Centre No.						Paper Reference							Surname	Initial(s)
Candidate No.						6	6	6	3	/	0	1	Signature	

Paper Reference(s)

6663/01

Edexcel GCE

Core Mathematics C1

Advanced Subsidiary

Monday 14 January 2013 – Morning

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
Total	

Materials required for examination
Mathematical Formulae (Pink)

Items included with question papers
Nil

Calculators may NOT be used in this examination.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer for each question in the space following the question.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 11 questions in this question paper. The total mark for this paper is 75.
There are 32 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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1. Factorise completely $x - 4x^3$

(3)

Q1

(Total 3 marks)

January 2013
6663 Core Mathematics C1
Mark Scheme

Question Number	Scheme	Marks
1.	$x(1-4x^2)$ Accept $x(-4x^2+1)$ or $-x(4x^2-1)$ or $-x(-1+4x^2)$ or even $4x(\frac{1}{4}-x^2)$ or equivalent	B1
	Factorises quadratic (or initial cubic) into two brackets	M1
	$x(1-2x)(1+2x)$ or $-x(2x-1)(2x+1)$ or $x(2x-1)(-2x-1)$	A1
		[3]
		3 marks
	Notes	
	B1: Takes out a factor of x or $-x$ or even $4x$. This line may be implied by correct final answer, but if this stage is shown it **must be correct**. So **B0** for $x(1+4x^2)$ **M1:** Factorises the quadratic resulting from their first factorisation using usual rules (see note 1 in General Principles). e.g. $x(1-4x)(x-1)$. Also allow attempts to factorise cubic such as $(x-2x^2)(1+2x)$ etc N.B. Should not be completing the square here. **A1:** Accept either $x(1-2x)(1+2x)$ or $-x(2x-1)(2x+1)$ or $x(2x-1)(-2x-1)$. (No fractions for this final answer)	
	Specific situations	
	Note: $x(1-4x^2)$ followed by $x(1-2x)^2$ scores B1M1A0 as factors follow quadratic factorisation criteria **And** $x(1-4x^2)$ followed by $x(1-4x)(1+4x)$ **B1M0A0.**	
	Answers with no working: Correct answer gets all three marks B1M1A1	
	: $x(2x-1)(2x+1)$ gets B0M1A0 if **no working** as $x(4x^2-1)$ would earn B0	
	Poor bracketing: e.g. $(-1+4x^2)-x$ gets B0 unless subsequent work implies bracket round the $-x$ in which case candidate may recover the mark by the following correct work.	
	N.B. If correct factors are followed by $x=0, x=\frac{1}{2}, x=-\frac{1}{2}$ then ignore this as subsequent work.	
	But these answers- $x=0, x=\frac{1}{2}, x=-\frac{1}{2}$ - with no working, or no factors, gets B0M0A0.	
	Ignore "=0" written at the end of lines and mark LHS as in the scheme above. Candidate who changes the question to $4x^3-x=x(4x^2-1)=x(2x-1)(2x+1)$ would earn B0 M1 A0 1/3	

Leave blank

2. Express 8^{2x+3} in the form 2^y, stating y in terms of x.

(2)

Q2

(Total 2 marks)

Question Number	Scheme	Marks
2.	$(8^{2x+3} = \left(2^3\right)^{2x+3}) = 2^{3(2x+3)}$ or 2^{ax+b} with $a = 6$ **or** $b = 9$	M1
	$= 2^{6x+9}$ *or* $= 2^{3(2x+3)}$ as **final** answer with no errors or $(y =)6x + 9$ or $3(2x + 3)$	A1 **[2]**
		2 marks
	Notes	
	M1: Uses $8 = 2^3$, and **multiplies** powers $3(2x + 3)$. Does **not** add powers. (Just $8 = 2^3$ or $8^{\frac{1}{3}} = 2$ is M0) **A1:** Either 2^{6x+9} *or* $= 2^{3(2x+3)}$ or $(y =)6x + 9$ or $3(2x + 3)$	
	Note: Examples: 2^{6x+3} scores M1A0 **:** $8^{2x+3} = \left(2^3\right)^{2x+3} = 2^{3+2x+3}$ gets M0A0 **Special case:** **:** $= 2^{6x}\,2^9$ without seeing as single power M1A0 **Alternative method using logs:** $8^{2x+3} = 2^y \Rightarrow (2x+3)\log 8 = y\log 2 \Rightarrow y = \dfrac{(2x+3)\log 8}{\log 2}$ **So** $(y =)6x + 9$ or $3(2x + 3)$	**M1** **A1 [2]**

Leave blank

3. (i) Express

$$\left(5-\sqrt{8}\right)\left(1+\sqrt{2}\right)$$

in the form $a + b\sqrt{2}$, where a and b are integers.

(3)

(ii) Express

$$\sqrt{80}+\frac{30}{\sqrt{5}}$$

in the form $c\sqrt{5}$, where c is an integer.

(3)

Question Number	Scheme	Marks
3. (i)	$\left(5-\sqrt{8}\right)\left(1+\sqrt{2}\right)$	
	$=5+5\sqrt{2}-\sqrt{8}-4$	M1
	$=5+5\sqrt{2}-2\sqrt{2}-4$ — $\sqrt{8}=2\sqrt{2}$, seen or implied at any point.	B1
	$=1+3\sqrt{2}$ — $1+3\sqrt{2}$ or $a=1$ and $b=3$.	A1 **[3]**
(ii)	Method 1: **Either** $\sqrt{80}+\dfrac{30}{\sqrt{5}}\left(\dfrac{\sqrt{5}}{\sqrt{5}}\right)$; Method 2: **Or** $\left(\dfrac{\sqrt{400}+30}{\sqrt{5}}\right)\dfrac{\sqrt{5}}{\sqrt{5}}$; Method 3: $\sqrt{80}+\dfrac{\sqrt{900}}{\sqrt{5}}=\sqrt{80}+\sqrt{180}$	M1
	Method 1: $=4\sqrt{5}+...$; Method 2: $=\left(\dfrac{20+..}{..}\right)\dfrac{..}{..}$; Method 3: $=4\sqrt{5}+...$	B1
	Method 1: $=4\sqrt{5}+6\sqrt{5}$; Method 2: $=\left(\dfrac{50\sqrt{5}}{5}\right)$; Method 3: $=4\sqrt{5}+6\sqrt{5}$	
	$=10\sqrt{5}$	A1
		[3]
Alternative for (i)	$\left(5-2\sqrt{2}\right)\left(1+\sqrt{2}\right)$ — This earns the B1 mark and is entered on epen as B1	
	$=5+5\sqrt{2}-2\sqrt{2}-2\sqrt{2}\sqrt{2}$ — Multiplies out correctly with $2\sqrt{2}$. This may be seen or implied and may be simplified e.g. $=5+3\sqrt{2}-2\sqrt{4}$ o.e.	M1
	For earlier use of $2\sqrt{2}$	B1
	$=1+3\sqrt{2}$ — $1+3\sqrt{2}$ or $a=1$ and $b=3$.	A1 **[3]**
		6 marks
	Notes	
(i)	**M1:** Multiplies out brackets correctly giving four correct terms or simplifying to correct expansion. (This may be implied by correct answer) – can appear as table **B1:** $\sqrt{8}=2\sqrt{2}$, seen or implied at any point **A1:** Fully and correctly simplified to $1+3\sqrt{2}$ or $a=1$ and $b=3$.	
(ii)	**M1:** Rationalises denominator i.e. Multiplies $\left(\dfrac{k}{\sqrt{5}}\right)$ by $\left(\dfrac{\sqrt{5}}{\sqrt{5}}\right)$ or $\left(\dfrac{-\sqrt{5}}{-\sqrt{5}}\right)$, seen or implied or uses Method 3 or similar e.g. $\left(\dfrac{30}{\sqrt{5}}\right)=\dfrac{6\times 5}{\sqrt{5}}=6\sqrt{5}$ **B1**: (Independent mark) States $\sqrt{80}=4\sqrt{5}$ Or either $\sqrt{400}=20$ *or* $\sqrt{80}\sqrt{5}=20$ at any point if they use Method 2. **A1:** $10\sqrt{5}$ or $c=10$.	
	N.B There are other methods e.g. $\sqrt{80}=\dfrac{20}{\sqrt{5}}$ (B1) then add $\dfrac{20}{\sqrt{5}}+\dfrac{30}{\sqrt{5}}=\dfrac{50}{\sqrt{5}}$ then M1 A1as before Those who multiply initial expression by $\sqrt{5}$ to obtain $\sqrt{400}+30=20+30=50$ earn M0 B1 A0	

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4. A sequence $u_1, u_2, u_3, \ldots$ satisfies

$$u_{n+1} = 2u_n - 1, \quad n \geqslant 1$$

Given that $u_2 = 9$,

(a) find the value of u_3 and the value of u_4,

(2)

(b) evaluate $\displaystyle\sum_{r=1}^{4} u_r$.

(3)

Question Number	Scheme		Marks
4.	$u_2 = 9,\ u_{n+1} = 2u_n - 1,\ \ n \geqslant 1$		
(a)	$u_3 = 2u_2 - 1 = 2(9) - 1 \quad (=17)$	$u_3 = 2(9) - 1.$	M1
	$u_4 = 2u_3 - 1 = 2(17) - 1 = 33$	Can be implied by $u_3 = 17$	
		Both $u_3 = 17$ and $u_4 = 33$	A1
			[2]
(b)	$\sum_{r=1}^{4} u_r = u_1 + u_2 + u_3 + u_4$		
	$(u_1) = 5$	$(u_1) = 5$	B1 (M1 on epen)
	$\sum_{r=1}^{4} u_r = "5" + 9 + "17" + "33" = 64$	Adds their first four terms obtained legitimately (see notes below)	M1
		64	A1
			[3]
			5 marks
	Notes		
(a)	**M1**: Substitutes 9 into RHS of iteration formula **A1**: Needs **both** 17 and 33 (but **allow if either or both seen in part (b)**)		
(b)	**B1: (Appears as M1 on epen) for** u_1=5 (however obtained – may appear in (a)) May be called *a=5* **M1:** Uses their u_1 found from $u_2 = 2u_1 - 1$ stated explicitly, or uses $u_1 = 4$ *or* $5\frac{1}{2}$, and adds it to u_2, their u_3 and their u_4 only. (See special cases below). There should be no fifth term included. Use of sum of AP is irrelevant and scores M0 **A1**: 64		
	Note: Special cases: A candidate who adds u_2, u_3, u_4 and u_5 scores B0M0A0. (M0M0A0 on epen) Such candidates will usually give a final answer of $9 + 17 + 33 + 65 = 124$. Candidates **who invent** an arbitrary (wrong) value for u_1 will also score B0 M0 A0. (M0M0A0 on epen) Uses $u_1 = 4$ to obtain sum (usually 63) get B0 M1 A0 (M0 M1 A0 on epen) Uses $u_1 = 5\frac{1}{2}$ to obtain sum (usually $64\frac{1}{2}$) also get B0 M1 A0 (M0 M1 A0 on epen)		

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5. The line l_1 has equation $y = -2x + 3$

The line l_2 is perpendicular to l_1 and passes through the point (5, 6).

(a) Find an equation for l_2 in the form $ax + by + c = 0$, where a, b and c are integers.
(3)

The line l_2 crosses the x-axis at the point A and the y-axis at the point B.

(b) Find the x-coordinate of A and the y-coordinate of B.
(2)

Given that O is the origin,

(c) find the area of the triangle OAB.
(2)

Question Number	Scheme	Marks
5.		
(a)	Gradient of l_2 is $\frac{1}{2}$ or 0.5 or $\frac{-1}{-2}$	B1
	Either $y-6 = "\tfrac{1}{2}"(x-5)$ or $y = "\tfrac{1}{2}"x + c$ and $6 = "\tfrac{1}{2}"(5) + c \Rightarrow c = ("\tfrac{7}{2}")$	M1
	$x-2y+7=0$ or $-x+2y-7=0$ or $k(x-2y+7) = 0$ with ***k* an integer**	A1 **[3]**
(b)	Puts x = 0, or y = 0 in **their** equation and solves to find appropriate co-ordinate	M1
	x-coordinate of A is -7 and y-coordinate of B is $\tfrac{7}{2}$.	A1 cao **[2]**
(c)	Area $OAB = \frac{1}{2}(7)\left(\frac{7}{2}\right) = \frac{49}{4}$ (units)^2 — Applies $\pm\tfrac{1}{2}$(base)(height)	M1
	$\frac{49}{4}$	A1**cso** **[2]**
		7 marks
	Notes	
(a)	**B1**: Must have ½ or 0.5 or $\frac{-1}{-2}$ o.e. stated and stops, or used in their line equation **M1:** Full method to obtain an equation of the line through (5,6) with their "m". So $y - 6 = m(x-5)$ with their gradient or uses $y = mx + c$ with $(5, 6)$ and their gradient to find c. Allow any numerical gradient here including –2 or –1 but **not zero** . (Allow (6,5) as a slip **if** $y - y_1 = m(x - x_1)$ is quoted first) **A1:** Accept any multiple of the correct equation, provided that the coefficients are integers and equation = 0 e.g. $-x+2y-7 = 0$ or $k(x-2y+7) = 0$ or even $2y - x - 7 = 0$	
(b)	**M1:** Either one of the x or y coordinates using their equation **A1:** Needs both correct values. Accept any correct equivalent.. Need not be written as co-ordinates. **Even** just –7 and 3.5 with **no** indication which is which may be awarded the A1.	
(c)	**M1:** Any correct method for area of triangle AOB, with their values for co-ordinates of A and B (may include negatives) *Method usually half base times height but determinants could be used.* **A1**: Any exact equivalent to 49/4, e.g. 12.25. (**negative final** answer is A0 but replacing by **positive** is A1) Do not need units. **c.s.o.** implies if A0 is scored in (b) then A0 is scored in (c) as well**. However** if candidate has correct line equation in (a) of wrong form may score A0 in (a) and A1 in (b) and (c)	
	Note: Special cases: $\frac{1}{2}(-7)\left(+\frac{7}{2}\right) = -\frac{49}{4}$ (units)^2 is M1 A0 but changing sign to area = $+\frac{49}{4}$ gets M1A1 (recovery) **N.B.** Candidates making sign errors in (b) and obtaining +7 and **-**$\tfrac{7}{2}$. may also get $\frac{49}{4}$ as their answer following previous errors. They should be awarded A0 as this answer is not ft and is for correct **solution** only **Special Case**: In (a) and (b): Produces parallel line instead of perpendicular line: So uses m = -2 This is not treated as a misread as it simplifies the question. The marks will usually be B0 M1 A0, M1 A0, M1 A0 i.e. maximum of 3/7	

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6.

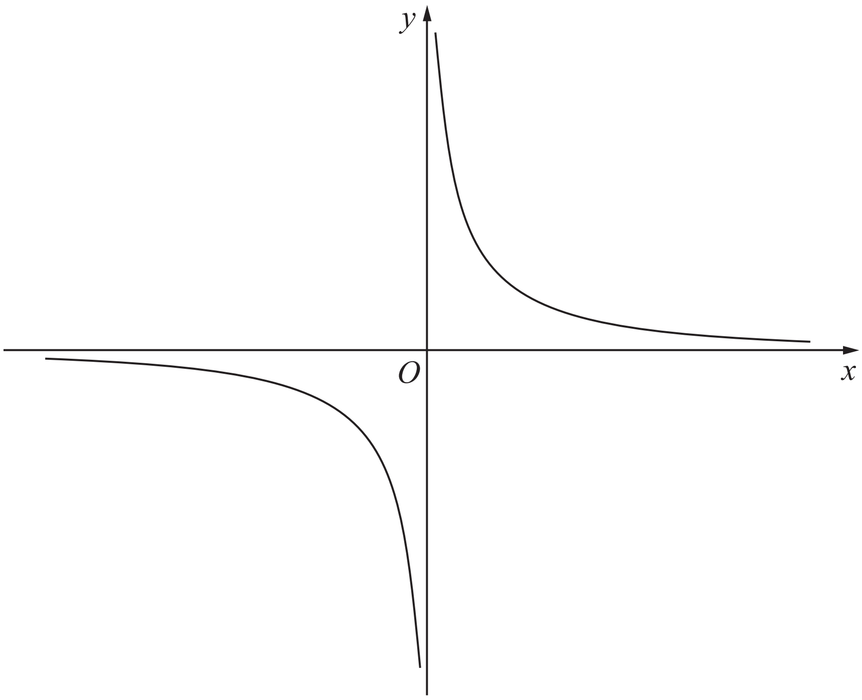


Figure 1

Figure 1 shows a sketch of the curve with equation $y = \dfrac{2}{x}, \quad x \neq 0$

The curve C has equation $y = \dfrac{2}{x} - 5, \quad x \neq 0,$ and the line l has equation $y = 4x + 2$

(a) Sketch and clearly label the graphs of C and l on a single diagram.

On your diagram, show clearly the coordinates of the points where C and l cross the coordinate axes.

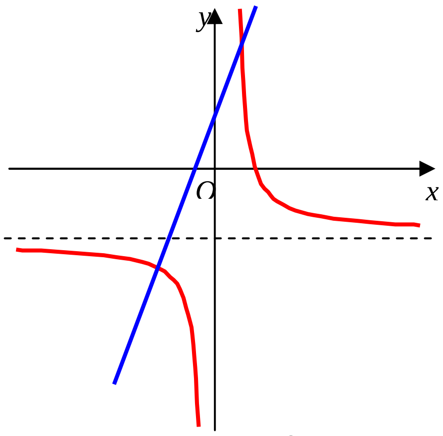
(5)

(b) Write down the equations of the asymptotes of the curve C.

(2)

(c) Find the coordinates of the points of intersection of $y = \dfrac{2}{x} - 5$ and $y = 4x + 2$

(5)

Question Number	Scheme		Marks
6. (a)		$y=\frac{2}{x}$ is translated up or down.	M1
		$y=\frac{2}{x}-5$ is in the correct position.	A1
		Intersection with x-axis at $\left(\frac{2}{5},\{0\}\right)$ **only** Independent mark.	B1
		$y=4x+2$: attempt at straight line, with positive gradient with positive y intercept.	B1
	Check graph in question for possible answers and space below graph for answers to part (b)	Intersection with x-axis at $\left(-\frac{1}{2},\{0\}\right)$ **and** y-axis at $\left(\{0\},2\right)$.	B1 **[5]**
(b)	Asymptotes : $x=0$ (or y-axis) and $y=-5$.	An asymptote stated correctly. Independent of (a)	B1
	(Lose second B mark for extra asymptotes)	These two lines only. Not ft their graph.	B1 **[2]**
(c)	Method 1: $\frac{2}{x}-5=4x+2$	Method 2: $\frac{y-2}{4}=\frac{2}{y+5}$	M1
	$4x^2+7x-2=0\Rightarrow x=$	$y^2+3y-18=0\rightarrow y=$	dM1
	$x=-2,\frac{1}{4}$	$y=-6,3$	A1
	When $x=-2,\ y=-6$, When $x=\frac{1}{4},\ y=3$	When $y=-6,\ x=-2$ When $y=3,\ x=\frac{1}{4}$.	M1A1 **[5]**
			12 marks

Notes

(a) **M1:** Curve implies y axis as asymptote and does not change shape significantly. Changed curve needs horizontal asymptote (roughly) Asymptote(s) need not be **shown** but shape of curve should be implying asymptote(s) parallel to x axis. Curve should not remain where it was in the given figure. Both sections move in the same direction. There should be no reflection

A1: Crosses positive x axis. Hyperbola has moved down. Both sections move by **almost** same amount. See sheet on page 19 for guidance.

B1: Check diagram and text of answer. Accept 2/5 or 0.4 shown on x -axis or x = 2/5, or (2/5 , 0) stated clearly in text or on graph. This is **independent** of the graph. Accept (0, 2/5) if clearly on x axis. Ignore any intersection points with y axis. Do not credit work in table of values for this mark.

B1: Must be attempt at astraight line, with positive gradient & with positive y intercept (need not cross x axis)

B1: Accept x = -1/2, or –0.5 shown on x -axis or (-1/2 , 0) or (-0.5,0) in text or on graph and similarly accept 2 on y axis or y = 2 or (0, 2) in text or on graph. **Need not cross curve and allow on separate axes.**

(b) **B1:** For either correct asymptote equation. Second **B1**: For both correct (lose this if extras e.g. $x=\pm1$ are given also). These asymptotes may follow correctly from equation after wrong graph in (a)

Just y = -5 **is B1 B0** **This may be awarded if given on the graph.** However for other B mark it must be clear that $x=0$ (or the y-axis) is an asymptote. NB $x\neq0,\ y\neq-5$ is B1B0

(c) **M1**: Either of these equations is enough for the method mark (May appear labelled as part (b))

dM1: Attempt to solve a 3 term quadratic by factorising, formula, completion of square or implied by correct answers. (see note 1) This mark depends on previous mark.

A1: Need both correct x answers (Accept equivalents e.g. 0.25) or both correct y values (Method 2)

M1: At least one attempt to find *second variable* (usually y) using their *first variable* (usually x) related to line meeting curve. Should not be substituting x or y values from part (a) or (b). This mark is **independent** of previous marks. Candidate may substitute in equation of line or equation of curve.

A1: Need both correct *second variable* answers Need not be written as co-ordinates (allow as in the scheme)

Note: Special case: Answer only with no working in part (c) can have 5 marks if completely correct, with **both** points found. If co-ordinates of just one of the points is correct – with no working – this earns M0 M0 A0 M1 A0 (i.e. 1 / 5)

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7. Lewis played a game of space invaders. He scored points for each spaceship that he captured.

Lewis scored 140 points for capturing his first spaceship.

He scored 160 points for capturing his second spaceship, 180 points for capturing his third spaceship, and so on.

The number of points scored for capturing each successive spaceship formed an arithmetic sequence.

(a) Find the number of points that Lewis scored for capturing his 20th spaceship.
(2)

(b) Find the total number of points Lewis scored for capturing his first 20 spaceships.
(3)

Sian played an adventure game. She scored points for each dragon that she captured. The number of points that Sian scored for capturing each successive dragon formed an arithmetic sequence.

Sian captured n dragons and the total number of points that she scored for capturing all n dragons was 8500.

Given that Sian scored 300 points for capturing her first dragon and then 700 points for capturing her nth dragon,

(c) find the value of n.
(3)

Question Number	Scheme	Marks
7.	Lewis; arithmetic series, $a = 140$, $d = 20$.	
(a)	$T_{20} = 140 + (20 - 1)(20); = 520$ Or lists 20 terms to get to 520	M1; A1
	OR 120 + (20)(20)	**[2]**
	Method 1 / Method 2	
(b)	**Either:** Uses $\frac{1}{2}n(2a + (n-1)d)$ / **Or:** Uses $\frac{1}{2}n(a + l)$	M1
	$\frac{20}{2}\left(2\times140 + (20-1)(20)\right)$ / $\frac{20}{2}\left(140 + \text{"520"}\right)$ ft 520	A1
	6600	A1
		[3]
(c)	Sian; arithmetic series, $a = 300$, $l = 700$, $S_n = 8500$	
	Either: Attempt to use $8500 = \frac{n}{2}(a + l)$ / **Or:** May use both $8500 = \frac{1}{2}n(2a + (n-1)d)$ and $l = a + (n-1)d$ **and eliminate** d	M1
	$8500 = \frac{n}{2}(300 + 700)$ / $8500 = \frac{n}{2}(600 + 400)$	A1
	$\Rightarrow n = 17$	A1
		[3]
		8 marks

	Notes
(a)	**M1:** Attempt to use formula for **20th term** of Arithmetic series with first term **140** and d = **20. Normal formula rules** apply – see General principles at the start of the mark scheme re "Method Marks" **Or:** uses 120 + 20n with n = 20 **Or: Listing method :** Lists 140, 160, 180, 200, 220, 240, 260, 280, … 520. M1A1 if correct M0A0 if wrong. (So 2 marks or zero) **A1:** For 520
(b)	**M1:** An attempt to apply $\frac{1}{2}n(2a + (n-1)d)$ **or** $\frac{1}{2}n(a + l)$ with their values for a, n, d and l **A1:** Uses a = 140, d = 20, n = 20 in their formula (two alternatives given above) but **ft on their value of** l from (a) if they use Method 2. **A1:** 6600 cao **Or: Listing method :** Lists 140, 160, 180, 200, 220, 240, 260, 280, … 520 and adds **6600** gets M1A1A1- any other answer gets M1 A0A0 provided there are 20 numbers, the first is 140 and the last is 520.
(c) **First method**	**M1:** Attempt to use $S_n = \frac{n}{2}(a + l)$ with their values for a, and l and S =8500 **A1:** Uses formula with correct values **A1:** Finds exact value 17
Alternative method	**M1:** If both formulae $8500 = \frac{1}{2}n(2a + (n-1)d)$ and $l = a + (n-1)d$ are used, then d **must be eliminated** before this mark is awarded by valid work. Should not be using d = 400. This would be **M0**. **A1:** Correct equation in n only then **A1** for 17 exactly **Trial and error methods:** Finds d = 25 and n = 17 and list from 300 to 700 with total checked – 3/3 BUT: Just n = 17 no working – send to review.

Leave blank

8.
$$\frac{dy}{dx} = -x^3 + \frac{4x - 5}{2x^3}, \qquad x \neq 0$$

Given that $y = 7$ at $x = 1$, find y in terms of x, giving each term in its simplest form.

(6)

Question Number	Scheme		Marks
8.	$(\frac{dy}{dx} =)\quad -x^3 + "2"x^{-2} - "\left(\frac{5}{2}\right)"x^{-3}$		M1
	$(y =)\quad -\frac{1}{4}x^4 + \frac{"2"x^{-1}}{(-1)} - "\left(\frac{5}{2}\right)"\frac{x^{-2}}{(-2)} (+c)$	Raises power correctly on any one term. Any two follow through terms correct.	M1 A1ft
	$(y =)\quad -\frac{1}{4}x^4 + \frac{2x^{-1}}{(-1)} - \frac{5}{2}\frac{x^{-2}}{(-2)} (+c)$	This is not follow through – must be correct	A1
	Given that $y = 7$, at $x = 1$, then $7 = -\frac{1}{4} - 2 + \frac{5}{4} + c \Rightarrow c =$		M1
	So, $(y =)\quad -\frac{1}{4}x^4 - 2x^{-1} + \frac{5}{4}x^{-2} + c$, $c = 8$ **or** $(y =) -\frac{1}{4}x^4 - 2x^{-1} + \frac{5}{4}x^{-2} + 8$		A1 **[6]**
			6 marks
	Notes		
	M1: Expresses as three term polynomial with powers 3, -2 and –3. Allow slips in coefficients. This may be implied by later integration having all three powers 4, -1 and -2. **M1:** An attempt to integrate at least one term so $x^n \to x^{n+1}$ (not a term in the numerator or denominator) **A1ft:** Any two integrations are correct – coefficients may be unsimplified (follow through errors in **coefficients only here**) so should have two of the powers 4, -1 and –2 after integration – depends on 2nd method mark only. There should be a maximum of three terms here. **A1:** Correct three terms – coefficients **may be unsimplified-** do not need constant for this mark Depends on both Method marks **M1: Need constant for this mark.** Uses $y = 7$ and $x = 1$ in their changed expression in order to find *c*, and attempt to find *c*. *This mark is available even after there is suggestion of differentiation.* **A1:** Need **all four** correct terms to be **simplified** and need $c = 8$ here.		

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9. The equation

$$(k+3)x^2 + 6x + k = 5, \text{ where } k \text{ is a constant,}$$

has two distinct real solutions for x.

(a) Show that k satisfies

$$k^2 - 2k - 24 < 0$$

(4)

(b) Hence find the set of possible values of k.

(3)

Question Number	Scheme	Marks
9. (a)	**Method 1:** Attempts $b^2 - 4ac$ for $a = (k+3)$, $b = 6$ and their c. $c \neq k$	M1
	$b^2 - 4ac = 6^2 - 4(k+3)(k-5)$	A1
	$(b^2 - 4ac =)$ $-4k^2 + 8k + 96$ or $-(b^2 - 4ac =)$ $4k^2 - 8k - 96$ (with no prior algebraic errors)	B1 (M1 on epen)
	As $b^2 - 4ac > 0$, then $-4k^2 + 8k + 96 > 0$ and so, $k^2 - 2k - 24 < 0$	A1 *
	Method 2: Considers $b^2 > 4ac$ for $a = (k+3)$, $b = 6$ and their c. $c \neq k$	M1
	$6^2 > 4(k+3)(k-5)$	A1
	$4k^2 - 8k - 96 < 0$ or $-4k^2 + 8k + 96 > 0$ or $9 > (k+3)(k-5)$ (with no prior algebraic errors)	B1 (M1 on epen)
	and so, $k^2 - 2k - 24 < 0$ following correct work	A1 *
		[4]
(b)	Attempts to solve $k^2 - 2k - 24 = 0$ to give $k =$ ($\Rightarrow$ Critical values, $k = 6, -4$.)	M1
	$k^2 - 2k - 24 < 0$ gives $-4 < k < 6$	M1 A1
		[3] **7 marks**
	Notes	
(a)	**Method 1: M1:** Attempts $b^2 - 4ac$ for $a = (k+3)$, $b = 6$ and their c. $c \neq k$ or uses quadratic formula and has this expression under square root. (ignore > 0, < 0 or = 0 for first 3 marks) **A1:** Correct expression for $b^2 - 4ac$ - need not be simplified (may be under root sign) **B1:** Uses algebra to manipulate result **without error** into **one of these** three term quadratics. Again may be under root sign in quadratic formula. (This mark is given as second M on epen). If inequality is used early in "proof" may see $4k^2 - 8k - 96 < 0$ and B1 would be given for $4k^2 - 8k - 96$ correctly stated. **A1:** Applies $b^2 - 4ac > 0$ correctly (**or writes** $b^2 - 4ac > 0$) to achieve the **result given** in the question. No errors should be seen. Any incorrect line of argument should be penalised here. There are several ways of reaching the answer; either multiplication of both sides of inequality by –1, or taking every term to other side of inequality. If doubtful send to review. **Need conclusion i.e. printed answer.** **Method 2: M1:** Allow $b^2 > 4ac$, $b^2 < 4ac$ or $b^2 = 4ac$ for $a = (k+3)$, $b = 6$ and their c. $c \neq k$ **A1:** Correct expressions on either side (ignore >, < or =). **B1:** Uses algebra to manipulate result into one of the two three term quadratics or divides both sides by 4 **again without error** **A1:** Produces result with no errors seen from initial consideration of $b^2 > 4ac$.	
(b)	**M1:** Uses factorisation, formula, completion of square method to find two values for k , or finds two **correct** answers with no obvious method **M1:** Their Lower Limit $< k <$ Their Upper Limit. Allow the M mark mark for $\leq$. (Allow k < upper and k > lower) **A1:** $-4 < k < 6$ Lose this mark for $\leq$ Allow (-4, 6) [not square brackets] or $k > -4$ **and** $k < 6$ (must be **and** not **or**) Can also use intersection symbol $\cap$ **NOT** $k > -4$, $k < 6$ (M1A0)	
	Special case : **In part (a)** uses $c = k$ instead of $k - 5$ - scores 0 . Allow $k + 5$ for method marks	
	Special Case: **In part (b)** Obtaining $-6 < k < 4$ This is a common wrong answer. Give M1 M1 A0 special case.	
	Special Case: **In part (b)** Use of x instead of k – M1M1A0	
	Special Case: $-4 < k < 6$ and $k < -4$, $k > 6$ both given is M0A0 for last two marks. Do not treat as isw.	

Leave blank

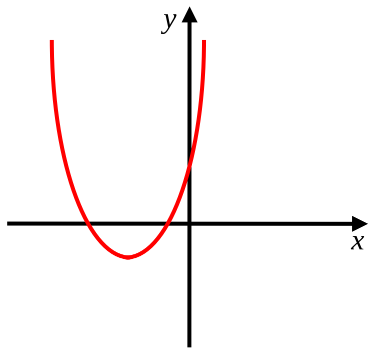
10. $$4x^2 + 8x + 3 \equiv a(x+b)^2 + c$$

(a) Find the values of the constants a, b and c.

(3)

(b) On the axes on page 27, sketch the curve with equation $y = 4x^2 + 8x + 3$, showing clearly the coordinates of any points where the curve crosses the coordinate axes.

(4)

Question Number	Scheme	Marks
10. (a)	This may be done by completion of square or by expansion and comparing coefficients	
	$a = 4$	B1 (M1 on epen)
	$b = 1$	B1 (A1 on epen)
	All three of $a = 4$, $b = 1$ and $c = -1$	B1 (A1 on epen) **[3]**
(b)	 U shaped quadratic graph.	M1
	The curve is correctly positioned with the minimum in the third quadrant. . It crosses x axis twice on negative x axis and y axis once on positive y axis.	A1
	Curve cuts y-axis at $(\{0\}, 3)$.only	B1
	Curve cuts x-axis at $\left(-\frac{3}{2}, \{0\}\right)$ and $\left(-\frac{1}{2}, \{0\}\right)$.	B1 **[4]**
		7 marks
	Notes	
(a)	**B1: (M1 on epen)** States $a = 4$ or obtains $4(x+b)^2 + c$, **B1: (A1 on epen)** States $b = 1$ or obtains $a(x+1)^2 + c$, **B1: (A1 on epen)** States $a = 4$, $b = 1$ **and** $c = -1$ or $4(x+1)^2 - 1$ (Needs all 3 correct for final mark) **Special cases:** If answer is left as $(2x+2)^2 - 1$ treat as misread B1B0B0 or as $2(x+1)^2 - 1$ then the mark is B0B1B0 from scheme	
(b)	**M1:** Any position provided U shaped (be generous in interpretation of U shape but V shape is M0) **A1 :** The curve is correctly positioned with the minimum in the third quadrant. It crosses x axis twice on negative x axis and y axis once on positive y axis. **B1:** Allow 3 on y axis and allow either $y = 3$ or (0 , 3) if given in text Curve does not need to pass through this point and this mark may be given even if there is no curve at all or if it is drawn as a line. **B1:** Allow –3/2 and –1/2 if given on x axis – need co-ordinates if given in text or x = -3/2, x = -1/2 . Accept decimal equivalents. Curve does not need to pass through these points and this mark may be given even if there is no curve. Ignore third point of intersection and allow touching instead of cutting. So even a cubic curve *might* get M0A0 B1 B1. A V shape with two ruled lines for example might get M0A0B1B1	

Leave blank

11. The curve C has equation

$$y = 2x - 8\sqrt{x} + 5, \quad x \geqslant 0$$

(a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$, giving each term in its simplest form.

(3)

The point P on C has x-coordinate equal to $\frac{1}{4}$

(b) Find the equation of the tangent to C at the point P, giving your answer in the form $y = ax + b$, where a and b are constants.

(4)

The tangent to C at the point Q is parallel to the line with equation $2x - 3y + 18 = 0$

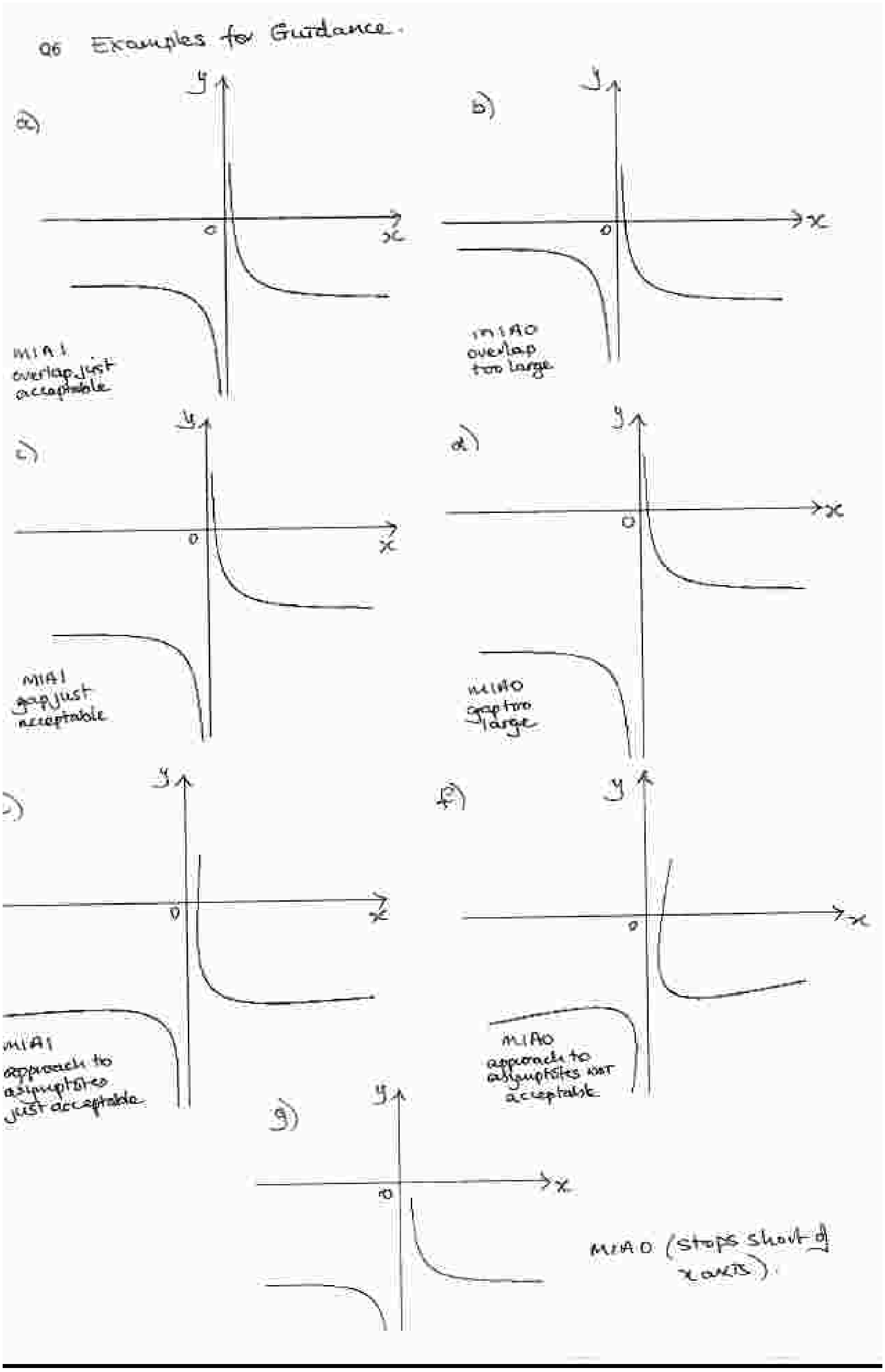
(c) Find the coordinates of Q.

(5)

Question Number	Scheme	Marks
11.	$C: y = 2x - 8\sqrt{x} + 5, \quad x \geqslant 0$	
(a)	So, $y = 2x - 8x^{\frac{1}{2}} + 5$	
	$\frac{dy}{dx} = 2 - 4x^{-\frac{1}{2}} + \{0\} \qquad (x > 0)$	M1 A1 A1
		[3]
(b)	(When $x = \frac{1}{4}$, $y = 2\left(\frac{1}{4}\right) - 8\sqrt{\left(\frac{1}{4}\right)} + 5$ so) $y = \frac{3}{2}$	B1
	(gradient $= \frac{dy}{dx} =$) $2 - \frac{4}{\sqrt{\left(\frac{1}{4}\right)}}$ $\{= -6\}$	M1
	Either : $y - "\frac{3}{2}" = "-6"\left(x - \frac{1}{4}\right)$ **or**: $y = "-6"x + c$ and $"\frac{3}{2}" = "-6"\left(\frac{1}{4}\right) + c \Rightarrow c = "3"$	dM1
	So $\underline{y = -6x + 3}$	A1
		[4]
(c)	Tangent at Q is parallel to $2x - 3y + 18 = 0$	
	($y = \frac{2}{3}x + 6 \Rightarrow$) Gradient $= \frac{2}{3}$. so tangent gradient is $\frac{2}{3}$	B1
	So, $"2 - \frac{4}{\sqrt{x}}" = "\frac{2}{3}"$ Sets their gradient function = their numerical gradient.	M1
	$\Rightarrow \frac{4}{3} = \frac{4}{\sqrt{x}} \Rightarrow x = 9$ Ignore extra answer x = -9	A1
	When $x = 9$, $y = 2(9) - 8\sqrt{9} + 5 = -1$ Substitutes their found x into equation of curve. $y = -1$.	dM1 A1
		[5]
		12 marks
	Notes	
(a)	**M1:** Evidence of differentiation, so $x^n \to x^{n-1}$ at least once so $x^1 \to 1$ or x^0 or $x^{\frac{1}{2}} \to x^{-\frac{1}{2}}$ not just $5 \to 0$ **A1:** Any two of the three terms correct – do not need to see zero – the 5 disappearing is sufficient; need not be simplified. **A1:** $2 - 4x^{-\frac{1}{2}}$ Both terms correct, and simplified. Do not need to include domain $x > 0$	
(b)	**B1:** Obtaining y = 3/2 or fractional or decimal equivalent (no working need be seen) **M1:** An attempt to substitute $x = \frac{1}{4}$ into $\frac{dy}{dx}$ to establish **gradient**. This may be implied by –6 or m = -6 but not y = - 6. Can earn this M mark if they go on to use $m = \frac{1}{6}$ or use their numerical value of $\frac{dy}{dx}$. **dM1: This depends on previous method mark.** Complete method for obtaining the equation of the tangent**,** using their **tangent** gradient and their value for y_1 (obtained from x = ¼, allow slip) i.e. $y - y_1 = m_T\left(x - \frac{1}{4}\right)$ with their tangent gradient and their y_1 or uses $y = mx + c$ with $\left(\frac{1}{4}, \text{ their } y_1\right)$ and their tangent gradient. **A1:** y = -6x + 3 or y= 3 – 6x or a = -6 and b = 3	
(c)	**B1:** For the value 2/3 **not** 2/3 x **not** -3/2 **M1:** Sets their gradient function dy/dx = their numerical gradient **A1:** Obtains x = 9 **dM1:** Substitutes their x (from gradient equation) into original equation of curve C i.e. original expression y = **A1:** (9, -1) or x = 9, y = -1, or just y = -1	
	Special Cases: In (b) Finds normal could get B1 M1 M0 A0 i.e. max of 2/4	
	In (c) Uses **perpendicular** instead of **parallel** then award B0 M1 A0 M1 A0 i.e max 2/5 – see over	

(c)	**Special case: Erroneous method** Tangent at Q is **perpendicular** to $2x - 3y + 18 = 0$	
	Uses – 3/2	B0
	So, $"2 - \frac{4}{\sqrt{x}}" = "-\frac{3}{2}"$ — Sets their gradient function = their numerical gradient.	M1
	$\Rightarrow \frac{7}{2} = \frac{4}{\sqrt{x}} \Rightarrow x = \frac{64}{49}$ **…**	A0
	When $x = \frac{64}{49}$, $y = 2\left(\frac{64}{49}\right) - 8 \times \frac{8}{7} + 5 = \ldots$ — Substitutes their found x into y.	dM1
	…	A0
		[2/5]

See next page for notes on graphs in qu 6:


Q6 Examples for Guidance.
a)
M1 A1
overlap just acceptable
b)
M1 A0
overlap too large
c)
M1 A1
gap just acceptable
d)
M1 A0
gap too large
e)
M1 A1
approach to asymptotes just acceptable
f)
M1 A0
approach to asymptotes NOT acceptable
g)
M1 A0 (stops short of x axis).