Centre No.						Paper Reference	Surname	Initial(s)
Candidate No.						6 6 6 4 / 0 1	Signature	

Paper Reference(s)

6664/01

Edexcel GCE

Core Mathematics C2

Advanced Subsidiary

Monday 22 May 2006 – Morning

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
1	
2	
3	
4	
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6	
7	
8	
9	
10	
Total	

Materials required for examination
Mathematical Formulae (Green)

Items included with question papers
Nil

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initial(s) and signature.
Check that you have the correct question paper.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.
You must write your answer for each question in the space following the question.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 10 questions in this question paper. The total mark for this paper is 75.
There are 20 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the examiner. Answers without working may gain no credit.

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1. Find the first 3 terms, in ascending powers of x, of the binomial expansion of $(2 + x)^6$, giving each term in its simplest form.

(4)

Q1

(Total 4 marks)

June 2006
6664 Core Mathematics C2
Mark Scheme

Question number	Scheme	Marks
1.	$(2+x)^6 = \ 64 \ldots$	B1
	$+\left(6\times 2^5\times x\right)+\left(\frac{6\times 5}{2}\times 2^4\times x^2\right)$, $\quad +192x, \quad +240x^2$	M1, A1, A1 (4)
		4
	The terms can be 'listed' rather than added. M1: Requires correct structure: 'binomial coefficients' (perhaps from Pascal's triangle), increasing powers of one term, decreasing powers of the other term (this may be 1 if factor 2 has been taken out). Allow 'slips'. $\binom{6}{1}$ and $\binom{6}{2}$ or equivalent are acceptable, or even $\left(\frac{6}{1}\right)$ and $\left(\frac{6}{2}\right)$. <u>Decreasing powers of x:</u> Can score only the M mark. $64(1+\ldots\ldots..)$, even if all terms in the bracket are correct, scores max. B1M1A0A0.	

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2. Use calculus to find the exact value of $\int_1^2 \left(3x^2 + 5 + \frac{4}{x^2}\right) \mathrm{d}x$.

(5)

Q2

(Total 5 marks)

Question number	Scheme	Marks
2.	$\int \left(3x^2 + 5 + 4x^{-2}\right)\mathrm{d}x = \dfrac{3x^3}{3} + 5x + \dfrac{4x^{-1}}{-1} \qquad \left(= x^3 + 5x - 4x^{-1}\right)$	M1 A1 A1
	$\left[x^3 + 5x - 4x^{-1}\right]_1^2 = \left(8 + 10 - 2\right) - \left(1 + 5 - 4\right), \quad = 14$	M1, A1 (5)
		5
	Integration: Accept any correct version, simplified or not. All 3 terms correct: M1 A1 A1, Two terms correct: M1 A1 A0, One power correct: M1 A0 A0. The given function must be integrated to score M1, and not e.g. $3x^4 + 5x^2 + 4$. Limits: M1: Substituting 2 and 1 into a 'changed function' and subtracting, either way round.	

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3. (i) Write down the value of $\log_6 36$.

(1)

(ii) Express $2 \log_a 3 + \log_a 11$ as a single logarithm to base a.

(3)

Q3

(Total 4 marks)

Question number	Scheme	Marks	
3.	(i) 2	B1	(1)
	(ii) $2\log 3 = \log 3^2$ (or $2\log p = \log p^2$)	B1	
	$\log_a p + \log_a 11 = \log_a 11p$, $= \log_a 99$ (Allow e.g. $\log_a(3^2 \times 11)$)	M1, A1	(3)
			4
	(ii) Ignore 'missing base' or wrong base. The correct answer with no working scores full marks. $\log_a 9 \times \log_a 11 = \log_a 99$, or similar mistakes, score M0 A0.		

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4. $$f(x) = 2x^3 + 3x^2 - 29x - 60.$$

(a) Find the remainder when f(x) is divided by (x + 2).
(2)

(b) Use the factor theorem to show that (x + 3) is a factor of f(x).
(2)

(c) Factorise f(x) completely.
(4)

N 2 3 5 5 8 A 0 6 2 0

Question number	Scheme	Marks
4.	(a) $f(-2) = 2(-2)^3 + 3(-2)^2 - 29(-2) - 60$ M: Attempt f(2) or f(–2)	M1
	$= -16 + 12 + 58 - 60 = -6$	A1 (2)
	(b) $f(-3) = 2(-3)^3 + 3(-3)^2 - 29(-3) - 60$ M: Attempt f(3) or f(–3)	M1
	$(= -54 + 27 + 87 - 60)$ $= 0$ $\therefore (x + 3)$ is a factor	A1 (2)
	(c) $(x+3)(2x^2 - 3x - 20)$	M1 A1
	$= (x+3)(2x+5)(x-4)$	M1 A1 (4)
		8
	(a) <u>Alternative (long division):</u> Divide by (x + 2) to get $(2x^2 + ax + b)$, $a \neq 0, b \neq 0$. [M1] $(2x^2 - x - 27)$, remainder = – 6 [A1] (b) A1 requires zero and a simple conclusion (even just a tick, or Q.E.D.). (c) First M requires division by (x+ 3) to get $(2x^2 + ax + b)$, $a \neq 0, b \neq 0$. Second M for the attempt to factorise their quadratic. Usual rule: $(2x^2 + ax + b) = (2x + c)(x + d)$, where $\|cd\| = \|b\|$. <u>Alternative (first 2 marks):</u> $(x+3)(2x^2 + ax + b) = 2x^3 + (6+a)x^2 + (3a+b)x + 3b = 0$, then compare coefficients to find <u>values</u> of a and b. [M1] $a = -3,\ b = -20$ [A1] <u>Alternative:</u> Factor theorem: Finding that $f\left(-\frac{5}{2}\right) = 0$ $\therefore$ factor is, $(2x + 5)$ [M1, A1] Finding that $f(4) = 0$ $\therefore$ factor is, $(x - 4)$ [M1, A1] "Combining" all 3 factors is <u>not</u> required. If just one of these is found, score the <u>first 2 marks</u> M1 A1 M0 A0. <u>Losing a factor of 2:</u> $(x+3)\left(x + \frac{5}{2}\right)(x-4)$ scores M1 A1 M1 A0. <u>Answer only, one sign wrong:</u> e.g. $(x+3)(2x-5)(x-4)$ scores M1 A1 M1 A0.	

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5. (a) In the space provided, sketch the graph of $y = 3^x$, $x \in \mathbb{R}$, showing the coordinates of the point at which the graph meets the y-axis.

(2)

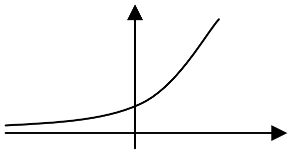
(b) Complete the table, giving the values of 3^x to 3 decimal places.

x	0	0.2	0.4	0.6	0.8	1
3^x		1.246	1.552			3

(2)

(c) Use the trapezium rule, with all the values from your table, to find an approximation for the value of $\int_0^1 3^x \, dx$.

(4)

Question number	Scheme	Marks
5.	(a)  Shape	B1
	(0, 1), or just 1 on the y-axis, or seen in table for (b)	B1 (2)
	(b) Missing values: 1.933, 2.408 (Accept awrt)	B1, B1 (2)
	(c) $\frac{1}{2}\times 0.2,\ \{(1+3)+2(1.246+1.552+1.933+2.408)\}$	B1, M1 A1ft
	= 1.8278 (awrt 1.83)	A1 (4)
		8
	Beware the order of marks! (a) Must be a curve (not a straight line). Curve must extend to the left of the y-axis, and must be increasing. Curve can 'touch' the x-axis, but must not go below it. Otherwise, be generous in cases of doubt. The B1 for (0, 1) is independent of the sketch. (c) Bracketing mistake: i.e. $\frac{1}{2}\times 0.2(1+3)+2(1.246+1.552+1.933+2.408)$ scores B1 M1 A0 A0 <u>unless</u> the final answer implies that the calculation has been done correctly (then full marks can be given).	

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6. (a) Given that $\sin \theta = 5\cos \theta$, find the value of $\tan \theta$.

(1)

(b) Hence, or otherwise, find the values of θ in the interval $0 \leqslant \theta < 360°$ for which

$$\sin \theta = 5\cos \theta,$$

giving your answers to 1 decimal place.

(3)

Question number	Scheme	Marks	
6.	(a) $\tan\theta = 5$	B1	(1)
	(b) $\tan\theta = k$ $\quad (\theta = \tan^{-1} k)$	M1	
	$\theta = 78.7, \quad 258.7$ (Accept awrt)	A1, A1ft	(3)
			4
	(a) Must be seen explicitly, e.g. $\tan\theta = \tan^{-1} 5 = 78.7$ or equiv. is B0, unless $\tan\theta = 5$ is also seen. (b) The M mark may be implied by working in (a). A1ft for 180 + α. ($\alpha \neq k$). Answers in radians would lose both the A marks. Extra answers between 0 and 360: Deduct the final mark. <u>Alternative:</u> Using $\cos^2\theta = 1 - \sin^2\theta$ (or equiv.) and proceeding to $\sin\theta = k$ (or equiv.): M1 then A marks as in main scheme.		

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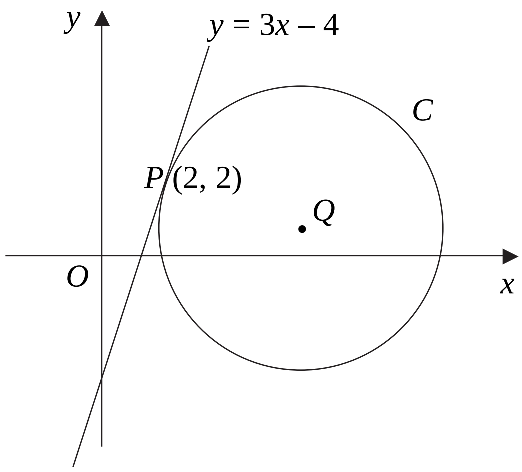
7.

Figure 1



The line $y = 3x - 4$ is a tangent to the circle C, touching C at the point $P\,(2, 2)$, as shown in Figure 1.

The point Q is the centre of C.

(a) Find an equation of the straight line through P and Q.

(3)

Given that Q lies on the line $y = 1$,

(b) show that the x-coordinate of Q is 5,

(1)

(c) find an equation for C.

(4)

N 2 3 5 5 8 A 0 1 2 2 0

Question number	Scheme	Marks
7.	(a) Gradient of PQ is $-\frac{1}{3}$	B1
	$y-2=-\frac{1}{3}(x-2)$ $\quad (3y+x=8)$	M1 A1 (3)
	(b) $y = 1$: $3 + x = 8$ $x = 5$ (*)	B1 (1)
	(c) $(\text{"5"}-2)^2+(1-2)^2$ M: Attempt PQ^2 or PQ	M1 A1
	$(x-5)^2+(y-1)^2=10$ M: $(x\pm a)^2+(y\pm b)^2=k$	M1 A1 (4)
		8
	(a) M1: eqn. of a straight line through (2, 2) with any gradient except 3, 0 or ∞. Alternative: Using (2, 2) in $y=mx+c$ to find a value of c scores M1, but an equation (general or specific) must be seen. If the given value $x = 5$ is used to find the gradient of PQ, maximum marks are (a) B0 M1 A1 (b) B0. (c) For the first M1, condone one slip, numerical or sign, inside a bracket. The first M1 can be scored if their x-coord. is used instead of 5. For the second M1, allow any equation in this form, with non-zero a, b and k.	

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8.

Figure 2

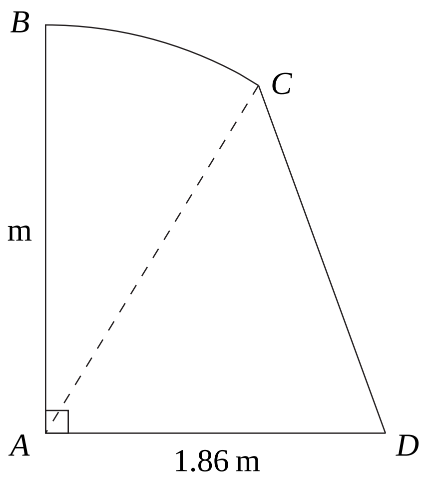

Figure 2 shows the cross section *ABCD* of a small shed.
The straight line *AB* is vertical and has length 2.12 m.
The straight line *AD* is horizontal and has length 1.86 m.
The curve *BC* is an arc of a circle with centre *A*, and *CD* is a straight line.
Given that the size of $\angle BAC$ is 0.65 radians, find

(a) the length of the arc *BC*, in m, to 2 decimal places,

(2)

(b) the area of the sector *BAC*, in m^2, to 2 decimal places,

(2)

(c) the size of $\angle CAD$, in radians, to 2 decimal places,

(2)

(d) the area of the cross section *ABCD* of the shed, in m^2, to 2 decimal places.

(3)

Question number	Scheme	Marks
8.	(a) $r\theta = 2.12 \times 0.65$ 1.38 (m)	M1 A1 (2)
	(b) $\frac{1}{2}r^2\theta = \frac{1}{2} \times 2.12^2 \times 0.65$ 1.46 (m^2)	M1 A1 (2)
	(c) $\frac{\pi}{2} - 0.65$ 0.92 (radians) (α)	M1 A1 (2)
	(d) ΔACD : $\frac{1}{2}(2.12)(1.86)\sin\alpha$ (With the value of α from part (c))	M1
	Area = "1.46" + "1.57", 3.03 (m^2)	M1 A1 (3)
		9
	(a) M1: Use of $r\theta$ with r = 2.12 or 1.86, and θ = 0.65, or equiv. method for the angle changed to degrees (allow awrt 37°). (b) M1: Use of $\frac{1}{2}r^2\theta$ with r = 2.12 or 1.86, and θ = 0.65, or equiv. method for the angle changed to degrees (allow awrt 37°). (c) M1: Subtracting 0.65 from $\frac{\pi}{2}$, or subtracting awrt 37 from 90 (degrees), (perhaps implied by awrt 53). <u>Angle changed to degrees wrongly and used throughout (a), (b) and (c):</u> Penalise 'method' only once, so could score M0A0, M1A0, M1A0. (d) First M1: Other area methods must be fully correct. Second M1: Adding answer to (b) to their ΔACD. <u>Failure to round to 2 d.p:</u> Penalise only once, on the first occurrence, then accept awrt. <u>If 0.65 is taken as degrees throughout:</u> Only award marks in part (d).	

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9. A geometric series has first term a and common ratio r.
The second term of the series is 4 and the sum to infinity of the series is 25.

(a) Show that $25r^2 - 25r + 4 = 0$.

(4)

(b) Find the two possible values of r.

(2)

(c) Find the corresponding two possible values of a.

(2)

(d) Show that the sum, S_n, of the first n terms of the series is given by

$$S_n = 25(1 - r^n).$$

(1)

Given that r takes the larger of its two possible values,

(e) find the smallest value of n for which S_n exceeds 24.

(2)

Question number	Scheme	Marks	
9.	(a) $ar = 4$, $\frac{a}{1-r} = 25$ (These can be seen elsewhere)	B1, B1	
	$a = 25(1-r)$ $25r(1-r) = 4$ M: Eliminate a	M1	
	$25r^2 - 25r + 4 = 0$ (*)	A1cso	(4)
	(b) $(5r-1)(5r-4) = 0$ $r = \dots$, $\frac{1}{5}$ or $\frac{4}{5}$	M1, A1	(2)
	(c) $r = \dots \Rightarrow a = \dots$, 20 or 5	M1, A1	(2)
	(d) $S_n = \frac{a(1-r^n)}{1-r}$, but $\frac{a}{1-r} = 25$, so $S_n = 25(1-r^n)$ (*)	B1	(1)
	(e) $25(1-0.8^n) > 24$ and proceed to $n = \dots$(or >, or <) with no unsound algebra.	M1	
	$\left(n > \frac{\log 0.04}{\log 0.8} \quad (= 14.425\ldots)\right)$ $n = 15$	A1	(2)
			11
	(a) The M mark is not dependent, but both expressions must contain both a and r. (b) Special case: One correct r value given, with no method (or perhaps trial and error): B1 B0. (c) M1: Substitute one r value back to find a value of a. (d) Sufficient here to verify with just one pair of values of a and r. (e) Accept "=" rather than inequalities throughout, and also allow the wrong inequality to be used at any stage. M1 requires use of their larger value of r. A correct answer with no working scores both marks. For "trial and error" methods, to score M1, a value of n between 12 and 18 (inclusive) must be tried.		

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10.

Figure 3

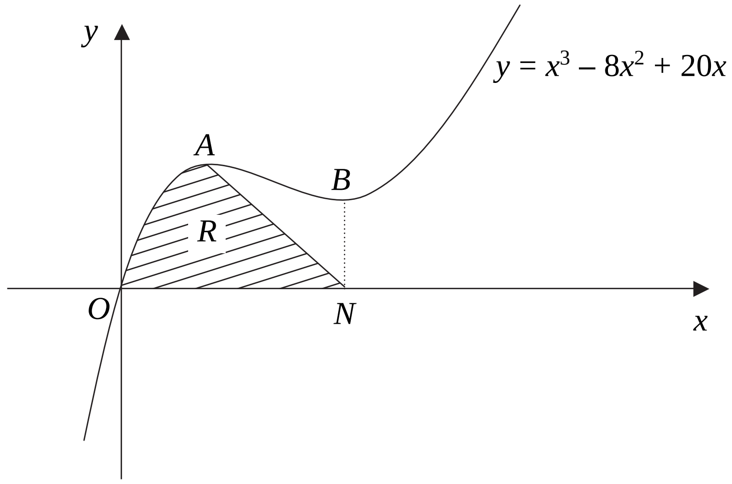

Figure 3 shows a sketch of part of the curve with equation $y = x^3 - 8x^2 + 20x$.
The curve has stationary points A and B.

(a) Use calculus to find the x-coordinates of A and B.

(4)

(b) Find the value of $\frac{d^2y}{dx^2}$ at A, and hence verify that A is a maximum.

(2)

The line through B parallel to the y-axis meets the x-axis at the point N.
The region R, shown shaded in Figure 3, is bounded by the curve, the x-axis and the line from A to N.

(c) Find $\int (x^3 - 8x^2 + 20x)\,dx$.

(3)

(d) Hence calculate the exact area of R.

(5)

Question number	Scheme	Marks	
10.	(a) $\frac{dy}{dx} = 3x^2 - 16x + 20$	M1 A1	
	$3x^2 - 16x + 20 = 0$ $(3x-10)(x-2) = 0$ $x = \ldots,$ $\frac{10}{3}$ and 2	dM1, A1	(4)
	(b) $\frac{d^2y}{dx^2} = 6x - 16$ At $x = 2$, $\frac{d^2y}{dx^2} = \ldots$	M1	
	-4 (or < 0, or both), therefore maximum	A1ft	(2)
	(c) $\int (x^3 - 8x^2 + 20x)dx = \frac{x^4}{4} - \frac{8x^3}{3} + \frac{20x^2}{2}$ $(+C)$	M1 A1 A1	(3)
	(d) $4 - \frac{64}{3} + 40$ $\left(= \frac{68}{3}\right)$	M1	
	A: $x = 2$: $y = 8 - 32 + 40 = 16$ (May be scored elsewhere)	B1	
	Area of $\Delta = \frac{1}{2}\left(\frac{10}{3} - 2\right) \times 16$ $\left(\frac{1}{2}(x_B - x_A) \times y_A\right)$ $\left(= \frac{32}{3}\right)$	M1	
	Shaded area $= \frac{68}{3} + \frac{32}{3} = \frac{100}{3}$ $\left(= 33\frac{1}{3}\right)$	M1 A1	(5)
			14
	(a) The second M is dependent on the first, and requires an attempt to solve a 3 term quadratic. (b) M1: Attempt second differentiation and substitution of one of the x values. A1ft: Requires correct second derivative and negative value of the second derivative, but ft from their x value. (c) All 3 terms correct: M1 A1 A1, Two terms correct: M1 A1 A0, One power correct: M1 A0 A0. (d) Limits M1: Substituting their lower x value into a 'changed' expression. Area of triangle M1: Fully correct method. Alternative for the triangle (finding an equation for the straight line then integrating) requires a fully correct method to score the M mark. Final M1: Fully correct method (beware valid alternatives!)		