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| Surname | Other names |
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**Pearson Edexcel GCE**

| Centre Number | Candidate Number |
|---|---|
| | |

# Core Mathematics C1

## Advanced Subsidiary

Wednesday 17 May 2017 – Morning
**Time: 1 hour 30 minutes**

Paper Reference
**6663/01**

**You must have:**
Mathematical Formulae and Statistical Tables (Pink)

Total Marks

**Calculators may NOT be used in this examination.**

## Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.

## Information

- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

## Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

*Turn over* ▶

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**1.** Find

$$\int \left( 2x^5 - \frac{1}{4x^3} - 5 \right) \mathrm{d}x$$

giving each term in its simplest form.

**(4)**

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| Question Number | Scheme | | Marks |
|---|---|---|---|
| **1.** | $\int\left(2x^5-\frac{1}{4}x^{-3}-5\right)\mathrm{d}x$ | | |
| | **Ignore any spurious integral signs throughout** | | |
| | $x^n \to x^{n+1}$ | Raises any of their powers by 1. E.g. $x^5 \to x^6$ or $x^{-3} \to x^{-2}$ or $k \to kx$ or $x^{\text{their}\,n} \to x^{\text{their}\,n+1}$. Allow the powers to be un-simplified e.g. $x^5 \to x^{5+1}$ or $x^{-3} \to x^{-3+1}$ or $kx^0 \to kx^{0+1}$. | M1 |
| | $2\times\frac{x^{5+1}}{6}$ or $-\frac{1}{4}\times\frac{x^{-3+1}}{-2}$ | Any one of the first two terms correct **<u>simplified or un-simplified</u>**. | A1 |
| | Two of: $\frac{1}{3}x^6,\ \frac{1}{8}x^{-2},\ -5x$ | Any two correct **<u>simplified</u>** terms. Accept $+\frac{1}{8x^2}$ for $+\frac{1}{8}x^{-2}$ but not $x^1$ for $x$. Accept 0.125 for $\frac{1}{8}$ but $\frac{1}{3}$ would clearly need to be identified as 0.3 recurring. | A1 |
| | $\frac{1}{3}x^6+\frac{1}{8}x^{-2}-5x+c$ | All correct and simplified and including + $c$ all on one line. Accept $+\frac{1}{8x^2}$ for $+\frac{1}{8}x^{-2}$ but not $x^1$ for $x$. Apply isw here. | A1 |
| | | | **(4 marks)** |

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**2.** Given

$$y = \sqrt{x} + \frac{4}{\sqrt{x}} + 4, \qquad x > 0$$

find the value of $\frac{dy}{dx}$ when $x = 8$, writing your answer in the form $a\sqrt{2}$, where $a$ is a rational number.

**(5)**

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| Question Number | Scheme | | Marks |
|---|---|---|---|
| **2.** | $y=\sqrt{x}+\dfrac{4}{\sqrt{x}}+4=x^{\frac{1}{2}}+4x^{-\frac{1}{2}}+4$ | | |
| | $x^n \to x^{n-1}$ | Decreases any power by 1. Either $x^{\frac{1}{2}} \to x^{-\frac{1}{2}}$ or $x^{-\frac{1}{2}} \to x^{-\frac{3}{2}}$ or $4 \to 0$ or $x^{\text{their}\,n} \to x^{\text{their}\,n-1}$ for fractional *n*. | M1 |
| | $\left(\dfrac{dy}{dx}=\right)\dfrac{1}{2}x^{-\frac{1}{2}}+4\times-\dfrac{1}{2}x^{-\frac{3}{2}}$ $\left(=\dfrac{1}{2}x^{-\frac{1}{2}}-2x^{-\frac{3}{2}}\right)$ | Correct derivative, simplified or un-simplified including indices. E.g. allow $\frac{1}{2}-1$ for $-\frac{1}{2}$ and allow $-\frac{1}{2}-1$ for $-\frac{3}{2}$ | A1 |
| | $x=8 \Rightarrow \dfrac{dy}{dx}=\dfrac{1}{2}8^{-\frac{1}{2}}+4\times-\dfrac{1}{2}8^{-\frac{3}{2}}$ | Attempts to substitute *x* = 8 into their 'changed' (even integrated) expression that is clearly not *y*. If they attempt algebraic manipulation of their d*y*/d*x* before substitution, this mark is still available. | M1 |
| | $=\dfrac{1}{2\sqrt{8}}-\dfrac{2}{\left(\sqrt{8}\right)^3}=\dfrac{1}{2\sqrt{8}}-\dfrac{2}{8\sqrt{8}}=\dfrac{1}{8\sqrt{2}}=\dfrac{1}{16}\sqrt{2}$ | B1: $\sqrt{8}=2\sqrt{2}$ seen or implied anywhere, including from substituting *x* = 8 into *y*. May be seen explicitly or implied from e.g. $8^{\frac{3}{2}}=16\sqrt{2}$ or $8^{\frac{5}{2}}=128\sqrt{2}$ or $4\sqrt{8}=8\sqrt{2}$<br>A1: cso $\dfrac{1}{16}\sqrt{2}$ or $\dfrac{\sqrt{2}}{16}$ and allow rational equivalents for $\dfrac{1}{16}$ e.g. $\dfrac{32}{512}$ Apply isw so award this mark as soon as a correct answer is seen. | B1A1 |
| | | | **(5 marks)** |

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**3.** A sequence $a_1, a_2, a_3, \ldots$ is defined by

$$a_1 = 1$$
$$a_{n+1} = \frac{k(a_n + 1)}{a_n}, \qquad n \geqslant 1$$

where $k$ is a positive constant.

(a) Write down expressions for $a_2$ and $a_3$ in terms of $k$, giving your answers in their simplest form.

**(3)**

Given that $\sum_{r=1}^{3} a_r = 10$

(b) find an exact value for $k$.

**(3)**

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| Question Number | Scheme | | Marks |
|---|---|---|---|
| **3.(a)** | $(a_2 =)2k$ | $2k$ only | B1 |
| | $(a_3 =)\dfrac{k("2k"+1)}{"2k"}$ | For substituting their $a_2$ into $a_3 = \dfrac{k(a_2+1)}{a_2}$ to find $a_3$ in terms of just $k$ | M1 |
| | $(a_3 =)\dfrac{2k+1}{2}$ | $(a_3 =)\dfrac{2k+1}{2}$ or exact simplified equivalent such as $(a_3 =)k+\dfrac{1}{2}$ or $\dfrac{1}{2}(2k+1)$ but not $k+\dfrac{k}{2k}$ **Must be seen in (a)** but isw once a correct simplified answer is seen. | A1 |
| | | | (3) |
| | **Note that there are <u>no</u> marks in (b) for using an AP (or GP) sum formula unless their terms do form an AP (or GP).** | | |
| **(b)** | $\sum_{r=1}^{3} a_r = 10 \Rightarrow 1+"2k"+"\dfrac{2k+1}{2}" = 10$ | Writes 1 + their $a_2$ + their $a_3$ = 10. E.g. $1+2k+\dfrac{2k^2+k}{2k}=10$. Must be a correct follow through equation in terms of $k$ only. | M1 |
| | $\Rightarrow 2+4k+2k+1=20 \Rightarrow k=...$ or e.g. $\Rightarrow 6k^2-17k=0 \Rightarrow k=...$ | Solves their equation in $k$ which has come from the sum of 3 terms = 10, and reaches $k=\ldots$ Condone poor algebra but if a quadratic is obtained then the usual rules apply for solving – see General Principles. (Note that it does not need to be a 3-term quadratic in this case) | M1 |
| | $(k =)\dfrac{17}{6}$ | $k=\dfrac{17}{6}$ or exact equivalent e.g. $2\dfrac{5}{6}$ Do **not** allow $k=\dfrac{8.5}{3}$ or $k=\dfrac{17/2}{3}$ Ignore any reference to $k$ = 0. Allow 2.83 recurring as long as the recurring is clearly indicated e.g. a dot over the 3. | A1 |
| | | | (3) |
| | | | **(6 marks)** |

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**4.** A company, which is making 140 bicycles each week, plans to increase its production. The number of bicycles produced is to be increased by $d$ each week, starting from 140 in week 1, to $140 + d$ in week 2, to $140 + 2d$ in week 3 and so on, until the company is producing 206 in week 12.

(a) Find the value of $d$.

**(2)**

After week 12 the company plans to continue making 206 bicycles each week.

(b) Find the total number of bicycles that would be made in the first 52 weeks starting from and including week 1.

**(5)**

| Question Number | Scheme | | Marks |
|---|---|---|---|
| **4. (a)** | $206=140+(12-1)\times d \Rightarrow d=...$ | Uses $206=140+(12-1)\times d$ and proceeds as far as $d=...$ | M1 |
| | $(d=)6$ | Correct answer only can score both marks. | A1 |
| | | | (2) |
| **(b)** | $S_{12}=\frac{12}{2}(140+206)$ or<br>$S_{12}=\frac{12}{2}(2\times140+(12-1)\times"6")$ or<br>$S_{11}=\frac{11}{2}(140+206-"6")$ or<br>$S_{11}=\frac{11}{2}(2\times140+(11-1)\times"6")$ | Attempts $S_n=\frac{n}{2}(a+l)$ or $S_n=\frac{n}{2}(2a+(n-1)d)$ with $n$ =12, $a=140, l=206, d='6'$ **WAY 1**<br>Or<br>Attempts $S_n=\frac{n}{2}(a+l)$ or $S_n=\frac{n}{2}(2a+(n-1)d)$ with $n$ =11, $a=140, l=206-'6', d='6'$ **WAY2**<br>If they are using $S_n=\frac{n}{2}(2a+(n-1)d)$, the $n$ must be used consistently. | M1 |
| | $S=2076$ **WAY1**<br>or<br>$S=1870$ **WAY 2** | Correct sum (may be implied) | A1 |
| | $(52-12)\times206=...$<br>or $(52-11)\times206=...$ | Attempts to find $(52-12)\times206$ or $(52-11)\times206$. Does **not** have to be consistent with their $n$ used for the first Method mark. | M1 |
| | Total $="2076"+"8240"=...$<br>**(WAY 1)**<br>or<br>Total $="1870"+"8446"=...$<br>**(WAY 2)** | Attempts to find the total by adding the sum to 12 terms with (52 - 12) lots of 206 or attempts to find the total by adding the sum to 11 terms with (52 - 11) lots of 206. I.e. consistency is now required for this mark. **Dependent on both previous method marks.** | **dd**M1 |
| | 10316 | cao | A1 |
| | | | (5) |
| | | | **(7 marks)** |

**Listing in (b):**

| Week | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| Bicycles | 140 | 146 | 152 | 158 | 164 | 170 | 176 |
| Total | 140 | 286 | 438 | 596 | 760 | 930 | 1106 |

| 8 | 9 | 10 | 11 | 12 | 13 | … | 52 |
|---|---|---|---|---|---|---|---|
| 182 | 188 | 194 | 200 | 206 | 206 | … | 206 |
| 1288 | 1476 | 1670 | 1870 | 2076 | 2282 | … | 10316 |

M1: Attempts the sum of either 12 or 11 terms of a series with first term 140 and their $d$ up to 140 +11$d$ or 140 + 10$d$.
A1: S = 2076 or 1870
Then follow the scheme

**Special case in (b) – Treats as single AP with $n$ = 52**

$$S_n = \frac{52}{2}\left(2\times 140+(52-1)\times "6"\right)=15236$$ Scores 11000

**M1:** $S_n = \frac{n}{2}\left(2a+(n-1)d\right)$ with $n$ = 52, $a$ = 140, $d$ = “6” **A1:** 15236

Leave blank

**5.** $$f(x) = x^2 - 8x + 19$$

(a) Express $f(x)$ in the form $(x + a)^2 + b$, where $a$ and $b$ are constants.

**(2)**

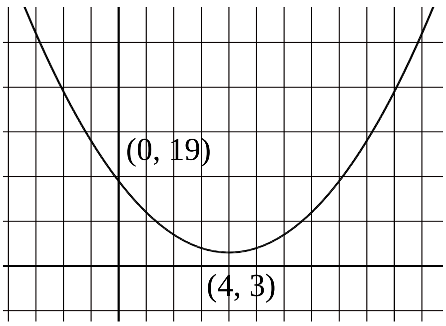
The curve $C$ with equation $y = f(x)$ crosses the $y$-axis at the point $P$ and has a minimum point at the point $Q$.

(b) Sketch the graph of $C$ showing the coordinates of point $P$ and the coordinates of point $Q$.

**(3)**

(c) Find the distance $PQ$, writing your answer as a simplified surd.

**(3)**

| Question Number | Scheme | | Marks |
|---|---|---|---|
| **5.(a)** | $\text{f}(x) = (x-4)^2 + 3$ | M1: $\text{f}(x) = (x \pm 4)^2 \pm \alpha,\ \alpha \neq 0$ (where $\alpha$ is a single number or a numerical expression $\neq 0$) | M1A1 |
| | | A1: Allow $(x +^{-} 4)^2 + 3$ and ignore any spurious "= 0" | |
| | **Allow *a* = -4, *b* = 3 to score both marks** | | |
| | | | **(2)** |
| **(b)** |  | B1: U shape anywhere even with no axes. Do not allow a "V" shape i.e. with an obvious vertex. | B1 |
| | | B1: *P*(0, 19). Allow (0, 19) or just 19 marked in the correct place as long as the curve (or straight line) passes through or touches here and allow (19, 0) as long as it is marked in the correct place. Correct coordinates may be seen in the body of the script as long as the curve (or straight line) passes through or touches here. If there is any ambiguity, the sketch has precedence. **(There must be a sketch to score this mark)** | B1 |
| | | B1: *Q*(4, 3). Correct coordinates that can be scored without a sketch but if a sketch is drawn then it must have a minimum in the first quadrant and no other turning points. May be seen in the body of the script. If there is any ambiguity, the sketch has precedence. Allow this mark if 4 is clearly marked on the *x*-axis below the minimum and 3 is marked clearly on the *y*-axis and corresponds to the minimum, | B1 |
| | | | **(3)** |

| | | | |
|---|---|---|---|
| **(c)** | $PQ^2 = (0-4)^2 + (19-3)^2$ | **Correct** use of Pythagoras' Theorem on 2 points of the form (0, $p$) and ($q$, $r$) where $q \neq 0$ and $p \neq r$ with $p$, $q$ and $r$ numeric. | M1 |
| | $PQ = \sqrt{4^2 + 16^2}$ | Correct un-simplified numerical expression for $PQ$ including the square root. **<u>This must come from a correct *P* and *Q*</u>**. Allow e.g $PQ = \sqrt{(0-4)^2 + (19-3)^2}$ . Allow $\pm\sqrt{(0-4)^2 + (19-3)^2}$ | A1 |
| | $PQ = 4\sqrt{17}$ | Cao and cso i.e. **<u>This must come from a correct *P* and *Q*</u>**. | A1 |
| | Note that it is possible to obtain the correct value for PQ from (-4,3) and (0, 19) and e.g. (0, 13) and (4, -3) but the A marks in (c) can only be awarded for the correct P and Q. | | |
| | | | **(3)** |
| | | | **(8 marks)** |

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**6.** (a) Given $y = 2^x$, show that

$$2^{2x+1} - 17(2^x) + 8 = 0$$

can be written in the form

$$2y^2 - 17y + 8 = 0$$

**(2)**

(b) Hence solve

$$2^{2x+1} - 17(2^x) + 8 = 0$$

**(4)**

<table>
<tr><th>Question Number</th><th colspan="2">Scheme</th><th>Marks</th></tr>
<tr><td>6.(a)</td><td>Replaces $2^{2x+1}$ with $2^{2x}\times 2$<br>or<br>states $2^{2x+1}=2^{2x}\times 2$<br>or<br>states $\left(2^x\right)^2=2^{2x}$</td><td>Uses the addition or power law of indices on $2^{2x}$ or $2^{2x+1}$. E.g.<br>$2^x\times 2^x=2^{2x}$ or $\left(2^x\right)^2=2^{2x}$ or<br>$2^{2x+1}=2\times 2^{2x}$ or $2^{x+0.5}=2^x\times\sqrt{2}$<br>or $2^{2x+1}=\left(2^{x+0.5}\right)^2$.</td><td>M1</td></tr>
<tr><td></td><td>$2^{2x+1}-17\times 2^x+8=0$<br>$\Rightarrow 2y^2-17y+8=0$*</td><td>Cso. Complete proof that includes explicit statements for the addition and power law of indices on $2^{2x+1}$ with no errors. The equation needs to be as printed including the "= 0". If they work backwards, they do not need to write down the printed answer first but must end with the version in $2^x$ including '= 0'.</td><td>A1*</td></tr>
<tr><td></td><td colspan="2">The following are examples of acceptable proofs.</td><td></td></tr>
<tr><td></td><td colspan="2">$2^{2x+1}=\left(2^{x+0.5}\right)^2=\left(2^x\sqrt{2}\right)^2=\left(y\sqrt{2}\right)^2=2y^2$<br>$\Rightarrow 2^{2x+1}-17\left(2^x\right)+8=2y^2-17y+8=0$</td><td></td></tr>
<tr><td></td><td colspan="2">$2y^2=2\times 2^x\times 2^x=2^{2x+1}$<br>$\Rightarrow 2^{2x+1}-17\left(2^x\right)+8=2y^2-17y+8=0$</td><td></td></tr>
<tr><td></td><td colspan="2">$2y^2-17y+8=0\Rightarrow 2\left(2^x\right)^2-17\left(2^x\right)+8=0$<br>$\Rightarrow 2\times 2^{2x}-17\left(2^x\right)+8=0\Rightarrow 2^{2x+1}-17\left(2^x\right)+8=0$</td><td></td></tr>
<tr><td></td><td colspan="2">$2^{2x+1}=2\times 2^{2x}\Rightarrow 2\times 2^{2x}-17\left(2^x\right)+8=0$<br>$\Rightarrow 2y^2-17y+8=0$<br>Scores M1A0 as $2^{2x}=\left(2^x\right)^2$ has not been shown explicitly</td><td></td></tr>
<tr><td></td><td colspan="2">Special Case:<br>$2^{2x+1}=2^1\times\left(2^x\right)^2$ or $2^{2x+1}=\left(2^x\right)^2\times 2^1$<br>With or without the multiplication signs and with no subsequent explicit evidence of the power law scores M1A0</td><td></td></tr>
<tr><td></td><td colspan="2">Example of insufficient working:<br>$2^{2x+1}=2\left(2^x\right)^2=2y^2$<br>scores no marks as neither rule has been shown explicitly.</td><td></td></tr>
<tr><td></td><td colspan="2"></td><td>(2)</td></tr>
</table>

<table>
<tr><td rowspan="5">(b)</td><td colspan="2">$2y^2-17y+8=0\Rightarrow(2y-1)(y-8)(=0)\Rightarrow y=...$<br>or<br>$2(2^x)^2-17(2^x)+8=0\Rightarrow(2(2^x)-1)((2^x)-8)(=0)\Rightarrow 2^x=...$<br>Solves the given quadratic either in terms of $y$ or in terms of $2^x$<br>See General Principles for solving a 3 term quadratic<br>Note that completing the square on e.g. $y^2-\frac{17}{2}y+4=0$ requires<br>$\left(y\pm\frac{17}{4}\right)^2\pm q\pm 4=0\Rightarrow y=...$</td><td>M1</td></tr>
<tr><td>$(y=)\frac{1}{2},8$ or $(2^x=)\frac{1}{2},8$</td><td>Correct values</td><td>A1</td></tr>
<tr><td>$\Rightarrow 2^x=\frac{1}{2},8\Rightarrow x=-1,3$</td><td>M1: Either finds one correct value of $x$ for their $2^x$ or obtains a correct numerical expression in terms of logs e.g. for $k>0$<br>$2^x=k\Rightarrow x=\log_2 k$ or $\frac{\log k}{\log 2}$<br><br>A1: $x=-1,3$ only. Must be values of $x$.</td><td>M1 A1</td></tr>
<tr><td colspan="2"></td><td>(4)</td></tr>
<tr><td colspan="2"></td><td>(6 marks)</td></tr>
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**7.** The curve $C$ has equation $y = \mathrm{f}(x)$, $x > 0$, where

$$\mathrm{f}'(x) = 30 + \frac{6 - 5x^2}{\sqrt{x}}$$

Given that the point $P(4, -8)$ lies on $C$,

(a) find the equation of the tangent to $C$ at $P$, giving your answer in the form $y = mx + c$, where $m$ and $c$ are constants.

**(4)**

(b) Find $\mathrm{f}(x)$, giving each term in its simplest form.

**(5)**

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P 4 8 7 6 0 A 0 1 4 2 8

| Question Number | Scheme | | Marks |
|---|---|---|---|
| **7.(a)** | $\mathrm{f}'(4)=30+\dfrac{6-5\times4^2}{\sqrt{4}}$ | Attempts to substitutes $x=4$ into $\mathrm{f}'(x)=30+\dfrac{6-5x^2}{\sqrt{x}}$ or their algebraically manipulated $\mathrm{f}'(x)$ | M1 |
| | $\mathrm{f}'(4)=-7$ | Gradient = $-7$ | A1 |
| | $y-(-8)="-7"\times(x-4)$ or $y="-7"x+c\Rightarrow-8="-7"\times4+c$ $\Rightarrow c=...$ | Attempts an equation of a tangent using their numeric $\mathrm{f}'(4)$ which has come from substituting $x$ = 4 into the given $\mathrm{f}'(x)$ or their algebraically manipulated $\mathrm{f}'(x)$ and $(4,-8)$ with the 4 and -8 correctly placed. If using $y = mx + c$, must reach as far as $c = \ldots$ | M1 |
| | $y=-7x+20$ | Cao. Allow $y=20-7x$ and allow the "$y$ =" to become "detached" but it must be present at some stage. E.g. $y = ...$ $=-7x+20$ | A1 |
| | | | (4) |
| **(b)** | **Allow the marks in (b) to score in (a) i.e. <u>mark (a) and (b) together</u>** | | |
| | $\Rightarrow\mathrm{f}(x)=30x+6\dfrac{x^{\frac{1}{2}}}{0.5}-5\dfrac{x^{\frac{5}{2}}}{2.5}(+c)$ | M1: $30\rightarrow30x$ or $\dfrac{6}{\sqrt{x}}\rightarrow\alpha x^{\frac{1}{2}}$ or $-\dfrac{5x^2}{\sqrt{x}}\rightarrow\beta x^{\frac{5}{2}}$ (these cases only)<br>A1: Any 2 correct terms which can be simplified or un-simplified. This includes the powers – so allow $-\frac{1}{2}+1$ for $\frac{1}{2}$ and allow $\frac{3}{2}+1$ for $\frac{5}{2}$ (With or without + $c$)<br>A1: All 3 terms correct which can be simplified or un-simplified. (With or without + $c$) | M1A1A1 |
| | **Ignore any spurious integral signs** | | |
| | $x=4,\mathrm{f}(x)=-8\Rightarrow$ $-8=120+24-64+c\Rightarrow c=...$ | Substitutes $x=4,\mathrm{f}(x)=-8$ into their f($x$) (not $\mathrm{f}'(x)$) i.e. a changed $\mathrm{f}'(x)$ containing $+c$ and rearranges to obtain a value or numerical expression for $c$. | M1 |
| | $\Rightarrow(\mathrm{f}(x)=)30x+12x^{\frac{1}{2}}-2x^{\frac{5}{2}}-88$ | Cao and cso (Allow $\sqrt{x}$ for $x^{\frac{1}{2}}$ and e.g. $\sqrt{x^5}$ or $x^2\sqrt{x}$ for $x^{\frac{5}{2}}$). Isw here so as soon as you see the correct answer, award this mark. Note that the "f($x$) =" is not needed. | A1 |
| | | | (5) |
| | | | **(9 marks)** |

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**8.**

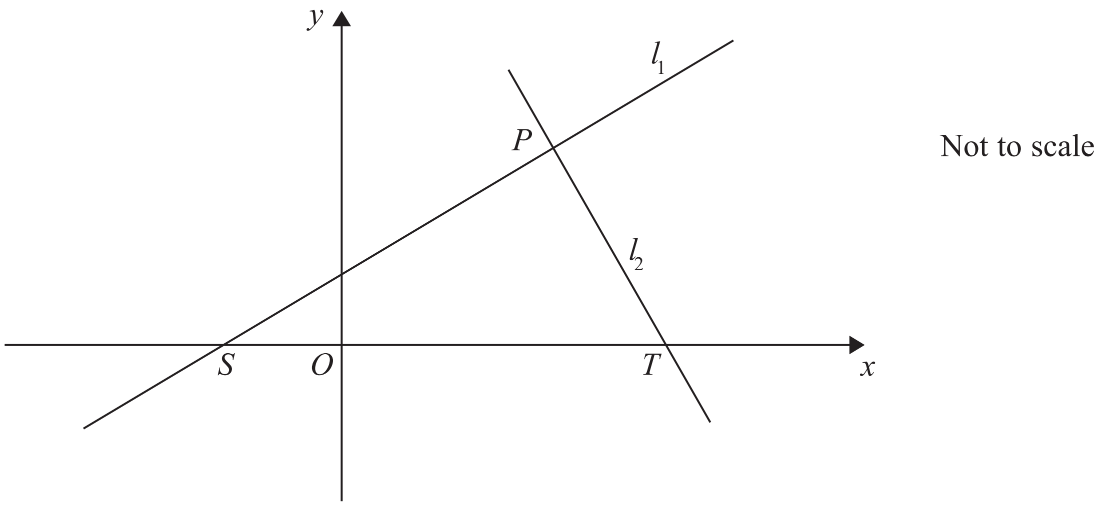


**Figure 1**

The straight line $l_1$, shown in Figure 1, has equation $5y = 4x + 10$

The point $P$ with $x$ coordinate 5 lies on $l_1$

The straight line $l_2$ is perpendicular to $l_1$ and passes through $P$.

(a) Find an equation for $l_2$, writing your answer in the form $ax + by + c = 0$ where $a$, $b$ and $c$ are integers.

**(4)**

The lines $l_1$ and $l_2$ cut the $x$-axis at the points $S$ and $T$ respectively, as shown in Figure 1.

(b) Calculate the area of triangle $SPT$.

**(4)**

| Question Number | Scheme | | Marks |
|---|---|---|---|
| **8.(a)** | Gradient of $l_1 = \frac{4}{5}$ oe | States or implies that the gradient of $l_1 = \frac{4}{5}$. E.g. may be implied by a perpendicular gradient of $-\frac{5}{4}$. Do not award this mark for just rearranging to $y = \frac{4}{5}x + ...$ unless they then state e.g. $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{4}{5}$ | B1 |
| | Point $P$ = (5, 6) | States or implies that $P$ has coordinates (5, 6). $y$ = 6 is sufficient. **May be seen on the diagram.** | B1 |
| | $-\frac{5}{4} = \frac{y - "6"}{x - 5}$ or $y - "6" = -\frac{5}{4}(x-5)$ or $"6" = -\frac{5}{4}(5) + c \Rightarrow c = ...$ | Correct straight line method using P(5, "6") and gradient of $-\frac{1}{\text{grad } l_1}$. Unless $-\frac{5}{4}$ or $-\frac{1}{4}$ is being used as the gradient here, the gradient of $l_1$ clearly needs to have been identified and its negative reciprocal attempted to score this mark. | M1 |
| | $5x + 4y - 49 = 0$ | Accept any integer multiple of this equation including "= 0" | A1 |
| | | | **(4)** |

<table>
<tr><td>8(b)</td><td>$y=0 \Rightarrow 5x+4(0)-49=0 \Rightarrow x=...$<br>or<br>$y=0 \Rightarrow 5(0)=4x+10 \Rightarrow x=...$</td><td>Substitutes $y = 0$ into their $l_2$ to find a value for $x$ or substitutes $y = 0$ into $l_1$ or their rearrangement of $l_1$ to find a value for $x$. This may be implied by a correct value on the diagram.</td><td>M1</td></tr>
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<tr><td></td><td colspan="2">(Note that at $T$, $x = 9.8$ and at $S$, $x = -2.5$)</td><td></td></tr>
<tr><td></td><td colspan="2">Fully correct method using their values to find the area of triangle $SPT$ with vertices at points of the form (5, “6”), ($p$, 0) and ($q$, 0) where $p \neq q$<br>Attempts to use integration should be sent to your team leader<br><br><u>Method 1:</u> $\frac{1}{2}ST \times "6"$<br>$\frac{1}{2}\times('9.8'-'-2.5')\times'6'=...$<br><br><u>Method 2:</u> $\frac{1}{2}SP \times PT$<br>$\frac{1}{2}\times\sqrt{(5-'-2.5')^2+('6')^2}\times\sqrt{('9.8'-5)^2+('6')^2}=...$<br>$\left(=\frac{1}{2}\times\frac{3\sqrt{41}}{2}\times\frac{6\sqrt{41}}{5}\right)$<br>Note that if the method is correct but slips are made when simplifying any of the calculations, the method mark can still be awarded<br><br><u>Method 3:</u> 2 Triangles<br>$\frac{1}{2}\times(5+'2.5')\times'6'+\frac{1}{2}\times('9.8'-5)\times'6'=...$<br><br><u>Method 4:</u> Shoelace method<br>$\frac{1}{2}\begin{vmatrix}5 & 9.8 & -2.5 & 5\\ 6 & 0 & 0 & 6\end{vmatrix}=\frac{1}{2}\left|(0+0-15)-(58.8+0+0)\right|=\frac{1}{2}\left|-73.8\right|=...$<br>(must see a correct calculation i.e. the middle expression for this determinant method)<br><br><u>Method 5:</u> Trapezium + 2 triangles<br>$\frac{1}{2}\times('2.5')\times'2'+\frac{1}{2}("2"+"6")\times5+\frac{1}{2}\times("9.8"-5')\times'6'=...$</td><td>ddM1</td></tr>
<tr><td></td><td>$=36.9$</td><td>36.9 cso oe e.g $\frac{369}{10}$, $36\frac{9}{10}$, $\frac{738}{20}$<br>but not e.g. $\frac{73.8}{2}$</td><td>A1</td></tr>
<tr><td></td><td colspan="2">Note that the final mark is cso so beware of any errors that have fortuitously resulted in a correct area.</td><td></td></tr>
<tr><td></td><td></td><td></td><td>(4)</td></tr>
<tr><td></td><td colspan="2"></td><td>(8 marks)</td></tr>
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**9.** (a) On separate axes sketch the graphs of

(i) $y = -3x + c$, where $c$ is a positive constant,

(ii) $y = \dfrac{1}{x} + 5$

On each sketch show the coordinates of any point at which the graph crosses the $y$-axis and the equation of any horizontal asymptote.

**(4)**

Given that $y = -3x + c$, where $c$ is a positive constant, meets the curve $y = \dfrac{1}{x} + 5$ at two distinct points,

(b) show that $(5 - c)^2 > 12$

**(3)**

(c) Hence find the range of possible values for $c$.

**(4)**

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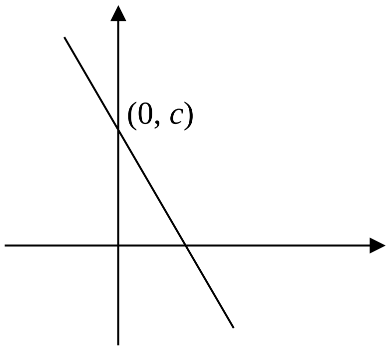
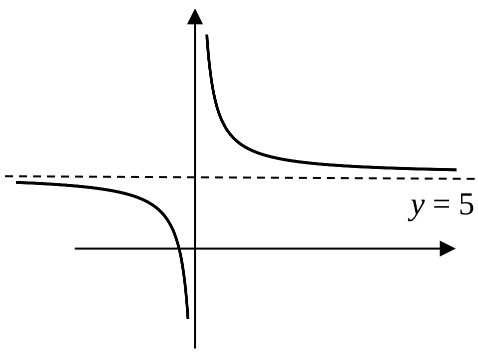
| Question Number | Scheme | | Marks |
|---|---|---|---|
| **9.(a)(i)** |  | B1: Straight line with negative gradient anywhere even with no axes. | B1 |
| | | B1: Straight line with an intercept at (0, *c*) or just *c* marked on the positive *y*-axis provided the line passes through the positive *y*-axis. Allow (*c*, 0) as long as it is marked in the correct place. Allow (0, *c*) in the body of the script but in any ambiguity, the sketch has precedence. **Ignore any intercepts with the *x*-axis.** | B1 |
| **(a)(ii)** |  | <u>**Either:**</u> For the shape of a $y = \frac{1}{x}$ curve in any position. It must have two branches and be asymptotic horizontally and vertically with no obvious "overlap" with the asymptotes, but otherwise be generous. The curve may bend away from the asymptote a little at the end. Sufficient curve must be seen to suggest the asymptotic behaviour, both vertically and horizontally and the branches must approach the same asymptote <u>**Or**</u> the **equation** $y = 5$ seen independently i.e. whether the sketch has an asymptote here or not. Do not allow $y \neq 5$ or $x = 5$. | B1 |
| | | B1: Fully correct graph **and** with a horizontal asymptote on the positive *y*-axis. The asymptote does not have to be drawn but the **equation** $y = 5$ must be seen. The shape needs to be reasonably accurate with the "ends" not bending away significantly from the asymptotes and the branches must approach the same asymptote. Ignore $x = 0$ given as an asymptote. | B1 |
| | **Allow sketches to be on the same axes.** | | |
| | | | **(4)** |

| | | | |
|---|---|---|---|
| **(b)** | $\frac{1}{x}+5=-3x+c \Rightarrow 1+5x=-3x^2+cx$ <br> $\Rightarrow 3x^2+5x-cx+1=0$ | Sets $\frac{1}{x}+5=-3x+c$, attempts to multiply by $x$ and collects terms (to one side). Allow e.g. ">" or "<" for "=". At least 3 of the terms must be multiplied by $x$, e.g. allow one slip. The ' = 0' may be implied by subsequent work and provided correct work follows, full marks are still possible in (b). | M1 |
| | $b^2-4ac=(5-c)^2-4\times1\times3$ | Attempts to use $b^2-4ac$ with **their** $a$, $b$ and $c$ from **their equation** where $a=\pm3$, $b=\pm5\pm c$ and $c=\pm1$. This could be as part of the quadratic formula or as $b^2<4ac$ or as $b^2>4ac$ or as $\sqrt{b^2-4ac}$ etc. If it is part of the quadratic formula only look for use of $b^2-4ac$. There must be no $x$'s. | M1 |
| | $(5-c)^2>12$ * | Completes proof with **no errors or incorrect statements** and with the ">" appearing correctly before the final answer, which could be from $b^2-4ac>0$. Note that the statement $3x^2+5x-cx+1>0$ or starting with e.g. $\frac{1}{x}+5>-3x+c$ would be an error. | A1* |
| | **Note: A minimum for (b) could be,** <br> $\frac{1}{x}+5=-3x+c \Rightarrow 3x^2+5x-cx+1(=0)$ (M1) <br> $b^2>4ac \Rightarrow (5-c)^2>12$ (M1A1) <br> If $b^2>4ac$ is not seen then $4\times3\times1$ needs to be seen explicitly. | | |
| | | | (3) |

| | | | |
|---|---|---|---|
| **(c)** | $(5-c)^2=12 \Rightarrow (c=)5\pm\sqrt{12}$<br>or<br>$(5-c)^2=12 \Rightarrow c^2-10c+13=0$<br>$\Rightarrow (c=)\dfrac{-{}^{-}10\pm\sqrt{(-10)^2-4\times 13}}{2}$ | M1: Attempts to find at least one critical value using the result in (b) or by expanding and solving a 3TQ (See General Principles) (the "= 0" may be implied)<br>A1: Correct critical values in any form. Note that $\sqrt{12}$ may be seen as $2\sqrt{3}$. | M1A1 |
| | $c<"5-\sqrt{12}",\ c>"5+\sqrt{12}"$ | Chooses outside region.<br>The '0 <' can be ignored for **this** mark. So look for $c$ < their $5-\sqrt{12}$, $c$ > their $5+\sqrt{12}$. This could be scored from $5+\sqrt{12}<c<5-\sqrt{12}$ or $5-\sqrt{12}>c>5+\sqrt{12}$. Evidence is to be taken from their answers not from a diagram. | M1 |
| | $0<c<5-\sqrt{12},\ c>5+\sqrt{12}$ | Correct ranges including the '0 <' e.g. answer as shown or each region written separately or e.g. $(0,5-\sqrt{12}),\ (5+\sqrt{12},\infty)$. The critical values may be un-simplified but must be at least $\dfrac{10+\sqrt{48}}{2},\ \dfrac{10-\sqrt{48}}{2}$. Note that $0<c<5-\sqrt{12}$ **and** $c>5+\sqrt{12}$ would score M1A0. | A1 |
| | **Allow the use of $x$ rather than $c$ in (c) but the final answer must be in terms of $c$.** | | |
| | | | (4) |
| | | | **(11 marks)** |

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**10.**

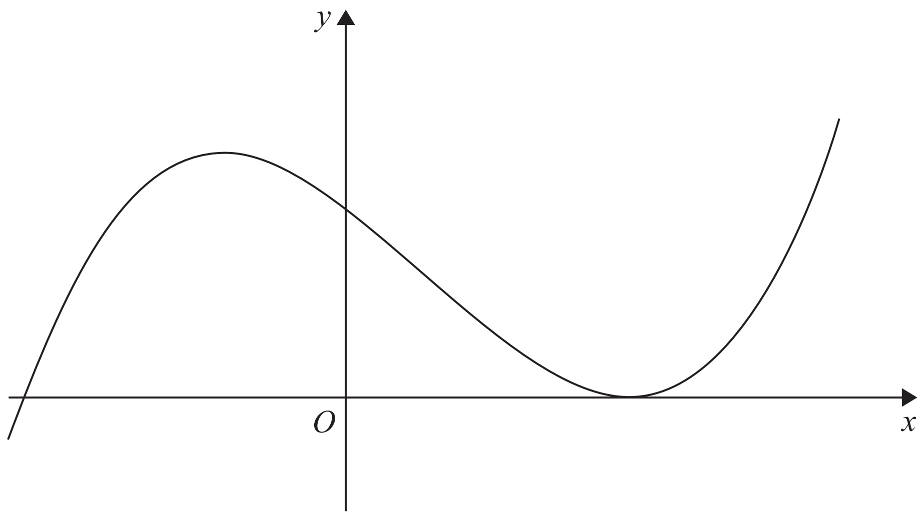


**Figure 2**

Figure 2 shows a sketch of part of the curve $y = \mathrm{f}(x)$, $x \in \mathbb{R}$, where

$$\mathrm{f}(x) = (2x - 5)^2(x + 3)$$

(a) Given that

(i) the curve with equation $y = \mathrm{f}(x) - k$, $x \in \mathbb{R}$, passes through the origin, find the value of the constant $k$,

(ii) the curve with equation $y = \mathrm{f}(x + c)$, $x \in \mathbb{R}$, has a minimum point at the origin, find the value of the constant $c$.

**(3)**

(b) Show that $\mathrm{f}'(x) = 12x^2 - 16x - 35$

**(3)**

Points $A$ and $B$ are distinct points that lie on the curve $y = \mathrm{f}(x)$.

The gradient of the curve at $A$ is equal to the gradient of the curve at $B$.

Given that point $A$ has $x$ coordinate 3

(c) find the $x$ coordinate of point $B$.

**(5)**

| Question Number | Scheme | | Marks |
|---|---|---|---|
| **10.(a)(i)** | $k=(-5)^2\times 3=75$ | M1: Attempts to find the '$y$' intercept. Accept as evidence $(-5)^2\times 3$ with or without the bracket. If they expand f($x$) to polynomial form here **then they must then select their constant to score this mark**. May be implied by sight of 75 on the diagram.<br>A1: $k=75$. Must clearly be identified as $k$. Allow this mark even from an incorrect or incomplete expansion as long as the constant $k$ = 75 is obtained. Do not isw e.g. if 75 is seen followed by $k$ = -75 score M1A0. | M1A1 |
| **(ii)** | $c=\frac{5}{2}$ only | $c=\frac{5}{2}$ oe (and no other values). Do not award just from the diagram – must be stated as the value of $c$. | B1 |
| | | | (3) |
| **(b)** | $\mathrm{f}(x)=(2x-5)^2(x+3)=\left(4x^2-20x+25\right)(x+3)=4x^3-8x^2-35x+75$<br>Attempts f($x$) as a cubic polynomial by attempting to square the first bracket and multiply by the linear bracket or expands $(2x-5)(x+3)$ and then multiplies by $2x-5$<br>**Must be seen or used in (b) but may be done in part (a).**<br>Allow poor squaring e.g. $(2x-5)^2=4x^2\pm 25$ | | M1 |
| | $(\mathrm{f}'(x)=)12x^2-16x-35$* | M1: Reduces powers by 1 in **all** terms including any constant $\to 0$<br>A1: Correct proof. Withhold this mark if there have been any errors including missing brackets earlier e.g. $(2x-5)^2(x+3)=4x^2-20x+25(x+3)=...$ | M1A1* |
| | | | (3) |

| | | | |
|---|---|---|---|
| **(c)** | $f'(3)=12\times3^2-16\times3-35$ | Substitutes $x$ = 3 into **their** $f'(x)$ **or the given** $f'(x)$. Must be a changed function i.e. not into f($x$). | M1 |
| | $12x^2-16x-35='25'$ | Sets **their** $f'(x)$ **or the given** $f'(x)=$ their $f'(3)$ with a consistent $f'$. **Dependent on the previous method mark.** | **d**M1 |
| | $12x^2-16x-60=0$ | $12x^2-16x-60=0$ or equivalent 3 term quadratic e.g. $12x^2-16x=60$. (A correct quadratic equation may be implied by later work). **This is cso so must come from correct work – i.e. they must be using the <u>given</u>** $f'(x)$. | A1 cso |
| | $(x-3)(12x+20)=0\Rightarrow x=...$ | Solves 3 term quadratic by suitable method – see General Principles. **Dependent on both previous method marks.** | **dd**M1 |
| | $x=-\frac{5}{3}$ | $x=-\frac{5}{3}$ oe clearly identified. If $x$ = 3 is also given and not rejected, this mark is withheld. (allow -1.6 recurring as long as it is clear i.e. a dot above the 6). **This is cso and must come from correct work – i.e. they must be using the <u>given</u>** $f'(x)$. | A1 cso |
| | | | (5) |
| | | | **(11 marks)** |
| **Alt (b) Product rule.** | $f(x)=(2x-5)^2(x+3)\Rightarrow f'(x)=(2x-5)^2\times1+(x+3)\times4(2x-5)$<br>M1: Attempts product rule to give an expression of the form<br>$p(2x-5)^2+q(x+3)(2x-5)$<br>M1: Multiplies out and collects terms<br>A1: $f'(x)=12x^2-16x-35*$ | | M1<br>M1A1* |