Centre No.						Paper Reference							Surname	Initial(s)
Candidate No.						6	6	6	4	/	0	1	Signature	

Paper Reference(s)

6664/01

Examiner's use only

Team Leader's use only

Edexcel GCE

Core Mathematics C2

Advanced Subsidiary

Monday 21 May 2007 – Morning

Time: 1 hour 30 minutes

Question Number	Leave Blank
1	
2	
3	
4	
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7	
8	
9	
10	
Total	

Materials required for examination
Mathematical Formulae (Green)

Items included with question papers
Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions. Write your answers in the spaces provided in this question paper.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 10 questions in this question paper. The total mark for this paper is 75.
There are 24 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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1. Evaluate $\int_1^8 \frac{1}{\sqrt{x}} \mathrm{d}x$, giving your answer in the form $a + b\sqrt{2}$, where a and b are integers.

(4)

Q1

(Total 4 marks)

June 2007
6664 Core Mathematics C2
Mark Scheme

Question number	Scheme	Marks
1.	$\int x^{-\frac{1}{2}}\,dx = \dfrac{x^{\frac{1}{2}}}{\left(\frac{1}{2}\right)}$ (Or equivalent, such as $2x^{\frac{1}{2}}$, or $2\sqrt{x}$)	M1 A1
	$\left[\dfrac{x^{\frac{1}{2}}}{\left(\frac{1}{2}\right)}\right]_1^8 = 2\sqrt{8}-2 = -2+4\sqrt{2}$ [or $4\sqrt{2}-2$, or $2(2\sqrt{2}-1)$, or $2(-1+2\sqrt{2})$]	M1 A1
		(4) **4**
	1^{st} M: $x^{-\frac{1}{2}} \to kx^{\frac{1}{2}},\ k \neq 0$. 2^{nd} M: Substituting limits 8 and 1 into a 'changed' function (i.e. not $\dfrac{1}{\sqrt{x}}$ or $x^{-\frac{1}{2}}$), and subtracting, either way round. 2^{nd} A: This final mark is still scored if $-2+4\sqrt{2}$ is reached via a decimal. N.B. Integration constant +C may appear, e.g. $\left[\dfrac{x^{\frac{1}{2}}}{\left(\frac{1}{2}\right)}+C\right]_1^8 = (2\sqrt{8}+C)-(2+C) = -2+4\sqrt{2}$ (Still full marks) But… a final answer such as $-2+4\sqrt{2}+C$ is A0. N.B. It will sometimes be necessary to 'ignore subsequent working' (isw) after a correct form is seen, e.g. $\int x^{-\frac{1}{2}}\,dx = \dfrac{x^{\frac{1}{2}}}{\left(\frac{1}{2}\right)}$ (M1 A1), followed by incorrect simplification $\int x^{-\frac{1}{2}}\,dx = \dfrac{x^{\frac{1}{2}}}{\left(\frac{1}{2}\right)} = \dfrac{1}{2}x^{\frac{1}{2}}$ (still M1 A1)…. The second M mark is still available for substituting 8 and 1 into $\dfrac{1}{2}x^{\frac{1}{2}}$ and subtracting.	

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2. $$f(x) = 3x^3 - 5x^2 - 16x + 12.$$

(a) Find the remainder when f(x) is divided by (x – 2).

(2)

Given that (x + 2) is a factor of f(x),

(b) factorise f(x) completely.

(4)

Q2

(Total 6 marks)

Question number	Scheme	Marks
2.	(a) f(2) = 24 – 20 – 32 + 12 = –16 (M: Attempt f(2) or f(–2)) (If continues to say 'remainder = 16', isw) Answer must be seen in part (a), not part (b). (b) $(x+2)(3x^2-11x+6)$ $(x+2)(3x-2)(x-3)$ (If continues to 'solve an equation', isw)	M1 A1 (2) M1 A1 M1 A1 (4) **6**
	(a) Answer only (if correct) scores both marks. (16 as 'answer only' is M0 A0). Alternative (long division): Divide by $(x-2)$ to get $(3x^2+ax+b)$, $a \neq 0, b \neq 0$. [M1] $(3x^2+x-14)$, and – 16 seen. [A1] (If continues to say 'remainder = 16', isw) (b) First M requires division by $(x+2)$ to get $(3x^2+ax+b)$, $a \neq 0, b \neq 0$. Second M for attempt to factorise their quadratic, even if wrongly obtained, perhaps with a remainder from their division. Usual rule: $(kx^2+ax+b)=(px+c)(qx+d)$, where $\lvert pq\rvert=\lvert k\rvert$ and $\lvert cd\rvert=\lvert b\rvert$. Just solving their quadratic by the formula is M0. "Combining" all 3 factors is not required. Alternative (first 2 marks): $(x+2)(3x^2+ax+b)=3x^3+(6+a)x^2+(2a+b)x+2b=0$, then compare coefficients to find values of a and b. [M1] $a=-11,\ b=6$ [A1] Alternative: Factor theorem: Finding that $f(3)=0$ $\therefore$ factor is, $(x-3)$ [M1, A1] Finding that $f\left(\frac{2}{3}\right)=0$ $\therefore$ factor is, $(3x-2)$ [M1, A1] If just one of these is found, score the first 2 marks M1 A1 M0 A0. Losing a factor of 3: $(x+2)\left(x-\frac{2}{3}\right)(x-3)$ scores M1 A1 M1 A0. Answer only, one sign wrong: e.g. $(x+2)(3x-2)(x+3)$ scores M1 A1 M1 A0.	

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3. (a) Find the first four terms, in ascending powers of x, in the binomial expansion of $(1+kx)^6$, where k is a non-zero constant.

(3)

Given that, in this expansion, the coefficients of x and x^2 are equal, find

(b) the value of k,

(2)

(c) the coefficient of x^3.

(1)

Question number	Scheme	Marks
3.	(a) $1+6kx$ [Allow unsimplified versions, e.g. $1^6+6(1^5)kx$, ${}^6C_0+{}^6C_1kx$]	B1
	$+\frac{6\times5}{2}(kx)^2+\frac{6\times5\times4}{3\times2}(kx)^3$ [See below for acceptable versions]	M1 A1 (3)
	N.B. THIS NEED NOT BE SIMPLIFIED FOR THE A1 (isw is applied)	
	(b) $6k=15k^2$ $k=\frac{2}{5}$ (or equiv. fraction, or 0.4) (Ignore k = 0, if seen)	M1 A1cso (2)
	(c) $c=\frac{6\times5\times4}{3\times2}\left(\frac{2}{5}\right)^3=\frac{32}{25}$ (or equiv. fraction, or 1.28)	A1cso (1)
	(Ignore x^3, so $\frac{32}{25}x^3$ is fine)	**6**
	(a) The terms can be 'listed' rather than added.	
	M1: Requires correct structure: 'binomial coefficients' (perhaps from Pascal's triangle), increasing powers of x. Allow a 'slip' or 'slips' such as: $+\frac{6\times5}{2}kx^2+\frac{6\times5\times4}{3\times2}kx^3$, $+\frac{6\times5}{2}(kx)^2+\frac{6\times5}{3\times2}(kx)^3$ $+\frac{5\times4}{2}kx^2+\frac{5\times4\times3}{3\times2}kx^3$, $+\frac{6\times5}{2}x^2+\frac{6\times5\times4}{3\times2}x^3$	
	But: $15+k^2x^2+20+k^3x^3$ or similar is M0.	
	Both x^2 and x^3 terms must be seen.	
	$\binom{6}{2}$ and $\binom{6}{3}$ or equivalent such as ${}^6C_2$ and ${}^6C_3$ are acceptable, and even $\left(\frac{6}{2}\right)$ and $\left(\frac{6}{3}\right)$ are acceptable for the method mark.	
	A1: Any correct (possibly unsimplified) version of these 2 terms. $\binom{6}{2}$ and $\binom{6}{3}$ or equivalent such as ${}^6C_2$ and ${}^6C_3$ are acceptable.	
	Descending powers of x: Can score the M mark if the required first 4 terms are not seen.	
	Multiplying out $(1+kx)(1+kx)(1+kx)(1+kx)(1+kx)(1+kx)$: M1: A full attempt to multiply out (power 6) B1 and A1 as on the main scheme.	
	(b) M: Equating the coefficients of x and x^2 (even if trivial, e.g. $6k = 15k$). Allow this mark also for the 'misread': equating the coefficients of x^2 and x^3. An equation in k alone is required for this M mark, although… …condone $6kx=15k^2x^2\Rightarrow\left(6k=15k^2\Rightarrow\right)k=\frac{2}{5}$.	

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4.

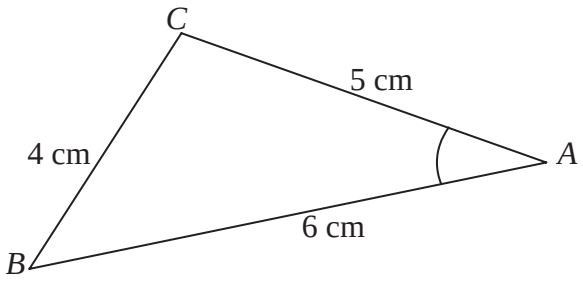

Figure 1

Figure 1 shows the triangle ABC, with $AB = 6$ cm, $BC = 4$ cm and $CA = 5$ cm.

(a) Show that $\cos A = \frac{3}{4}$.

(3)

(b) Hence, or otherwise, find the exact value of sin A.

(2)

Question number	Scheme	Marks
4.	(a) $4^2 = 5^2 + 6^2 - (2 \times 5 \times 6 \cos\theta)$	M1
	$\cos\theta = \dfrac{5^2 + 6^2 - 4^2}{2 \times 5 \times 6}$	A1
	$\left(= \dfrac{45}{60}\right) = \dfrac{3}{4}$ (*)	A1cso (3)
	(b) $\sin^2 A + \left(\dfrac{3}{4}\right)^2 = 1$ (or equiv. Pythag. method)	M1
	$\left(\sin^2 A = \dfrac{7}{16}\right)$ $\sin A = \dfrac{1}{4}\sqrt{7}$ or equivalent exact form, e.g. $\sqrt{\dfrac{7}{16}}$, $\sqrt{0.4375}$	A1 (2)
		5
	(a) M: Is also scored for $5^2 = 4^2 + 6^2 - (2 \times 4 \times 6 \cos\theta)$ or $6^2 = 5^2 + 4^2 - (2 \times 5 \times 4 \cos\theta)$ or $\cos\theta = \dfrac{4^2 + 6^2 - 5^2}{2 \times 4 \times 6}$ or $\cos\theta = \dfrac{5^2 + 4^2 - 6^2}{2 \times 5 \times 4}$. 1st A: Rearranged correctly and numerically correct (possibly unsimplified), in the form $\cos\theta = ...$ or $60\cos\theta = 45$ (or equiv. in the form $p\cos\theta = q$). Alternative (verification): $4^2 = 5^2 + 6^2 - \left(2 \times 5 \times 6 \times \dfrac{3}{4}\right)$ [M1] Evaluate correctly, at least to $16 = 25 + 36 - 45$ [A1] Conclusion (perhaps as simple as a tick). [A1cso] (Just achieving 16 = 16 is insufficient without at least a tick). (b) M: Using a correct method to find an equation in $\sin^2 A$ or $\sin A$ which would give an exact value. Correct answer without working (or with unclear working or decimals): Still scores both marks.	

Leave blank

5. The curve C has equation

$$y = x\sqrt{(x^3 + 1)}, \qquad 0 \leqslant x \leqslant 2.$$

(a) Complete the table below, giving the values of y to 3 decimal places at $x = 1$ and $x = 1.5$.

x	0	0.5	1	1.5	2
y	0	0.530			6

(2)

(b) Use the trapezium rule, with all the y values from your table, to find an approximation for the value of $\int_0^2 x\sqrt{(x^3+1)}\,dx$, giving your answer to 3 significant figures.

(4)

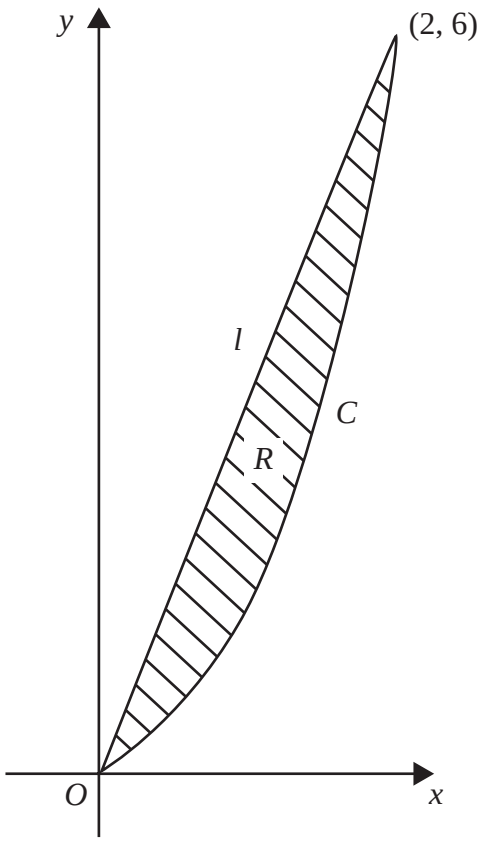

Figure 2

Figure 2 shows the curve C with equation $y = x\sqrt{(x^3 + 1)}, 0 \leqslant x \leqslant 2$, and the straight line segment l, which joins the origin and the point (2, 6). The finite region R is bounded by C and l.

(c) Use your answer to part (b) to find an approximation for the area of R, giving your answer to 3 significant figures.

(3)

Question number	Scheme	Marks
5.	(a) 1.414 (allow also exact answer $\sqrt{2}$), 3.137 Allow awrt	B1, B1 (2)
	(b) $\frac{1}{2}(0.5)$ ….	B1
	…. $\{0+6+2(0.530+1.414+3.137)\}$	M1 A1ft
	= 4.04 (Must be 3 s.f.)	A1 (4)
	(c) Area of triangle = $\frac{1}{2}(2\times 6)$	B1
	(Could also be found by integration, or even by the trapezium rule on $y = 3x$)	
	Area required = Area of triangle – Answer to (b) (Subtract either way round)	M1
	6 – 4.04 = 1.96 Allow awrt	A1ft (3)
	(ft from (b), dependent on the B1, and on answer to (b) less than 6)	**9**
	(a) If answers are given to only 2 d.p. (1.41 and 3.14), this is B0 B0, but full marks can be given in part (b) if 4.04 is achieved. (b) Bracketing mistake: i.e. $\frac{1}{2}(0.5)(0+6)+2(0.530+1.414+3.137)$ scores B1 M1 A0 A0 unless the final answer implies that the calculation has been done correctly (then full marks can be given). Alternative (finding and adding separate areas): $\frac{1}{2}\times\frac{1}{2}$ (Triangle/trapezium formulae, and height of triangle/trapezium)[B1] Fully correct method for total area, with values from table. [M1, A1ft] 4.04 [A1] (c) B1: Can be given for 6 with no working, but should not be given for 6 obtained from wrong working. A1ft: This is a dependent follow-through: the B1 for 6 must have been scored, and the answer to (b) must be less than 6.	

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6. (a) Find, to 3 significant figures, the value of x for which $8^x = 0.8$.

(2)

(b) Solve the equation

$$2\log_3 x - \log_3 7x = 1.$$

(4)

Question number	Scheme	Marks
6.	(a) $x = \dfrac{\log 0.8}{\log 8}$ or $\log_8 0.8$, $= -0.107$ Allow awrt	M1, A1 (2)
	(b) $2\log x = \log x^2$	B1
	$\log x^2 - \log 7x = \log \dfrac{x^2}{7x}$	M1
	"Remove logs" to form equation in x, using the base correctly: $\dfrac{x^2}{7x} = 3$	M1
	$x = 21$ (Ignore $x = 0$, if seen)	A1cso (4) **6**

(a) Allow also the 'implicit' answer $8^{-0.107}$ (M1 A1).

Answer only: –0.107 or awrt: Full marks.

Answer only: –0.11 or awrt (insufficient accuracy): M1 A0

Trial and improvement: Award marks as for "answer only".

(b) Alternative:

$2\log x = \log x^2$ B1

$\log 7x + 1 = \log 7x + \log 3 = \log 21x$ M1

"Remove logs" to form equation in x: $x^2 = 21x$ M1

$x = 21$ (Ignore $x = 0$, if seen) A1

Alternative:

$\log 7x = \log 7 + \log x$ B1

$2\log x - (\log 7 + \log x) = 1$

$\log_3 x = 1 + \log_3 7$ M1

$x = 3^{(1+\log_3 7)}$ $\left(= 3^{2.771\ldots}\right)$ or $\log_3 x = \log_3 3 + \log_3 7$ M1

$x = 21$ A1

Attempts using change of base will usually require the same steps as in the main scheme or alternatives, so can be marked equivalently.

A common mistake:

$\log x^2 - \log 7x = \dfrac{\log x^2}{\log 7x}$ B1 M0

$\dfrac{x^2}{7x} = 3$ $x = 21$ M1('Recovery'), but A0

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7.

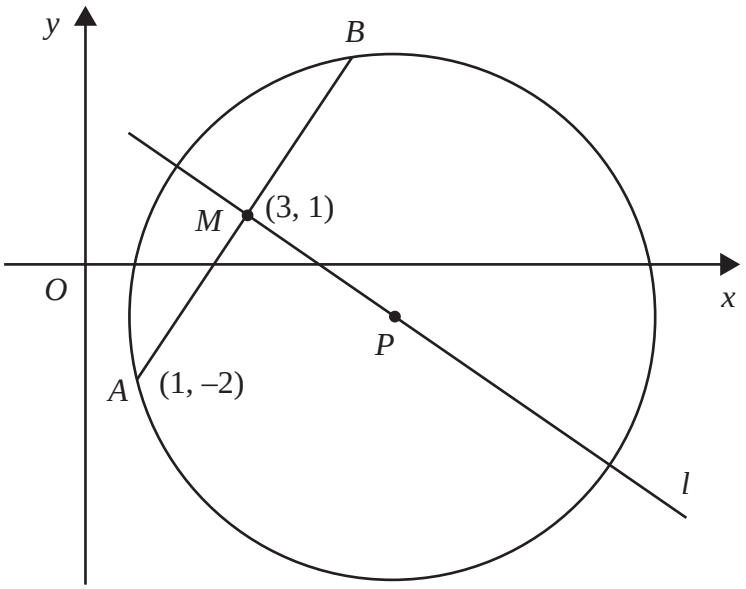

Figure 3

The points A and B lie on a circle with centre P, as shown in Figure 3.
The point A has coordinates $(1, -2)$ and the mid-point M of AB has coordinates $(3, 1)$.
The line l passes through the points M and P.

(a) Find an equation for l.

(4)

Given that the x-coordinate of P is 6,

(b) use your answer to part (a) to show that the y-coordinate of P is -1,

(1)

(c) find an equation for the circle.

(4)

Question number	Scheme	Marks
7.	(a) Gradient of AM: $\frac{1-(-2)}{3-1}=\frac{3}{2}$ or $\frac{-3}{-2}$	B1
	Gradient of l: $=-\frac{2}{3}$ M: use of $m_1m_2=-1$, or equiv.	M1
	$y-1=-\frac{2}{3}(x-3)$ or $\frac{y-1}{x-3}=-\frac{2}{3}$ $\left[3y=-2x+9\right]$ (Any equiv. form)	M1 A1 (4)
	(b) $x = 6$: $3y = –12 + 9 = –3$ $y = –1$ (or show that for $y = –1$, $x = 6$) (*) (A conclusion is not required).	B1 (1)
	(c) $\left(r^2=\right)$ $(6-1)^2+(-1-(-2))^2$ M: Attempt r^2 or r	M1 A1
	N.B. Simplification is not required to score M1 A1	
	$(x\pm6)^2+(y\pm1)^2=k$, $k\neq0$ (Value for k not needed, could be r^2 or r)	M1
	$(x-6)^2+(y+1)^2=26$ (or equiv.)	A1 (4)
	Allow $\left(\sqrt{26}\right)^2$ or other exact equivalents for 26. (But… $(x-6)^2+(y--1)^2=26$ scores M1 A0)	
	(Correct answer with no working scores full marks)	**9**
	(a) 2nd M1: eqn. of a straight line through (3, 1) with any gradient except 0 or ∞.	
	Alternative: Using (3, 1) in $y=mx+c$ to find a value of c scores M1, but an equation (general or specific) must be seen.	
	Having coords the wrong way round, e.g. $y-3=-\frac{2}{3}(x-1)$, loses the 2nd M mark unless a correct general formula is seen, e.g. $y-y_1=m(x-x_1)$.	
	If the point $P(6,-1)$ is used to find the gradient of MP, maximum marks are (a) B0 M0 M1 A1 (b) B0.	
	(c) 1st M1: Condone one slip, numerical or sign, inside a bracket.	
	Must be attempting to use points $P(6, –1)$ and $A(1, –2)$, or perhaps P and B. (Correct coordinates for B are (5, 4)).	
	1st M alternative is to use a complete Pythag. method on triangle MAP, n.b. $MP = MA = \sqrt{13}$.	
	Special case: If candidate persists in using their value for the y-coordinate of P instead of the given –1, allow the M marks in part (c) if earned.	

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8. A trading company made a profit of £50 000 in 2006 (Year 1).

A model for future trading predicts that profits will increase year by year in a geometric sequence with common ratio r, $r > 1$.

The model therefore predicts that in 2007 (Year 2) a profit of £50 000r will be made.

(a) Write down an expression for the predicted profit in Year n.

(1)

The model predicts that in Year n, the profit made will exceed £200 000.

(b) Show that $n > \dfrac{\log 4}{\log r} + 1$.

(3)

Using the model with $r = 1.09$,

(c) find the year in which the profit made will first exceed £200 000,

(2)

(d) find the total of the profits that will be made by the company over the 10 years from 2006 to 2015 inclusive, giving your answer to the nearest £10 000.

(3)

Question number	Scheme	Marks
8.	(a) $50\,000r^{n-1}$ (or equiv.) (Allow ar^{n-1} if $50\,000r^{n-1}$ is seen in (b))	B1 (1)
	(b) $50\,000r^{n-1} > 200\,000$ (Using answer to (a), which must include *r* and *n*, and 200 000) (Allow equals sign or the wrong inequality sign) (Condone 'slips' such as omitting a zero)	M1
	$r^{n-1} > 4 \quad \Rightarrow \quad (n-1)\log r > \log 4$ (Introducing logs and dealing correctly with the power) (Allow equals sign or the wrong inequality sign)	M1
	$n > \dfrac{\log 4}{\log r} + 1$ (*)	A1cso (3)
	(c) $r = 1.09$: $n > \dfrac{\log 4}{\log 1.09} + 1$ or $n - 1 > \dfrac{\log 4}{\log 1.09}$ $(n > 17.086...)$ (Allow equality)	M1
	Year 18 or 2023 (If one of these is correct, ignore the other)	A1 (2)
	(d) $S_n = \dfrac{a(1-r^n)}{1-r} = \dfrac{50\,000(1-1.09^{10})}{1-1.09}$	M1 A1
	£760 000 (Must be this answer… nearest £10000)	A1 (3)
		9
	(b) Incorrect inequality sign at any stage loses the A mark. Condone missing brackets if otherwise correct, e.g $n-1\log r > \log 4$. A common mistake: $50\,000r^{n-1} > 200\,000$ M1 $(n-1)\log 50\,000r > \log 200\,000$ M0 ('Recovery' from here is not possible).	
	(c) Correct answer with no working scores full marks. Year 17 (or 2022) with no working scores M1 A0. Treat other methods (e.g. "year by year" calculation) as if there is no working.	
	(d) M1: Use of the correct formula with *a* = 50000, 5000 or 500000, and *n* = 9, 10, 11 or 15. M1 can also be scored by a "year by year" method, with terms added. (Allow the M mark if there is evidence of adding 9, 10, 11 or 15 terms). 1st A1 is scored if 10 correct terms have been added (allow "nearest £100"). (50000, 54500, 59405, 64751, 70579, 76931, 83855, 91402, 99628, 108595) No working shown: Special case: 760 000 scores 1 mark, scored as 1, 0, 0. (Other answers with no working score no marks).	

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9. (a) Sketch, for $0 \leqslant x \leqslant 2\pi$, the graph of $y = \sin\left(x + \frac{\pi}{6}\right)$.

(2)

(b) Write down the exact coordinates of the points where the graph meets the coordinate axes.

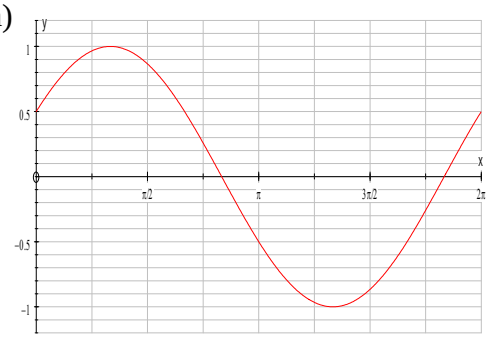
(3)

(c) Solve, for $0 \leqslant x \leqslant 2\pi$, the equation

$$\sin\left(x + \frac{\pi}{6}\right) = 0.65,$$

giving your answers in radians to 2 decimal places.

(5)

Question number	Scheme	Marks
9.	(a)  Sine wave (anywhere) with at least 2 turning points.	M1
	Starting on positive *y*-axis, going up to a max., then min. below *x*-axis, no further turning points in range, finishing above *x*-axis at $x = 2\pi$ or 360°. There must be <u>some</u> indication of scale on the *y*-axis… (e.g. 1, –1 or 0.5) Ignore parts of graph outside 0 to 2π.	A1 (2)
	n.b. Give credit if necessary for what is seen on an initial sketch (before any transformation has been performed).	
	(b) $\left(0,\frac{1}{2}\right)$, $\left(\frac{5\pi}{6},0\right)$, $\left(\frac{11\pi}{6},0\right)$ (Ignore any extra solutions) (<u>Not</u> 150°, 330°)	B1, B1, B1 (3)
	$\left(\pi-\frac{\pi}{6}\right)$ and $\left(2\pi-\frac{\pi}{6}\right)$ are insufficient, but if <u>both</u> are seen allow B1 B0.	
	(c) awrt 0.71 radians (0.70758…), or awrt 40.5° (40.5416…) (α)	B1
	$(\pi-\alpha)$ (2.43…) or $(180-\alpha)$ <u>if α is in degrees</u>. $\left[\underline{\text{NOT}}\ \pi-\left(\alpha-\frac{\pi}{6}\right)\right]$	M1
	Subtract $\frac{\pi}{6}$ from α (or from $(\pi-\alpha)$)… or subtract 30 <u>if α is in degrees</u>	M1
	0.18 (or 0.06π), 1.91 (or 0.61π) Allow awrt (The 1st A mark is dependent on just the 2nd M mark)	A1, A1 (5)
		10
	(b) The zeros are not required, i.e. allow 0.5, etc. (and also allow coordinates the wrong way round). These marks are also awarded if the exact intercept values are seen in part (a), but if values in (b) and (a) are contradictory, (b) takes precedence.	
	(c) B1: If the required value of α is <u>not seen</u>, this mark can be given by implication if a final answer rounding to 0.18 or 0.19 (or a final answer rounding to 1.91 or 1.90) is achieved. (Also see premature approx. note*).	
	<u>Special case:</u> $\sin\left(x+\frac{\pi}{6}\right)=0.65 \Rightarrow \sin x+\sin\frac{\pi}{6}=0.65 \Rightarrow \sin x=0.15$ $x=\arcsin 0.15=0.15056...$ and $x=\pi-0.15056=2.99$ (B0 M1 M0 A0 A0) (This special case mark is also available for degrees… 180 – 8.62…)	
	Extra solutions outside 0 to 2π : Ignore. Extra solutions between 0 and 2π : Loses the final A mark. *<u>Premature approximation</u> in part (c): e.g. $\alpha = 41°$, 180 – 41 = 139, 41 – 30 = 11 and 139 – 30 = 109 Changing to radians: 0.19 and 1.90 This would score B1 (required value of α not seen, but there is a final answer 0.19 (or 1.90)), M1 M1 A0 A0.	

Leave blank

10.

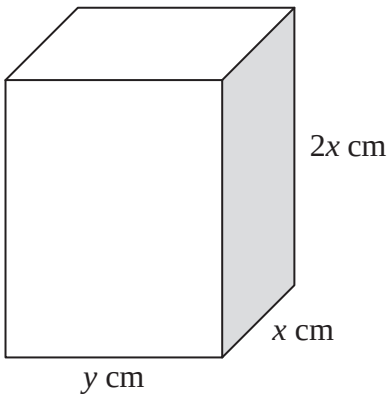


Figure 4

Figure 4 shows a solid brick in the shape of a cuboid measuring $2x$ cm by x cm by y cm.

The total surface area of the brick is 600 cm^2.

(a) Show that the volume, V cm^3, of the brick is given by

$$V = 200x - \frac{4x^3}{3}.$$

(4)

Given that x can vary,

(b) use calculus to find the maximum value of V, giving your answer to the nearest cm^3.

(5)

(c) Justify that the value of V you have found is a maximum.

(2)

Question number	Scheme	Marks
10.	(a) $4x^2 + 6xy = 600$	M1 A1
	$V = 2x^2y = 2x^2\left(\frac{600-4x^2}{6x}\right)$ $\quad V = 200x - \frac{4x^3}{3}$ (*)	M1 A1cso (4)
	(b) $\frac{dV}{dx} = 200 - 4x^2$	B1
	Equate their $\frac{dV}{dx}$ to 0 and solve for x^2 or x : $x^2 = 50$ or $x = \sqrt{50}$ (7.07…)	M1 A1
	Evaluate V: $V = 200(\sqrt{50}) - \frac{4}{3}(50\sqrt{50}) = 943$ cm^3 Allow awrt	M1 A1 (5)
	(c) $\frac{d^2V}{dx^2} = -8x$ Negative, $\quad \therefore$ Maximum	M1, A1ft (2)
		11

(a) 1st M: Attempting an expression in terms of x and y for the total surface area (the expression should be dimensionally correct).

1st A: Correct expression (not necessarily simplified), equated to 600.

2nd M: Substituting their y into $2x^2y$ to form an expression in terms of x only. (Or substituting y from $2x^2y$ into their area equation).

(b) 1st A: Ignore $x = -\sqrt{50}$, if seen.

The 2nd M mark (for substituting their x value into the given expression for V) is dependent on the 1st M.

Final A: Allow also exact value $\frac{400\sqrt{50}}{3}$ or $\frac{2000\sqrt{2}}{3}$ or equiv. single term.

(c) Allow marks if the work for (c) is seen in (b) (or vice-versa).

M: Find second derivative and consider its sign.

A: Second derivative following through correctly from their $\frac{dV}{dx}$, and correct reason/conclusion (it must be a maximum, not a minimum).
An actual value of x does not have to be used… this mark can still be awarded if no x value has been found or if a wrong x value is used.

Alternative:

M: Find value of $\frac{dV}{dx}$ on each side of "$x = \sqrt{50}$" and consider sign.

A: Indicate sign change of positive to negative for $\frac{dV}{dx}$, and conclude max.

Alternative:

M: Find value of V on each side of "$x = \sqrt{50}$" and compare with "943".

A: Indicate that both values are less than 943, and conclude max.