

January 2013
6664 Core Mathematics C2
Mark Scheme

Question Number	Scheme	Marks
1.	$(2 - 5x)^6$	
	$(2^6 =) 64$	Award this when first seen (not $64x^0$)
	$+6 \times (2)^5 (-5x) + \frac{6 \times 5}{2} (2)^4 (-5x)^2$	Attempt binomial expansion with correct structure for at least one of these terms. E.g. a term of the form: $\binom{6}{p} \times (2)^{6-p} (-5x)^p$ with $p = 1$ or $p = 2$ consistently. Condone sign errors. Condone missing brackets if later work implies correct structure and allow alternative forms for binomial coefficients e.g. 6C_1 or $\binom{6}{1}$ or even $\left(\frac{6}{1}\right)$
	$-960x$	Do not allow $+ -960x$
	$(+)6000x^2$	Allow this to come from $(5x)^2$
	Ignore any extra terms and isw e.g. divides all terms by 2 The terms do not have to form a sum i.e. they can be listed with commas or given on separate lines.	
	Special Case - decreasing powers can score M1 with the conditions as above for the second and third terms.	
	$(2 - 5x)^6 = 64 + \binom{6}{1}(2^5 - 5x) + \binom{6}{2}(2^4 + (-5x)^2)$ scores B1 only as the powers of 2 and $(-5x)$ are being added not multiplied.	
	Fully correct answer with no working can score full marks. If either the second or third term is correct, the M1 can be implied and the A1 scored for that term.	
		(4)
Way 2	$64(1 \pm \dots\dots\dots)$	64 and $(1 \pm \dots -$ Award when first seen.
	$\left(1 - \frac{5x}{2}\right)^6 = 1 - 6 \times \frac{5x}{2} + \frac{6 \times 5}{2} \left(\frac{-5x}{2}\right)^2$	Correct structure for at least one of the underlined terms. E.g. a term of the form: $\binom{6}{p} \times (kx)^p$ with $p = 1$ or $p = 2$ consistently and $k \neq \pm 5$ Condone sign errors. Condone missing brackets if later work implies correct structure but it must be an expansion of $(1 - kx)^6$ where $k \neq \pm 5$
	$-960x$	Do not allow $+ -960x$
	$(+)6000x^2$	Allow this to come from $\left(\frac{5x}{2}\right)^2$
		(4)

Leave blank

2. $f(x) = ax^3 + bx^2 - 4x - 3$, where a and b are constants.

Given that $(x - 1)$ is a factor of $f(x)$,

(a) show that

$$a + b = 7$$

(2)

Given also that, when $f(x)$ is divided by $(x + 2)$, the remainder is 9,

(b) find the value of a and the value of b , showing each step in your working.

(4)



Leave blank

3. A company predicts a yearly profit of £120 000 in the year 2013. The company predicts that the yearly profit will rise each year by 5%. The predicted yearly profit forms a geometric sequence with common ratio 1.05

(a) Show that the predicted profit in the year 2016 is £138 915 **(1)**

(b) Find the first year in which the yearly predicted profit exceeds £200 000 **(5)**

(c) Find the total predicted profit for the years 2013 to 2023 inclusive, giving your answer to the nearest pound. **(3)**



Question Number	Scheme		Marks
3.			
(a)	$120000 \times (1.05)^3 = 138915 *$	Or $120000 \times 1.05 \times 1.05 \times 1.05 = 138915$ Or 120000, 126000, 132000, 138915 Or $a = 120000$ and $a \times (1.05)^3 = 138915$	B1
			(1)
(b)	$120000 \times (1.05)^{n-1} > 200000$	Allow n or $n - 1$ and “>”, “<”, or “=” etc.	M1
	$\log 1.05^{n-1} > \log \left(\frac{5}{3} \right)$	Takes logs correctly Allow n or $n - 1$ and “>”, “<”, or “=” etc.	M1
	$(n - 1 >) \frac{\log \left(\frac{5}{3} \right)}{\log 1.05}$ or equivalent e.g. $(n >) \frac{\log \left(\frac{7}{4} \right)}{\log 1.05}$	Allow n or $n - 1$ and “>”, “<”, or “=” etc. Allow 1.6 or awrt 1.67 for 5/3.	A1
	2024	M1: Identifies a calendar year using their value of n or $n - 1$ A1: 2024 only cso	M1A1
2024 with no working = no marks			
See appendix for alternative taking logs base 1.05 and mis-read as total profit			
			(5)
(c)	$\frac{a(1 - r^n)}{1 - r} = \frac{120000(1 - 1.05^{11})}{1 - 1.05}$	M1: Correct sum formula with $n = 10, 11$ or 12 A1: Correct numerical expression with $n = 11$	M1 A1
	1704814	Cao (Allow 1704814.00)	A1
			(3)
			[9]
Listing or trial/improvement in (b)			
	$U_{10} = 186\ 159.39, U_{11} = 195\ 467.36, U_{12} = 205\ 240.72$		
	Attempt to find at least the 10 th or 11 th or 12 th terms correctly using a common ratio of 1.05 (all the terms need not be listed)		M1
	Forms the geometric progression correctly to reach a term > 200 000 (May be implied e.g. reaches 195 467.36 – Hence the next year)		M1
	Obtains an “11 th ” term of awrt 195 500 and a “12 th ” term of awrt 205 200		A1
	Uses their number of terms to identify a calendar year		M1
	2024		A1
If you are not sure how to award the marks please consult your Team Leader			
			(5)

Question Number	Scheme		Marks
4.			
	$\cos^{-1}(-0.4) = 113.58 (\alpha)$	Awrt 114	B1
	$3x - 10 = \alpha \Rightarrow x = \frac{\alpha + 10}{3}$	Uses their α to find x . Allow $x = \frac{\alpha \pm 10}{3}$ not $\frac{\alpha}{3} \pm 10$	M1
	Note: If $x = \frac{\alpha \pm 10}{3}$ is not clearly applied from their first angle it may be recovered if applied to their second or third angle.		
	$x = 41.2$	Awrt	A1
	$(3x - 10 =) 360 - \alpha$ (246.4....)	$360 - \alpha$ (can be implied by 246.4...)	M1
	$x = 85.5$	Awrt	A1
	$(3x - 10 =) 360 + \alpha$ (=473.57....)	$360 + \alpha$ (Can be implied by 473.57...)	M1
	$x = 161.2$	Awrt	A1
	Note 1: Do not penalise incorrect accuracy more than once and penalise it the first time it occurs. E.g if answers are only given to the nearest integer (41, 85, 161) only the first A mark that would otherwise be scored is lost.		
	Note 2: Ignore any answers outside the range. For extra answers in range in an otherwise fully correct solution lose final A1		
	Note 3: Lack of working means that it is sometimes not clear where their intermediate angles are coming from. In these cases, if the final answers are incorrect score M0.		
	Note 4: Candidates are unlikely to be working in radians <u>deliberately</u> but may have their calculator in radian mode (gives $\alpha = 1.98$). In such cases the main scheme should be applied and the method marks are available. If you suspect that the candidate is working in radians correctly then please use the review mechanism and/or consult your team leader.		
Way 2	$\cos^{-1}(0.4) = 66.42 (\alpha)$		
	$180 - 66.42 = 113.58$	Awrt 114	B1
	$3x - 10 = 113.58 \Rightarrow x = \frac{113.58 + 10}{3}$	Uses their 113.58 to find x	M1
	$x = 41.2$	Awrt	A1
	$3x - 10 = 180 + \alpha$ (246.4....)	$180 + \alpha$	M1
	to give $x = 85.5$		A1
	$3x - 10 = 540 - \alpha$ (473.57....)	$540 - \alpha$	M1
	to give $x = 161.2$		A1
	Special case - takes 0.4 as -0.4		
	$\cos^{-1}(0.4) = 66.42 (\alpha)$		B0
	$3x - 10 = 66.4 \Rightarrow x = \frac{66.4 \pm 10}{3}$		M1
	$x = 41.2$		A0
	$3x - 10 = 360 - \alpha$ (293.6....)		M1
	$x = 101.2$		A0
	$3x - 10 = 360 + \alpha$ (426.4....)		M1
	$x = 145.5$		A0
			(3/7)

5. The circle C has equation

$$x^2 + y^2 - 20x - 24y + 195 = 0$$

The centre of C is at the point M .

(a) Find

- (i) the coordinates of the point M ,
- (ii) the radius of the circle C .

(5)

N is the point with coordinates $(25, 32)$.

(b) Find the length of the line MN .

(2)

The tangent to C at a point P on the circle passes through point N .

(c) Find the length of the line NP .

(2)



Question Number	Scheme		Marks
5.			
(a)	Parts (i) and (ii) are likely to be solved together so mark as one part		
(i)	The centre is at (10, 12)	B1: $x = 10$ B1: $y = 12$	B1 B1
(ii)	Uses $(x-10)^2 + (y-12)^2 = -195 + 100 + 144 \Rightarrow r = \dots$		M1
	Completes the square for both x and y in an attempt to find r . $(x \pm "10")^2 \pm a$ and $(y \pm "12")^2 \pm b$ and $+195 = 0, (a, b \neq 0)$ Allow slips in obtaining their r^2 but must find square root		
	$r = \sqrt{10^2 + 12^2 - 195}$	A correct numerical expression for r including the square root and can implied by a correct value for r	A1
	$r = 7$	Not $r = \pm 7$ unless -7 is rejected	A1
			(5)
(a) Way 2	Compares the given equation with $x^2 + y^2 + 2gx + 2fy + c = 0$ to write down centre $(-g, -f)$ i.e. (10, 12)	B1: $x = 10$ B1: $y = 12$	B1B1
	Uses $r = \sqrt{(\pm "10")^2 + (\pm "12")^2 - c}$		M1
	$r = \sqrt{10^2 + 12^2 - 195}$	A correct numerical expression for r	A1
	$r = 7$		A1
			(5)
	Note that although the marks for the centre are B marks, they do need to come from correct work. E.g. $(x+10)^2, (y+12)^2$ giving a centre of (10, 12) scores B0 B0 but could score the M1A1ftA1ft for the radius as a special case. Similarly $(x+10)^2, (y-12)^2$ giving a centre of (-10, 12) scores B0 B1, $(x-10)^2, (y+12)^2$ giving a centre of (10, -12) scores B1 B0 but both could score M1A1ftA1ft for the radius as a special case also.		
(b)	$MN = \sqrt{(25 - "10")^2 + (32 - "12")^2}$	Correct use of Pythagoras	M1
	$MN (= \sqrt{625}) = 25$		A1
			(2)
(c)	$NP = \sqrt{"25"{}^2 - "7"{}^2}$	$NP = \sqrt{(MN^2 - r^2)}$	M1
	$NP = \sqrt{(25^2 + 7^2)}$ is M0 (Quite common)		
	$NP (= \sqrt{576}) = 24$		A1
			(2)
(c) Way 2	$\cos(NMP) = \frac{7}{"25"} \Rightarrow NP = "25" \sin(NMP)$	Correct strategy for finding NP	M1
	$NP = 24$		A1
			(2)
			[9]

Question Number	Scheme		Marks
6.			
(a)	$2\log(x+15) = \log(x+15)^2$		B1
	$\log(x+15)^2 - \log x = \log \frac{(x+15)^2}{x}$	Correct use of $\log a - \log b = \log \frac{a}{b}$	M1
	$2\log(x+15) - \log x = 6 \Rightarrow \log\left(\frac{(x+15)^2}{x}\right) = 6$ with no incorrect work scores B1M1 together		
	$2\log_2(x+15) - \log_2 x = 2\log_2 \frac{(x+15)}{x}$ is M0		
	$2^6 = 64$ or $\log_2 64 = 6$	64 used in the correct context	B1
	$\log_2 \frac{(x+15)^2}{x} = 6 \Rightarrow \frac{(x+15)^2}{x} = 64$	Removes logs correctly	M1
	$2\log(x+15) - \log x = 6 \Rightarrow \log(x+15)^2 - \log x = 6 \Rightarrow \frac{(x+15)^2}{x} = 64$ Is acceptable for the first 4 marks		
	This method mark should only be awarded for the removal of logs in an appropriate way. Some examples are below, <div style="display: flex; justify-content: space-around; margin-top: 10px;"> <div style="border: 1px solid black; padding: 5px;">$\frac{\log(x+15)^2}{\log x} = 6 \Rightarrow \frac{(x+15)^2}{x} = 6$ M0</div> <div style="border: 1px solid black; padding: 5px;">$\log \frac{(x+15)^2}{x} = 6 \Rightarrow \frac{(x+15)^2}{x} = 6$ M0</div> </div> <div style="margin-top: 10px; border: 1px solid black; padding: 5px; display: inline-block;">$\log \frac{(x+15)^2}{x} = 6 \Rightarrow \frac{(x+15)^2}{x} = \log_2 6$ M0</div> <div style="margin-top: 10px; border: 1px solid black; padding: 5px; display: inline-block;">$\log \frac{(x+15)^2}{x} = 6 \Rightarrow \frac{(x+15)^2}{x} = 6^2$ M0</div> <div style="margin-top: 10px; border: 1px solid black; padding: 5px; display: inline-block;">$\log\left(\frac{(x+15)}{x}\right)^2 = 6 \Rightarrow \left(\frac{(x+15)}{x}\right)^2 = 64$ M1</div>		
	$\Rightarrow x^2 + 30x + 225 = 64x$ or $x + 30 + 225x^{-1} = 64$	Must see expansion of $(x+15)^2$ to score the final mark.	
	$\therefore x^2 - 34x + 225 = 0$ *	Correct completion to printed answer with no errors but allow recovery from 'invisible' brackets e.g. $x + 15^2 \rightarrow x^2 + 30x + 225$	A1
			(5)
(b)	$(x-25)(x-9) = 0 \Rightarrow x = 25$ or $x = 9$	M1: Correct attempt to solve the given quadratic as far as $x = \dots$ <u>It must be an attempt at solving the given quadratic but allow mis-copy e.g. 255 for 225</u> A1: Both 25 and 9	M1 A1
			(2)
			[7]
	See appendix for some alternative correct and incorrect methods for (a)		

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7.

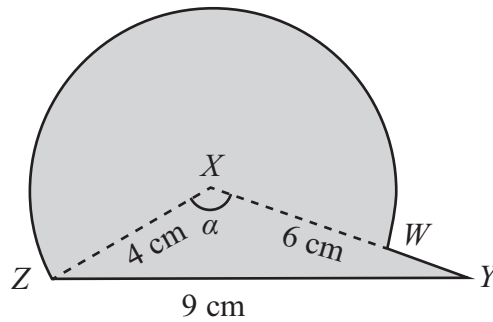


Figure 1

The triangle XYZ in Figure 1 has $XY = 6$ cm, $YZ = 9$ cm, $ZX = 4$ cm and angle $ZXY = \alpha$. The point W lies on the line XY .

The circular arc ZW , in Figure 1 is a major arc of the circle with centre X and radius 4 cm.

- (a) Show that, to 3 significant figures, $\alpha = 2.22$ radians. (2)

- (b) Find the area, in cm^2 , of the major sector $XZWX$. (3)

The region enclosed by the major arc ZW of the circle and the lines WY and YZ is shown shaded in Figure 1.

Calculate

- (c) the area of this shaded region, (3)

- (d) the perimeter $ZWYZ$ of this shaded region. (4)



Question Number	Scheme		Marks
7.			
(a)	$9^2 = 4^2 + 6^2 - 2 \times 4 \times 6 \cos \alpha \Rightarrow \cos \alpha = \dots$	Correct use of cosine rule leading to a value for $\cos \alpha$	M1
	$\cos \alpha = \frac{4^2 + 6^2 - 9^2}{2 \times 4 \times 6} \left(= -\frac{29}{48} = -0.604.. \right)$		
	$\alpha = 2.22$ *	Cso (2.22 must be seen here)	A1
	(NB $\alpha = 2.219516005$)		(2)
(a) Way 2	$XY^2 = 4^2 + 6^2 - 2 \times 4 \times 6 \cos 2.22 \Rightarrow XY^2 = ..$	Correct use of cosine rule leading to a value for XY^2	M1
	$XY^2 = 81.01....$		
	$XY = 9.00....$		A1
			(2)
(b)	$2\pi - 2.22 (= 4.06366.....)$	$2\pi - 2.22$ or awrt 4.06 or $2\pi - 2.2$ or awrt 4.08 (May be implied)	B1
	$\frac{1}{2} \times 4^2 \times "4.06"$	Correct method for major sector area. Allow $\pi - 2.22$ for the major sector angle.	M1
	32.5	Awrt 32.5	A1
	Finding the minor sector area here (17.8) is 0/3		(3)
(b) Way2	Circle – Minor sector		
	$\pi \times 4^2$	Correct expression for circle area	B1
	$\pi \times 4^2 - \frac{1}{2} \times 4^2 \times 2.22 = 32.5$	Correct method for circle - minor sector area	M1
	$= 32.5$	Awrt 32.5	A1
			(3)
(c)	Area of triangle = $\frac{1}{2} \times 4 \times 6 \times \sin 2.22 (= 9.56)$	Correct expression for the area of triangle XYZ (allow 2.2 or awrt 2.22)	B1
	So area required = "9.56" + "32.5"	Their Triangle XYZ (Not triangle ZXW) + (part (b) answer or correct attempt at major sector)	M1
	$= 42.1 \text{ cm}^2$ or 42.0 cm^2	Awrt 42.1 or 42.0 (Or just 42)	A1
			(3)
	Note: The minor sector area (17.76) + the triangle (9.56) = 27.32 which looks like the answer to (d) – beware!		
(d)	Arc length = $4 \times 4.06 (= 16.24)$ Or $8\pi - 4 \times 2.22$	M1: $4 \times \textit{their} (2\pi - 2.22)$ Or circumference – minor arc A1: Correct ft expression	M1A1ft
	Perimeter = $ZY + WY + \text{Arc Length}$	$9 + 2 + \text{Any Arc}$	M1
	Perimeter = 27.2 or 27.3	Awrt 27.2 or awrt 27.3	A1
	Note the order of marks on Epen is M1M1A1A1 – the M's and A's must correspond so that the second mark on Epen is the second M1 on the scheme		
			(4)
	(Generally do not apply isw in this question and mark their final answer unless a correct answer is subsequently rounded incorrectly)		[12]
	In this question we will need to be careful with labelling as each part has clear demands and must be marked as labelled by the candidate.		

Question Number	Scheme		Marks
8.	$y = 6 - 3x - \frac{4}{x^3}$		
(a)	$\frac{dy}{dx} = -3 + \frac{12}{x^4}$ or $-3 + 12x^{-4}$	M1: $x^n \rightarrow x^{n-1}$ ($x^{-1} \rightarrow x^0$ or $x^{-3} \rightarrow x^{-4}$ or $6 \rightarrow 0$)	M1 A1
		A1: Correct derivative	
	$\frac{dy}{dx} = 0 \Rightarrow -3 + \frac{12}{x^4} = 0 \Rightarrow x = \dots$ or $\frac{dy}{dx} = -3 + \frac{12}{\sqrt{2}^4}$	$y' = 0$ and attempt to solve for x May be implied by $\frac{dy}{dx} = -3 + \frac{12}{x^4} = 0 \Rightarrow \frac{12}{x^4} = 3 \Rightarrow x = \dots$ or Substitutes $x = \sqrt{2}$ into their y'	M1
	So $x^4 = 4$ and $x = \sqrt{2}$ or $\frac{dy}{dx} = -3 + \frac{12}{(\sqrt{2})^4}$ or $-3 + 12(\sqrt{2})^{-4} = 0$	Correct completion to printed answer with no errors by solving their $y' = 0$ or substituting $x = \sqrt{2}$ into their y'	A1
	For solving, allow e.g. $x^{-4} = \frac{1}{4} \Rightarrow x = \left(\frac{1}{4}\right)^{-\frac{1}{4}} = \sqrt{2}$ The minimum for verification is as in the scheme which could be implied by $-3 + 3 = 0$		
	Do not allow $x^4 = 4 \Rightarrow x = 1.41\dots = \sqrt{2}$ for the final A1		(4)
(b)	$x = -\sqrt{2}$	Awrt -1.41	B1
			(1)
(c)	$\frac{d^2y}{dx^2} = \frac{-48}{x^5}$ or $-48x^{-5}$	Follow through their first derivative from part (a)	B1ft
			(1)
(d)	An appreciation that either $y'' > 0 \Rightarrow$ a minimum or $y'' < 0 \Rightarrow$ a maximum	A generous mark that is independent of any previous work	B1
	Maximum at P as $y'' < 0$	Cso	B1
	Need a fully correct solution for this mark. y'' need not be evaluated but must be correct and there must be reference to P or to $\sqrt{2}$ and negative or < 0 and maximum. There must be no incorrect or contradictory statements (NB allow $y'' =$ awrt-8 or -9)		
	Minimum at Q as $y'' > 0$	Cso	B1
	Need a fully correct solution for this mark. y'' need not be evaluated but must be correct and part (b) must be correct and there must be reference to P or to $-\sqrt{2}$ and positive or > 0 and minimum. There must be no incorrect or contradictory statements (NB allow $y'' =$ awrt 8 or 9)		
			(3)
			[9]
	Other methods for identifying the nature of the turning points are acceptable. The first B1 is for finding values of y or dy/dx either side of $\sqrt{2}$ or their x at Q and the second and third B1's for fully correct solutions to identify the maximum/minimum.		

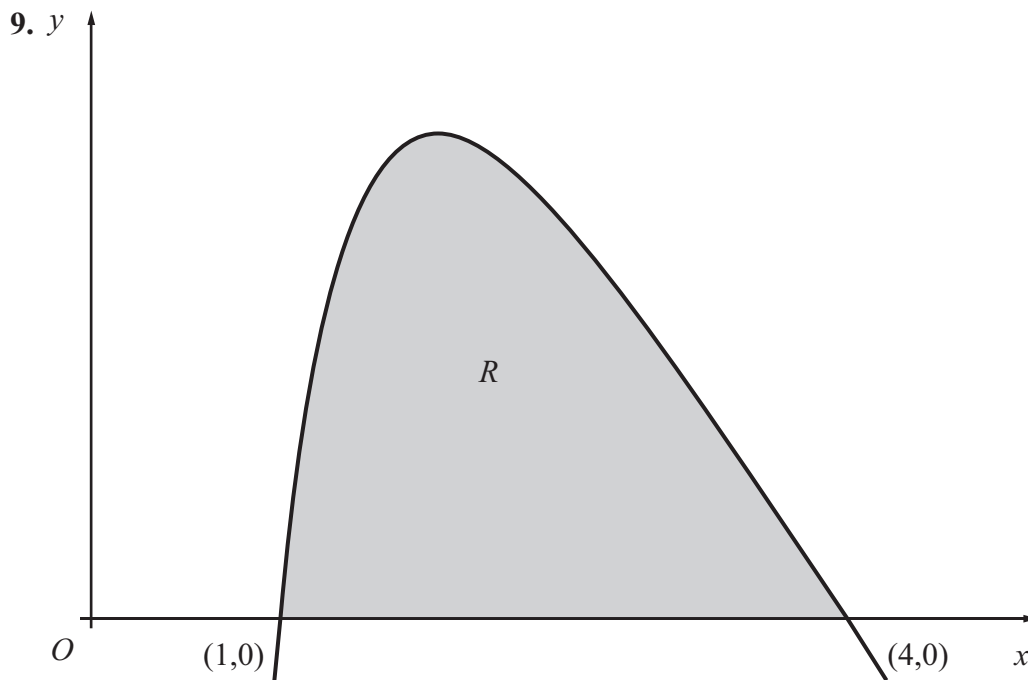


Figure 2

The finite region R , as shown in Figure 2, is bounded by the x -axis and the curve with equation

$$y = 27 - 2x - 9\sqrt{x} - \frac{16}{x^2}, \quad x > 0$$

The curve crosses the x -axis at the points $(1, 0)$ and $(4, 0)$.

(a) Complete the table below, by giving your values of y to 3 decimal places.

x	1	1.5	2	2.5	3	3.5	4
y	0	5.866		5.210		1.856	0

(2)

(b) Use the trapezium rule with all the values in the completed table to find an approximate value for the area of R , giving your answer to 2 decimal places.

(4)

(c) Use integration to find the exact value for the area of R .

(6)



Question Number	Scheme		Marks
9.	$y = 27 - 2x - 9\sqrt{x} - \frac{16}{x^2}$		
(a)	6.272 , 3.634	Awrt in each case	B1, B1
	Special case 6.27 and 3.63 scores B1B0		
			(2)
(b)	$\frac{1}{2} \times \frac{1}{2}$ or $\frac{1}{4}$		B1
 $\{(0 + 0) + 2(5.866 + "6.272" + 5.210 + "3.634" + 1.856)\}$	Need {} or implied later for A1ft	M1A1ft
	(0 + 0) may be implied if omitted and follow through their f(2) and f(3) in an otherwise correct expression and allow one missing or mis-copied term in the 2(...) bracket for the method mark		
	$\frac{1}{2} \times 0.5(0 + 0) + 2(5.866 + "6.272" + 5.210 + "3.634" + 1.856)$ Unless followed by an answer that implies correct (missing) brackets, scores B1M1A0A0 (Usually implied by an answer of 45.676)		
	$\frac{1}{2} \times 0.5\{(0 + 0) + 2(5.866 + "6.272" + 5.210 + "3.634" + 1.856)\}$ $= \frac{1}{4} \times 45.676$		
	= 11.42	cao	A1
	Separate trapezia may be used : B1 for 0.25, M1 for $\frac{1}{2}h(a+b)$ used 5 or 6 times (and A1ft all correct)		
	NB $\frac{1}{2} \times 0.5\{(0 + 0) + 2(0 + 5.866 + "6.272" + 5.210 + "3.634" + 1.856 + 0)\}$ Scores B1M0A0A0		
	Correct answer with no working scores 0/4		
			(4)
(c)	$\int y dx = 27x - x^2 - 6x^{\frac{3}{2}} + 16x^{-1} (+c)$	M1: $x^n \rightarrow x^{n+1}$ on any term	M1A1A1A1
		A1: $27x - x^2$	
		A1: $-6x^{\frac{3}{2}}$	
		A1: $+16x^{-1}$	
	Accept any correct and possibly unsimplified versions for the terms and mark in this order on Epen		
	$(27(4) - (4)^2 - 6(4)^{\frac{3}{2}} + 16(4)^{-1})$ $- (27(1) - (1)^2 - 6(1)^{\frac{3}{2}} + 16(1)^{-1})$	Attempt to subtract either way round using the limits 4 and 1. Dependent on the previous M1. May be implied by 48 - 36 but you may need to check both their values if the integration has errors.	dM1
	$= (48 - 36)$		
	12	Cao (Penalise -12)	A1
			(6)
			[12]