Centre No.						Paper Reference							Surname	Initial(s)
Candidate No.						6	6	6	3	/	0	1	Signature	

Paper Reference(s)

6663/01

Edexcel GCE

Core Mathematics C1

Advanced Subsidiary

Monday 22 May 2006 – Morning

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
1	
2	
3	
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8	
9	
10	
11	
Total	

Materials required for examination
Mathematical Formulae (Green)

Items included with question papers
Nil

Calculators may NOT be used in this examination.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initial(s) and signature.
Check that you have the correct question paper.
You must write your answer for each question in the space following the question.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 11 questions in this question paper. The total mark for this paper is 75.
There are 24 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the examiner. Answers without working may gain no credit.

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1. Find $\int \left(6x^2 + 2 + x^{-\frac{1}{2}}\right) \mathrm{d}x$, giving each term in its simplest form.

(4)

Q1

(Total 4 marks)

June 2006
6663 Core Mathematics C1
Mark Scheme

Question number	Scheme	Marks
1.	$\frac{6x^3}{3} + 2x + \frac{x^{\frac{1}{2}}}{\frac{1}{2}}$ $(+c)$	M1
		A1
	$= 2x^3 + 2x + 2x^{\frac{1}{2}}$	A1
	$+c$	B1
		4

M1 for some attempt to integrate $x^n \rightarrow x^{n+1}$

1st A1 for either $\frac{6}{3}x^3$ or $\frac{x^{\frac{1}{2}}}{\frac{1}{2}}$ or better

2nd A1 for all terms in x correct. Allow $2\sqrt{x}$ and $2x^1$.

B1 for $+ c$, when first seen with a changed expression.

Leave blank

2. Find the set of values of x for which

$$x^2 - 7x - 18 > 0.$$

(4)

Q2

(Total 4 marks)

Question number	Scheme	Marks
2.	Critical Values	
	$(x \pm a)(x \pm b)$ with ab=18 or $x = \frac{7 \pm \sqrt{49 - -72}}{2}$ or $\left(x - \frac{7}{2}\right)^2 \pm \left(\frac{7}{2}\right)^2 - 18$	M1
	$(x - 9)(x + 2)$ or $x = \frac{7 \pm 11}{2}$ or $x = \frac{7}{2} \pm \frac{11}{2}$	A1
	Solving Inequality $x > 9$ or $x < -2$ Choosing "outside"	M1
		A1 **4**

1st M1 For attempting to find critical values.
Factors alone are OK for M1, x = appearing somewhere for the formula and as written for completing the square

1st A1. Factors alone are OK . Formula or completing the square need x = as written.

2nd M1 For choosing outside region. Can f.t. their critical values.
They must have two different critical values.

$-2 > x > 9$ is M1A0 but ignore if it follows a correct version

$-2 < x < 9$ is M0A0 whatever the diagram looks like.

2nd A1 Use of $\geq$ in final answer gets A0

Leave blank

3. On separate diagrams, sketch the graphs of

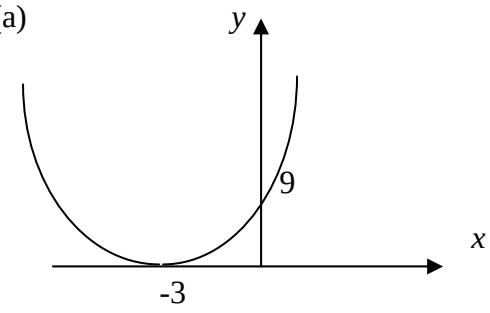
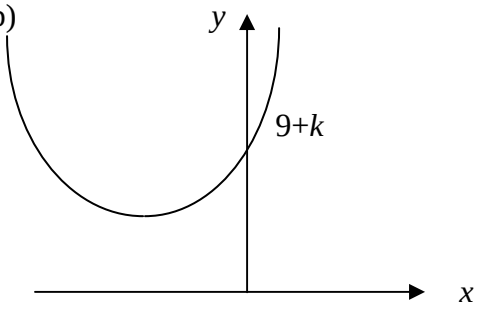
(a) $y = (x + 3)^2$,

(3)

(b) $y = (x + 3)^2 + k$, where k is a positive constant.

(2)

Show on each sketch the coordinates of each point at which the graph meets the axes.

Question number	Scheme	Marks
3.	(a)  U shape touching x-axis (- 3, 0) (0, 9)	B1 B1 B1 (3)
	(b)  Translated parallel to y-axis up (0, 9+k)	M1 B1f.t. (2) **5**

(a)	2nd B1	They can score this even if other intersections with the x-axis are given.
	2nd B1 & 3rd B1	The -3 and 9 can appear on the sketch as shown
(b)	M1	Follow their curve in (a) up only. If it is not obvious do not give it. e.g. if it cuts y-axis in (a) but doesn't in (b) then it is M0.
	B1f.t.	Follow through their 9

Leave blank

4. A sequence $a_1, a_2, a_3, \ldots$ is defined by

$$a_1 = 3,$$
$$a_{n+1} = 3a_n - 5, \quad n \geqslant 1.$$

(a) Find the value of a_2 and the value of a_3.

(2)

(b) Calculate the value of $\sum_{r=1}^{5} a_r$.

(3)

Question number	Scheme	Marks
4. (a)	$a_2 = 4$ $a_3 = 3 \times a_2 - 5 = 7$	B1 B1f.t. (2)
(b)	$a_4 = 3a_3 - 5 (= 16)$ and $a_5 = 3a_4 - 5 (= 43)$	M1
	3 + 4 + 7 + 16 + 43	M1
	= 73	A1c.a.o. (3)
		5

(a)	2nd B1f.t.	Follow through their a_2 but it must be a value. $3 \times 4 - 5$ is B0 Give wherever it is first seen.
(b)	1st M1	For two further attempts to use of $a_{n+1} = 3a_n - 5$, wherever seen. Condone arithmetic slips
	2nd M1	For attempting to add 5 relevant terms (i.e. terms derived from an attempt to use the recurrence formula) or an expression. Follow through their values for $a_2 - a_5$ Use of formulae for arithmetic series is M0A0 but could get 1st M1 if a_4 and a_5 are correctly attempted.

Leave blank

5. Differentiate with respect to x

(a) $x^4 + 6\sqrt{x}$,

(3)

(b) $\dfrac{(x+4)^2}{x}$.

(4)

Question number	Scheme	Marks
5. (a)	$(y = x^4 + 6x^{\frac{1}{2}} \Rightarrow y' =)\ 4x^3 + 3x^{-\frac{1}{2}}$ or $4x^3 + \frac{3}{\sqrt{x}}$	M1A1A1 (3)
(b)	$(x+4)^2 = x^2 + 8x + 16$	M1
	$\frac{(x+4)^2}{x} = x + 8 + 16x^{-1}$ (allow 4+4 for 8)	A1
	$(y = \frac{(x+4)^2}{x} \Rightarrow y' =)\ 1 - 16x^{-2}$ o.e.	M1A1 (4)
		7

(a) M1 For some attempt to differentiate $x^n \rightarrow x^{n-1}$

1st A1 For one correct term as printed.

2nd A1 For both terms correct as printed.

$4x^3 + 3x^{-\frac{1}{2}} + c$ scores M1A1A0

(b) 1st M1 For attempt to expand $(x+4)^2$, must have x^2, x, x^0 terms and at least 2 correct

e.g. $x^2 + 8x + 8$ or $x^2 + 2x + 16$

1st A1 Correct expression for $\frac{(x+4)^2}{x}$. As printed but allow $\frac{16}{x}$ and $8x^0$.

2nd M1 For some correct differentiation, any term. Can follow through their simplification.

N.B. $\frac{x^2 + 8x + 16}{x}$ giving rise to (2x + 8)/1 is M0A0

ALT <u>Product or Quotient rule</u> (If in doubt send to review)

M2 For correct use of product or quotient rule. Apply usual rules on formulae.

1st A1 For $\frac{2(x+4)}{x}$ or $\frac{2x(x+4)}{x^2}$

2nd A1 for $-\frac{(x+4)^2}{x^2}$

Leave blank

6. (a) Expand and simplify $(4 + \sqrt{3})(4 - \sqrt{3})$.

(2)

(b) Express $\frac{26}{4+\sqrt{3}}$ in the form $a + b\sqrt{3}$, where a and b are integers.

(2)

Question number	Scheme	Marks
6. (a)	$16+4\sqrt{3}-4\sqrt{3}-\left(\sqrt{3}\right)^2$ or $16-3$	M1
	= 13	A1c.a.o (2)
(b)	$\frac{26}{4+\sqrt{3}}\times\frac{4-\sqrt{3}}{4-\sqrt{3}}$	M1
	$=\frac{26(4-\sqrt{3})}{13}=\underline{8-2\sqrt{3}}$ or $8+(-2)\sqrt{3}$ or $a=8$ and $b=-2$	A1 (2)
		4

(a) M1 For 4 terms, at least 3 correct

e.g. $8+4\sqrt{3}-4\sqrt{3}-\left(\sqrt{3}\right)^2$ or $16\pm8\sqrt{3}-\left(\sqrt{3}\right)^2$ or 16 + 3

4^2 instead of 16 is OK

$(4+\sqrt{3})(4+\sqrt{3})$ scores M0A0

(b) M1 For a correct attempt to rationalise the denominator

Can be implied

NB $\frac{-4+\sqrt{3}}{-4+\sqrt{3}}$ is OK

Leave blank

7. An athlete prepares for a race by completing a practice run on each of 11 consecutive days. On each day after the first day, he runs further than he ran on the previous day. The lengths of his 11 practice runs form an arithmetic sequence with first term a km and common difference d km.

He runs 9 km on the 11th day, and he runs a total of 77 km over the 11 day period.

Find the value of a and the value of d.

(7)

Question number	Scheme	Marks
7.	$a+(n-1)d=k$ $\quad k$ = 9 or 11	M1
	$(u_{11}=)\ a+10d=9$	A1c.a.o.
	$\frac{n}{2}[2a+(n-1)d]=77$ or $\frac{(a+l)}{2}\times n=77$ $\quad l$ = 9 or 11	M1
	$(S_{11}=)\ \frac{11}{2}(2a+10d)=77$ or $\frac{(a+9)}{2}\times 11=77$	A1
	e.g. $a+10d=9$, $a+5d=7$ or $a+9=14$	M1
	$a=5$ and $d=0.4$ or exact equivalent	A1 A1
		7

1st M1	Use of u_n to form a linear equation in a and d. $a + nd$ =9 is M0A0
1st A1	For $a + 10d = 9$.
2nd M1	Use of S_n to form an equation for a and d (LHS) or in a (RHS)
2nd A1	A correct equation based on S_n.
	For 1st 2 Ms they must write n or use $n = 11$.
3rd M1	Solving (LHS simultaneously) or (RHS a linear equation in a)
	Must lead to a = … or d = …. and depends on one previous M
3rd A1	for $a = 5$
4th A1	for $d = 0.4$ (o.e.)
ALT	Uses $\frac{(a+l)}{2}\times n=77$ to get $a = 5$, gets second and third M1A1 i.e. 4/7
	Then uses $\frac{n}{2}[2a+(n-1)d]=77$ to get d, gets 1st M1A1 and 4th A1
MR	Consistent MR of 11 for 9 leading to $a = 3$, $d = 0.8$ scores M1A0M1A0M1A1ftA1ft

Leave blank

8. The equation $x^2 + 2px + (3p + 4) = 0$, where p is a positive constant, has equal roots.

(a) Find the value of p.

(4)

(b) For this value of p, solve the equation $x^2 + 2px + (3p + 4) = 0$.

(2)

Question number	Scheme	Marks
8. (a)	$b^2 - 4ac = 4p^2 - 4(3p+4) = 4p^2 - 12p - 16\ (=0)$	M1, A1
	or $(x+p)^2 - p^2 + (3p+4) = 0 \Rightarrow p^2 - 3p - 4(=0)$	
	$(p-4)(p+1) = 0$	M1
	$p = (-1 \text{ or } 4)$	A1c.s.o. (4)
(b)	$x = \frac{-b}{2a}$ or $(x+p)(x+p) = 0 \Rightarrow x = ...$	M1
	$x\ (= -p) = \underline{-4}$	A1f.t. (2)
		6

(a)

1st M1 For use of $b^2 - 4ac$ or a full attempt to complete the square leading to a 3TQ in p.
May use $b^2 = 4ac$. One of b or c must be correct.

1st A1 For a correct 3TQ in p. Condone missing "=0" but all 3 terms must be on one side.

2nd M1 For attempt to solve their 3TQ leading to $p = \ldots$

2nd A1 For $p = 4$ (ignore $p = -1$).

$b^2 = 4ac$ leading to $p^2 = 4(3p+4)$ and then "spotting" $p = 4$ scores 4/4.

(b)

M1 For a full method leading to a repeated root $x = \ldots$

A1f.t. For $x = -4$ (- their p)

Trial and Improvement

M2 For substituting values of p into the equation and attempting to factorize.
(Really need to get to $p = 4$ or -1)

A2c.s.o. Achieve $p = 4$. Don't give without valid method being seen.

Leave blank

9. Given that $f(x) = (x^2 - 6x)(x - 2) + 3x$,

(a) express $f(x)$ in the form $x(ax^2 + bx + c)$, where a, b and c are constants.

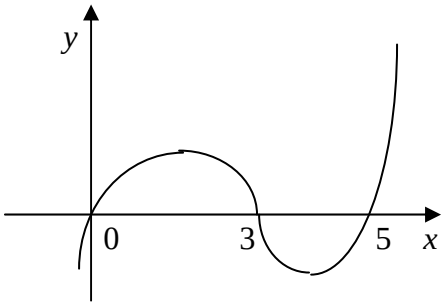
(3)

(b) Hence factorise $f(x)$ completely.

(2)

(c) Sketch the graph of $y = f(x)$, showing the coordinates of each point at which the graph meets the axes.

(3)

Question number	Scheme	Marks
9. (a)	$f(x) = x[(x-6)(x-2)+3]$ or $x^3-6x^2-2x^2+12x+3x = x($	M1
	$f(x) = x(x^2-8x+15)$ $b = -8$ or $c = 15$	A1
	both and $a = 1$	A1 (3)
(b)	$(x^2-8x+15) = (x-5)(x-3)$	M1
	$f(x) = x(x-5)(x-3)$	A1 (2)
(c)	 Shape	B1
	their 3 <u>or</u> their 5	B1f.t.
	<u>both</u> their 3 <u>and</u> their 5 and (0,0) by implication	B1f.t. (3)
		8

(a) M1 for a correct method to get the factor of x. $x($ as printed is the minimum.

1st A1 for $b = -8$ or $c = 15$.

-8 comes from -6-2 and must be coefficient of x, and 15 from 6x2+3 and must have no xs.

2nd A1 for $a = 1$, $b = -8$ and $c = 15$. Must have $x(x^2-8x+15)$.

(b) M1 for attempt to factorise their 3TQ from part (a).

A1 for all 3 terms correct. They must include the x.

For part (c) they must have <u>at most</u> 2 non-zero roots of their $f(x) = 0$ to ft their 3 and their 5.

(c) 1st B1 for correct shape (i.e. from bottom left to top right and two turning points.)

2nd B1f.t. for crossing at their 3 or their 5 indicated on graph or in text.

3rd B1f.t. if graph passes through (0, 0) [needn't be marked] and both their 3 and their 5.

Leave blank

10. The curve C with equation $y = f(x)$, $x \neq 0$, passes through the point $(3, 7\frac{1}{2})$.

Given that $f'(x) = 2x + \frac{3}{x^2}$,

(a) find $f(x)$.

(5)

(b) Verify that $f(-2) = 5$.

(1)

(c) Find an equation for the tangent to C at the point $(-2, 5)$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(4)

Question number	Scheme	Marks
10.(a)	$f(x) = \frac{2x^2}{2} + \frac{3x^{-1}}{-1}(+c)$ $-\frac{3}{x}$ is OK	M1A1
	$(3, 7\frac{1}{2})$ gives $\frac{15}{2} = 9 - \frac{3}{3} + c$ 3^2 or 3^{-1} are OK instead of 9 or $\frac{1}{3}$	M1A1f.t.
	$c = -\frac{1}{2}$	A1 (5)
(b)	$f(-2) = 4 + \frac{3}{2} - \frac{1}{2}$ (*)	B1c.s.o. (1)
(c)	m = - 4 + $\frac{3}{4}$, = -3.25	M1,A1
	Equation of tangent is: $y - 5 = -3.25(x + 2)$	M1
	$\underline{4y + 13x + 6 = 0}$ o.e.	A1 (4)
		10

(a)
1st M1 for some attempt to integrate $x^n \rightarrow x^{n+1}$
1st A1 for both x terms as printed or better. Ignore $(+c)$ here.
2nd M1 for use of $(3, 7\frac{1}{2})$ or (-2, 5) to form an equation for c. There must be some correct substitution. No $+c$ is M0. Some changes in x terms of function needed.
2nd A1f.t. for a correct equation for c. Follow through their integration. They must tidy up fraction/fraction and signs (e.g. - - to +).

(b)
B1cso If (-2, 5) is used to find c in (a) B0 here unless they verify f(3)=7.5.

(c)
1st M1 for attempting $m = f'(\pm 2)$
1st A1 for $-\frac{13}{4}$ or -3.25
2nd M1 for attempting equation of tangent at (-2, 5), f.t. their m, based on $\frac{dy}{dx}$.
2nd A1 o.e. must have a, b and c integers and = 0.

Treat (a) and (b) together as a batch of 6 marks.

Leave blank

11. The line l_1 passes through the points $P(-1, 2)$ and $Q(11, 8)$.

(a) Find an equation for l_1 in the form $y = mx + c$, where m and c are constants.

(4)

The line l_2 passes through the point $R(10, 0)$ and is perpendicular to l_1. The lines l_1 and l_2 intersect at the point S.

(b) Calculate the coordinates of S.

(5)

(c) Show that the length of RS is $3\sqrt{5}$.

(2)

(d) Hence, or otherwise, find the exact area of triangle PQR.

(4)

Question number	Scheme	Marks
11.(a)	$m = \frac{8-2}{11+1}$ $(=\frac{1}{2})$	M1 A1
	$y-2 = \frac{1}{2}(x--1)$ or $y-8 = \frac{1}{2}(x-11)$ o.e.	M1
	$y = \frac{1}{2}x + \frac{5}{2}$ accept exact equivalents e.g. $\frac{6}{12}$	A1c.a.o. (4)
(b)	Gradient of $l_2 = -2$	M1
	Equation of l_2: $y - 0 = -2(x - 10)$ [$y = -2x + 20$]	M1
	$\frac{1}{2}x + \frac{5}{2} = -2x + 20$	M1
	$x = 7$ and $y = 6$ depend on all 3 Ms	A1, A1 (5)
(c)	$RS^2 = (10-7)^2 + (0-6)^2 (= 3^2 + 6^2)$	M1
	$RS = \sqrt{45} = 3\sqrt{5}$ (*)	A1c.s.o. (2)
(d)	$PQ = \sqrt{12^2 + 6^2}, = 6\sqrt{5}$ or $\sqrt{180}$ or $PS = 4\sqrt{5}$ and $SQ = 2\sqrt{5}$	M1,A1
	Area $= \frac{1}{2} PQ \times RS = \frac{1}{2} 6\sqrt{5} \times 3\sqrt{5}$	dM1
	$= 45$	A1 c.a.o. (4)
		15

(a) 1st M1 for attempting $\frac{y_1 - y_2}{x_1 - x_2}$, must be y over x. No formula condone one sign slip, but if formula is quoted then there must be some correct substitution.
1st A1 for a fully correct expression, needn't be simplified.
2nd M1 for attempting to find equation of l_1.

(b) 1st M1 for using the perpendicular gradient rule
2nd M1 for attempting to find equation of l_2. Follow their gradient provided different.
3rd M1 for forming a suitable equation to find S.

(c) M1 for expression for RS or RS^2. Ft their S coordinates

(d) 1st M1 for expression for PQ or PQ^2. $PQ^2 = 12^2 + 6^2$ is M1 but $PQ = 12^2 + 6^2$ is M0 Allow one numerical slip.
2nd dM1 for a full, correct attempt at area of triangle. Dependent on previous M1.