theta}$ are both correct.
	<b>1<sup>st</sup> A1</b>	Correct $\frac{dy}{dx}$ i.e. $\frac{-8\cos\theta\sin\theta}{3\sec^2\theta}$ or $-\frac{8}{3}\cos^3\theta\sin\theta$ or $-\frac{4}{3}\sin 2\theta\cos^2\theta$ or any equivalent form.
	<b>2<sup>nd</sup> M1</b>	<i>Some evidence</i> of substituting $\theta = \frac{\pi}{4}$ or $\theta = 45^\circ$ into their $\frac{dy}{dx}$
	<b>Note</b>	For 3 <sup>rd</sup> M1 and 4 <sup>th</sup> M1, $m(\mathbf{T})$ must be found by using $\frac{dy}{dx}$ .
	<b>3<sup>rd</sup> M1</b>	applies $m(\mathbf{N}) = \frac{-1}{m(\mathbf{T})}$ . Numerical value for $m(\mathbf{N})$ is required here.
	<b>4<sup>th</sup> M1</b>	<ul style="list-style-type: none"> <li>Applies <math>y - 2 = (\text{their } m_N)(x - 3)</math>, where <math>m(\mathbf{N})</math> is a numerical value,</li> <li>or <b>finds c</b> by solving <math>2 = (\text{their } m_N)3 + c</math>, where <math>m(\mathbf{N})</math> is a numerical value,</li> </ul> and $m_N = -\frac{1}{\text{their } m(\mathbf{T})}$ or $m_N = \frac{1}{\text{their } m(\mathbf{T})}$ or $m_N = -\text{their } m(\mathbf{T})$ .
	<b>Note</b>	This mark can be implied by subsequent working.
(b)	<b>2<sup>nd</sup> A1</b>	$x = \frac{5}{3}$ or $1\frac{2}{3}$ or awrt 1.67 <b>from a correct solution only</b> .
	<b>1<sup>st</sup> M1</b>	Applying $\int y^2 dx$ as $y^2 \frac{dx}{d\theta}$ with their $\frac{dx}{d\theta}$ . Ignore $\pi$ or $\frac{1}{3}\pi$ outside integral.
	<b>Note</b>	You can ignore the omission of an integral sign and/or $d\theta$ for the 1 <sup>st</sup> M1.
	<b>Note</b>	Allow 1 <sup>st</sup> M1 for $\int (\cos^2\theta)^2 \times \text{"their } 3\sec^2\theta"$ $d\theta$ or $\int 4(\cos^2\theta)^2 \times \text{"their } 3\sec^2\theta"$ $d\theta$
	<b>1<sup>st</sup> A1</b>	Correct expression $\left\{ \pi \int y^2 dx \right\} = \pi \int (4\cos^2\theta)^2 3\sec^2\theta \{d\theta\}$ (Allow the omission of $d\theta$ )
	<b>Note</b>	<b>IMPORTANT:</b> The $\pi$ can be recovered later, <b>but as a correct statement only</b> .
	<b>2<sup>nd</sup> A1</b>	$\left\{ \int y^2 dx \right\} = \int 48\cos^2\theta \{d\theta\}$ . (Ignore $d\theta$ ). <b>Note:</b> 48 can be written as 24(2) for example.
	<b>2<sup>nd</sup> M1</b>	Applies $\cos 2\theta = 2\cos^2\theta - 1$ to their integral. (Seen or <b>implied</b> .)
	<b>3<sup>rd</sup> dM1*</b>	<b>which is dependent on the 1<sup>st</sup> M1 mark.</b> Integrating $\cos^2\theta$ to give $\pm\alpha\theta \pm \beta\sin 2\theta$ , $\alpha \neq 0$ , $\beta \neq 0$ , un-simplified or simplified.
	<b>3<sup>rd</sup> A1</b>	<b>which is dependent on the 3<sup>rd</sup> M1 mark and the 1<sup>st</sup> M1 mark.</b> Integrating $\cos^2\theta$ to give $\frac{1}{2}\theta + \frac{1}{4}\sin 2\theta$ , un-simplified or simplified.  This can be implied by $k\cos^2\theta$ giving $\frac{k}{2}\theta + \frac{k}{4}\sin 2\theta$ , un-simplified or simplified.
	<b>4<sup>th</sup> dM1</b>	<b>which is dependent on the 3<sup>rd</sup> M1 mark and the 1<sup>st</sup> M1 mark.</b> Some evidence of applying limits of $\frac{\pi}{4}$ and 0 (0 can be implied) to an integrated function in $\theta$
	<b>5<sup>th</sup> M1</b>	Applies $V_{\text{cone}} = \frac{1}{3}\pi(2)^2(3 - \text{their part (a) answer})$ .
	<b>Note</b>	Also allow the 5 <sup>th</sup> M1 for $V_{\text{cone}} = \pi \int_{\text{their } \frac{5}{3}}^3 \left( \frac{3}{2}x - \frac{5}{2} \right)^2 \{dx\}$ , which includes the correct limits.
	<b>4<sup>th</sup> A1</b>	$\frac{92}{9}\pi + 6\pi^2$ or $10\frac{2}{9}\pi + 6\pi^2$
	<b>Note</b>	A decimal answer of 91.33168464... (without a correct <b>exact</b> answer) is A0.
	<b>Note</b>	The $\pi$ in the volume formula is only needed for the 1 <sup>st</sup> A1 mark and the final accuracy mark.

7.		<p><b>Working with a Cartesian Equation</b></p> <p>A cartesian equation for <math>C</math> is <math>y = \frac{36}{x^2 + 9}</math></p>
(a)	<p><b>1<sup>st</sup> M1</b></p> <p><b>1<sup>st</sup> A1</b></p> <p><b>2<sup>nd</sup> dM1</b></p>	<p><math>\frac{dy}{dx} = \pm \lambda x (\pm \alpha x^2 \pm \beta)^{-2}</math> or <math>\frac{dy}{dx} = \frac{\pm \lambda x}{(\pm \alpha x^2 \pm \beta)^2}</math></p> <p><math>\frac{dy}{dx} = -36(x^2 + 9)^{-2}(2x)</math> or <math>\frac{dy}{dx} = \frac{-72x}{(x^2 + 9)^2}</math> un-simplified or simplified.</p> <p><b>Dependent on the 1<sup>st</sup> M1 mark if a candidate uses this method</b></p> <p>For substituting <math>x = 3</math> into their <math>\frac{dy}{dx}</math></p> <p>i.e. at <math>P(3, 2)</math>, <math>\frac{dy}{dx} = \frac{-72(3)}{(3^2 + 9)^2} \left\{ = -\frac{2}{3} \right\}</math></p> <p>From this point onwards the original scheme can be applied.</p>
(b)	<p><b>1<sup>st</sup> M1</b></p> <p><b>A1</b></p>	<p>For <math>\int \left( \frac{\pm \lambda}{\pm \alpha x^2 \pm \beta} \right)^2 \{dx\}</math> (<math>\pi</math> not required for this mark)</p> <p>For <math>\pi \int \left( \frac{36}{x^2 + 9} \right)^2 \{dx\}</math> (<math>\pi</math> required for this mark)</p> <p>To integrate, a substitution of <math>x = 3 \tan \theta</math> is required which will lead to <math>\int 48 \cos^2 \theta d\theta</math> and so from this point onwards the original scheme can be applied.</p>
(a)	<p><b>1<sup>st</sup> M1</b></p> <p><b>1<sup>st</sup> A1</b></p> <p><b>2<sup>nd</sup> dM1</b></p>	<p>Another cartesian equation for <math>C</math> is <math>x^2 = \frac{36}{y} - 9</math></p> <p><math>\pm \alpha x = \pm \frac{\beta}{y^2} \frac{dy}{dx}</math> or <math>\pm \alpha x \frac{dx}{dy} = \pm \frac{\beta}{y^2}</math></p> <p><math>2x = -\frac{36}{y^2} \frac{dy}{dx}</math> or <math>2x \frac{dx}{dy} = -\frac{36}{y^2}</math></p> <p><b>Dependent on the 1<sup>st</sup> M1 mark if a candidate uses this method</b></p> <p>For substituting <math>x = 3</math> to find <math>\frac{dy}{dx}</math></p> <p>i.e. at <math>P(3, 2)</math>, <math>2(3) = -\frac{36}{4} \frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \dots</math></p> <p>From this point onwards the original scheme can be applied.</p>

8. Relative to a fixed origin  $O$ , the point  $A$  has position vector  $\begin{pmatrix} -2 \\ 4 \\ 7 \end{pmatrix}$

and the point  $B$  has position vector  $\begin{pmatrix} -1 \\ 3 \\ 8 \end{pmatrix}$

The line  $l_1$  passes through the points  $A$  and  $B$ .

(a) Find the vector  $\vec{AB}$ . (2)

(b) Hence find a vector equation for the line  $l_1$ . (1)

The point  $P$  has position vector  $\begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix}$

Given that angle  $PBA$  is  $\theta$ ,

(c) show that  $\cos \theta = \frac{1}{3}$ . (3)

The line  $l_2$  passes through the point  $P$  and is parallel to the line  $l_1$

(d) Find a vector equation for the line  $l_2$ . (2)

The points  $C$  and  $D$  both lie on the line  $l_2$

Given that  $AB = PC = DP$  and the  $x$  coordinate of  $C$  is positive,

(e) find the coordinates of  $C$  and the coordinates of  $D$ . (3)

(f) find the exact area of the trapezium  $ABCD$ , giving your answer as a simplified surd. (4)

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Question Number	Scheme	Marks
8.	$\overline{OA} = -2\mathbf{i} + 4\mathbf{j} + 7\mathbf{k}$ , $\overline{OB} = -\mathbf{i} + 3\mathbf{j} + 8\mathbf{k}$ & $\overline{OP} = 0\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$	
(a)	$\overline{AB} = -\mathbf{i} + 3\mathbf{j} + 8\mathbf{k} - (-2\mathbf{i} + 4\mathbf{j} + 7\mathbf{k}) = \mathbf{i} - \mathbf{j} + \mathbf{k}$	M1; A1 [2]
(b)	$\{l_1 : \mathbf{r}\} = \begin{pmatrix} -2 \\ 4 \\ 7 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$ or $\{\mathbf{r}\} = \begin{pmatrix} -1 \\ 3 \\ 8 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$	B1ft [1]
(c)	$\overline{PB} = \overline{OB} - \overline{OP} = \begin{pmatrix} -1 \\ 3 \\ 8 \end{pmatrix} - \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ 5 \end{pmatrix}$ or $\overline{BP} = \begin{pmatrix} 1 \\ -1 \\ -5 \end{pmatrix}$	M1
	$\{\cos \theta =\} \frac{\overline{AB} \cdot \overline{PB}}{ \overline{AB}   \overline{PB} } = \frac{\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 1 \\ 5 \end{pmatrix}}{\sqrt{(1)^2 + (-1)^2 + (1)^2} \cdot \sqrt{(-1)^2 + (1)^2 + (5)^2}}$	M1 Applies dot product formula between their $(\overline{AB}$ or $\overline{BA})$ and their $(\overline{PB}$ or $\overline{BP})$ .
	$\{\cos \theta\} = \frac{-1-1+5}{\sqrt{3} \cdot \sqrt{27}} = \frac{3}{9} = \frac{1}{3}$	A1 cso Correct proof
(d)	$\{l_2 : \mathbf{r}\} = \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$	M1 A1 ft [2] $\mathbf{p} + \lambda \mathbf{d}$ or $\mathbf{p} + \mu \mathbf{d}$ , $\mathbf{p} \neq 0$ , $\mathbf{d} \neq 0$ with either $\mathbf{p} = 0\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ or $\mathbf{d} =$ their $\overline{AB}$ , or a multiple of their $\overline{AB}$ . Correct vector equation.
(e)	$\overline{OC} = \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix} + \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 4 \end{pmatrix}$ or $\overline{OD} = \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix} - \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 3 \\ 2 \end{pmatrix}$ $\{C(1, 1, 4), D(-1, 3, 2)\}$	M1 A1 ft A1 ft [3] Either $\overline{OP} +$ their $\overline{AB}$ or $\overline{OP} -$ their $\overline{AB}$ At least one set of coordinates are correct. Both sets of coordinates are correct.
Way 1 (f)	$\frac{h}{\sqrt{(-1)^2 + (1)^2 + (5)^2}} = \sin \theta$ $h = \sqrt{27} \sin(70.5\dots) \left\{ = \sqrt{27} \frac{\sqrt{8}}{3} = 2\sqrt{6} = \text{awrt } 4.9 \right\}$ $\text{Area } ABCD = \frac{1}{2} 2\sqrt{6} (\sqrt{3} + 2\sqrt{3})$ $\left\{ = \frac{1}{2} 2\sqrt{6} (3\sqrt{3}) = 3\sqrt{18} \right\} = 9\sqrt{2}$	M1 A1 oe dM1 A1 cao [4] 15 $\frac{h}{\text{their }  \overline{PB} } = \sin \theta$ $\sqrt{27} \sin(70.5\dots)$ or $\sqrt{27} \cdot \frac{\sqrt{8}}{3}$ or $2\sqrt{6}$ or awrt 4.9 or equivalent $\frac{1}{2}(\text{their } h)(\text{their } AB + \text{their } CD)$

<p>8. (f)</p>	<p><b>Helpful Diagram!</b></p> <p>Area <math>\triangle APB = 4.2426\dots</math></p> <p><math>\vec{DA} = \vec{PB} = \begin{pmatrix} -1 \\ 1 \\ 5 \end{pmatrix}</math></p> <p><math>\vec{PA} = \vec{CB} = \begin{pmatrix} -2 \\ 2 \\ 4 \end{pmatrix}</math></p>	
	<p><math>\vec{PA} = \vec{CB} = \begin{pmatrix} -2 \\ 2 \\ 4 \end{pmatrix}</math> and <math>\vec{AB} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}</math>, so <math>BC \perp AB</math></p>	<p>Candidates do not need to prove this result for part (f)</p>
<p>8. (f) Way 2</p>	<p><math>h =  \vec{CB}  = \sqrt{(-2)^2 + (2)^2 + (4)^2} = \sqrt{24} = 2\sqrt{6} = 4.8989\dots</math></p> <p>Area <math>ABCD = \frac{1}{2} \sqrt{24} (\sqrt{3} + 2\sqrt{3})</math> or <math>\frac{1}{2} \sqrt{24} \sqrt{3} + \sqrt{24} \sqrt{3}</math>  <math>= 9\sqrt{2}</math></p>	<p>Attempts <math> \vec{PA} </math> or <math> \vec{CB} </math> M1  <math> \vec{PA}  =  \vec{CB}  = \sqrt{24}</math> A1 oe  <math>\frac{1}{2}h(\text{their } AB + \text{their } CD)</math> dM1 oe  <math>9\sqrt{2}</math> A1 cso</p>
<p>Way3 8. (f)</p>	<p><b>Finds the area of either triangle APB or APD or BCP and triples the result.</b></p> <p>Area <math>\triangle APB = \frac{1}{2} \sqrt{3} (3\sqrt{3}) \sin \theta</math>  <math>= \frac{1}{2} \sqrt{3} (3\sqrt{3}) \sin(70.5\dots)</math></p> <p>Area <math>ABCD = 3 (3\sqrt{2})</math>  <math>= 9\sqrt{2}</math></p>	<p>Attempts <math>\frac{1}{2}(\text{their } AB)(\text{their } PB) \sin \theta</math> M1  <math>\frac{1}{2} \sqrt{3} (3\sqrt{3}) \sin(70.5\dots)</math> or <math>3\sqrt{2}</math> A1          or awrt 4.24 or equivalent  <math>3 \times \text{Area of } \triangle APB</math> dM1  <math>9\sqrt{2}</math> A1 cso</p>



Question 8 Notes	
8. (a)	<p><b>M1</b> Finding the difference (either way) between <math>\overline{OB}</math> and <math>\overline{OA}</math>. If no “subtraction” seen, you can award M1 for 2 out of 3 correct components of the difference.</p> <p><b>A1</b> <math>\mathbf{i} - \mathbf{j} + \mathbf{k}</math> or <math>\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}</math> or <math>(1, -1, 1)</math> or benefit of the doubt <math>\begin{matrix} 1 \\ -1 \\ 1 \end{matrix}</math></p>
(b)	<p><b>B1ft</b> <math>\{\mathbf{r}\} = \begin{pmatrix} -2 \\ 4 \\ 7 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}</math> or <math>\{\mathbf{r}\} = \begin{pmatrix} -1 \\ 3 \\ 8 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}</math>, with <math>\overline{AB}</math> or <math>\overline{BA}</math> correctly followed through from (a).</p> <p><b>Note</b> <math>\mathbf{r} =</math> is not needed.</p>
(c)	<p><b>M1</b> An attempt to find either the vector <math>\overline{PB}</math> or <math>\overline{BP}</math>. If no “subtraction” seen, you can award M1 for 2 out of 3 correct components of the difference.</p> <p><b>M1</b> Applies dot product formula between their <math>(\overline{AB}</math> or <math>\overline{BA})</math> and their <math>(\overline{PB}</math> or <math>\overline{BP})</math>.</p> <p><b>A1</b> Obtains <math>\{\cos \theta\} = \frac{1}{3}</math> <i>by correct solution only</i>.</p> <p><b>Note</b> If candidate starts by applying <math>\frac{\overline{AB} \cdot \overline{PB}}{ \overline{AB}  \cdot  \overline{PB} }</math> correctly (without reference to <math>\cos \theta = \dots</math>) they can gain both 2<sup>nd</sup> M1 and A1 mark.</p> <p><b>Note</b> Award the final A1 mark if candidate achieves <math>\{\cos \theta\} = \frac{1}{3}</math> by either taking the dot product between</p> <p>(i) <math>\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}</math> and <math>\begin{pmatrix} -1 \\ 1 \\ 5 \end{pmatrix}</math> or (ii) <math>\begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}</math> and <math>\begin{pmatrix} 1 \\ -1 \\ -5 \end{pmatrix}</math>. Ignore if any of these vectors are labelled incorrectly.</p> <p><b>Note</b> Award final A0, cso for those candidates who take the dot product between</p> <p>(iii) <math>\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}</math> and <math>\begin{pmatrix} 1 \\ -1 \\ -5 \end{pmatrix}</math> or (iv) <math>\begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}</math> and <math>\begin{pmatrix} -1 \\ 1 \\ 5 \end{pmatrix}</math></p> <p>They will usually find <math>\{\cos \theta\} = -\frac{1}{3}</math> or may fudge <math>\{\cos \theta\} = \frac{1}{3}</math>.</p> <p>If these candidates give a convincing detailed explanation which must include reference to the direction of their vectors then this can be given A1 cso</p>
(c)	<p><b>Alternative Method 1: The Cosine Rule</b></p> <p><math>\overline{PB} = \overline{OB} - \overline{OP} = \begin{pmatrix} -1 \\ 3 \\ 8 \end{pmatrix} - \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ 5 \end{pmatrix}</math> or <math>\overline{BP} = \begin{pmatrix} 1 \\ -1 \\ -5 \end{pmatrix}</math></p> <p>Note <math> \overline{PB}  = \sqrt{27}</math>, <math> \overline{AB}  = \sqrt{3}</math> and <math> \overline{PA}  = \sqrt{24}</math></p> <p><math>(\sqrt{24})^2 = (\sqrt{27})^2 + (\sqrt{3})^2 - 2(\sqrt{27})(\sqrt{3})\cos \theta</math></p> <p><math>\cos \theta = \frac{27 + 3 - 24}{18} = \frac{1}{3}</math></p> <p style="text-align: right;">Mark in the same way as the main scheme. M1</p> <p style="text-align: right;">Applies the cosine rule the correct way round M1 oe</p> <p style="text-align: right;">Correct proof A1 cso</p> <p style="text-align: right;"><b>[3]</b></p>

<p>8. (c)</p>	<p><b>Alternative Method 2: Right-Angled Trigonometry</b></p> $\overline{PB} = \overline{OB} - \overline{OP} = \begin{pmatrix} -1 \\ 3 \\ 8 \end{pmatrix} - \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ 5 \end{pmatrix} \text{ or } \overline{BP} = \begin{pmatrix} 1 \\ -1 \\ -5 \end{pmatrix}$ <p>Either <math>(\sqrt{24})^2 + (\sqrt{3})^2 = (\sqrt{27})^2</math></p> <p>or <math>\overline{AB} \cdot \overline{PA} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} -2 \\ 2 \\ 4 \end{pmatrix} = -2 - 2 + 4 = 0</math></p> <p>So, <math>\left\{ \cos \theta = \frac{AB}{PB} \Rightarrow \right\} \cos \theta = \frac{\sqrt{3}}{\sqrt{27}} = \frac{1}{3}</math></p>	<p>Mark in the same way as the main scheme. M1</p> <p>Confirms <math>\Delta PAB</math> is right-angled M1</p> <p>Correct proof A1 cso</p> <p style="text-align: right;"><b>[3]</b></p>
<p>(d)</p>	<p><b>M1</b> Writing down a line in the form <math>\mathbf{p} + \lambda \mathbf{d}</math> or <math>\mathbf{p} + \mu \mathbf{d}</math> with either <math>\mathbf{a} = \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix}</math> or <math>\mathbf{d} =</math> their <math>\overline{AB}</math> <math>\mathbf{d} =</math> their <math>\overline{AB}</math>, or a multiple of their <math>\overline{AB}</math> found in part (a).</p> <p><b>A1ft</b> Writing <math>\begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}</math> or <math>\begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix} + \mu \mathbf{d}</math>, where <math>\mathbf{d} =</math> their <math>\overline{AB}</math> or a multiple of their <math>\overline{AB}</math> found in part (a).</p> <p><b>Note</b> <math>\mathbf{r} =</math> is not needed.</p> <p><b>Note</b> Using the same scalar parameter as in part (b) is fine for A1.</p>	
<p>(e)</p>	<p><b>M1</b> Either <math>\overline{OP} +</math> their <math>\overline{AB}</math> or <math>\overline{OP} -</math> their <math>\overline{AB}</math>.</p> <p><b>Note</b> This can be implied at least two out of three correct components for either their <math>C</math> or their <math>D</math>.</p> <p><b>A1ft</b> At least one set of coordinates are correct. Ignore labelling of <math>C, D</math></p> <p><b>A1ft</b> Both sets of coordinates are correct. Ignore labelling of <math>C, D</math></p> <p><b>Note</b> You can follow through either or both accuracy marks in this part using their <math>\overline{AB}</math> from part (a).</p>	
<p>(f)</p>	<p><b>M1</b> Way 1: <math>\frac{h}{\text{their }  \overline{PB} } = \sin \theta</math></p> <p>Way 2: Attempts <math> \overline{PA} </math> or <math> \overline{CB} </math></p> <p>Way 3: Attempts <math>\frac{1}{2} (\text{their } PB)(\text{their } AB) \sin \theta</math></p> <p><b>Note</b> Finding <math>AD</math> by itself is M0.</p>	
	<p><b>A1</b> Either</p> <ul style="list-style-type: none"> <li><math>h = \sqrt{27} \sin(70.5\dots)</math> or <math> \overline{PA}  =  \overline{CB}  = \sqrt{24}</math> or equivalent. (See Way 1 and Way 2)</li> </ul> <p>or</p> <ul style="list-style-type: none"> <li>the area of either triangle <math>APB</math> or <math>APD</math> or <math>BDP = \frac{1}{2} \sqrt{3} (3\sqrt{3}) \sin(70.5\dots)</math> o.e. (See Way 3).</li> </ul>	
	<p><b>dM1</b> which is dependent on the 1<sup>st</sup> M1 mark.</p> <p>A full method to find the area of trapezium <math>ABCD</math>. (See Way 1, Way 2 and Way 3).</p> <p><b>A1</b> <math>9\sqrt{2}</math> from a correct solution only.</p> <p><b>Note</b> A decimal answer of 12.7279... (without a correct <b>exact</b> answer) is A0.</p>	