

June 2006
6663 Core Mathematics C1
Mark Scheme

Question number	Scheme	Marks
1.	$\frac{6x^3}{3} + 2x + \frac{x^{\frac{1}{2}}}{\frac{1}{2}} \quad (+c)$ $= 2x^3 + 2x + 2x^{\frac{1}{2}} + c$	<p>M1</p> <p>A1</p> <p>A1</p> <p>B1</p> <p style="text-align: right;">4</p>
	<p>M1 for some attempt to integrate $x^n \rightarrow x^{n+1}$</p> <p>1st A1 for either $\frac{6}{3}x^3$ or $\frac{x^{\frac{1}{2}}}{\frac{1}{2}}$ or better</p> <p>2nd A1 for all terms in x correct. Allow $2\sqrt{x}$ and $2x^1$.</p> <p>B1 for $+c$, when first seen with a changed expression.</p>	

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2. Find the set of values of x for which

$$x^2 - 7x - 18 > 0.$$

(4)

Q2

(Total 4 marks)



Question number	Scheme	Marks
2.	<p><u>Critical Values</u></p> <p>$(x \pm a)(x \pm b)$ with $ab=18$ or $x = \frac{7 \pm \sqrt{49 - 72}}{2}$ or $(x - \frac{7}{2})^2 \pm (\frac{7}{2})^2 - 18$</p> <p>$(x - 9)(x + 2)$ or $x = \frac{7 \pm 11}{2}$ or $x = \frac{7}{2} \pm \frac{11}{2}$</p> <p><u>Solving Inequality</u> $x > 9$ or $x < -2$ Choosing "outside"</p>	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>4</p>
	<p>1st M1 For attempting to find critical values. Factors alone are OK for M1, $x =$ appearing somewhere for the formula and as written for completing the square</p> <p>1st A1. Factors alone are OK. Formula or completing the square need $x =$ as written.</p> <p>2nd M1 For choosing outside region. Can f.t. their critical values. They must have two different critical values.</p> <p>- $2 > x > 9$ is M1A0 but ignore if it follows a correct version</p> <p>- $2 < x < 9$ is M0A0 whatever the diagram looks like.</p> <p>2nd A1 Use of \geq in final answer gets A0</p>	

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3. On separate diagrams, sketch the graphs of

(a) $y = (x + 3)^2$,

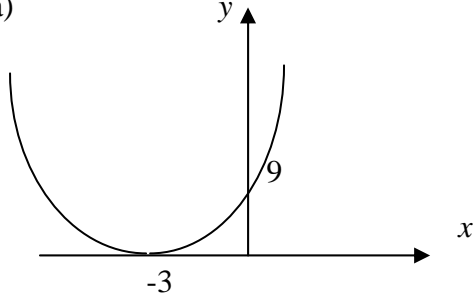
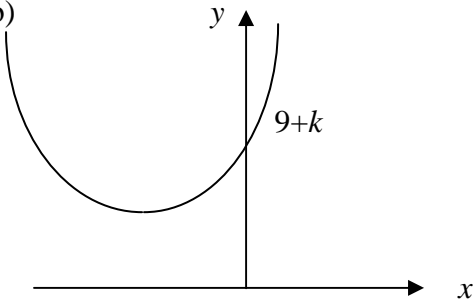
(3)

(b) $y = (x + 3)^2 + k$, where k is a positive constant.

(2)

Show on each sketch the coordinates of each point at which the graph meets the axes.



Question number	Scheme	Marks
3.	<p>(a)</p>  <p>(b)</p>  <p>U shape touching x-axis $(-3, 0)$ $(0, 9)$</p> <p>Translated parallel to y-axis up $(0, 9+k)$</p>	<p>B1 B1 B1 (3)</p> <p>M1 B1f.t. (2)</p> <p>5</p>
(a)	<p>2nd B1 2nd B1 & 3rd B1</p> <p>They can score this even if other intersections with the x-axis are given. The -3 and 9 can appear on the sketch as shown</p>	
(b)	<p>M1 B1f.t.</p> <p>Follow their curve in (a) up only. If it is not obvious do not give it. e.g. if it cuts y-axis in (a) but doesn't in (b) then it is M0. Follow through their 9</p>	

Question number	Scheme	Marks
<p>4. (a)</p> <p>(b)</p>	<p>$a_2 = 4$ $a_3 = 3 \times a_2 - 5 = 7$</p> <p>$a_4 = 3a_3 - 5 (= 16)$ and $a_5 = 3a_4 - 5 (= 43)$</p> <p>$3 + 4 + 7 + 16 + 43$ $= 73$</p>	<p>B1 B1f.t. (2)</p> <p>M1</p> <p>M1</p> <p>A1c.a.o. (3)</p> <p>5</p>
<p>(a)</p> <p>(b)</p>	<p>2nd B1f.t. Follow through their a_2 but it must be a value. $3 \times 4 - 5$ is B0 Give wherever it is first seen.</p> <p>1st M1 For two further attempts to use of $a_{n+1} = 3a_n - 5$, wherever seen. Condone arithmetic slips</p> <p>2nd M1 For attempting to add 5 relevant terms (i.e. terms derived from an attempt to use the recurrence formula) or an expression. Follow through their values for $a_2 - a_5$</p> <p>Use of formulae for arithmetic series is M0A0 but could get 1st M1 if a_4 and a_5 are correctly attempted.</p>	

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5. Differentiate with respect to x

(a) $x^4 + 6\sqrt{x}$,

(3)

(b) $\frac{(x+4)^2}{x}$.

(4)



Question number	Scheme	Marks
5. (a)	$(y = x^4 + 6x^{\frac{1}{2}} \Rightarrow y' =) 4x^3 + 3x^{-\frac{1}{2}} \quad \text{or} \quad 4x^3 + \frac{3}{\sqrt{x}}$	M1A1A1 (3)
(b)	$(x+4)^2 = x^2 + 8x + 16$ $\frac{(x+4)^2}{x} = x + 8 + 16x^{-1}$ (allow 4+4 for 8)	M1 A1
	$(y = \frac{(x+4)^2}{x} \Rightarrow y' =) 1 - 16x^{-2} \quad \text{o.e.}$	M1A1 (4) 7
(a)	M1 For some attempt to differentiate $x^n \rightarrow x^{n-1}$ 1 st A1 For one correct term as printed. 2 nd A1 For both terms correct as printed. $4x^3 + 3x^{-\frac{1}{2}} + c$ scores M1A1A0	
(b)	1 st M1 For attempt to expand $(x+4)^2$, must have x^2, x, x^0 terms and at least 2 correct e.g. $x^2 + 8x + 8$ or $x^2 + 2x + 16$ 1 st A1 Correct expression for $\frac{(x+4)^2}{x}$. As printed but allow $\frac{16}{x}$ and $8x^0$. 2 nd M1 For some correct differentiation, any term. Can follow through their simplification. N.B. $\frac{x^2 + 8x + 16}{x}$ giving rise to $(2x + 8)/1$ is M0A0	
ALT	<u>Product or Quotient rule (If in doubt send to review)</u> M2 For correct use of product or quotient rule. Apply usual rules on formulae. 1 st A1 For $\frac{2(x+4)}{x}$ or $\frac{2x(x+4)}{x^2}$ 2 nd A1 for $-\frac{(x+4)^2}{x^2}$	

Question number	Scheme	Marks
<p>6. (a)</p> <p>(b)</p>	<p>$16 + 4\sqrt{3} - 4\sqrt{3} - (\sqrt{3})^2$ or $16 - 3$ $= 13$</p> <p>$\frac{26}{4 + \sqrt{3}} \times \frac{4 - \sqrt{3}}{4 - \sqrt{3}}$</p> <p>$= \frac{26(4 - \sqrt{3})}{13} = \underline{8 - 2\sqrt{3}}$ or $8 + (-2)\sqrt{3}$ or $a = 8$ and $b = -2$</p>	<p>M1 A1c.a.o (2)</p> <p>M1 A1 (2)</p> <p>4</p>
<p>(a)</p> <p>(b)</p>	<p>M1 For 4 terms, at least 3 correct e.g. $8 + 4\sqrt{3} - 4\sqrt{3} - (\sqrt{3})^2$ or $16 \pm 8\sqrt{3} - (\sqrt{3})^2$ or $16 + 3$ 4^2 instead of 16 is OK $(4 + \sqrt{3})(4 + \sqrt{3})$ scores M0A0</p> <p>M1 For a correct attempt to rationalise the denominator Can be implied NB $\frac{-4 + \sqrt{3}}{-4 + \sqrt{3}}$ is OK</p>	

Question number	Scheme	Marks
7.	$a + (n - 1)d = k \qquad k = 9 \text{ or } 11$ $(u_{11} =) a + 10d = 9$ $\frac{n}{2}[2a + (n - 1)d] = 77 \text{ or } \frac{(a + l)}{2} \times n = 77 \qquad l = 9 \text{ or } 11$ $(S_{11} =) \frac{11}{2}(2a + 10d) = 77 \text{ or } \frac{(a + 9)}{2} \times 11 = 77$ <p>e.g. $a + 10d = 9$ or $a + 9 = 14$ $a + 5d = 7$</p> $a = 5 \text{ and } d = 0.4 \text{ or exact equivalent}$	<p>M1 A1c.a.o. M1 A1 M1 A1 A1</p> <p style="text-align: right;">7</p>
	<p>1st M1 Use of u_n to form a linear equation in a and d. $a + nd = 9$ is M0A0</p> <p>1st A1 For $a + 10d = 9$.</p> <p>2nd M1 Use of S_n to form an equation for a and d (LHS) or in a (RHS)</p> <p>2nd A1 A correct equation based on S_n. For 1st 2 Ms they must write n or use $n = 11$.</p> <p>3rd M1 Solving (LHS simultaneously) or (RHS a linear equation in a) Must lead to $a = \dots$ or $d = \dots$ and depends on one previous M</p> <p>3rd A1 for $a = 5$</p> <p>4th A1 for $d = 0.4$ (o.e.)</p> <p><u>ALT</u> Uses $\frac{(a + l)}{2} \times n = 77$ to get $a = 5$, gets second and third M1A1 i.e. 4/7 Then uses $\frac{n}{2}[2a + (n - 1)d] = 77$ to get d, gets 1st M1A1 and 4th A1</p> <p><u>MR</u> Consistent MR of 11 for 9 leading to $a = 3$, $d = 0.8$ scores M1A0M1A0M1A1ftA1ft</p>	

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8. The equation $x^2 + 2px + (3p + 4) = 0$, where p is a positive constant, has equal roots.

(a) Find the value of p .

(4)

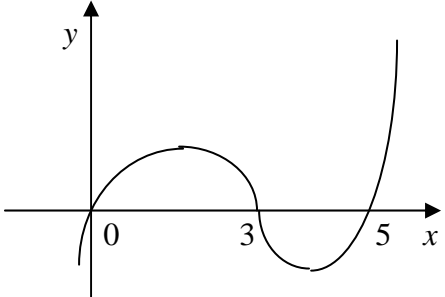
(b) For this value of p , solve the equation $x^2 + 2px + (3p + 4) = 0$.

(2)



Question number	Scheme	Marks
8. (a)	$b^2 - 4ac = 4p^2 - 4(3p + 4) = 4p^2 - 12p - 16 (=0)$ <p>or $(x + p)^2 - p^2 + (3p + 4) = 0 \Rightarrow p^2 - 3p - 4 (=0)$</p> $(p - 4)(p + 1) = 0$ $p = (-1 \text{ or } 4)$	M1, A1 M1 A1c.s.o. (4)
(b)	$x = \frac{-b}{2a} \text{ or } (x + p)(x + p) = 0 \Rightarrow x = \dots$ $x (= -p) = \underline{-4}$	M1 A1f.t. (2)
(a)	<p>1st M1 For use of $b^2 - 4ac$ or a full attempt to complete the square leading to a 3TQ in p. May use $b^2 = 4ac$. One of b or c must be correct.</p> <p>1st A1 For a correct 3TQ in p. Condone missing “=0” but all 3 terms must be on one side.</p> <p>2nd M1 For attempt to solve their 3TQ leading to $p = \dots$</p> <p>2nd A1 For $p = 4$ (ignore $p = -1$). $b^2 = 4ac$ leading to $p^2 = 4(3p + 4)$ and then "spotting" $p = 4$ scores 4/4.</p>	
(b)	<p>M1 For a full method leading to a repeated root $x = \dots$</p> <p>A1f.t. For $x = -4$ (- their p)</p> <p><u>Trial and Improvement</u></p> <p>M2 For substituting values of p into the equation and attempting to factorize. (Really need to get to $p = 4$ or -1)</p> <p>A2c.s.o. Achieve $p = 4$. Don't give without valid method being seen.</p>	

6

Question number	Scheme	Marks
9. (a)	$f(x) = x[(x-6)(x-2)+3]$ or $x^3 - 6x^2 - 2x^2 + 12x + 3x = x(x^2 - 8x + 15)$ $f(x) = x(x^2 - 8x + 15)$	M1 A1 both and $a = 1$ A1 (3)
(b)	$(x^2 - 8x + 15) = (x-5)(x-3)$ $f(x) = x(x-5)(x-3)$	M1 A1 (2)
(c)		Shape their 3 <u>or</u> their 5 B1f.t. both their 3 <u>and</u> their 5 and (0,0) by implication B1f.t. (3)
8		
(a)	M1 for a correct method to get the factor of x . $x($ as printed is the minimum. 1 st A1 for $b = -8$ or $c = 15$. -8 comes from $-6-2$ and must be coefficient of x , and 15 from $6x^2+3$ and must have no x s. 2 nd A1 for $a = 1$, $b = -8$ and $c = 15$. Must have $x(x^2 - 8x + 15)$.	
(b)	M1 for attempt to factorise their 3TQ from part (a). A1 for all 3 terms correct. They must include the x . For part (c) they must have <u>at most</u> 2 non-zero roots of their $f(x) = 0$ to fit their 3 and their 5.	
(c)	1 st B1 for correct shape (i.e. from bottom left to top right and two turning points.) 2 nd B1f.t. for crossing at their 3 or their 5 indicated on graph or in text. 3 rd B1f.t. if graph passes through (0, 0) [needn't be marked] and both their 3 and their 5.	

Question number	Scheme	Marks
10.(a)	$f(x) = \frac{2x^2}{2} + \frac{3x^{-1}}{-1} (+c)$ $(3, 7\frac{1}{2}) \text{ gives } \frac{15}{2} = 9 - \frac{3}{3} + c$ $c = -\frac{1}{2}$	<p>M1A1</p> <p>M1A1f.t.</p> <p>A1 (5)</p>
(b)	$f(-2) = 4 + \frac{3}{2} - \frac{1}{2} \quad (*)$	B1c.s.o. (1)
(c)	$m = -4 + \frac{3}{4}, = -3.25$ <p>Equation of tangent is: $y - 5 = -3.25(x + 2)$</p> <p><u>$4y + 13x + 6 = 0$</u></p>	<p>M1,A1</p> <p>M1</p> <p>A1 (4)</p> <p style="text-align: right;">o.e.</p>
10		
(a)	<p>1st M1 for some attempt to integrate $x^n \rightarrow x^{n+1}$</p> <p>1st A1 for both x terms as printed or better. Ignore $(+c)$ here.</p> <p>2nd M1 for use of $(3, 7\frac{1}{2})$ or $(-2, 5)$ to form an equation for c. There must be some correct substitution. No $+c$ is M0. Some changes in x terms of function needed.</p> <p>2nd A1f.t. for a correct equation for c. Follow through their integration. They must tidy up fraction/fraction and signs (e.g. - - to +).</p>	
(b)	<p>B1cso If $(-2, 5)$ is used to find c in (a) B0 here unless they verify $f(3)=7.5$.</p>	
(c)	<p>1st M1 for attempting $m = f'(\pm 2)$</p> <p>1st A1 for $-\frac{13}{4}$ or -3.25</p> <p>2nd M1 for attempting equation of tangent at $(-2, 5)$, f.t. their m, based on $\frac{dy}{dx}$.</p> <p>2nd A1 o.e. must have a, b and c integers and $= 0$.</p>	
Treat (a) and (b) together as a batch of 6 marks.		

Question number	Scheme	Marks
11.(a)	$m = \frac{8-2}{11+1} \quad (= \frac{1}{2})$ $y - 2 = \frac{1}{2}(x - -1) \quad \text{or} \quad y - 8 = \frac{1}{2}(x - 11) \quad \text{o.e.}$ $y = \frac{1}{2}x + \frac{5}{2} \quad \text{accept exact equivalents e.g. } \frac{6}{12}$ <p>(b) Gradient of $l_2 = -2$</p> <p>Equation of l_2: $y - 0 = -2(x - 10)$ [$y = -2x + 20$]</p> $\frac{1}{2}x + \frac{5}{2} = -2x + 20$ $\underline{x = 7 \quad \text{and} \quad y = 6}$ <p>(c) $RS^2 = (10 - 7)^2 + (0 - 6)^2 (= 3^2 + 6^2)$</p> $RS = \sqrt{45} = 3\sqrt{5} \quad (*)$ <p>(d) $PQ = \sqrt{12^2 + 6^2} = 6\sqrt{5}$ or $\sqrt{180}$ or $PS = 4\sqrt{5}$ and $SQ = 2\sqrt{5}$</p> $\text{Area} = \frac{1}{2}PQ \times RS = \frac{1}{2}6\sqrt{5} \times 3\sqrt{5}$ $\underline{= 45}$	<p>M1 A1</p> <p>M1</p> <p>A1c.a.o. (4)</p> <p>M1</p> <p>M1</p> <p>M1</p> <p>A1, A1 (5)</p> <p>M1</p> <p>A1c.s.o. (2)</p> <p>M1,A1</p> <p>dM1</p> <p>A1 c.a.o. (4)</p> <p>15</p>
(a)	<p>1st M1 for attempting $\frac{y_1 - y_2}{x_1 - x_2}$, must be y over x. No formula condone one sign slip, but if formula is quoted then there must be some correct substitution.</p> <p>1st A1 for a fully correct expression, needn't be simplified.</p> <p>2nd M1 for attempting to find equation of l_1.</p>	
(b)	<p>1st M1 for using the perpendicular gradient rule</p> <p>2nd M1 for attempting to find equation of l_2. Follow their gradient provided different.</p> <p>3rd M1 for forming a suitable equation to find S.</p>	
(c)	<p>M1 for expression for RS or RS^2. Ft their S coordinates</p>	
(d)	<p>1st M1 for expression for PQ or PQ^2. $PQ^2 = 12^2 + 6^2$ is M1 but $PQ = 12^2 + 6^2$ is M0</p> <p>Allow one numerical slip.</p> <p>2nd dM1 for a full, correct attempt at area of triangle. Dependent on previous M1.</p>	