Centre No.						Paper Reference							Surname	Initial(s)
Candidate No.						6	6	6	7	/	0	1	Signature	

Paper Reference(s)

6667/01

Edexcel GCE

Further Pure Mathematics FP1

Advanced/Advanced Subsidiary

Monday 31 January 2011 – Afternoon

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
Total	

Materials required for examination
Mathematical Formulae (Pink)

Items included with question papers
Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer to each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 10 questions in this question paper. The total mark for this paper is 75.
There are 32 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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1. $$z = 5 - 3\text{i}, \quad w = 2 + 2\text{i}$$

Express in the form $a + b\text{i}$, where a and b are real constants,

(a) z^2,

(2)

(b) $\dfrac{z}{w}$.

(3)

Q1

(Total 5 marks)

January 2011
Further Pure Mathematics FP1 6667
Mark Scheme

Question Number	Scheme		Marks
1.	$z = 5 - 3\text{i}, \quad w = 2 + 2\text{i}$		
(a)	$z^2 = (5 - 3\text{i})(5 - 3\text{i})$		
	$= 25 - 15\text{i} - 15\text{i} + 9\text{i}^2$ $= 25 - 15\text{i} - 15\text{i} - 9$	An attempt to multiply out the brackets to give four terms (or four terms implied). zw is M0	M1
	$= 16 - 30\text{i}$	$16 - 30\text{i}$ Answer only 2/2	A1 (2)
(b)	$\frac{z}{w} = \frac{(5 - 3\text{i})}{(2 + 2\text{i})}$		
	$= \frac{(5 - 3\text{i})}{(2 + 2\text{i})} \times \frac{(2 - 2\text{i})}{(2 - 2\text{i})}$	Multiplies $\frac{z}{w}$ by $\frac{(2 - 2\text{i})}{(2 - 2\text{i})}$	M1
	$= \frac{10 - 10\text{i} - 6\text{i} - 6}{4 + 4}$	Simplifies realising that a real number is needed on the denominator and applies $\text{i}^2 = -1$ on their numerator expression and denominator expression.	M1
	$= \frac{4 - 16\text{i}}{8}$		
	$= \frac{1}{2} - 2\text{i}$	$\frac{1}{2} - 2\text{i}$ or $a = \frac{1}{2}$ and $b = -2$ or equivalent Answer as a single fraction A0	A1
			(3) **[5]**

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2.

$$\mathbf{A} = \begin{pmatrix} 2 & 0 \\ 5 & 3 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} -3 & -1 \\ 5 & 2 \end{pmatrix}$$

(a) Find **AB**.

(3)

Given that

$$\mathbf{C} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

(b) describe fully the geometrical transformation represented by **C**,

(2)

(c) write down $\mathbf{C}^{100}$.

(1)

Q2

(Total 6 marks)

Question Number	Scheme		Marks
2.			
(a)	$\mathbf{A} = \begin{pmatrix} 2 & 0 \\ 5 & 3 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} -3 & -1 \\ 5 & 2 \end{pmatrix}$		
	$\mathbf{AB} = \begin{pmatrix} 2 & 0 \\ 5 & 3 \end{pmatrix}\begin{pmatrix} -3 & -1 \\ 5 & 2 \end{pmatrix}$		
	$= \begin{pmatrix} 2(-3) + 0(5) & 2(-1) + 0(2) \\ 5(-3) + 3(5) & 5(-1) + 3(2) \end{pmatrix}$	A correct method to multiply out two matrices. Can be implied by two out of four correct elements.	M1
	$= \begin{pmatrix} -6 & -2 \\ 0 & 1 \end{pmatrix}$	Any three elements correct	A1
		Correct answer	A1
		Correct answer only 3/3	(3)
(b)	Reflection; about the *y*-axis.	<u>Reflection</u>	M1
		<u>*y*-axi</u>s (or $x = 0$.)	A1
			(2)
(c)	$\mathbf{C}^{100} = \mathbf{I} = \underline{\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}}$	$\underline{\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}}$ or $\mathbf{I}$	B1
			(1)
			[6]

Leave blank

3. $$f(x) = 5x^2 - 4x^{\frac{3}{2}} - 6, \quad x \geqslant 0$$

The root α of the equation $f(x) = 0$ lies in the interval $[1.6, 1.8]$.

(a) Use linear interpolation once on the interval $[1.6, 1.8]$ to find an approximation to α. Give your answer to 3 decimal places.
(4)

(b) Differentiate $f(x)$ to find $f'(x)$.
(2)

(c) Taking 1.7 as a first approximation to α, apply the Newton-Raphson process once to $f(x)$ to obtain a second approximation to α. Give your answer to 3 decimal places.
(4)

Question Number	Scheme		Marks
3.	$f(x) = 5x^2 - 4x^{\frac{3}{2}} - 6, \quad x \geqslant 0$		
(a)	$f(1.6) = -1.29543081...$	awrt -1.30	B1
	$f(1.8) = 0.5401863372...$	awrt 0.54	B1
	$\dfrac{\alpha - 1.6}{"1.29543081..."} = \dfrac{1.8 - \alpha}{"0.5401863372..."}$	Correct linear interpolation method with signs correct. Can be implied by working below.	M1
	$\alpha = 1.6 + \left(\dfrac{"1.29543081..."}{"0.5401863372..." + "1.29543081..."}\right)0.2$		
	$= 1.741143899...$	awrt 1.741	A1
		Correct answer seen 4/4	(4)
(b)	$f'(x) = 10x - 6x^{\frac{1}{2}}$	At least one of $\pm ax$ or $\pm bx^{\frac{1}{2}}$ correct.	M1
		Correct differentiation.	A1
			(2)
(c)	$f(1.7) = -0.4161152711...$	$f(1.7) = \text{awrt } -0.42$	B1
	$f'(1.7) = 9.176957114...$	$f'(1.7) = \text{awrt } 9.18$	B1
	$\alpha_2 = 1.7 - \left(\dfrac{"-0.4161152711..."}{"9.176957114..."}\right)$	Correct application of Newton-Raphson formula using their values.	M1
	$= 1.745343491...$		
	$= 1.745 \ \ (3\text{dp})$	1.745	A1 **cao**
		Correct answer seen 4/4	(4)
			[10]

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4. Given that 2 – 4i is a root of the equation

$$z^2 + pz + q = 0,$$

where p and q are real constants,

(a) write down the other root of the equation,

(1)

(b) find the value of p and the value of q.

(3)

Question Number	Scheme		Marks
4.	$z^2 + pz + q = 0,\ \ z_1 = 2 - 4\text{i}$		
(a)	$z_2 = 2 + 4\text{i}$	$2 + 4\text{i}$	B1 (1)
(b)	$(z - 2 + 4\text{i})(z - 2 - 4\text{i}) = 0$		
	$\Rightarrow z^2 - 2z - 4\text{i}z - 2z + 4 - 8\text{i} + 4\text{i}z - 8\text{i} + 16 = 0$	An attempt to multiply out brackets of two complex factors and no i^2.	M1
	$\Rightarrow z^2 - 4z + 20 = 0$	Any one of $p = -4,\ q = 20$.	A1
		Both $p = -4,\ q = 20$.	A1
		$\Rightarrow z^2 - 4z + 20 = 0$ only 3/3	(3)
			[4]

Leave blank

5. (a) Use the results for $\sum_{r=1}^{n} r$, $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r^3$, to prove that

$$\sum_{r=1}^{n} r(r+1)(r+5) = \frac{1}{4}n(n+1)(n+2)(n+7)$$

for all positive integers n.

(5)

(b) Hence, or otherwise, find the value of

$$\sum_{r=20}^{50} r(r+1)(r+5)$$

(2)

Question Number	Scheme		Marks
5	$\sum_{r=1}^{n} r(r+1)(r+5)$		
(a)	$= \sum_{r=1}^{n} r^3 + 6r^2 + 5r$	Multiplying out brackets and an attempt to use at least one of the standard formulae correctly.	M1
	$= \underline{\frac{1}{4}n^2(n+1)^2 + 6.\frac{1}{6}n(n+1)(2n+1) + 5.\frac{1}{2}n(n+1)}$	Correct expression.	A1
	$= \frac{1}{4}n^2(n+1)^2 + n(n+1)(2n+1) + \frac{5}{2}n(n+1)$		
	$= \frac{1}{4}n(n+1)\left(n(n+1) + 4(2n+1) + 10\right)$	Factorising out at least $n(n+1)$	dM1
	$= \frac{1}{4}n(n+1)\left(n^2 + n + 8n + 4 + 10\right)$		
	$= \frac{1}{4}n(n+1)\left(n^2 + 9n + 14\right)$	Correct 3 term quadratic factor	A1
	$= \frac{1}{4}n(n+1)(n+2)(n+7)$ *	Correct proof. No errors seen.	A1 (5)
(b)	$S_n = \sum_{r=20}^{50} r(r+1)(r+5)$		
	$= S_{50} - S_{19}$		
	$= \frac{1}{4}(50)(51)(52)(57) - \frac{1}{4}(19)(20)(21)(26)$	Use of $S_{50} - S_{19}$	M1
	$= 1889550 - 51870$		
	$= 1837680$	1837680 Correct answer only 2/2	A1 (2) **[7]**

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6.

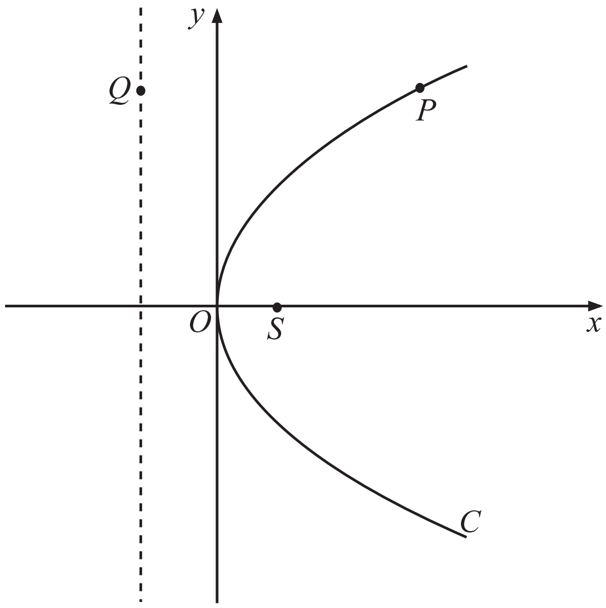


Figure 1

Figure 1 shows a sketch of the parabola C with equation $y^2 = 36x$.
The point S is the focus of C.

(a) Find the coordinates of S.

(1)

(b) Write down the equation of the directrix of C.

(1)

Figure 1 shows the point P which lies on C, where $y > 0$, and the point Q which lies on the directrix of C. The line segment QP is parallel to the x-axis.

Given that the distance PS is 25,

(c) write down the distance QP,

(1)

(d) find the coordinates of P,

(3)

(e) find the area of the trapezium $OSPQ$.

(2)

Question Number	Scheme		Marks
6.	$C: y^2 = 36x \Rightarrow a = \frac{36}{4} = 9$		
(a)	$S(9, 0)$	$(9, 0)$	B1 (1)
(b)	$x + 9 = 0$ or $x = -9$	$x + 9 = 0$ or $x = -9$ or ft using their a from part (a).	B1 $\sqrt{}$ (1)
(c)	$PS = 25 \Rightarrow \underline{QP = 25}$	Either 25 by itself or $PQ = 25$. Do not award if just $PS = 25$ is seen.	B1 (1)
(d)	x-coordinate of $P \Rightarrow x = 25 - 9 = 16$	$x = 16$	B1 $\sqrt{}$
	$y^2 = 36(16)$	Substitutes their x-coordinate into equation of C.	M1
	$\underline{y} = \sqrt{576} = \underline{24}$	$\underline{y = 24}$	A1 (3)
	Therefore $P(16, 24)$		
(e)	Area $OSPQ = \frac{1}{2}(9 + 25)24$	$\frac{1}{2}(\text{their } a + 25)(\text{their } y)$ or rectangle and 2 distinct triangles, correct for their values.	M1
	$= \underline{408}$ (units)^2	408	A1 (2) **[8]**

Leave blank

7. $$z = -24 - 7\text{i}$$

(a) Show z on an Argand diagram.
(1)

(b) Calculate arg z, giving your answer in radians to 2 decimal places.
(2)

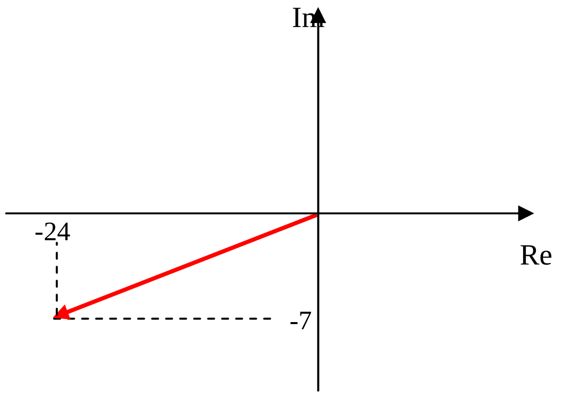
It is given that

$$w = a + b\text{i}, \quad a \in \mathbb{R},\ b \in \mathbb{R}$$

Given also that $|w| = 4$ and $\arg w = \dfrac{5\pi}{6}$,

(c) find the values of a and b,
(3)

(d) find the value of $|zw|$.
(3)

Question Number	Scheme		Marks
7. (a)		Correct quadrant with $(-24,-7)$ indicated.	B1 (1)
(b)	$\arg z = -\pi + \tan^{-1}\left(\frac{7}{24}\right)$	$\tan^{-1}\left(\frac{7}{24}\right)$ or $\tan^{-1}\left(\frac{24}{7}\right)$	M1
	$= -2.857798544... = -2.86$ (2 dp)	awrt -2.86 or awrt 3.43	A1 (2)
(c)	$\lvert w\rvert = 4$, $\arg w = \frac{5\pi}{6} \Rightarrow r = 4$, $\theta = \frac{5\pi}{6}$		
	$w = r\cos\theta + \mathrm{i}\, r\sin\theta$		
	$w = 4\cos\left(\frac{5\pi}{6}\right) + 4\mathrm{i}\sin\left(\frac{5\pi}{6}\right)$	Attempt to apply $r\cos\theta + \mathrm{i}\, r\sin\theta$. Correct expression for w.	M1 A1
	$= 4\left(\frac{-\sqrt{3}}{2}\right) + 4\mathrm{i}\left(\frac{1}{2}\right)$		
	$= -2\sqrt{3} + 2\mathrm{i}$	either $-2\sqrt{3} + 2\mathrm{i}$ or awrt $-3.5 + 2\mathrm{i}$	A1 (3)
	$a = -2\sqrt{3},\ b = 2$		
(d)	$\lvert z\rvert = \sqrt{(-24)^2 + (-7)^2} = \underline{25}$	$\underline{\lvert z\rvert = 25}$ or $zw = (48\sqrt{3}+14) + (14\sqrt{3}-48)\mathrm{i}$ or awrt 97.1-23.8i	B1
	$\lvert zw\rvert = \lvert z\rvert \times \lvert w\rvert = (25)(4)$	Applies $\lvert z\rvert \times \lvert w\rvert$ or $\lvert zw\rvert$	M1
	$= \underline{100}$	$\underline{100}$	A1 (3) **[9]**

Leave blank

8.

$$\mathbf{A} = \begin{pmatrix} 2 & -2 \\ -1 & 3 \end{pmatrix}$$

(a) Find det **A**.

(1)

(b) Find $\mathbf{A}^{-1}$.

(2)

The triangle *R* is transformed to the triangle *S* by the matrix **A**.
Given that the area of triangle *S* is 72 square units,

(c) find the area of triangle *R*.

(2)

The triangle *S* has vertices at the points $(0, 4)$, $(8, 16)$ and $(12, 4)$.

(d) Find the coordinates of the vertices of *R*.

(4)

Question Number	Scheme		Marks
8.	$\mathbf{A} = \begin{pmatrix} 2 & -2 \\ -1 & 3 \end{pmatrix}$		
(a)	$\det \mathbf{A} = 2(3) - (-1)(-2) = 6 - 2 = \underline{4}$	$\underline{4}$	B1 (1)
(b)	$\mathbf{A}^{-1} = \frac{1}{4}\begin{pmatrix} 3 & 2 \\ 1 & 2 \end{pmatrix}$	$\frac{1}{\det \mathbf{A}}\begin{pmatrix} 3 & 2 \\ 1 & 2 \end{pmatrix}$	M1
		$\frac{1}{4}\begin{pmatrix} 3 & 2 \\ 1 & 2 \end{pmatrix}$	A1 (2)
(c)	$\text{Area}(R) = \frac{72}{4} = \underline{18}\ (\text{units})^2$	$\frac{72}{\text{their}\ \det \mathbf{A}}$ or $72(\text{their}\ \det \mathbf{A})$	M1
		$\underline{18}$ or ft answer.	A1 $\sqrt{}$ (2)
(d)	$\mathbf{AR} = \mathbf{S} \Rightarrow \mathbf{A}^{-1}\mathbf{AR} = \mathbf{A}^{-1}\mathbf{S} \Rightarrow \mathbf{R} = \mathbf{A}^{-1}\mathbf{S}$		
	$\mathbf{R} = \frac{1}{4}\begin{pmatrix} 3 & 2 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 0 & 8 & 12 \\ 4 & 16 & 4 \end{pmatrix}$	At least one attempt to apply $\mathbf{A}^{-1}$ by any of the three vertices in $\mathbf{S}$.	M1
	$= \frac{1}{4}\begin{pmatrix} 8 & 56 & 44 \\ 8 & 40 & 20 \end{pmatrix}$		
	$= \begin{pmatrix} 2 & 14 & 11 \\ 2 & 10 & 5 \end{pmatrix}$	At least one correct **column** o.e.	A1 $\sqrt{}$
		At least two correct **columns** o.e.	A1
	Vertices are $(2, 2)$, $(14, 10)$ and $(11, 5)$.	All three **coordinates** correct.	A1 (4) **[9]**

Leave blank

9. A sequence of numbers $u_1, u_2, u_3, u_4, \ldots$ is defined by

$$u_{n+1} = 4u_n + 2, \quad u_1 = 2$$

Prove by induction that, for $n \in \mathbb{Z}^+$,

$$u_n = \frac{2}{3}\left(4^n - 1\right)$$

(5)

Question Number	Scheme		Marks
9.	$u_{n+1} = 4u_n + 2$, $u_1 = 2$ and $u_n = \frac{2}{3}(4^n - 1)$		
	$n = 1$; $u_1 = \frac{2}{3}(4^1 - 1) = \frac{2}{3}(3) = 2$ So u_n is true when $n = 1$.	Check that $u_n = \frac{2}{3}(4^n - 1)$ yields 2 when $n = 1$.	B1
	Assume that for $n = k$ that, $u_k = \frac{2}{3}(4^k - 1)$ is true for $k \in \mathbb{Z}^+$.		
	Then $u_{k+1} = 4u_k + 2$		
	$= 4\left(\frac{2}{3}(4^k - 1)\right) + 2$	Substituting $u_k = \frac{2}{3}(4^k - 1)$ into $u_{n+1} = 4u_n + 2$.	M1
	$= \frac{8}{3}(4)^k - \frac{8}{3} + 2$	An attempt to multiply out the brackets by 4 or $\frac{8}{3}$	M1
	$= \frac{2}{3}(4)(4)^k - \frac{2}{3}$		
	$= \frac{2}{3}4^{k+1} - \frac{2}{3}$		
	$= \frac{2}{3}(4^{k+1} - 1)$	$\frac{2}{3}(4^{k+1} - 1)$	A1
	Therefore, the general statement, $u_n = \frac{2}{3}(4^n - 1)$ is true when $n = k+1$. (As u_n is true for $n = 1$,) then u_n is true for all positive integers by mathematical induction	Require 'True when n=1', 'Assume true when *n*=*k*' and 'True when $n = k+1$' then true for all *n* o.e.	A1
			(5) **[5]**

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10. The point $P\left(6t, \frac{6}{t}\right), t \neq 0$, lies on the rectangular hyperbola H with equation $xy = 36$.

(a) Show that an equation for the tangent to H at P is

$$y = -\frac{1}{t^2}x + \frac{12}{t}$$

(5)

The tangent to H at the point A and the tangent to H at the point B meet at the point $(-9, 12)$.

(b) Find the coordinates of A and B.

(7)

Question Number	Scheme		Marks
10.	$xy = 36$ at $\left(6t, \frac{6}{t}\right)$.		
(a)	$y = \frac{36}{x} = 36x^{-1} \Rightarrow \frac{dy}{dx} = -36x^{-2} = -\frac{36}{x^2}$	An attempt at $\frac{dy}{dx}$. or $\frac{dy}{dt}$ and $\frac{dx}{dt}$	M1
	At $\left(6t, \frac{6}{t}\right)$, $\frac{dy}{dx} = -\frac{36}{(6t)^2}$	An attempt at $\frac{dy}{dx}$. in terms of t	M1
	So, $m_T = \frac{dy}{dx} = -\frac{1}{t^2}$	$\frac{dy}{dx} = -\frac{1}{t^2}$ * **Must see working to award here**	A1
	T: $y - \frac{6}{t} = -\frac{1}{t^2}(x - 6t)$	Applies $y - \frac{6}{t} =$ their $m_T(x - 6t)$	M1
	T: $y - \frac{6}{t} = -\frac{1}{t^2}x + \frac{6}{t}$		
	T: $y = -\frac{1}{t^2}x + \frac{6}{t} + \frac{6}{t}$		
	T: $y = -\frac{1}{t^2}x + \frac{12}{t}$ *	Correct solution .	A1 **cso**
			(5)
(b)	Both **T** meet at $(-9, 12)$ gives		
	$12 = -\frac{1}{t^2}(-9) + \frac{12}{t}$	Substituting (-9,12) into **T**.	M1
	$12 = \frac{9}{t^2} + \frac{12}{t}$ $\left(\times t^2\right)$		
	$12t^2 = 9 + 12t$		
	$12t^2 - 12t - 9 = 0$	An attempt to form a "3 term quadratic"	M1
	$4t^2 - 4t - 3 = 0$		
	$(2t - 3)(2t + 1) = 0$	An attempt to factorise.	M1
	$t = \frac{3}{2}, -\frac{1}{2}$	$t = \frac{3}{2}, -\frac{1}{2}$	A1
	$t = \frac{3}{2} \Rightarrow x = 6\left(\frac{3}{2}\right) = 9$, $y = \frac{6}{\left(\frac{3}{2}\right)} = 4 \Rightarrow (9, 4)$	An attempt to substitute either their $t = \frac{3}{2}$ or their $t = -\frac{1}{2}$ into x and y.	M1
	$t = -\frac{1}{2} \Rightarrow x = 6\left(-\frac{1}{2}\right) = -3$,	At least one of $(9, 4)$ or $(-3, -12)$.	A1
	$y = \frac{6}{\left(-\frac{1}{2}\right)} = -12 \Rightarrow (-3, -12)$	Both $(9, 4)$ and $(-3, -12)$.	A1
			(7) **[12]**