

Centre No.						Paper Reference							Surname	Initial(s)
Candidate No.						6	6	6	4	/	0	1	Signature	

Paper Reference(s)

6664/01

Edexcel GCE

Core Mathematics C2

Advanced Subsidiary

Thursday 24 May 2012 – Morning

Time: 1 hour 30 minutes

Examiner's use only

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Team Leader's use only

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[illegible]

Materials required for examination

Mathematical Formulae (Pink)

Items included with question papers

Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature. Check that you have the correct question paper.

Answer ALL the questions.

You must write your answer for each question in the space following the question.

When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet ‘Mathematical Formulae and Statistical Tables’ is provided.

Full marks may be obtained for answers to ALL questions.

The marks for individual questions and the parts of questions are shown in round brackets: e.g. (2).

There are 9 questions in this question paper. The total mark for this paper is 75.

There are 28 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.

You should show sufficient working to make your methods clear to the Examiner.

Answers without working may not gain full credit.

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1. Find the first 3 terms, in ascending powers of x , of the binomial expansion of

$$(2 - 3x)^5$$

giving each term in its simplest form.

(4)

Q1

(Total 4 marks)



Summer 2012

6664 Core Mathematics 2

Mark Scheme

Question number	Scheme	Marks
1	$[(2-3x)^5] = \dots + \binom{5}{1} 2^4 (-3x) + \binom{5}{2} 2^3 (-3x)^2 + \dots$ $= 32, -240x, +720x^2$	M1 B1, A1, A1
Notes	<p>M1: The method mark is awarded for an attempt at Binomial to get the second and/or third term – need correct binomial coefficient combined with correct power of x. Ignore errors (or omissions) in powers of 2 or 3 or sign or bracket errors. Accept any notation for 5C_1 and 5C_2, e.g. $\binom{5}{1}$ and $\binom{5}{2}$ (unsimplified) or 5 and 10 from Pascal's triangle This mark may be given if no working is shown, but either or both of the terms including x is correct.</p> <p>B1: must be simplified to 32 (writing just 2^5 is B0). 32 must be the only constant term in the final answer- so $32 + 80 - 3x + 80 + 9x^2$ is B0 but may be eligible for M1A0A0 .</p> <p>A1: is cao and is for $-240x$. (not $+240x$) The x is required for this mark</p> <p>A1: is c.a.o and is for $720x^2$ (can follow omission of negative sign in working)</p> <p>A list of correct terms may be given credit i.e. series appearing on different lines</p> <p>Ignore extra terms in x^3 and/or x^4 (isw)</p>	Total 4
Special Case	<p>Special Case: <i>Descending powers</i> of x would be</p> $(-3x)^5 + 2 \times 5 \times (-3x)^4 + 2^2 \times \binom{5}{3} \times (-3x)^3 + \dots$ <p>i.e. $-243x^5 + 810x^4 - 1080x^3 + \dots$ This is a misread but award as s.c. M1B1A0A0 if completely “correct” or M1 B0A0A0 for <u>correct</u> binomial coefficient in any form with the correct power of x</p>	
Alternative Method	<p>Method 1: $[(2-3x)^5] = 2^5 (1 + \binom{5}{1}(-\frac{3x}{2}) + \binom{5}{2}(\frac{-3x}{2})^2 + \dots)$ is M1B0A0A0 { The M1 is for the expression in the bracket and as in first method– need correct binomial coefficient combined with correct power of x. Ignore bracket errors or errors (or omissions) in powers of 2 or 3 or sign or bracket errors }</p> <p>– answers must be simplified to $= 32, -240x, +720x^2$ for full marks (awarded as before)</p> $[(2-3x)^5] = 2(1 + \binom{5}{1}(-\frac{3x}{2}) + \binom{5}{2}(\frac{-3x}{2})^2 + \dots)$ <p>would also be awarded M1B0A0A0</p> <p>Method 2: Multiplying out : B1 for 32 and M1A1A1 for other terms with M1 awarded if x or x^2 term is correct. Completely correct is 4/4</p>	

2. Find the values of x such that

$$2\log_3 x - \log_3(x - 2) = 2$$

(5)



Question number	Scheme	Marks
2	$2 \log x = \log x^2$ $\log_3 x^2 - \log_3 (x-2) = \log_3 \frac{x^2}{x-2}$ $\frac{x^2}{x-2} = 9$ <p>Solves $x^2 - 9x + 18 = 0$ to give $x = \dots$</p> <p>$x = 3, x = 6$</p>	<p>B1</p> <p>M1</p> <p>A1 o.e.</p> <p>M1</p> <p>A1</p> <p>Total 5</p>
Notes	<p>B1 for this correct use of power rule (may be implied)</p> <p>M1: for correct use of subtraction rule (or addition rule) for logs</p> <p>N.B. $2 \log_3 x - \log_3 (x-2) = 2 \log_3 \frac{x}{x-2}$ is M0</p> <p>A1. for correct equation without logs (Allow any correct equivalent including 3^2 instead of 9.)</p> <p>M1 for attempting to solve $x^2 - 9x + 18 = 0$ to give $x =$ (see notes on marking quadratics)</p> <p>A1 for these two correct answers</p>	
Alternative Method	<p>$\log_3 x^2 = 2 + \log_3 (x-2)$ is B1,</p> <p>so $x^2 = 3^{2+\log_3 (x-2)}$ needs to be followed by $(x^2) = 9(x-2)$ for M1 A1</p> <p>Here M1 is for complete method i.e. correct use of powers after logs are used correctly</p>	
Common Slips	<p>$2 \log x - \log x + \log 2 = 2$ may obtain B1 if $\log x^2$ appears but the statement is M0 and so leads to no further marks</p> <p>$2 \log_3 x - \log_3 (x-2) = 2$ so $\log_3 x - \log_3 (x-2) = 1$ and $\log_3 \frac{x}{x-2} = 1$ can earn M1 for <i>correct</i> subtraction rule following error, but no other marks</p>	
Special Case	<p>$\frac{\log x^2}{\log(x-2)} = 2$ leading to $\frac{x^2}{x-2} = 9$ and then to $x=3, x=6$, usually earns B1M0A0, but may then earn M1A1 (special case) so 3/5 [This <i>recovery</i> after uncorrected error is very common]</p> <p>Trial and error, Use of a table or just stating answer with both $x=3$ and $x=6$ should be awarded B0M0A0 then final M1A1 i.e. 2/5</p>	

3.

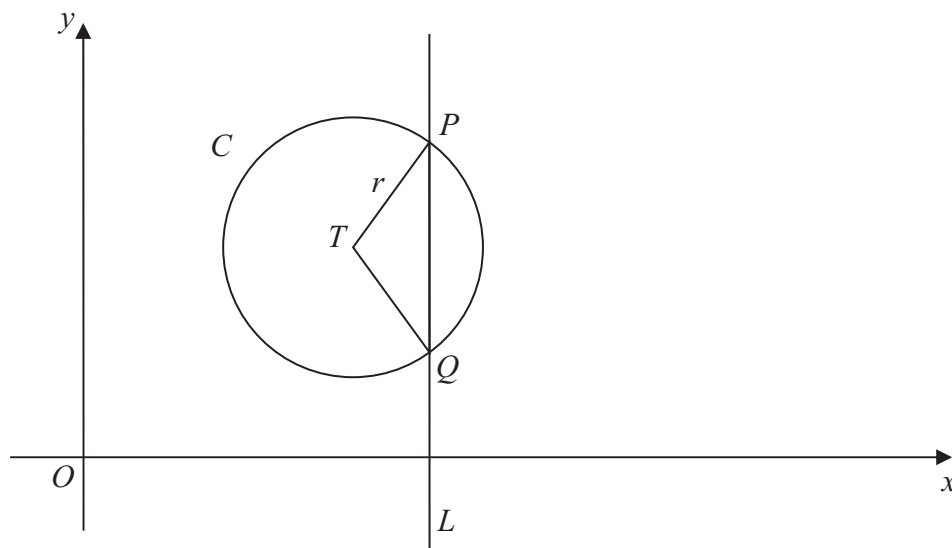


Figure 1

The circle C with centre T and radius r has equation

$$x^2 + y^2 - 20x - 16y + 139 = 0$$

- (a) Find the coordinates of the centre of C .

(3)

- (b) Show that $r = 5$

(2)

The line L has equation $x = 13$ and crosses C at the points P and Q as shown in Figure 1.

- (c) Find the y coordinate of P and the y coordinate of Q .

(3)

Given that, to 3 decimal places, the angle PTQ is 1.855 radians,

- (d) find the perimeter of the sector PTQ .

(3)



Question number	Scheme	Marks
3	Obtain $\underline{(x \pm 10)^2}$ and $\underline{(y \pm 8)^2}$	M1
(a)	Obtain $\underline{(x - 10)^2}$ and $\underline{(y - 8)^2}$	A1
	Centre is (10, 8). N.B. This may be indicated on diagram only as (10, 8)	A1
		(3)
(b)	See $\underline{(x \pm 10)^2} + \underline{(y \pm 8)^2} = 25 (= r^2)$ or $(r^2 =) "100" + "64" - 139$	M1
	$r = 5$ * (this is a printed answer so need one of the above two reasons)	A1
		(2)
(c)	Use $x = 13$ in either form of equation of circle and solve resulting quadratic to give $y =$ e.g. $x = 13 \Rightarrow (13 - 10)^2 + (y - 8)^2 = 25 \Rightarrow (y - 8)^2 = 16$ so $y =$ or $13^2 + y^2 - 20 \times 13 - 16y + 139 = 0 \Rightarrow y^2 - 16y + 48 = 0$ so $y =$ $y = 4$ or 12 (on EPEN mark one correct value as A1A0 and both correct as A1 A1)	M1
		A1, A1
		(3)
(d)	Use of $r\theta$ with $r = 5$ and $\theta = 1.855$ (may be implied by 9.275)	M1
	Perimeter $PTQ = 2r +$ their arc PQ (Finding perimeter of triangle is M0 here)	M1
	$= 19.275$ or 19.28 or 19.3	A1
		(3)
		11 marks
Alternatives	<i>Method 2:</i> From $x^2 + y^2 + 2gx + 2fy + c = 0$ centre is $(\pm g, \pm f)$	M1
(a)	Centre is $(-g, -f)$, and so centre is (10, 8).	A1, A1
OR	<i>Method 3:</i> Use any value of y to give two points (L and M) on circle. x co-ordinate of mid point of LM is "10" and Use any value of x to give two points (P and Q) on circle. y co-ordinate of mid point of PQ is "8" (Centre – chord theorem) . (10,8) is M1A1A1	M1
		A1 A1
		(3)
(b)	<i>Method 2:</i> Using $\sqrt{g^2 + f^2 - c}$ or $(r^2 =) "100" + "64" - 139$	M1
	$r = 5$ *	A1
OR	<i>Method 3:</i> Use point on circle with centre to find radius. Eg $\sqrt{(13 - 10)^2 + (12 - 8)^2}$	M1
	$r = 5$ *	A1
		cao
		(2)
(c)	Divide triangle PTQ and use Pythagoras with $r^2 - (13 - "10")^2 = h^2$, then evaluate $"8 \pm h"$ - (N.B. Could use 3,4,5 Triangle and 8 ± 4).	M1
	Accuracy as before	
Notes	Mark (a) and (b) together	
(a)	M1 as in scheme and can be <u>implied</u> by $(\pm 10, \pm 8)$. Correct centre (10, 8) implies M1A1A1	
(b)	M1 for a correct method leading to $r = \dots$, or $r^2 = "100" + "64" - 139$ (not $139 - "100" - "64"$) or for using equation of circle in $\underline{(x \pm 10)^2} + \underline{(y \pm 8)^2} = k^2$ form to identify $r =$ 3rd A1 $r = 5$ (NB This is a given answer so should follow $k^2 = 25$ or $r^2 = 100 + 64 - 139$) Special case: if centre is given as (-10, -8) or (10, -8) or (-10, 8) allow M1A1 for $r = 5$ worked correctly as $r^2 = 100 + 64 - 139$	
(d)	Full marks available for calculation using major sector so Use of $r\theta$ with $r = 5$ and $\theta = 4.428$ leading to perimeter of 32.14 for major sector	

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$$f(x) = 2x^3 - 7x^2 - 10x + 24$$

(a) Use the factor theorem to show that $(x + 2)$ is a factor of $f(x)$.

(2)

(b) Factorise $f(x)$ completely.

(4)



Question number	Scheme	Marks
4 (a)	$f(-2) = 2.(-2)^3 - 7.(-2)^2 - 10.(-2) + 24$ $= 0 \text{ so } (x+2) \text{ is a factor}$	M1 A1 (2)
(b)	$f(x) = (x+2)(2x^2 - 11x + 12)$ $f(x) = (x+2)(2x-3)(x-4)$	M1 A1 dM1 A1 (4)
		6 marks
Notes (a)	<p>M1 : Attempts $f(\pm 2)$ (Long division is M0) A1 : is for $=0$ and conclusion Note: Stating “hence factor” or “it is a factor” or a “√” (tick) or “QED” is fine for the conclusion. Note also that a conclusion can be implied from a <u>preamble</u>, eg: “If $f(-2) = 0$, $(x+2)$ is a factor...” (Not just $f(-2)=0$)</p>	
(b)	<p>1st M1: Attempts long division by correct factor or other method leading to obtaining $(2x^2 \pm ax \pm b)$, $a \neq 0$, $b \neq 0$, even with a remainder. Working need not be seen as could be done “by inspection.” Or <i>Alternative Method</i> : 1st M1: Use $(x+2)(ax^2 + bx + c) = 2x^3 - 7x^2 - 10x + 24$ with expansion and comparison of coefficients to obtain $a = 2$ and to obtain values for b and c 1st A1: For seeing $(2x^2 - 11x + 12)$. [Can be seen here in (b) after work done in (a)] 2nd M1: Factorises quadratic. (see rule for factorising a quadratic). This is dependent on the previous method mark being awarded and needs factors 2nd A1: is cao and needs all three factors together. Ignore subsequent work (such as a solution to a quadratic equation.)</p> <p>Note: Some candidates will go from $\{(x+2)\}(2x^2 - 11x + 12)$ to $\{x = -2\}$, $x = \frac{3}{2}$, 4, and not list all three factors. Award these responses M1A1M0A0.</p> <p>Finds $x = 4$ and $x = 1.5$ by factor theorem, formula or calculator and produces factors M1 $f(x) = (x+2)(2x-3)(x-4)$ or $f(x) = 2(x+2)(x-1.5)(x-4)$ o.e. is full marks $f(x) = (x+2)(x-1.5)(x-4)$ loses last A1</p>	

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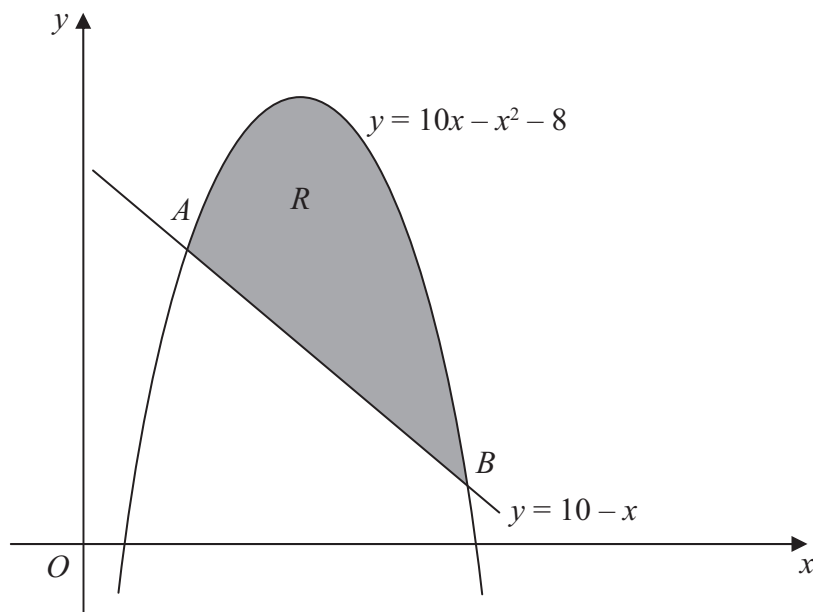


Figure 2

Figure 2 shows the line with equation $y = 10 - x$ and the curve with equation $y = 10x - x^2 - 8$

The line and the curve intersect at the points A and B , and O is the origin.

- (a) Calculate the coordinates of A and the coordinates of B .

(5)

The shaded area R is bounded by the line and the curve, as shown in Figure 2.

- (b) Calculate the exact area of R .

(7)

[illegible]

Question number	Scheme		Marks
Method 1 5 (a)	<p>Puts $10 - x = 10x - x^2 - 8$ and rearranges to give three term quadratic</p> <p>Solves their "$x^2 - 11x + 18 = 0$" using acceptable method as in general principles to give $x =$</p> <p>Obtains $x = 2$, $x = 9$ (may be on diagram or in part (b) in limits)</p> <p>Substitutes their x into a given equation to give $y =$ (may be on diagram)</p> <p>$y = 8, y = 1$</p>	<p>Or puts $y = 10(10 - y) - (10 - y)^2 - 8$ and rearranges to give three term quadratic</p> <p>Solves their "$y^2 - 9y + 8 = 0$" using acceptable method as in general principles to give $y =$</p> <p>Obtains $y = 8, y = 1$ (may be on diagram)</p> <p>Substitutes their y into a given equation to give $x =$ (may be on diagram or in part (b))</p> <p>$x = 2, x = 9$</p>	<p>M1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1 (5)</p>
(b)	$\int (10x - x^2 - 8) dx = \frac{10x^2}{2} - \frac{x^3}{3} - 8x \{+ c\}$ $\left[\frac{10x^2}{2} - \frac{x^3}{3} - 8x \right]_2^9 = (\dots) - (\dots)$ $= 90 - \frac{4}{3} = 88\frac{2}{3} \text{ or } \frac{266}{3}$ <p>Area of trapezium $= \frac{1}{2}(8+1)(9-2) = 31.5$</p> <p>So area of R is $88\frac{2}{3} - 31.5 = 57\frac{1}{6} \text{ or } \frac{343}{6}$</p>		<p>M1 A1 A1</p> <p>dM1</p> <p>B1</p> <p>M1A1 cao (7)</p>
Notes (a)	<p>First M1: See scheme Second M1: See notes relating to solving quadratics</p> <p>Third M1: This may be awarded if one substitution is made</p> <p>Two correct Answers following tables of values, or from Graphical calculator are 5/5</p> <p>Just one pair of correct coordinates – no working or from table is M0M0A0M1A0</p>		
(b)	<p>M1: $x^n \rightarrow x^{n+1}$ for any one term.</p> <p>1st A1: at least two out of three terms correct 2nd A1: All three correct</p> <p>dM1: Substitutes 9 and 2 (or limits from part(a)) into an "integrated function" and subtracts, either way round</p> <p>(NB: If candidate changes all signs to get $\int (-10x + x^2 + 8) dx = -\frac{10x^2}{2} + \frac{x^3}{3} + 8x \{+ c\}$ This is M1 A1 A1</p> <p>Then uses limits dM1 and trapezium is B1</p> <p>Needs to <i>change sign of value obtained</i> from integration for final M1A1 so $-88\frac{2}{3} - 31.5$ is M0A0)</p> <p>B1: Obtains 31.5 for area under line using any correct method (could be integration) or triangle minus triangle $\frac{1}{2} \times 8 \times 8 - \frac{1}{2}$ or rectangle plus triangle [may be implied by correct 57 1/6]</p> <p>M1: Their Area under curve – Their Area under line (if integrate both need same limits)</p> <p>A1: Accept 57.16recurring but not 57.16</p> <p>PTO for Alternative method</p>		12 marks

Method 2 for (b)	<p>Area of R</p> $= \int_2^9 (10x - x^2 - 8) - (10 - x) \, dx$ $\int_2^9 -x^2 + 11x - 18 \, dx$ $= -\frac{x^3}{3} + \frac{11x^2}{2} - 18x \{+c\}$ $\left[-\frac{x^3}{3} + \frac{11x^2}{2} - 18x \right]_2^9 = (\dots) - (\dots)$ <p>This mark is implied by final answer which rounds to 57.2</p> <p><i>See above working(allow bracketing errors) to decide to award 3rd M1 mark for (b) here:</i></p> $40.5 - (-16\frac{2}{3}) = 57\frac{1}{6} \text{ cao}$	<p>3rd M1 (in (b)): Uses difference between two functions in integral. M: $x^n \rightarrow x^{n+1}$ for any one term. A1 at least two out of these three simplified terms Correct integration. (Ignore + c). Substitutes 9 and 2 (or limits from part(a)) into an “integrated function” and subtracts, either way round.</p>	<p>M1</p> <p>A1</p> <p>A1</p> <p>dM1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>(7)</p>
Special case of above method	$\int_2^9 x^2 - 11x + 18 \, dx = \frac{x^3}{3} - \frac{11x^2}{2} + 18x \{+c\}$ $\left[\frac{x^3}{3} - \frac{11x^2}{2} + 18x \right]_2^9 = (\dots) - (\dots)$ <p>This mark is implied by final answer which rounds to 57.2 (not -57.2)</p> <p>Difference of functions implied (see above expression)</p> $40.5 - (-16\frac{2}{3}) = 57\frac{1}{6} \text{ cao}$		<p>M1A1A1</p> <p>DM1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>(7)</p>
Special Case 2	Integrates expression in y e.g. " $y^2 - 9y + 8 = 0$ ": This can have first M1 in part (b) and no other marks. (It is not a method for finding this area)		
Notes	Take away trapezium again having used Method 2 loses last two marks Common Error: Integrates $-x^2 + 9x - 18$ is likely to be M1A1A0dM1B0M1A0 Integrates $2 - 11x - x^2$ is likely to be M1A0A0dM1B0M1A0 Writing $\int_2^9 (10x - x^2 - 8) - (10 - x) \, dx$ only earns final M mark		

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- $$\tan 2x = 5 \sin 2x$$

can be written in the form

$$(1 - 5 \cos 2x) \sin 2x = 0$$

(2)

- $$\tan 2x = 5 \sin 2x$$

giving your answers to 1 decimal place where appropriate. You must show clearly how you obtained your answers.

(5)



Question number	Scheme		Marks
6(a)	States or uses $\tan 2x = \frac{\sin 2x}{\cos 2x}$		M1
	$\frac{\sin 2x}{\cos 2x} = 5 \sin 2x \Rightarrow \sin 2x - 5 \sin 2x \cos 2x = 0 \Rightarrow \sin 2x(1 - 5 \cos 2x) = 0$ *		A1 (2)
(b)	$\sin 2x = 0$ gives $2x = 0, 180, 360$ so $x = 0, 90, 180$ $\cos 2x = \frac{1}{5}$ gives $2x = 78.46$ (or 78.5 or 78.4) or $2x = 281.54$ (or 281.6) $x = 39.2$ (or 39.3), 140.8 (or 141)	B1 for two correct answers, second B1 for all three correct. Excess in range – lose last B1	B1, B1
			M1
		A1, A1 (5)	
7 marks			
Notes	<p>(a) M1: Statement that $\tan \theta = \frac{\sin \theta}{\cos \theta}$ or Replacement of tan (wherever it appears). Must be a correct statement but may involve θ instead of $2x$. A1: the answer is given so all steps should be given.</p> <p>N.B. $\sin 2x - 5 \sin 2x \cos 2x = 0$ or $-5 \sin 2x \cos 2x + \sin 2x = 0$ or $\sin 2x(\frac{1}{\cos 2x} - 5) = 0$ o.e. must be seen and be followed by printed answer for A1 mark $\sin 2x = 5 \sin 2x \cos 2x$ is not sufficient.</p> <p>(b) Statement of 0 and 180 with no working gets B1 B0 (bod) as it is two solutions M1: This mark for one of the two statements given (must relate to $2x$ not just to x) A1, A1: first A1 for 39.2, second for 140.8 <i>Special case</i> solving $\cos 2x = -1/5$ giving $2x = 101.5$ or 258.5 is awarded M1A0A0 140.8 omitted would give M1A1A0 Allow answers which round to 39.2 or 39.3 and which round to 140.8 and allow 141 Answers in radians lose last A1 awarded (These are 0, 0.68, 1.57, 2.46 and 3.14) Excess answers in range lose last A1 Ignore excess answers outside range. All 5 correct answers with no extras and no working gets full marks in part (b). The answers imply the method here</p>		

7.

$$y = \sqrt[3]{(3^x + x)}$$

- (a) Complete the table below, giving the values of y to 3 decimal places.

x	0	0.25	0.5	0.75	1
y	1	1.251			2

(2)

- (b) Use the trapezium rule with all the values of y from your table to find an approximation

for the value of $\int_0^1 \sqrt[3]{(3^x + x)} \, dx$

You must show clearly how you obtained your answer.

(4)



Question number	Scheme	Marks												
7 (a)	<table><tr><td>x</td><td>0</td><td>0.25</td><td>0.5</td><td>0.75</td><td>1</td></tr><tr><td>y</td><td>1</td><td>1.251</td><td>1.494</td><td>1.741</td><td>2</td></tr></table>	x	0	0.25	0.5	0.75	1	y	1	1.251	1.494	1.741	2	B1, B1 (2)
x	0	0.25	0.5	0.75	1									
y	1	1.251	1.494	1.741	2									
(b)	$\frac{1}{2} \times 0.25, \{(1+2) + 2(1.251+1.494+1.741)\} \text{ o.e.}$ $=1.4965$	B1, M1,A1 ft A1 (4)												
		6 marks												
Notes	<p>(a) first B1 for 1.494 and second B1 for 1.741 (1.740 is B0) Wrong accuracy e.g. 1.49, 1.74 is B1B0</p> <p>(b) B1: Need $\frac{1}{2}$ of 0.25 or 0.125 o.e. M1: requires first bracket to contain first plus last values and second bracket to include no additional values from the three in the table. If the only mistake is to omit one value from second bracket this may be regarded as a slip and M mark can be allowed (An extra repeated term forfeits the M mark however) x values: M0 if values used in brackets are x values instead of y values</p> <p>A1ft follows their answers to part (a) and is for {correct expression} Final A1: Accept 1.4965, 1.497. or 1.50 only after correct work. (No follow through except one special case below following 1.740 in table) Separate trapezia may be used : B1 for 0.125, M1 for $\frac{1}{2}h(a+b)$ used 3 or 4 times (and A1ft if it is all correct) e.g.. $0.125(1+ 1.251) + 0.125(1.251+1.494) + 0.125(1.741 + 2)$ is M1 A0 equivalent to missing one term in { } in main scheme</p> <p>Special Case: Bracketing mistake: i.e. $0.125(1+2) + 2(1.251+1.494+1.741)$ scores B1 M1 A0 A0 for 9.347 If the final answer implies that the calculation has been done correctly i.e. 1.4965 (then full marks can be given). Need to see trapezium rule – answer only (with no working) is 0/4 any doubts send to review</p> <p>Special Case; Uses 1.740 to give 1.49625 or 1.4963 or 1.496 or 1.50 gets, B1 B0 B1M1A1ft then A1 (lose 1 mark)</p> <p>NB Bracket is 11.972</p>													

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A diagram of a cylinder. The height is indicated by a vertical double-headed arrow on the right side, labeled $h \text{ mm}$. The radius is indicated by a horizontal double-headed arrow at the bottom, labeled $x \text{ mm}$. The bottom circular face is shaded grey.

Figure 3

Given that the volume of each tablet has to be 60 mm^3 ,

- (b) show that the surface area, $A \text{ mm}^2$, of a tablet is given by $A = 2\pi x^2 + \frac{120}{x}$ (3)

The manufacturer needs to minimise the surface area $A \text{ mm}^2$, of a tablet.

- (d) Calculate the minimum value of A , giving your answer to the nearest integer. (2)

- (e) Show that this value of A is a minimum. (2)



Question number	Scheme	Marks
8		
(a)	$(h =) \frac{60}{\pi x^2}$ or equivalent exact (not decimal) expression e.g. $(h =) 60 \div \pi x^2$	B1 (1)
(b)	$(A =) 2\pi x^2 + 2\pi xh$ or $(A =) 2\pi r^2 + 2\pi rh$ or $(A =) 2\pi r^2 + \pi dh$ may not be simplified and may appear on separate lines Either $(A) = 2\pi x^2 + 2\pi x \left(\frac{60}{\pi x^2} \right)$ or As $\pi xh = \frac{60}{x}$ then $(A =) 2\pi x^2 + 2 \left(\frac{60}{x} \right)$	B1 M1
(c)	$A = 2\pi x^2 + \left(\frac{120}{x} \right)$ *	A1 cso (3)
	$\left(\frac{dA}{dx} \right) = 4\pi x - \frac{120}{x^2}$ or $= 4\pi x - 120x^{-2}$	M1 A1
	$4\pi x - \frac{120}{x^2} = 0$ implies $x^3 =$ (Use of > 0 or < 0 is M0 then M0A0)	M1
	$x = \sqrt[3]{\frac{120}{4\pi}}$ or answers which round to 2.12 (-2.12 is A0)	dM1 A1 (5)
(d)	$A = 2\pi(2.12)^2 + \frac{120}{2.12}, = 85$ (only ft $x = 2$ or 2.1 – both give 85)	M1, A1 (2)
(e)	Either $\frac{d^2 A}{dx^2} = 4\pi + \frac{240}{x^3}$ and sign considered (May appear in (c)) Or (method 2) considers gradient to left and right of their 2.12 (e.g at 2 and 2.5) Or (method 3) considers value of A either side	M1
	which is > 0 and therefore minimum (most substitute 2.12 but it is not essential to see a substitution) (may appear in (c)) Finds numerical values for gradients and observes gradients go from negative to zero to positive so concludes minimum OR finds numerical values of A , observing greater than minimum value and draws conclusion	A1 (2)
		13 marks
Notes	<p>(a) B1: This expression must be correct and in part (a) $\frac{60}{\pi r^2}$ is B0</p> <p>(b) B1: Accept any equivalent correct form – may be on two or more lines. M1 : substitute their expression for h in terms of x into Area formula of the form $kx^2 + cxh$ A1: There should have been no errors in part (b) in obtaining this printed answer</p> <p>(c) M1: At least one power of x decreased by 1 A1 accept any equivalent correct answer</p> <p>M1: Setting $\frac{dA}{dx} = 0$ and finding a value for x^3 ($x^3 =$ may be implied by answer). Allow $\frac{dy}{dx} = 0$</p> <p>dM1: Using cube root to find x A1 : For any equivalent correct answer (need 3sf or more) Correct answer implies previous M mark</p> <p>(d) M1 : Substitute the (+ve) x value found in (c) into equation for A and evaluate . A1 is for 85 only</p> <p>(e) M1: Complete method, usually one of the three listed in the scheme. For first method $A''(x)$ must be attempted and sign considered A1: Clear statements and conclusion. (numerical substitution of x is not necessary in first method shown, and x or calculation could be wrong but $A''(x)$ must be correct . Must not see 85 substituted)</p>	

9. A geometric series is $a + ar + ar^2 + \dots$

(a) Prove that the sum of the first n terms of this series is given by

$$S_n = \frac{a(1-r^n)}{1-r} \quad (4)$$

The third and fifth terms of a geometric series are 5.4 and 1.944 respectively and all the terms in the series are positive.

For this series find,

(b) the common ratio,

(2)

(c) the first term, (2)

(d) the sum to infinity.



Question	Scheme	Marks
9 (a)	$(S_n =) a + ar + (ar^2) + \dots + ar^{n-1}$ and $rS_n = ar + ar^2 + (ar^3) \dots + ar^n$ $S_n - rS_n = a - ar^n$ $S_n(1-r) = a(1-r^n)$ And so result $S_n = \frac{a(1-r^n)}{(1-r)}$ *	M1 M1 dM1 A1 (4)
(b)	Divides one term by other (either way) to give $r^2 = \dots$ then square roots to give $r =$ $r^2 = \frac{1.944}{5.4}$, $r = 0.6$ (ignore -0.6)	Or: (Method 2) Finds geometric mean i.e 3.24 and divides one term by 3.24 or 3.24 by one term $r = 0.6$ (ignore -0.6) M1 A1 (2)
(c)	Uses $5.4 \div r^2$ or $1.944 \div r^4$, to give $a =$ $a = 15$	M1, A1ft (2)
(d)	Uses $S = \frac{15}{1-0.6}$, to obtain 37.5	M1A1 ,A1 (3)
		11 marks
Notes	(a) M1: Lists both of these sums ($S_n =$) may be omitted, rS_n (or rS) must be stated 1 st two terms must be correct in each series. Last term must be ar^{n-1} or ar^n in first series and the corresponding ar^n or ar^{n+1} in second series. Must be n and not a number. Reference made to other terms e.g. space or dots to indicate missing terms M1: Subtracts series for rS from series for S (or other way round) to give $RHS = \pm(a - ar^n)$. This may have been obtained by following a pattern. If wrong power stated on line 1 M0 here. (Ignore LHS)M0M0M0A0 dM1: Factorises both sides correctly– must follow from a previous M1 (It is possible to obtain M0M1M1A0 or M1M0M1A0) A1: completes the proof with no errors seen No errors seen: First line absolutely correct , omission of second line, third and fourth lines correct: M1M0M1A1 See next sheet of common errors. Refer any attempts involving sigma notation , or any proofs by induction to team leader. Also attempts which begin with the answer and work backwards .	
Special Case	(b) M1: Deduces r^2 by dividing either term by other and attempts square root A1: any correct equivalent for r e.g. $3/5$ Answer only is $2/2$ (Method 2) Those who find fourth term must use \sqrt{ab} and not $\frac{1}{2}(a+b)$ then must use it in a division with given term to obtain $r =$ (c) M1: May be done in two steps or more e.g. $5.4 \div r$ then divided by r again A1ft: follow through their value of r . Just $a = 15$ with no wrong working implies M1A1 (d) M1: States sum to infinity formula with values of a and r found earlier, provided $ r < 1$ A1: uses 15 and 0.6 (or $3/5$) (This is not a ft mark) A1: 37.5 or exact equivalent	
Common errors	(i) Fraction inverted in (b) $r^2 = \frac{5.4}{1.944}$ and $r = 1\frac{2}{3}$, then correct ft gives M1A0 M1 A1ft M0A0A0 i.e. $3/7$ (ii) Uses $r = 0.36$: (b)M0A0 (c)M1A1ft (d) M1A0A0 i.e. $3/7$ (iii) Uses $ar^3 = 5.4$, $ar^5 = 1.944$ Likely to have (b)M1A1 (c)M0A0 (d) M1A0A0 i.e. $3/7$	