

Leave blank

1. Find the value of

(a) $25^{\frac{1}{2}}$

(1)

(b) $25^{-\frac{3}{2}}$

(2)

Lined area for student answers, consisting of approximately 20 horizontal lines.

(Total 3 marks)

Q1

Small empty rectangular box for marking.



June 2011
Core Mathematics C1 6663
Mark Scheme

Question Number	Scheme	Marks
1. (a)	5 (or ± 5)	B1 (1)
(b)	$25^{-\frac{3}{2}} = \frac{1}{25^{\frac{3}{2}}}$ or $25^{\frac{3}{2}} = 125$ or better $\frac{1}{125}$ or 0.008 (or $\pm \frac{1}{125}$)	M1 A1 (2) 3
Notes		
(a) Give B1 for 5 or ± 5 Anything else is B0 (including just -5) (b) M: Requires reciprocal OR $25^{\frac{3}{2}} = 125$ Accept $\frac{1}{5^3}$, $\frac{1}{\sqrt{15625}}$, $\frac{1}{25 \times 5}$, $\frac{1}{25\sqrt{25}}$, $\frac{1}{\sqrt{25^3}}$ for M1 Correct answer with no working (or notation errors in working) scores both marks i.e. M1 A1 M1A0 for $-\frac{1}{125}$ without $+\frac{1}{125}$		

Leave blank

2. Given that $y = 2x^5 + 7 + \frac{1}{x^3}$, $x \neq 0$, find, in their simplest form,

(a) $\frac{dy}{dx}$, (3)

(b) $\int y \, dx$. (4)

Handwritten area with horizontal lines for student response.



Question Number	Scheme	Marks
<p>2.</p> <p>(a)</p>	$\frac{dy}{dx} = 10x^4 - 3x^{-4} \quad \text{or} \quad 10x^4 - \frac{3}{x^4}$	<p>M1 A1 A1</p> <p>(3)</p>
<p>(b)</p>	$\left(\int =\right) \frac{2x^6}{6} + 7x + \frac{x^{-2}}{-2} = \frac{x^6}{3} + 7x - \frac{x^{-2}}{2} + C$	<p>M1 A1 A1</p> <p>B1</p> <p>(4)</p> <p>7</p>
<p style="text-align: center;"><u>Notes</u></p> <p>(a) M1: Attempt to differentiate $x^n \rightarrow x^{n-1}$ (for any of the 3 terms) i.e. ax^4 or ax^{-4}, where a is any non-zero constant or the 7 differentiated to give 0 is sufficient evidence for M1 1st A1: One correct (non-zero) term, possibly unsimplified. 2nd A1: Fully correct simplified answer.</p> <p>(b) M1: Attempt to integrate $x^n \rightarrow x^{n+1}$ (i.e. ax^6 or ax or ax^{-2}, where a is any non-zero constant). 1st A1: Two correct terms, possibly unsimplified. 2nd A1: All three terms correct and simplified.</p> <p>Allow correct equivalents to printed answer, e.g. $\frac{x^6}{3} + 7x - \frac{1}{2x^2}$ or $\frac{1}{3}x^6 + 7x - \frac{1}{2}x^{-2}$</p> <p>Allow $\frac{1x^6}{3}$ or $7x^1$</p> <p>B1: + C appearing at any stage in part (b) (independent of previous work)</p>		

Question Number	Scheme	Marks
3.	<p>Mid-point of PQ is $(4, 3)$</p> $PQ: m = \frac{0-6}{9-(-1)}, \left(= -\frac{3}{5} \right)$ <p>Gradient perpendicular to $PQ = -\frac{1}{m} \left(= \frac{5}{3} \right)$</p> $y-3 = \frac{5}{3}(x-4)$ $5x-3y-11=0 \text{ or } 3y-5x+11=0 \text{ or multiples e.g. } 10x-6y-22=0$	<p>B1</p> <p>B1</p> <p>M1</p> <p>M1</p> <p>A1 (5)</p> <p>5</p>
	<p style="text-align: center;">Notes</p> <p>B1: correct midpoint. B1: correct numerical expression for gradient – need not be simplified 1st M: Negative reciprocal of their numerical value for m 2nd M: Equation of a line through their $(4, 3)$ with any gradient except 0 or ∞.</p> <p>If the 4 and 3 are the wrong way round the 2nd M mark can still be given if a correct formula (e.g. $y - y_1 = m(x - x_1)$) is seen, otherwise M0. If $(4, 3)$ is substituted into $y = mx + c$ to find c, the 2nd M mark is for attempting this.</p> <p>A1: Requires integer form with an = zero (see examples above)</p>	

Leave blank

Question 3 continued

Lined area for writing the answer to Question 3 continued.

(Total 5 marks)

Q3



Question Number	Scheme	Marks		
<p>4.</p>	<table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 50%; border-right: 1px solid black; padding: 5px;"> <p>Either</p> $y^2 = 4 - 4x + x^2$ $4(4 - 4x + x^2) - x^2 = 11$ <p>or $4(2 - x)^2 - x^2 = 11$</p> $3x^2 - 16x + 5 = 0$ $(3x - 1)(x - 5) = 0, \quad x =$ $x = \frac{1}{3} \quad x = 5$ $y = \frac{5}{3} \quad y = -3$ </td> <td style="width: 50%; padding: 5px;"> <p>Or</p> $x^2 = 4 - 4y + y^2$ $4y^2 - (4 - 4y + y^2) = 11$ <p>or $4y^2 - (2 - y)^2 = 11$</p> $3y^2 + 4y - 15 = 0 \quad \text{Correct 3 terms}$ $(3y - 5)(y + 3) = 0, \quad y = \dots$ $y = \frac{5}{3} \quad y = -3$ $x = \frac{1}{3} \quad x = 5$ </td> </tr> </table>	<p>Either</p> $y^2 = 4 - 4x + x^2$ $4(4 - 4x + x^2) - x^2 = 11$ <p>or $4(2 - x)^2 - x^2 = 11$</p> $3x^2 - 16x + 5 = 0$ $(3x - 1)(x - 5) = 0, \quad x =$ $x = \frac{1}{3} \quad x = 5$ $y = \frac{5}{3} \quad y = -3$	<p>Or</p> $x^2 = 4 - 4y + y^2$ $4y^2 - (4 - 4y + y^2) = 11$ <p>or $4y^2 - (2 - y)^2 = 11$</p> $3y^2 + 4y - 15 = 0 \quad \text{Correct 3 terms}$ $(3y - 5)(y + 3) = 0, \quad y = \dots$ $y = \frac{5}{3} \quad y = -3$ $x = \frac{1}{3} \quad x = 5$	<p>M1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>M1 A1</p> <p style="text-align: right;">(7) 7</p>
<p>Either</p> $y^2 = 4 - 4x + x^2$ $4(4 - 4x + x^2) - x^2 = 11$ <p>or $4(2 - x)^2 - x^2 = 11$</p> $3x^2 - 16x + 5 = 0$ $(3x - 1)(x - 5) = 0, \quad x =$ $x = \frac{1}{3} \quad x = 5$ $y = \frac{5}{3} \quad y = -3$	<p>Or</p> $x^2 = 4 - 4y + y^2$ $4y^2 - (4 - 4y + y^2) = 11$ <p>or $4y^2 - (2 - y)^2 = 11$</p> $3y^2 + 4y - 15 = 0 \quad \text{Correct 3 terms}$ $(3y - 5)(y + 3) = 0, \quad y = \dots$ $y = \frac{5}{3} \quad y = -3$ $x = \frac{1}{3} \quad x = 5$			
	<p style="text-align: center;">Notes</p> <p>1st M: Squaring to give 3 or 4 terms (need a middle term)</p> <p>2nd M: Substitute to give quadratic in one variable (may have just two terms)</p> <p>3rd M: Attempt to solve a 3 term quadratic.</p> <p>4th M: Attempt to find at least one y value (or x value). (The second variable)</p> <p>This will be by substitution or by starting again.</p> <p>If y solutions are given as x values, or vice-versa, penalise accuracy, so that it is possible to score M1 M1A1 M1 A0 M1 A0.</p> <p><u>“Non-algebraic” solutions:</u></p> <p>No working, and only one correct solution pair found (e.g. $x = 5, y = -3$): M0 M0 A0 M1 A0 M1 A0</p> <p>No working, and both correct solution pairs found, but not demonstrated: M0 M0 A0 M1 A1 M1 A1</p> <p>Both correct solution pairs found, and demonstrated: Full marks are possible (send to review)</p>			

Leave blank

5. A sequence a_1, a_2, a_3, \dots is defined by

$$\begin{aligned} a_1 &= k, \\ a_{n+1} &= 5a_n + 3, \quad n \geq 1, \end{aligned}$$

where k is a positive integer.

(a) Write down an expression for a_2 in terms of k .

(1)

(b) Show that $a_3 = 25k + 18$.

(2)

(c) (i) Find $\sum_{r=1}^4 a_r$ in terms of k , in its simplest form.

(ii) Show that $\sum_{r=1}^4 a_r$ is divisible by 6.

(4)



Question Number	Scheme	Marks
5. (a)	$(a_2 =) 5k + 3$	B1 (1)
(b)	$(a_3 =) 5(5k + 3) + 3$ $= 25k + 18$ (*)	M1 A1 cso (2)
(c) (i)	$a_4 = 5(25k + 18) + 3$ (= $125k + 93$) $\sum_{r=1}^4 a_r = k + (5k + 3) + (25k + 18) + (125k + 93)$ $= 156k + 114$	M1 M A A : ao (4)
(ii)	$= 6(26k + 19)$ (or explain each term is divisible by 6)	7
Notes		
<p>(a) $5k + 3$ must be seen in (a) to gain the mark</p> <p>(b) 1st M: Substitutes their a_2 into $5a_2 + 3$ - note the answer is given so working must be seen.</p> <p>(c) 1st M1: Substitutes their a_3 into $5a_3 + 3$ or uses $125k + 93$</p> <p>2nd M1: for their sum $k + a_2 + a_3 + a_4$ - must see evidence of four terms with plus signs and must not be sum of AP</p> <p>1st A1: All correct so far</p> <p>2nd A1ft: Limited ft – previous answer must be divisible by 6 (eg $156k + 42$). This is dependent on second M mark in (c)</p> <p>Allow $\frac{156k + 114}{6} = 26k + 19$ without explanation. No conclusion is needed.</p>		

Leave blank

6. Given that $\frac{6x+3x^{\frac{5}{2}}}{\sqrt{x}}$ can be written in the form $6x^p + 3x^q$,

(a) write down the value of p and the value of q .

(2)

Given that $\frac{dy}{dx} = \frac{6x+3x^{\frac{5}{2}}}{\sqrt{x}}$, and that $y = 90$ when $x = 4$,

(b) find y in terms of x , simplifying the coefficient of each term.

(5)



Question Number	Scheme	Marks
<p>6.</p> <p>(a)</p>	$p = \frac{1}{2}, q = 2 \quad \text{or} \quad 6x^{\frac{1}{2}}, 3x^2$	<p>B1, B1</p> <p>(2)</p>
<p>(b)</p>	$\frac{6x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)} + \frac{3x^3}{3} \quad \left(= 4x^{\frac{3}{2}} + x^3 \right)$ <p>$x = 4, y = 90: 32 + 64 + C = 90 \Rightarrow C = -6$</p> <p>$y = 4x^{\frac{3}{2}} + x^3 + \text{"their"} - 6$</p>	<p>M1 A1ft</p> <p>M1 A1</p> <p>A1</p> <p>(5)</p> <p>7</p>
Notes		
<p>(a) Accept any equivalent answers, e.g. $p = 0.5, q = 4/2$</p> <p>(b) 1st M: Attempt to integrate $x^n \rightarrow x^{n+1}$ (for either term)</p> <p>1st A: fit their p and q, but terms need not be simplified (+C not required for this mark)</p> <p>2nd M: Using $x = 4$ <u>and</u> $y = 90$ to form an equation in C.</p> <p>2nd A: cao</p> <p>3rd A: answer as shown with simplified correct coefficients and powers – but follow through their value for C</p> <p>If there is a 'restart' in part (b) it can be marked independently of part (a), but marks for part (a) cannot be scored for work seen in (b).</p> <p><u>Numerator and denominator integrated separately:</u></p> <p>First M mark cannot be awarded so only mark available is second M mark. So 1 out of 5 marks.</p>		

Question Number	Scheme	Marks
7. (a)	Discriminant: $b^2 - 4ac = (k + 3)^2 - 4k$ or equivalent	M1 A1 (2)
(b)	$(k + 3)^2 - 4k = k^2 + 2k + 9 = (k + 1)^2 + 8$	M1 A1 (2)
(c)	For real roots, $b^2 - 4ac \geq 0$ or $b^2 - 4ac > 0$ or $(k + 1)^2 + 8 > 0$ $(k + 1)^2 \geq 0$ for all k , so $b^2 - 4ac > 0$, so roots are real for all k (or equiv.)	M1 A1 cso (2) 6
Notes		
<p>(a) M1: attempt to find discriminant – substitution is required If formula $b^2 - 4ac$ is seen at least 2 of a, b and c must be correct If formula $b^2 - 4ac$ is not seen all 3 of a, b and c must be correct Use of $b^2 + 4ac$ is M0 A1: correct unsimplified</p> <p>(b) M1: Attempt at completion of square (see earlier notes) A1: both correct (no ft for this mark)</p> <p>(c) M1: States condition as on scheme or attempts to explain that their $(k + 1)^2 + 8$ is greater than 0 A1: The final mark (A1cso) requires $(k + 1)^2 \geq 0$ and conclusion. We will allow $(k + 1)^2 > 0$ (or word positive) also allow $b^2 - 4ac \geq 0$ and conclusion.</p>		

8.

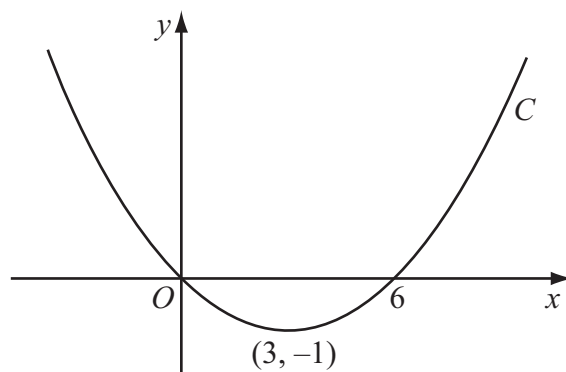
**Figure 1**

Figure 1 shows a sketch of the curve C with equation $y = f(x)$.
The curve C passes through the origin and through $(6, 0)$.
The curve C has a minimum at the point $(3, -1)$.

On separate diagrams, sketch the curve with equation

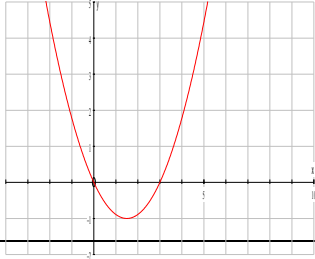

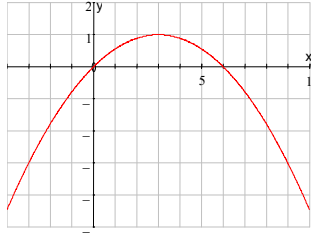

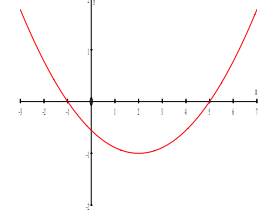

(a) $y = f(2x)$, **(3)**

(b) $y = -f(x)$, **(3)**

(c) $y = f(x + p)$, where p is a constant and $0 < p < 3$. **(4)**

On each diagram show the coordinates of any points where the curve intersects the x -axis and of any minimum or maximum points.



Question Number	Scheme	Marks
<p>8. (a)</p>	 <p>Shape  through (0, 0) (3, 0) (1.5, -1)</p>	<p>B1 B1 B1 (3)</p>
<p>(b)</p>	 <p>Shape  (0, 0) and (6, 0) (3, 1)</p>	<p>B1 B1 B1 (3)</p>
<p>(c)</p>	 <p>Shape , <u>not</u> through (0, 0) Minimum in 4th quadrant (-p, 0) and (6 - p, 0) (3 - p, -1)</p>	<p>M1 A1 B1 B1 (4) 10</p>
Notes		
<p>(a) B1: U shaped parabola through origin B1: (3,0) stated or 3 labelled on x axis B1: (1.5, -1) or equivalent e.g. (3/2, -1) (b) B1: Cap shaped parabola in any position B1: through origin (may not be labelled) and (6,0) stated or 6 labelled on x - axis B1: (3,1) shown (c) M1: U shaped parabola not through origin A1: Minimum in 4th quadrant (depends on M mark having been given) B1: Coordinates stated or shown on x axis B1: Coordinates stated Note: If values are taken for p, then it is possible to give M1A1B0B0 even if there are several attempts. (In this case all minima should be in fourth quadrant)</p>		

Question Number	Scheme	Marks
<p>9.</p> <p>(a)</p>	<p>Series has 50 terms</p> $S = \frac{1}{2}(50)(2 + 100) = 2550 \quad \text{or} \quad S = \frac{1}{2}(50)(4 + 49 \times 2) = 2550$	<p>B1</p> <p>M1 A1</p> <p>(3)</p>
<p>(b)</p> <p>(i)</p> <p>(ii)</p>	$\frac{100}{k}$ <p>Sum: $\frac{1}{2}\left(\frac{100}{k}\right)(k + 100)$ or $\frac{1}{2}\left(\frac{100}{k}\right)\left(2k + \left(\frac{100}{k} - 1\right)k\right)$</p> $= 50 + \frac{5000}{k} \quad (*)$	<p>B1</p> <p>M1 A1</p> <p>A1 cso</p> <p>(4)</p>
<p>(c)</p>	$50^{\text{th}} \text{ term} = a + (n - 1)d$ $= (2k + 1) + 49(2k + 3)$ $= 100k + 148$ <div style="display: inline-block; vertical-align: middle; border-left: 1px solid black; padding-left: 10px;"> <p>Or $2k + 49(2k) + 1 + 49(3)$</p> $= 100k + 148$ </div>	<p>M1</p> <p>A1</p> <p>(2)</p> <p>9</p>
<p style="text-align: center;">Notes</p> <p>(a) B for seeing attempt to use $n = 50$ or $n = 50$ stated M for attempt to use $\frac{1}{2}n(a + l)$ or $\frac{1}{2}n(2a + (n - 1)d)$ with $a = 2$ and values for other variables (Using $n = 100$ may earn B0 M1A0)</p> <p>(b) M for use of $a = k$ and $d = k$ or $l = 100$ with their value for n, could be numerical or even letter n in correct formula for sum. A1: Correct formula with $n = 100/k$ A1: NB Answer is printed – so no slips should have appeared in working</p> <p>(c) M for use of formula $a + 49d$ with $a = 2k + 1$ and with d obtained from difference of terms A1: Requires this simplified answer</p>		

Leave
blank

10. The curve C has equation

$$y = (x+1)(x+3)^2$$

(a) Sketch C , showing the coordinates of the points at which C meets the axes. (4)

(b) Show that $\frac{dy}{dx} = 3x^2 + 14x + 15$. (3)


The point A , with x -coordinate -5 , lies on C .

(c) Find the equation of the tangent to C at A , giving your answer in the form $y = mx + c$, where m and c are constants. (4)

Another point B also lies on C . The tangents to C at A and B are parallel.

(d) Find the x -coordinate of B . (3)



Question Number	Scheme	Marks
<p>10. (a)</p>	 <p>Shape (cubic in this orientation) Touching x-axis at -3 Crossing at -1 on x-axis Intersection at 9 on y-axis</p>	<p>B1 B1 B1 B1 (4)</p>
<p>(b)</p>	<p>$y = (x + 1)(x^2 + 6x + 9) = x^3 + 7x^2 + 15x + 9$ or equiv. (possibly unsimplified) Differentiates their polynomial correctly – may be unsimplified $\frac{dy}{dx} = 3x^2 + 14x + 15$ (*)</p>	<p>B1 M1 A1 cso (3)</p>
<p>(c)</p>	<p>At $x = -5$: $\frac{dy}{dx} = 75 - 70 + 15 = 20$ At $x = -5$: $y = -16$ $y - (-16) = 20(x - (-5))$ or $y = 20x + c$ with $(-5, -16)$ used to find c $y = 20x + 84$</p>	<p>B1 B1 M1 A1 (4)</p>
<p>(d)</p>	<p>Parallel: $3x^2 + 14x + 15 = 20$ $(3x - 1)(x + 5) = 0$ $x = \dots$ $x = \frac{1}{3}$</p>	<p>M1 M1 A1 (3) 14</p>
<p style="text-align: center;">Notes</p> <p>(a) Crossing at -3 is B0. Touching at -1 is B0 (b) M: This needs to be correct differentiation here A1: Fully correct simplified answer. (c) M: If the -5 and -16 are the wrong way round or – omitted the M mark can still be given if a correct formula is seen, (e.g. $y - y_1 = m(x - x_1)$) otherwise M0. m should be numerical and not 0 or infinity and should not have involved negative reciprocal. (d) 1st M: Putting the derivative expression equal to their value for gradient 2nd M: Attempt to solve quadratic (see notes) This may be implied by correct answer.</p>		