

Centre No.						Paper Reference							Surname	Initial(s)
Candidate No.						6	6	6	4	/	0	1	Signature	

Paper Reference(s)

6664/01

Edexcel GCE

Core Mathematics C2

Advanced Subsidiary

Friday 5 June 2009 – Afternoon

Time: 1 hour 30 minutes

Examiner's use only

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Team Leader's use only

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[illegible]

Materials required for examination

Mathematical Formulae
(Orange or Green)

Items included with question papers

Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature. Check that you have the correct question paper.

Answer ALL the questions. Write your answers in the spaces provided in this question paper. When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.

Full marks may be obtained for answers to ALL questions.

The marks for individual questions and the parts of questions are shown in round brackets: e.g. (2).

There are 9 questions in this question paper. The total mark for this paper is 75.

There are 24 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.

You should show sufficient working to make your methods clear to the Examiner.

Answers without working may not gain full credit.

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1. Use calculus to find the value of

$$\int_1^4 (2x + 3\sqrt{x}) \, dx$$

(5)

Q1

(Total 5 marks)



June 2009
6664 Core Mathematics C2
Mark Scheme

Question Number	Scheme	Marks
Q1	$\int \left(2x + 3x^{\frac{1}{2}} \right) dx = \frac{2x^2}{2} + \frac{3x^{\frac{3}{2}}}{\frac{3}{2}}$ $\int_1^4 \left(2x + 3x^{\frac{1}{2}} \right) dx = \left[x^2 + 2x^{\frac{3}{2}} \right]_1^4 = 16 + 2 \times 8 - 1 + 2$ $= 29 \quad (29 + C \text{ scores A0})$	<p>M1 A1A1</p> <p>M1</p> <p>A1 (5) [5]</p>
	<p>1st M1 for attempt to integrate x or $x^{\frac{1}{2}}$.</p> <p>1st A1 for $\frac{2x^2}{2}$ or a simplified version.</p> <p>2nd A1 for $\frac{3x^{\frac{3}{2}}}{\frac{3}{2}}$ or $\frac{3\sqrt{x^3}}{2}$ or a simplified version.</p> <p>Ignore + C, if seen, but two correct terms and an <u>extra non-constant</u> term scores M1A1A0.</p> <p>2nd M1 for correct use of correct limits ('top' – 'bottom'). Must be used in a 'changed function', not just the original. (The changed function may have been found by differentiation).</p> <p>Ignore 'poor notation' (e.g. missing integral signs) if the intention is clear.</p> <p><u>No working:</u> The answer 29 with no working scores M0A0A0M1A0 (1 mark).</p>	

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- $$(2 + kx)^7$$

where k is a constant. Give each term in its simplest form.

(4)

(b) find the value of k .

(2)

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Question Number	Scheme	Marks
Q2 (a)	<p>$7 \cdot 1 \cdot 1$ or $1 \cdot 7 \cdot 1$ The 7 or 21 can be in 'unsimplified' form.</p> $2 + kx^7 = 2^7 + 2^6 \times 7 \times kx + 2^5 \times \binom{7}{2} k^2 x^2$ $= 128; + 448kx, + 672k^2 x^2 \text{ [or } 672(kx)^2 \text{]}$ <p>(If $672kx^2$ follows $672(kx)^2$, isw and allow A1)</p>	M1 B1; A1, A1 (4)
(b)	$6 \times 448k = 672k^2$ $k = 4$ (Ignore $k = 0$, if seen)	M1 A1 (2) [6]
(a)	<p>The terms can be 'listed' rather than added. Ignore any extra terms.</p> <p>M1 for <u>either</u> the x term <u>or</u> the x^2 term. Requires <u>correct</u> binomial coefficient in any form with the correct power of x, but the other part of the coefficient (perhaps including powers of 2 and/or k) may be wrong or missing.</p> <p><u>Allow</u> binomial coefficients such as $\binom{7}{1}$, $\binom{7}{2}$.</p> <p>However, $448 \cdot x$ or similar is M0.</p> <p>B1, A1, A1 for the <u>simplified</u> versions seen above.</p> <p><u>Alternative:</u></p> <p>Note that a factor 2^7 can be taken out first: $2^7 \cdot \frac{kx}{2}$, but the mark scheme still applies.</p> <p><u>Ignoring subsequent working (isw):</u></p> <p>Is w if necessary after correct working:</p> <p>e.g. $128 \cdot 448kx \cdot 672k^2 x^2$ M1 B1 A1 A1</p> <p>$4 \cdot 4kx \cdot 21k^2 x^2$ isw</p> <p>(Full marks are still available in part (b)).</p>	
(b)	<p>M1 for equating their coefficient of x^2 to 6 times that of x... to get an equation in k, ... <u>or</u> equating their coefficient of x to 6 times that of x^2, to get an equation in k.</p> <p>Allow this M mark even if the equation is trivial, providing their coefficients from part (a) have been used, e.g. $6 \cdot 448k \cdot 672k$, but beware $k = 4$ following from this, which is A0.</p> <p><u>An equation in k alone</u> is required for this M mark, so...</p> <p>e.g. $6 \cdot 448kx \cdot 672k^2 x^2$ or similar is M0 A0 (equation in coefficients only is never seen), but ...</p> <p>e.g. $6 \cdot 448kx \cdot 672k^2 x^2 \cdot 448k \cdot 672k^2$ will get M1 A1 (as coefficients rather than terms have now been considered).</p> <p>The mistake $2 \cdot \frac{kx}{2}$ would give a maximum of 3 marks: M1B0A0A0, M1A1</p>	

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$$f(x) = (3x - 2)(x - k) - 8$$

(a) Write down the value of $f(k)$.

(1)

When $f(x)$ is divided by $(x - 2)$ the remainder is 4

(b) Find the value of k .

(2)

(c) Factorise $f(x)$ completely.

(3)



Question Number	Scheme	Marks
Q3 (a)	$f(k) = -8$	B1 (1)
(b)	$f(2) = 4 \Rightarrow 4 = (6-2)(2-k) - 8$ So $k = -1$	M1 A1 (2)
(c)	$f(x) = 3x^2 - 2 + 3kx + 2k - 8$ $= (3x - 5)(x + 2)$	M1 M1A1 (3)
[6]		
(b)	<p>M1 for substituting $x = 2$ (not x) and equating to 4 to form an equation in k. If the expression is expanded in this part, condone 'slips' for this M mark. Treat the omission of the here as a 'slip' and allow the M mark.</p> <p><u>Beware:</u> Substituting x and equating to 0 (M0 A0) also gives k.</p> <p><u>Alternative;</u> M1 for dividing by (x), to get $3x$ (function of k), with remainder as a function of k, and equating the remainder to 4. [Should be $3x - 4 + k$], remainder k].</p> <p><u>No working:</u> k with no working scores M0 A0.</p>	
(c)	<p>1st M1 for multiplying out <u>and</u> substituting their (constant) value of k (in either order). The multiplying-out may occur earlier. Condone, for example, sign slips, but if the 4 (from part (b)) is included in the $f(x)$ expression, this is M0. The 2nd M1 is still available.</p> <p>2nd M1 for an attempt to factorise their three term quadratic (3TQ).</p> <p>A1 The correct answer, as a <u>product of factors</u>, is required. Allow $3(x - \frac{5}{3})(x + 2)$</p> <p>Ignore following work (such as a solution to a quadratic equation). If the 'equation' is solved but factors are never seen, the 2nd M is not scored.</p>	

4. (a) Complete the table below, giving values of $\sqrt[3]{(2^x + 1)}$ to 3 decimal places.

x	0	0.5	1	1.5	2	2.5	3
$\sqrt{(2^x + 1)}$	1.414	1.554	1.732	1.957			3

A graph in the xy -plane. The horizontal axis is the x -axis and the vertical axis is the y -axis. The origin is labeled O . A curve starts at a point on the y -axis and increases as it moves to the right. The region bounded by the y -axis, the x -axis, and the curve is shaded gray and labeled R . A vertical dashed line is drawn from the point where the curve intersects the x -axis (labeled 3) down to the x -axis.

Figure 1 shows the region R which is bounded by the curve with equation $y = \sqrt{2^x + 1}$, the x -axis and the lines $x = 0$ and $x = 3$

- (b) Use the trapezium rule, with all the values from your table, to find an approximation for the area of R .

(4)

- (c) By reference to the curve in Figure 1 state, giving a reason, whether your approximation in part (b) is an overestimate or an underestimate for the area of R .

(2)



Question Number	Scheme	Marks
Q4 (a)	$x = 2$ gives 2.236 (allow AWRT) Accept $\sqrt{5}$ $x = 2.5$ gives 2.580 (allow AWRT) Accept 2.58	B1 B1 (2)
(b)	$\frac{1}{2} \left(\frac{1}{2} + \frac{1}{2} \right) (1.414 + 2.580) + \frac{1}{2} (1.554 + 2.732) + \frac{1}{2} (1.957 + 2.236) + \frac{1}{2} (2.236 + 2.580)$ $= 6.133$ (AWRT 6.13, even following minor slips)	B1, [M1A1ft] A1 (4)
(c)	Overestimate 'Since the trapezia lie <u>above the curve</u> ', or an equivalent explanation, or sketch of (one or more) trapezia above the curve on a diagram (or on the given diagram, in which case there should be reference to this). (Note that there must be some reference to a trapezium or trapezia in the explanation or diagram).	B1 dB1 (2) [8]
(b)	B1 for $\frac{1}{2} \left(\frac{1}{2} + \frac{1}{2} \right)$ or equivalent. For the M mark, the first bracket must contain the 'first and last' values, and the second bracket (which must be multiplied by 2) must have no additional values. If the only mistake is to <u>omit</u> one of the values from the second bracket, this can be considered as a slip and the M mark can be allowed. Bracketing mistake: i.e. $\frac{1}{2} \left(\frac{1}{2} + \frac{1}{2} \right) (1.414 + 2.580) + \frac{1}{2} (1.554 + 2.732) + \frac{1}{2} (1.957 + 2.236) + \frac{1}{2} (2.236 + 2.580)$ scores B1 M1 A0 A0 <u>unless</u> the final answer implies that the calculation has been done correctly (then full marks can be given). <u>Alternative:</u> Separate trapezia may be used, and this can be marked equivalently. $\frac{1}{4} (1.414 + 1.554) + \frac{1}{4} (1.554 + 1.732) + \dots + \frac{1}{4} (2.580 + 2.732)$	1 st A1ft for correct expression, ft their 2.236 and their 2.580
(c)	1 st B1 for 'overestimate', ignoring earlier mistakes and ignoring any reasons given. 2 nd B1 is dependent upon the 1 st B1 (overestimate).	

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5. The third term of a geometric sequence is 324 and the sixth term is 96
- (a) Show that the common ratio of the sequence is $\frac{2}{3}$ (2)
- (b) Find the first term of the sequence. (2)
- (c) Find the sum of the first 15 terms of the sequence. (3)
- (d) Find the sum to infinity of the sequence. (2)

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

Question Number	Scheme	Marks
Q5 (a)	$324r^3 = 96$ or $r^3 = \frac{96}{324}$ or $r^3 = \frac{8}{27}$ $r = \frac{2}{3}$ (*)	M1 A1cso (2)
(b)	$a = \dots, 729$	M1, A1 (2)
(c)	$S_{15} = \frac{729 \left(1 - \left[\frac{2}{3}\right]^{15}\right)}{1 - \frac{2}{3}}, = 2182.00\dots$ (AWRT 2180)	M1A1ft, (3)
(d)	$S_{\infty} = \frac{729}{1 - \frac{2}{3}}, = 2187$	M1, A1 (2)
[9]		
(a)	<p>M1 for forming an equation for r^3 based on 96 and 324 (e.g. $96r^3 = 324$ scores M1). The equation must involve multiplication/division rather than addition/subtraction.</p> <p>A1 Do not penalise solutions with working in decimals, providing these are correctly rounded or truncated to at least 2dp <u>and</u> the final answer $2/3$ is seen.</p> <p><u>Alternative:</u> (verification)</p> <p>M1 Using $r^3 = \frac{8}{27}$ and multiplying 324 by this (or multiplying by $r = \frac{2}{3}$ three times).</p> <p>A1 Obtaining 96 (cso). (A conclusion is not required).</p> <p>$324 \times \frac{8}{27} = 96$ (no real evidence of calculation) is not quite enough and scores M1 A0.</p>	
(b)	<p>M1 for the use of a correct formula or for 'working back' by dividing by $\frac{2}{3}$ (or by their r) twice from 324 (or 5 times from 96).</p> <p>Exceptionally, allow M1 also for using $ar^3 = 324$ or $ar^6 = 96$ instead of $ar^2 = 324$ or $ar^5 = 96$, or for dividing by r three times from 324 (or 6 times from 96)... but no other exceptions are allowed.</p>	
(c)	<p>M1 for use of sum to 15 terms formula with values of a and r. If the wrong power is used, e.g. 14, the M mark is scored only if the correct sum formula is stated.</p> <p>1st A1ft for a correct expression or correct ft their a with $r = \frac{2}{3}$.</p> <p>2nd A1 for awrt 2180, even following 'minor inaccuracies'.</p> <p>Condone missing brackets round the $\frac{2}{3}$ for the marks in part (c).</p> <p><u>Alternative:</u></p>	
(d)	<p>M1 for adding 15 terms and 1st A1ft for adding the 15 terms that ft from their a and $r = \frac{2}{3}$.</p> <p>M1 for use of correct sum to infinity formula with their a. For this mark, if a value of r different from the given value is being used, M1 can still be allowed providing $r < 1$.</p>	

6. The circle C has equation

$$x^2 + y^2 - 6x + 4y = 12$$

- (a) Find the centre and the radius of C .

(5)

The point $P(-1, 1)$ and the point $Q(7, -5)$ both lie on C .

- (b) Show that PQ is a diameter of C .

(2)

The point R lies on the positive y -axis and the angle $PRQ = 90^\circ$.

- (c) Find the coordinates of R .

(4)



Question Number	Scheme	Marks
Q6 (a)	$x - 3^2 - 9 + y + 2^2 - 4 = 12$ Centre is (3, -2) $x - 3^2 + y + 2^2 = 12 + "9" + "4"$ $r = \sqrt{12 + "9" + "4"} = 5$ (or $\sqrt{25}$)	M1 A1, A1 M1 A1 (5)
(b)	$PQ = \sqrt{(7 - -1)^2 + (-5 - -1)^2}$ or $\sqrt{8^2 + 6^2}$ $= 10 = 2 \times \text{radius}, \therefore \text{diam.}$ (N.B. For A1, need a comment or conclusion) [ALT: midpt. of PQ $(\frac{7+1}{2}, \frac{1-5}{2})$: M1, $(3, -2) = \text{centre}$: A1] [ALT: eqn. of PQ $3x + 4y = 10$: M1, verify (3, -2) lies on this: A1] [ALT: find two grads, e.g. PQ and P to centre: M1, equal \therefore diameter: A1] [ALT: show that point S(7, 1) or (7, 1) lies on circle: M1 because $\angle PSQ = 90^\circ$, semicircle \therefore diameter: A1]	M1 A1 (2)
(c)	R must lie on the circle (angle in a semicircle theorem)... often <u>implied</u> by a <u>diagram with R on the circle</u> or by subsequent working $x = 0 \Rightarrow y^2 + 4y - 12 = 0$ $(y - 2)(y + 6) = 0$ $y = \dots$ (M is dependent on previous M) $y = -6$ or 2 (Ignore $y = -6$ if seen, and 'coordinates' are not required)	B1 M1 dM1 A1 (4) [11]
(a)	1 st M1 for attempt to complete square. Allow $(x \dots)^2 \dots$, or $(y \dots)^2 \dots$, $k \dots$. 1 st A1 x-coordinate 3, 2 nd A1 y-coordinate -2 2 nd M1 for a full method leading to $r = \dots$, with their 9 and their 4, 3 rd A1 5 or $\sqrt{25}$ The 1 st M can be <u>implied</u> by (\dots, \dots) but a full method must be seen for the 2 nd M. Where the 'diameter' in part (b) has <u>clearly</u> been used to answer part (a), no marks in (a), but in this case the M1 (<u>not</u> the A1) for part (b) can be given for work seen in (a). <u>Alternative</u> 1 st M1 for comparing with $x^2 \dots y^2 \dots gx \dots fy \dots$ to write down centre (\dots, \dots) directly. Condone sign errors for this M mark. 2 nd M1 for using $r \dots \sqrt{g^2 \dots f^2} \dots$. Condone sign errors for this M mark.	
(c)	1 st M1 for setting $x = 0$ and getting a 3TQ in y by using eqn. of circle. 2 nd M1 (dep.) for attempt to solve a 3TQ leading to <u>at least one</u> solution for y. <u>Alternative 1</u> : (Requires the B mark as in the main scheme) 1 st M for using (3, 4, 5) triangle with vertices (3, \dots), (0, \dots), (0, y) to get a linear or quadratic equation in y (e.g. $3^2 \dots y \dots$) ² $\dots 25$). 2 nd M (dep.) as in main scheme, but may be scored by simply solving a linear equation. <u>Alternative 2</u> : (Not requiring realisation that R is on the circle) B1 for attempt at $m_{PR} \dots n_{QR} \dots$, (<u>NOT</u> m_{PQ}) or for attempt at Pythag. in triangle PQR. 1 st M1 for setting $x = 0$, i.e. (0, y), and proceeding to get a 3TQ in y. Then main scheme. <u>Alternative 2 by 'verification'</u> : B1 for attempt at $m_{PR} \dots n_{QR} \dots$, (<u>NOT</u> m_{PQ}) or for attempt at Pythag. in triangle PQR. 1 st M1 for trying (0, 2). 2 nd M1 (dep.) for performing all required calculations. A1 for fully correct working and conclusion.	

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7. (i) Solve, for $-180^\circ \leq \theta < 180^\circ$,

$$(1 + \tan \theta)(5 \sin \theta - 2) = 0.$$

(4)

Leave
blank(ii) Solve, for $0 \leq x < 360^\circ$,

$$4\sin x = 3\tan x.$$

(6)

Q7

(Total 10 marks)



H 3 4 2 6 3 A 0 1 9 2 4

Question Number	Scheme	Marks
Q7 (i)	$\tan \theta = -1 \Rightarrow \theta = -45, 135$ $\sin \theta = \frac{2}{5} \Rightarrow \theta = 23.6, 156.4$ (AWRT: 24, 156)	B1, B1ft B1, B1ft (4)
(ii)	$4 \sin x = \frac{3 \sin x}{\cos x}$ $4 \sin x \cos x = 3 \sin x \Rightarrow \sin x(4 \cos x - 3) = 0$ Other possibilities (after squaring): $\sin^2 x(16 \sin^2 x \blacksquare) \blacksquare$, $(16 \cos^2 x \blacksquare)(\cos^2 x \blacksquare) \blacksquare$ $x = 0, 180$ <u>seen</u> $x = 41.4, 318.6$ (AWRT: 41, 319)	M1 M1 B1, B1 B1, B1ft (6) [10]
(i)	1 st B1 for -45 <u>seen</u> (α , where $\blacksquare \blacksquare 0$) 2 nd B1 for 135 <u>seen</u> , or <u>ft</u> $(180 + \alpha)$ if α is negative, or $(\alpha - 180)$ if α is positive. If $\tan \blacksquare$ is obtained from <u>wrong working</u> , 2 nd B1ft is still available. 3 rd B1 for awrt 24 (β , where $\blacksquare \blacksquare 0$) 4 th B1 for awrt 156, or <u>ft</u> $(180 - \beta)$ if β is positive, or $-(180 + \beta)$ if β is negative. If $\sin \blacksquare$ is obtained from <u>wrong working</u> , 4 th B1ft is still available.	
(ii)	1 st M1 for use of $\tan x = \frac{\sin x}{\cos x}$. Condone $\frac{3 \sin x}{3 \cos x}$. 2 nd M1 for correct work leading to 2 factors (may be implied). 1 st B1 for 0, 2 nd B1 for 180. 3 rd B1 for awrt 41 (γ , where $\blacksquare \blacksquare 80$) 4 th B1 for awrt 319, or <u>ft</u> $(360 - \gamma)$. If $\cos \blacksquare$ is obtained from <u>wrong working</u> , 4 th B1ft is still available. N.B. Losing $\sin x \blacksquare$ usually gives a maximum of 3 marks M1M0B0B0B1B1 <u>Alternative:</u> (squaring both sides) 1 st M1 for squaring both sides and using a 'quadratic' identity. e.g. $16 \sin^2 \blacksquare (\sec^2 \blacksquare)$ 2 nd M1 for reaching a factorised form. e.g. $(16 \cos^2 \blacksquare)(\cos^2 \blacksquare) \blacksquare$ Then marks are equivalent to the main scheme. Extra solutions, if not rejected, are penalised as in the main scheme. <u>For both parts of the question:</u> <u>Extra solutions outside required range:</u> Ignore <u>Extra solutions inside required range:</u> For each <u>pair</u> of B marks, the 2 nd B mark is lost if there are two correct values and one or more extra solution(s), e.g. $\tan \blacksquare \blacksquare 45, \blacksquare 45, 135$ is B1 B0 <u>Answers in radians:</u> Loses a maximum of 2 B marks in the whole question (to be deducted at the first and second occurrence).	

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- $$\log_2 y = -3$$

(2)

- $$\frac{\log_2 32 + \log_2 16}{\log_2 x} = \log_2 x$$

(5)



Question Number	Scheme	Marks
Q8 (a)	$\log_2 y = -3 \Rightarrow y = 2^{-3}$ $y = \frac{1}{8}$ or 0.125	M1 A1 (2)
(b)	$32 \blacksquare^5$ or $16 \blacksquare^4$ or $512 \blacksquare^9$ [or $\log_2 32 = 5\log_2 2$ or $\log_2 16 = 4\log_2 2$ or $\log_2 512 = 9\log_2 2$] [or $\log_2 32 \blacksquare \frac{\log_{10} 32}{\log_{10} 2}$ or $\log_2 16 \blacksquare \frac{\log_{10} 16}{\log_{10} 2}$ or $\log_2 512 \blacksquare \frac{\log_{10} 512}{\log_{10} 2}$] $\log_2 32 + \log_2 16 = 9$ $(\log x)^2 \blacksquare$ or $(\log x)(\log x) \blacksquare$. (May not be seen explicitly, so M1 may be implied by later work, and the base may be 10 rather than 2) $\log_2 x = 3 \Rightarrow x = 2^3 = 8$ $\log_2 x = -3 \Rightarrow x = 2^{-3} = \frac{1}{8}$	M1 A1 M1 A1 A1ft (5) [7]
(a)	M1 for <u>getting out of logs</u> correctly. If done by change of base, $\log_{10} y \blacksquare .903\dots$ is insufficient for the M1, but $y \blacksquare 0^{.903}$ scores M1. A1 for the <u>exact</u> answer, e.g. $\log_{10} y \blacksquare .903 \blacksquare \blacksquare .12502$. scores M1 (implied) A0. <u>Correct answer</u> with no working scores both marks. <u>Allow</u> both marks for implicit statements such as $\log_2 0.125 \blacksquare$.	
(b)	1 st M1 for expressing 32 or 16 or 512 as a power of 2, or for a change of base enabling evaluation of $\log_2 32$, $\log_2 16$ or $\log_2 512$ by calculator. (Can be implied by 5, 4 or 9 respectively). 1 st A1 for 9 (exact). 2 nd M1 for getting $\log_2 x \blacksquare = \text{constant}$. The constant can be a log or a sum of logs. If written as $\log_2 x^2$ instead of $\log_2 x \blacksquare$, allow the M mark <u>only</u> if subsequent work implies correct interpretation. 2 nd A1 for 8 (exact). Change of base methods leading to a non-exact answer score A0. 3 rd A1ft for an answer of $\frac{1}{\text{their } 8}$. An ft answer may be non-exact. <u>Possible mistakes:</u> $\log_2 \blacksquare^9 \blacksquare \log_2 \blacksquare^2 \blacksquare^2 \blacksquare^9 \blacksquare \blacksquare$. scores M1A1(implied by 9)M0A0A0 $\log_2 512 \blacksquare \log_2 x \blacksquare \log_2 x \blacksquare^2 \blacksquare 512 \blacksquare \blacksquare$. scores M0A0(9 never seen)M1A0A0 $\log_2 48 \blacksquare \log_2 x \blacksquare \blacksquare \log_2 x \blacksquare \blacksquare 5.585 \blacksquare \blacksquare 5.145$, $x \blacksquare 0.194$ scores M0A0M1A0A1ft <u>No working</u> (or 'trial and improvement'): $x = 8$ scores M0 A0 M1 A1 A0	

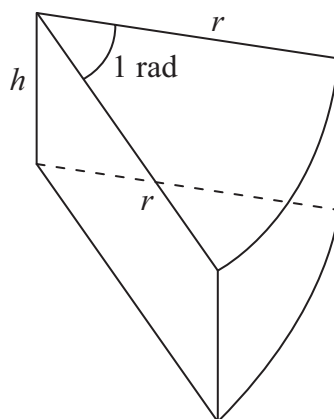


Figure 2

Figure 2 shows a closed box used by a shop for packing pieces of cake. The box is a right prism of height h cm. The cross section is a sector of a circle. The sector has radius r cm and angle 1 radian.

The volume of the box is 300 cm^3 .

- (a) Show that the surface area of the box, $S \text{ cm}^2$, is given by

$$S = r^2 + \frac{1800}{r} \quad (5)$$

- (b) Use calculus to find the value of r for which S is stationary.

- (c) Prove that this value of r gives a minimum value of S . (2)

- (d) Find, to the nearest cm^2 , this minimum value of S . (2)



Question Number	Scheme	Marks
Q9 (a)	<p>(Arc length =) $r\theta$. Can be awarded by implication from later work, e.g. $3rh$ or $(2rh + rh)$ in the S formula. (Requires use of $\theta = 1$).</p> <p>(Sector area =) $\frac{1}{2}r^2\theta$. Can be awarded by implication from later work, e.g. the correct volume formula. (Requires use of $\theta = 1$).</p> <p>Surface area = 2 sectors + 2 rectangles + curved face $(= r^2 + 3rh)$ (See notes below for what is allowed here)</p> <p>Volume = $300 = \frac{1}{2}r^2h$</p> <p>Sub for h: $S = r^2 + 3 \times \frac{600}{r} = r^2 + \frac{1800}{r}$ (*)</p>	<p>B1</p> <p>B1</p> <p>M1</p> <p>B1</p> <p>A1cso (5)</p>
(b)	<p>$\frac{dS}{dr} = 2r - \frac{1800}{r^2}$ or $2r - 1800r^{-2}$ or $2r - 1800r^{-2}$</p> <p>$\frac{dS}{dr} = 2r - 1800r^{-2}$, $r = \sqrt[3]{900}$, or AWR 9.7 (NOT 9.7 or 9.7)</p>	<p>M1A1</p> <p>M1, A1 (4)</p>
(c)	<p>$\frac{d^2S}{dr^2} = 2 + \frac{3600}{r^3} > 0$ so point is a minimum</p>	<p>M1, A1ft (2)</p>
(d)	<p>$S_{\min} = 9.65...^2 + \frac{1800}{9.65...}$</p> <p>(Using their value of r, however found, in the given S formula) $= 279.65...$ (AWRT: 280) (Dependent on full marks in part (b))</p>	<p>M1</p> <p>A1 (2)</p> <p>[13]</p>
(a)	<p>M1 for attempting a formula (with terms added) for surface area. May be incomplete or wrong and may have extra term(s), but must have an r^2 (or r^2) term and an rh (or rh) term.</p>	
(b)	<p><u>In parts (b), (c) and (d), ignore labelling of parts</u></p> <p>1st M1 for attempt at differentiation (one term is sufficient) r^n</p> <p>2nd M1 for setting their derivative (a 'changed function') = 0 and solving as far as r^3. (depending upon their 'changed function', this could be r or r^2, etc., but the algebra must deal with a negative power of r and should be sound apart from possible sign errors, so that r^n is consistent with their derivative).</p>	
(c)	<p>M1 for attempting second derivative (one term is sufficient) r^n, and considering its sign. Substitution of a value of r is not required. (Equating it to zero is M0).</p> <p>A1ft for a correct second derivative (or correct ft from their first derivative) and a valid reason (e.g. > 0), and conclusion. The actual value of the second derivative, if found, can be ignored. To score this mark as ft, their second derivative must indicate a minimum.</p> <p><u>Alternative:</u></p> <p>M1: Find value of $\frac{dS}{dr}$ on each side of their value of r and consider sign.</p> <p>A1ft: Indicate sign change of negative to positive for $\frac{dS}{dr}$, and conclude minimum.</p> <p><u>Alternative:</u></p> <p>M1: Find value of S on each side of their value of r and compare with their 279.65.</p> <p>A1ft: Indicate that both values are more than 279.65, and conclude minimum.</p>	