Write your name here

Surname	Other names

Pearson Edexcel
International Advanced Level

Centre Number	Candidate Number

Core Mathematics C2

Advanced Subsidiary

Monday 13 January 2014 – Morning
Time: 1 hour 30 minutes

Paper Reference
6664A/01

You must have:
Mathematical Formulae and Statistical Tables (Pink)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

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1. The first three terms in ascending powers of x in the binomial expansion of $(1 + px)^{12}$ are given by

$$1 + 18x + qx^2$$

where p and q are constants.

Find the value of p and the value of q.

(5)

Question Number	Scheme	Notes	Marks
1.	$(1+px)^{12}$		
	$1+\underline{\binom{12}{1}\times px+\binom{12}{2}\times(px)^2}$ or $1+\underline{12px+\frac{12.11}{2}(px)^2}$	Correct structure for at least 1 of the underlined terms, including coefficients. Could be implied by e.g. $12p = 18$	M1
	$=(1+12px+66p^2x^2)$		
	$12p=18\Rightarrow p=$	Compare coefficients of x and solve for p	M1
	$p=\frac{18}{12}\left(=\frac{3}{2}\right)$	Correct value for p	A1
	$q=66\times\left(\text{their } \frac{3}{2}\right)^2$	Substitutes their value of p into their coefficient of x^2 to find q	M1
	$q=148.5$ or equivalent	cao	A1
			(5)
	Note failing to square p in the x^2 term could score M1M1A1M1A0 (4/5) (Gives $q = 99$)		
			Total 5

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2. $f(x) = 2x^3 + x^2 + ax + b$, where a and b are constants.

Given that when $f(x)$ is divided by $(x - 2)$ the remainder is 25,

(a) show that $2a + b = 5$

(2)

Given also that $(x + 3)$ is a factor of $f(x)$,

(b) find the value of a and the value of b.

(4)

Question Number	Scheme	Notes	Marks
2	$\mathrm{f}(x) = 2x^3 + x^2 + ax + b$		
(a) Way 1	$\mathrm{f}(2) = 2(2)^3 + (2)^2 + 2a + b = 25$	$\mathrm{f}(\pm 2) = 25$	M1
	$16 + 4 + 2a + b = 25 \Rightarrow 2a + b = 5$ *	Correct completion to printed answer. If f(2) is not seen explicitly and "$16+4+2a+b=25$" is incorrect, score M0	A1
			(2)
(a) Way 2	Alternative by long division:		
	$2x^3 + x^2 + ax + b \div (x-2)$ Quotient $= 2x^2 + 5x + a + 10$ Remainder $= 2a + b + 20$	Attempt Quotient & Remainder: Needs a quotient of the form $2x^2 + kx + \mathrm{f}(a)$ and a remainder that is a function of *a* and *b*	M1
	$2a + b + 20 = 25 \Rightarrow 2a + b = 5$ *	Correct completion to printed answer	A1
			(2)
(b)	$\mathrm{f}(-3) = 2(-3)^3 + (-3)^2 - 3a + b = 0$	$\mathrm{f}(\pm 3) = 0$	M1
	$2a + b = 5, \quad b - 3a = 45 \rightarrow a = or\ b =$	Solves simultaneously to *a* = or *b* =	M1
	a = -8, *b* = 21	**First** A1: One correct constant **Second** A1: Both constants correct	A1, A1
			(4)
			Total 6

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3. The curve C has equation

$$y = 2\sqrt{x} + \frac{18}{\sqrt{x}} - 1, \qquad x > 0$$

(a) Find

(i) $\dfrac{dy}{dx}$

(ii) $\dfrac{d^2y}{dx^2}$

(5)

(b) Use calculus to find the coordinates of the stationary point of C.

(4)

(c) Determine whether the stationary point is a maximum or minimum, giving a reason for your answer.

(2)

Question Number	Scheme	Notes	Marks
3(a)	$y = 2\sqrt{x} + \frac{18}{\sqrt{x}} - 1 = 2x^{\frac{1}{2}} + 18x^{-\frac{1}{2}} - 1$		
(i)	$\frac{dy}{dx} = x^{-\frac{1}{2}} - 9x^{-\frac{3}{2}}$	M1: $x^n \to x^{n-1}$ A1: $x^{-\frac{1}{2}}$ A1: $-9x^{-\frac{3}{2}}$ and $-1 \to 0$	M1A1A1
(ii)	$\frac{d^2y}{dx^2} = -\frac{1}{2}x^{-\frac{3}{2}} + \frac{27}{2}x^{-\frac{5}{2}}$	M1: $x^n \to x^{n-1}$ A1: $-\frac{1}{2}x^{-\frac{3}{2}} + \frac{27}{2}x^{-\frac{5}{2}}$	M1A1ft
			(5)
(b)	$x^{-\frac{1}{2}} - 9x^{-\frac{3}{2}} = 0$	Set $\frac{dy}{dx} = 0$ and proceed to $x =$	M1
	$x - 9 = 0 \Rightarrow x = 9$ **only**	Cso	A1
	$y = 2\sqrt{"9"} + \frac{18}{\sqrt{"9"}} - 1$	Substitutes their x value(s) into the given equation	M1
	$y = 11$	Cao. **There must be no other turning points for this mark but allow recovery if x = 9 is obtained by the invalid method shown below**	A1
	Allow correct answers only from a correct derivative otherwise apply the scheme		
			(4)
(c)	$f''(9) = -\frac{1}{2}("9")^{-\frac{3}{2}} + \frac{27}{2}("9")^{-\frac{5}{2}}$	Substitutes their x value(s) into their second derivative	M1
	$f''(9) = \frac{1}{27} > 0 \therefore$ Minimum	Fully correct solution including a correct numerical second derivative (awrt 0.04) and a reference to positive or > 0 **There must be no other turning points for this mark i.e. x = 9 only used but allow recovery as above.**	A1
	Accept full valid alternative arguments for the minimum e.g. finds gradient either side of $x = 9$		(2)
			Total 11
(b)	$x^{-\frac{1}{2}} - 9x^{-\frac{3}{2}} = 0 \Rightarrow \frac{1}{\sqrt{x}} - \frac{9}{x\sqrt{x}} = 0$		
	$\Rightarrow \frac{1}{x} - \frac{81}{x^3} = 0 \Rightarrow x^2 = 81 \Rightarrow x = 9$		M1A0
	Then allow remaining marks to be recovered		

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4. The first term of a geometric series is 5 and the common ratio is 1.2

For this series find, to 1 decimal place,

(a) (i) the 20^{th} term,

(ii) the sum of the first 20 terms.

(4)

The sum of the first n terms of the series is greater than 3000

(b) Calculate the smallest possible value of n.

(4)

Question Number	Scheme	Notes	Marks
4(a)(i)	$t_{20} = 5\times 1.2^{19} = 159.7$	M1: Use of $t_n = ar^{n-1}$	M1A1
		A1: Cao	
(ii)	$S_{20} = \dfrac{5\left(1-1.2^{20}\right)}{1-1.2} = 933.4$	M1: Use of a correct sum formula with n = 19 or n = 20 NB if n = 19 is used and no formula is quoted, score M0	M1A1
		A1: Cao	
			(4)
(b)	$\dfrac{5\left(1-1.2^{n}\right)}{1-1.2}(> \text{or} =)3000$	Correct statement (allow 'a' and/or 'r' instead of 5 and 1.2)	B1
	$1.2^{n} > 121$	1.2^n (> or < or =) k	M1
	$\log 1.2^{n} > \log 121$ or $n > \log_{1.2} 121$	Takes logs correctly	M1
	$n > \dfrac{\log 121}{\log 1.2}$ i.e. $n = 27$	cao	A1
	Ignore symbols e.g. '=' throughout with no errors getting n = 27 scores full marks		
	In (b) Treat $5\times 1.2^{n-1} > 3000$ as a misread and allow the M's if scored (gives n = 37)		
			(4)
			Total 8

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5. The height of water, H metres, in a harbour on a particular day is given by the equation

$$H = 10 + 5\sin\left(\frac{\pi t}{6}\right), \qquad 0 \leqslant t < 24$$

where t is the number of hours after midnight.

(a) Show that the height of the water 1 hour after midnight is 12.5 metres.

(1)

(b) Find, to the nearest minute, the times before midday when the height of the water is 9 metres.

(6)

Question Number	Scheme	Notes	Marks
5(a)	$H=10+5\sin\left(\frac{\pi(1)}{6}\right)=12.5$ * Or just $H=10+5\sin\left(\frac{\pi}{6}\right)=12.5$ *	12.5 oe	B1
			(1)
(b)	$9=10+5\sin\left(\frac{\pi t}{6}\right)\Rightarrow 5\sin\left(\frac{\pi t}{6}\right)=-1$	Proceed to $5\sin\left(\frac{\pi t}{6}\right)=k$ May be implied by e.g. $\sin\left(\frac{\pi t}{6}\right)=-\frac{1}{5}$	M1
	$\sin\left(\frac{\pi t}{6}\right)=-\frac{1}{5}\Rightarrow\left(\frac{\pi t}{6}\right)=\arcsin\left(\pm\frac{1}{5}\right)$	$\arcsin\left(\pm\frac{k}{5}\right)$	M1
	$\alpha=\pm0.2(0135792)$ (or 11.536….degrees)	May be implied. Given the similarity between $-\frac{1}{5}$ and $\arcsin\left(-\frac{1}{5}\right)$ allow $\alpha=$ awrt ±0.2	B1
	$\left(\frac{\pi t}{6}\right)=\pi+0.201...$ or $\left(\frac{\pi t}{6}\right)=2\pi-0.201...$ $\left(\frac{\pi t}{6}\right)=3.34295...$ or $\left(\frac{\pi t}{6}\right)=6.08127...$ = 6.384565.... or 11.615434....	May be implied. Do not allow mixing of degrees and radians but allow working in just degrees.	M1
	$t=$ 0623, 1137	Accept 6hrs 23mins, 11hrs 37mins Or 5hrs 37mins, 23 mins before midday	A1, A1
			(6)
			Total 7

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6. Given that

$$\log_x(7y + 1) - \log_x(2y) = 1, \qquad x > 4, \qquad 0 < y < 1$$

express y in terms of x.

(5)

Question Number	Scheme	Notes	Marks
6	$\log_x(7y+1)-\log_x 2y=\log_x\left(\frac{7y+1}{2y}\right)$	Combines logs correctly	B1
	$1=\log_x x$	Correct statement (may be implied)	B1
	$\frac{7y+1}{2y}=x$	Remove logs to obtain this equation or equivalent.	M1
	$2yx=7y+1\Rightarrow y(2x-7)=1$	Isolate *y* correctly to give *y* as a function of *x*. Allow sign errors only. Dependent on the previous method mark.	dM1
	$y=\frac{1}{2x-7}$ or $\frac{-1}{7-2x}$	cao	A1
			(5)
			Total 5
Way 2	$\log_x(7y+1)=1+\log_x 2y$		
	$\log_x(7y+1)=\log_x x+\log_x 2y$	$1=\log_x x$ (may be implied)	B1
	$\log_x x+\log_x 2y=\log_x 2xy$	Combines logs correctly	B1
	$7y+1=2xy$	Remove logs to obtain this equation or equivalent.	M1
	$2yx=7y+1\Rightarrow y(2x-7)=1$	Isolate *y* correctly to give *y* as a function of *x*. Allow sign errors only. Dependent on the previous method mark.	dM1
	$y=\frac{1}{2x-7}$ or $\frac{-1}{7-2x}$	cao	A1
Way 3	$\log_x(7y+1)-\log_x 2y=\log_x\left(\frac{7y+1}{2y}\right)$	Combines logs correctly	B1
	$\log_x\left(\frac{7y+1}{2y}\right)=\frac{\log_{10}\left(\frac{7y+1}{2y}\right)}{\log_{10} x}$	Correct change of base	B1
	$\log_{10}\left(\frac{7y+1}{2y}\right)=\log_{10} x$		
	$\frac{7y+1}{2y}=x$	Remove logs to obtain this equation or equivalent.	M1
	Then as above		

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7.

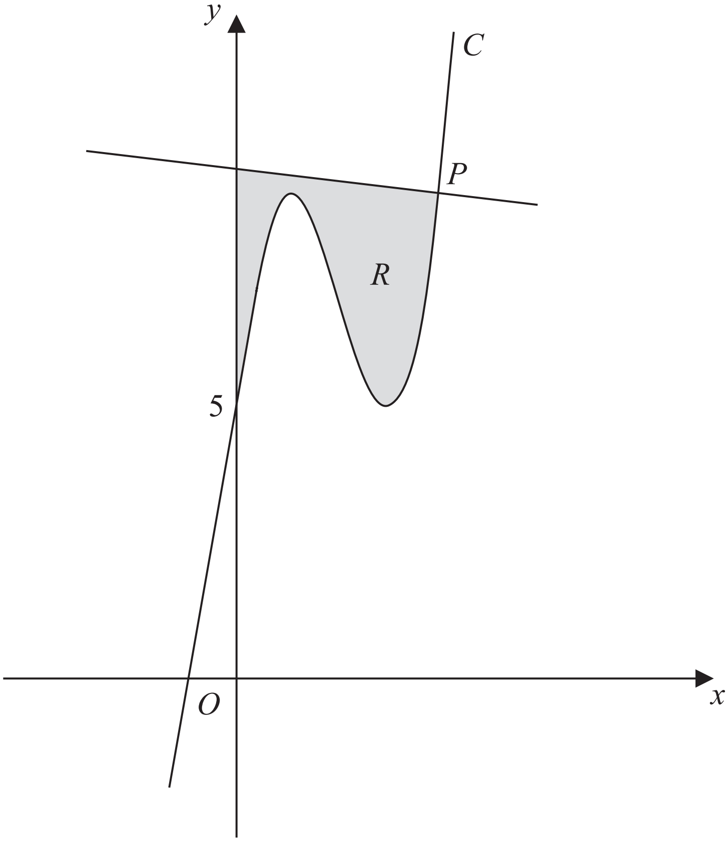

Figure 1

Figure 1 shows a sketch of part of the curve C with equation

$$y = x^3 - 6x^2 + 9x + 5$$

The point $P(4, 9)$ lies on C.

(a) Show that the normal to C at the point P has equation

$$x + 9y = 85$$

(6)

The region R, shown shaded in Figure 1, is bounded by the curve C, the y-axis and the normal to C at P.

(b) Showing all your working, calculate the exact area of R.

(7)

Question Number	Scheme	Notes	Marks
7(a)	$y = x^3 - 6x^2 + 9x + 5$		
	$\frac{dy}{dx} = 3x^2 - 12x + 9$	M1: $x^n \to x^{n-1}$ A1: Correct derivative	M1A1
	$f'(4) = 3(4)^2 - 12(4) + 9 = 9$	Finds $f'(4)$	M1
	$m_N = -\frac{1}{"9"}$	Perpendicular gradient rule applied to their $f'(4)$. Dependent on the previous method mark.	dM1
	$y - 9 = "-\frac{1}{9}"(x - 4)$	Correct straight line method as shown or $y = mx + c$ with an attempt to find c. Depends on both previous method marks.	ddM1
	$x + 9y = 85$ *	Correct completion to printed answer. Allow this from the previous line.	A1*
			(6)
(b) Way1	$x = 0 \Rightarrow y = \frac{85}{9}$		
	$Area\ trapezium = \frac{1}{2} \times 4 \times \left(9 + \frac{85}{9}\right) \quad \left(= \frac{332}{9} = 36.88...\right)$	M1: Correct method for trapezium A1: Correct numerical expression	M1A1
	$\int y\,dx = \frac{x^4}{4} - 2x^3 + \frac{9x^2}{2} + 5x$	M1: $x^n \to x^{n+1}$ A1: Correct integration	M1A1
	$\left[\frac{x^4}{4} - 2x^3 + \frac{9x^2}{2} + 5x\right]_0^4 = \frac{4^4}{4} - 2 \times 4^3 + \frac{9 \times 4^2}{2} + 5 \times 4(-0)$	Use of limits 0 and 4 in a changed function and subtracts (either way round) (- 0 may be implied)	M1
	$R = \frac{332}{9} - 28 = \frac{80}{9}$	M1: Their Trapezium – Their Integral or Their Integral – Their Trapezium A1: Cso	M1A1
			(7)
	See appendix for alternative methods for part (b)		
			Total 13

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8.

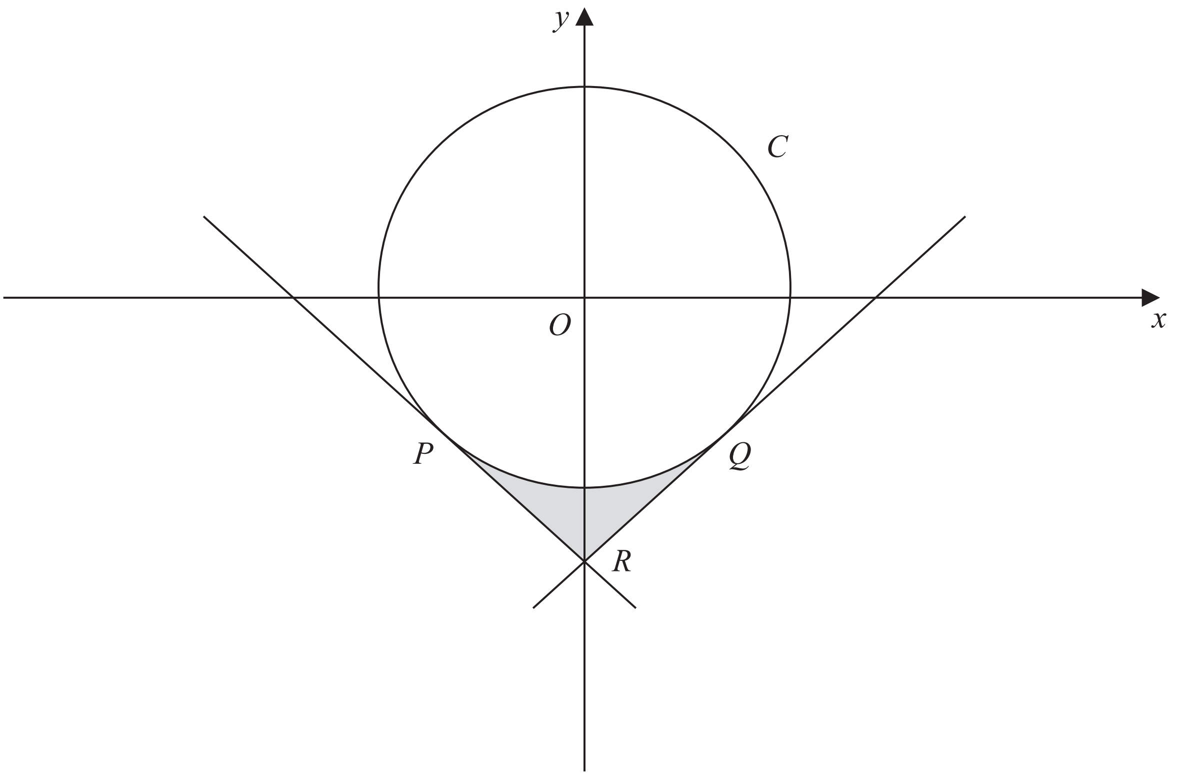

Figure 2

Figure 2 shows a circle *C* with centre *O* and radius 5

(a) Write down the cartesian equation of *C*.

(1)

The points *P*(–3, –4) and *Q*(3, –4) lie on *C*.

(b) Show that the tangent to *C* at the point *Q* has equation

$$3x - 4y = 25$$

(4)

(c) Show that, to 3 decimal places, angle *POQ* is 1.287 radians.

(2)

The tangent to *C* at *P* and the tangent to *C* at *Q* intersect on the *y*-axis at the point *R*.

(d) Find the area of the shaded region *PQR* shown in Figure 2.

(4)

Question Number	Scheme	Notes	Marks
8(a)	$x^2+y^2=25\left(\text{or } 5^2\right)$	Allow $(x-0)^2+(y-0)^2=25$	B1
			(1)
(b)	$\textit{Gradient } OQ=-\frac{4}{3}$	Correct gradient	B1
	$\textit{Tangent Gradient}=\frac{3}{4}$	Correct perpendicular gradient rule	M1
	$y+4=\frac{3}{4}(x-3)$	Correct straight line method using (3, -4) and their numerical gradient.	M1
	$3x-4y=25*$	Correct completion with no errors	A1
			(4)
(c)	$6^2=5^2+5^2-2\times5\times5\cos\theta$ or $\tan\frac{1}{2}\theta=\frac{3}{4}$	Correct statement for angle POQ	M1
	$\theta=\cos^{-1}\left(\frac{5^2+5^2-6^2}{2\times5\times5}\right)$ or $\theta=2\tan^{-1}\left(\frac{3}{4}\right)$		
	$\theta=1.287*$	cso	A1
			(2)
(d)	At R $y=-\frac{25}{4}$ or $OR=\frac{25}{4}$ or QR = $\frac{15}{4}$	May be implied	B1
	Area $POQR=\frac{25}{4}\times3(=18.75)$ $or\ \ OPQ+PQR=\frac{4\times6}{2}+\frac{6}{2}\left(\frac{25}{4}-4\right)(=18.75)$ $or\ 2\times OQR=2\times\frac{1}{2}\times5\times\frac{15}{4}(=18.75)$	Valid attempt at kite area	M1
	$\textit{Area Sector}=\frac{1}{2}\times5^2\times1.287$ (16.0875)	Attempt sector area	M1
	$18.75-\frac{1}{2}\times5^2\times1.287=2.6625$	Awrt 2.66	A1
			(4)
			Total 11

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9. (a) Show that the equation

$$5 \sin x - \cos^2 x + 2 \sin^2 x = 1$$

can be written in the form

$$3 \sin^2 x + 5 \sin x - 2 = 0$$

(2)

(b) Hence solve, for $-180° \leqslant \theta < 180°$, the equation

$$5 \sin 2\theta - \cos^2 2\theta + 2 \sin^2 2\theta = 1$$

giving your answers to 2 decimal places.

(7)

Question Number	Scheme	Notes	Marks
9(a)	$5\sin x - \cos^2 x + 2\sin^2 x = 1$		
	$5\sin x - (1-\sin^2 x) + 2\sin^2 x = 1$	Use of $\cos^2 x = 1 - \sin^2 x$	M1
	$3\sin^2 x + 5\sin x - 2 = 0$ *		A1
			(2)
(b)	$(3\sin 2\theta - 1)(\sin 2\theta + 2) = 0 \Rightarrow \sin 2\theta = ...$	Attempt to solve for sin2θ or sinθ	M1
	$\sin(2\theta) / \sin\theta = \frac{1}{3}(or -2)$		A1
	$2\theta / \theta = \sin^{-1}\left("\frac{1}{3}"\right)$		M1
	$2\theta = 19.47122....$		
	$\theta = 9.74$	Awrt	A1
	$2\theta / \theta = 180 - 19.47,\ -180 - 19.47...,\ -360 + 19.47...$	At least **one** of these	M1
	θ = 80.26, -99.74, -170.26 **Allow awrt 80.3, -99.7, -170.3**	A1: Any **two** of these to the awrt accuracy indicated A1: **All** values as shown to the awrt accuracy indicated and no other values in range.	A1,A1
	For use of radians allow the method marks		
			(7)
			Total 9
	If the quadratic is solved incorrectly, the M marks are available e.g.		
	$(3\sin 2\theta + 1)(\sin 2\theta - 2) = 0 \Rightarrow \sin 2\theta = ...$		M1
	$\sin(2\theta) / \sin\theta = -\frac{1}{3}(or + 2)$		A0
	$2\theta / \theta = \sin^{-1}\left("-\frac{1}{3}"\right)$		M1
	$2\theta = -19.47122....$		
	$\theta = -9.74$		A0
	$2\theta / \theta = -180 + 19.47,\ 180 + 19.47...,\ 360 - 19.47...$	At least **one** of these	M1
			A0,A0
			(3/7)