Centre No.						Paper Reference	Surname	Initial(s)
Candidate No.						6 6 6 7 / 0 1	Signature	

Paper Reference(s)

6667/01

Edexcel GCE

Further Pure Mathematics FP1

Advanced/Advanced Subsidiary

Wednesday 17 June 2009 – Morning

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
1	
2	
3	
4	
5	
6	
7	
8	
Total	

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Orange)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions. You must write your answer to each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.
There are 24 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

Printer's Log. No.
M35146A
W850/R6667/57570 3/5/5/3
M 3 5 1 4 6 A 0 1 2 4

Turn over

edexcel
advancing learning, changing lives

Leave blank

1. The complex numbers z_1 and z_2 are given by

$$z_1 = 2 - \mathrm{i} \quad \text{and} \quad z_2 = -8 + 9\mathrm{i}$$

(a) Show z_1 and z_2 on a single Argand diagram.
(1)

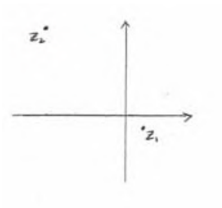
Find, showing your working,

(b) the value of $|z_1|$,
(2)

(c) the value of arg z_1, giving your answer in radians to 2 decimal places,
(2)

(d) $\dfrac{z_2}{z_1}$ in the form $a + b\mathrm{i}$, where a and b are real.
(3)

June 2009
6667 Further Pure Mathematics FP1 (new)
Mark Scheme

Question Number	Scheme	Marks
Q1 (a)		B1 (1)
(b)	$\lvert z_1 \rvert = \sqrt{2^2 + (-1)^2} = \sqrt{5}$ (or awrt 2.24)	M1 A1 (2)
(c)	$\alpha = \arctan\left(\frac{1}{2}\right)$ or $\arctan\left(-\frac{1}{2}\right)$ $\arg z_1 = -0.46$ or 5.82 (awrt) (answer in degrees is A0 unless followed by correct conversion)	M1 A1 (2)
(d)	$\frac{-8+9\text{i}}{2-\text{i}} \times \frac{2+\text{i}}{2+\text{i}}$ $= \frac{-16-8\text{i}+18\text{i}-9}{5} = -5+2\text{i}$ i.e. a = -5 and b =2 or $-\frac{2}{5}a$	M1 A1 A1ft (3) **[8]**
Notes	Alternative method to part (d) $-8+9\text{i} = (2-i)(a+b\text{i})$, and so $2a+b=-8$ and $2b-a=9$ and attempt to solve as far as equation in one variable So a = -5 and $b = 2$ (a) B1 needs both complex numbers as either points or vectors, in correct quadrants and with 'reasonably correct' relative scale (b) M1 Attempt at Pythagoras to find modulus of either complex number A1 condone correct answer even if negative sign not seen in (-1) term A0 for $\pm\sqrt{5}$ (c) arctan 2 is M0 unless followed by $\boxed{\frac{3\pi}{2}+\arctan 2}$ or $\boxed{\frac{\pi}{2}-\arctan 2}$ Need to be clear that argz = - 0.46 or 5.82 for A1 (d) M1 Multiply numerator and denominator by conjugate of their denominator A1 for –5 and A1 for 2i (should be simplified) Alternative scheme for (d) Allow slips in working for first M1	M1 A1 A1cao

Leave blank

2. (a) Using the formulae for $\sum_{r=1}^{n} r$, $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r^3$, show that

$$\sum_{r=1}^{n} r(r+1)(r+3) = \frac{1}{12}n(n+1)(n+2)(3n+k),$$

where k is a constant to be found.

(7)

(b) Hence evaluate $\sum_{r=21}^{40} r(r+1)(r+3)$.

(2)

Question Number	Scheme	Marks
Q2 (a)	$r(r+1)(r+3) = r^3 + 4r^2 + 3r$, so **use** $\sum r^3 + 4\sum r^2 + 3\sum r$	M1
	$= \frac{1}{4}n^2(n+1)^2 + 4\left(\frac{1}{6}n(n+1)(2n+1)\right) + 3\left(\frac{1}{2}n(n+1)\right)$	A1 A1
	$= \frac{1}{12}n(n+1)\{3n(n+1) + 8(2n+1) + 18\}$ or $= \frac{1}{12}n\{3n^3 + 22n^2 + 45n + 26\}$	
	or $= = \frac{1}{12}(n+1)\{3n^3 + 19n^2 + 26n\}$	M1 A1
	$= \frac{1}{12}n(n+1)\{3n^2 + 19n + 26\} = \frac{1}{12}n(n+1)(n+2)(3n+13)$ $(k = 13)$	M1 A1cao (7)
(b)	$\sum_{21}^{40} = \sum_{1}^{40} - \sum_{1}^{20}$	M1
	$= \frac{1}{12}(40 \times 41 \times 42 \times 133) - \frac{1}{12}(20 \times 21 \times 22 \times 73) = 763420 - 56210 = 707210$	A1 cao (2) **[9]**
Notes	(a) M1 expand and must start to use at least one standard formula First 2 A marks: One wrong term A1 A0, two wrong terms A0 A0. M1: Take out factor $kn(n+1)$ or kn or $k(n+1)$ directly or from quartic A1: See scheme (cubics must be simplified) M1: Complete method including a quadratic factor and attempt to factorise it A1 Completely correct work. Just gives $k = 13$, no working is **0** marks for the question. **Alternative method** Expands $(n+1)(n+2)(3n+k)$ and confirms that it equals $\{3n^3 + 22n^2 + 45n + 26\}$ together with statement $k = 13$ can earn last **M1A1** The previous **M1A1** can be implied if they are using a quartic. **(b)** M 1 is for substituting 40 and 20 into their **answer** to (a) and subtracting. (NB not 40 and 21) Adding terms is M0A0 as the question said "Hence"	

Leave blank

3. $$f(x) = (x^2 + 4)(x^2 + 8x + 25)$$

(a) Find the four roots of $f(x) = 0$.

(5)

(b) Find the sum of these four roots.

(2)

Question Number	Scheme	Marks
Q3 (a)	$x^2+4=0 \quad \Rightarrow \quad x=k\text{i}, \quad x=\pm 2\text{i}$	M1, A1
	Solving 3-term quadratic	M1
	$x=\dfrac{-8\pm\sqrt{64-100}}{2} \quad =-4+3\text{i}$ and $-4-3\text{i}$	A1 A1ft
		(5)
(b)	$2\text{i}+(-2\text{i})+(-4+3\text{i})+(-4-3\text{i})=-8$	M1 A1cso
		(2)
		[7]
	Alternative method : Expands f(x) as quartic and chooses $\pm$ coefficient of x^3	M1
	-8	A1 cso
Notes	(a) Just $x=2\text{i}$ is M1 A0 $x=\pm 2$ is M0A0 M1 for solving quadratic follows usual conventions, then A1 for a correct root (simplified as here) and A1ft for conjugate of first answer. Accept correct answers with no working here. Do not give accuracy marks for factors unless followed by roots. (b) M1 for adding four roots of which at least two are complex conjugates and getting a real answer. A1 for –8 following **correct** roots or the alternative method. If any incorrect working in part (a) this A mark will be A0	

Leave blank

4. Given that α is the only real root of the equation

$$x^3 - x^2 - 6 = 0$$

(a) show that $2.2 < \alpha < 2.3$

(2)

(b) Taking 2.2 as a first approximation to α, apply the Newton-Raphson procedure once to $f(x) = x^3 - x^2 - 6$ to obtain a second approximation to α, giving your answer to 3 decimal places.

(5)

(c) Use linear interpolation once on the interval [2.2, 2.3] to find another approximation to α, giving your answer to 3 decimal places.

(3)

Question Number	Scheme	Marks
Q4 (a)	$\text{f}(2.2) = 2.2^3 - 2.2^2 - 6 \quad (= -0.192)$ $\text{f}(2.3) = 2.3^3 - 2.3^2 - 6 \quad (= 0.877)$	M1
	Change of sign $\Rightarrow$ Root need numerical values correct (to 1 s.f.).	A1 (2)
(b)	$\text{f}'(x) = 3x^2 - 2x$	B1
	$\text{f}'(2.2) = 10.12$	B1
	$x_1 = x_0 - \frac{\text{f}(x_0)}{\text{f}'(x_0)} = 2.2 - \frac{-0.192}{10.12}$	M1 A1ft
	$= 2.219$	A1cao (5)
(c)	$\frac{\alpha - 2.2}{\pm'0.192'} = \frac{2.3 - \alpha}{\pm'0.877'}$ (or equivalent such as $\frac{k}{\pm'0.192'} = \frac{0.1 - k}{\pm'0.877'}$.)	M1
	$\alpha(0.877 + 0.192) = 2.3 \times 0.192 + 2.2 \times 0.877$ or $k(0.877 + 0.192) = 0.1 \times 0.192$, where $\alpha = 2.2 + k$	A1
	so $\alpha \approx 2.218$ (2.21796…) (Allow awrt)	A1 (3) **[10]**
Alternative	Uses equation of line joining (2.2, -0.192) to (2.3, 0.877) and substitutes $y = 0$	M1
	$y + 0.192 = \frac{0.192 + 0.877}{0.1}(x - 2.2)$ and $y = 0$, so $\alpha \approx 2.218$ or awrt as before (NB Gradient = 10.69)	A1, A1
Notes	(a) M1 for attempt at f(2.2) and f(2.3) A1 need indication that there is a change of sign – (could be –0.19<0, 0.88>0) and need conclusion. (These marks may be awarded in other parts of the question if not done in part (a)) (b) B1 for seeing correct derivative (but may be implied by later correct work) B1 for seeing 10.12 or this may be implied by later work M1 Attempt Newton-Raphson with their values A1ft may be implied by the following answer (but does not require an evaluation) Final A1 must 2.219 exactly as shown. So answer of 2.21897 would get 4/5 If done twice ignore second attempt (c) M1 Attempt at ratio with their values of ± f(2.2) and ± f(2.3). N.B. If you see $0.192 - \alpha$ or $0.877 - \alpha$ in the fraction then this is M0 A1 correct linear expression and definition of variable if not α (may be implied by final correct answer- does not need 3 dp accuracy) A1 for awrt 2.218 If done twice ignore second attempt	

Leave blank

5. $\mathbf{R} = \begin{pmatrix} a & 2 \\ a & b \end{pmatrix}$, where a and b are constants and $a > 0$.

(a) Find $\mathbf{R}^2$ in terms of a and b.

(3)

Given that $\mathbf{R}^2$ represents an enlargement with centre (0, 0) and scale factor 15,

(b) find the value of a and the value of b.

(5)

Question Number	Scheme	Marks
Q5 (a)	$\mathbf{R}^2 = \begin{pmatrix} a^2+2a & 2a+2b \\ a^2+ab & 2a+b^2 \end{pmatrix}$	M1 A1 A1 (3)
(b)	Puts their $a^2+2a = 15$ or their $2a+b^2 = 15$ or their $(a^2+2a)(2a+b^2)-(a^2+ab)(2a+2b) = 225$ (or to 15) ,	M1,
	Puts their $a^2+ab=0$ or their $2a+2b=0$	M1
	Solve to find either a or b	M1
	$a=3,\quad b=-3$	A1, A1 (5) **[8]**
Alternative for (b)	Uses $\mathbf{R}^2\times$ column vector = $15\times$ column vector, and equates rows to give two equations in a and b only Solves to find either a or b as above method	M1, M1 M1 A1 A1
Notes	(a) 1 term correct: M1 A0 A0 2 or 3 terms correct: M1 A1 A0 (b) M1 M1 as described in scheme (In the alternative scheme column vector can be general or specific for first M1 but must be specific for 2nd M1) M1 requires solving equations to find a and/or b (though checking that correct answer satisfies the equations will earn this mark) This mark can be given independently of the first two method marks. So solving $\mathbf{M}^2 = 15\mathbf{M}$ for example gives M0M0M1A0A0 in part (b) Also putting leading diagonal = 0 and other diagonal = 15 is M0M0M1A0A0 (No possible solutions as $a > 0$) A1 A1 for correct answers only **Any Extra answers given**, e.g. $a = -5$ and $b = 5$ or wrong answers – **deduct last A1 awarded** So the two sets of answers would be A1 A0 Just the answer . $a = -5$ and $b = 5$ is A0 A0 Stopping at two values for a or for b – no attempt at other is A0A0 Answer with no working at all is 0 marks	

Leave blank

6. The parabola C has equation $y^2 = 16x$.

(a) Verify that the point $P(4t^2, 8t)$ is a general point on C.
(1)

(b) Write down the coordinates of the focus S of C.
(1)

(c) Show that the normal to C at P has equation

$$y + tx = 8t + 4t^3$$
(5)

The normal to C at P meets the x-axis at the point N.

(d) Find the area of triangle PSN in terms of t, giving your answer in its simplest form.
(4)

Question Number	Scheme	Marks
Q6 (a)	$y^2 = (8t)^2 = 64t^2$ and $16x = 16 \times 4t^2 = 64t^2$ Or identifies that $a = 4$ and uses general coordinates $(at^2, 2at)$	B1 (1)
(b)	$(4, 0)$	B1 (1)
(c)	$y = 4x^{\frac{1}{2}}$ $\quad \frac{dy}{dx} = 2x^{-\frac{1}{2}}$	B1
	Replaces x by $4t^2$ to give **gradient** $\quad [2(4t^2)^{-\frac{1}{2}} = \frac{2}{2t} = \frac{1}{t}]$	M1,
	Uses Gradient of normal is $-\frac{1}{\text{gradient of curve}}$ $\quad [-t]$	M1
	$y - 8t = -t(x - 4t^2) \Rightarrow y + tx = 8t + 4t^3$ (*)	M1 A1cso (5)
(d)	At N, $y = 0$, so $x = 8 + 4t^2$ or $\frac{8t + 4t^3}{t}$	B1
	Base $SN = (8 + 4t^2) - 4 \;\; (= 4 + 4t^2)$	B1ft
	Area of $\Delta\, PSN = \frac{1}{2}(4 + 4t^2)(8t) = 16t(1 + t^2)$ or $16t + 16t^3$ for $t > 0$	M1 A1 (4) **[11]**
	{Also Area of $\Delta\, PSN = \frac{1}{2}(4 + 4t^2)(-8t) = -16t(1 + t^2)$ for $t < 0$ } *this is not required*	
	<u>Alternatives:</u> (c) $\frac{dx}{dt} = 8t$ and $\frac{dy}{dt} = 8$ B1 $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt} = \frac{1}{t}$ M1 , then as in main scheme. (c) $2y\frac{dy}{dx} = 16$ B1 (or uses $x = \frac{y^2}{8}$ to give $\frac{dx}{dy} = \frac{2y}{8}$) $\frac{dy}{dx} = \frac{8}{y} = \frac{8}{8t} = \frac{1}{t}$ M1 , then as in main scheme.	
Notes	(c) Second M1 – need not be function of t Third M1 requires linear equation (not fraction) and should include the parameter t but could be given for equation of tangent (So tangent equation loses 2 marks only and could gain B1M1M0M1A0) (d) Second B1 does not require simplification and may be a constant rather than an expression in t. M1 needs correct area of triangle formula using ½ 'their SN' $\times 8t$ Or may use two triangles in which case need $(4t^2 - 4)$ and $(4t^2 + 8 - 4t^2)$ for B1ft Then Area of $\Delta\, PSN = \frac{1}{2}(4t^2 - 4)(8t) + \frac{1}{2}(4t^2 + 8 - 4t^2)(8t) = 16t(1 + t^2)$ or $16t + 16t^3$	

Leave blank

7. $$\mathbf{A} = \begin{pmatrix} a & -2 \\ -1 & 4 \end{pmatrix}, \text{ where } a \text{ is a constant.}$$

(a) Find the value of a for which the matrix **A** is singular.
(2)

$$\mathbf{B} = \begin{pmatrix} 3 & -2 \\ -1 & 4 \end{pmatrix}$$

(b) Find $\mathbf{B}^{-1}$.
(3)

The transformation represented by **B** maps the point P onto the point Q.

Given that Q has coordinates $(k - 6, 3k + 12)$, where k is a constant,

(c) show that P lies on the line with equation $y = x + 3$.
(3)

Question Number	Scheme	Marks
Q7 (a)	Use $4a-(-2\times -1)=0 \quad \Rightarrow \quad a,=\frac{1}{2}$	M1, A1 (2)
(b)	Determinant: $(3\times 4)-(-2\times -1)=10 \quad (\Delta)$	M1
	$\mathbf{B}^{-1}=\frac{1}{10}\begin{pmatrix}4 & 2\\ 1 & 3\end{pmatrix}$	M1 A1cso (3)
(c)	$\frac{1}{10}\begin{pmatrix}4 & 2\\ 1 & 3\end{pmatrix}\begin{pmatrix}k-6\\ 3k+12\end{pmatrix}, \quad =\frac{1}{10}\begin{pmatrix}4(k-6)+2(3k+12)\\ (k-6)+3(3k+12)\end{pmatrix}$	M1, A1ft
	$\begin{pmatrix}k\\ k+3\end{pmatrix}$ Lies on $y=x+3$	A1 (3) **[8]**
	<u>Alternatives:</u> (c) $\begin{pmatrix}3 & -2\\ -1 & 4\end{pmatrix}\begin{pmatrix}x\\ x+3\end{pmatrix}, \quad =\begin{pmatrix}3x-2(x+3)\\ -x+4(x+3)\end{pmatrix}$,	M1, A1,
	$=\begin{pmatrix}x-6\\ 3x+12\end{pmatrix}$**,** which was of the form $(k - 6, 3k +12)$	A1
	Or $\begin{pmatrix}3 & -2\\ -1 & 4\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}, \quad =\begin{pmatrix}3x-2y\\ -x+4y\end{pmatrix}=\begin{pmatrix}k-6\\ 3k+12\end{pmatrix},$ and solves simultaneous equations	M1
	Both equations correct and eliminate one letter to get $x = k$ or $y = k + 3$ or $10x-10y=-30$ or equivalent.	A1
	Completely correct work (to $x = k$ and $y = k + 3$), and conclusion lies on $y=x+3$	A1
Notes	(a) Allow sign slips for first M1 (b) Allow sign slip for determinant for first M1 (This mark may be awarded for 1/10 appearing in inverse matrix.) Second M1 is for correctly treating the 2 by 2 matrix, ie for $\begin{pmatrix}4 & 2\\ 1 & 3\end{pmatrix}$ Watch out for determinant (3 + 4) – (-1 + -2) = 10 – M0 then final answer is A0 (c) M1 for multiplying matrix by appropriate column vector A1 correct work (ft wrong determinant) A1 for conclusion	

Leave blank

8. Prove by induction that, for $n \in \mathbb{Z}^+$,

(a) $f(n) = 5^n + 8n + 3$ is divisible by 4,

(7)

(b) $\begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix}^n = \begin{pmatrix} 2n+1 & -2n \\ 2n & 1-2n \end{pmatrix}$

(7)

Question Number	Scheme	Marks
Q8 (a)	$f(1) = 5 + 8 + 3 = 16$, (which is divisible by 4). ($\therefore$ True for n = 1).	B1
	Using the formula to write down f(k + 1), $f(k+1) = 5^{k+1} + 8(k+1) + 3$	M1 A1
	$f(k+1) - f(k) = 5^{k+1} + 8(k+1) + 3 - 5^k - 8k - 3$	M1
	$= 5(5^k) + 8k + 8 + 3 - 5^k - 8k - 3 = 4(5^k) + 8$	A1
	$f(k+1) = 4(5^k + 2) + f(k)$, **which is divisible by** 4	A1ft
	$\therefore$ True for $n = k + 1$ **if** true for $n = k$. True for n = 1, $\therefore$ true for all n.	A1cso (7)
(b)	For n = 1, $\begin{pmatrix} 2n+1 & -2n \\ 2n & 1-2n \end{pmatrix} = \begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix} = \begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix}^1$ ($\therefore$ True for n = 1.)	B1
	$\begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix}^{k+1} = \begin{pmatrix} 2k+1 & -2k \\ 2k & 1-2k \end{pmatrix}\begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix} = \begin{pmatrix} 2k+3 & -2k-2 \\ 2k+2 & -2k-1 \end{pmatrix}$	M1 A1 A1
	$= \begin{pmatrix} 2(k+1)+1 & -2(k+1) \\ 2(k+1) & 1-2(k+1) \end{pmatrix}$	M1 A1
	$\therefore$ True for $n = k + 1$ **if** true for $n = k$. **True for *n*** = 1, $\therefore$ **true for all *n***	A1 cso (7) **[14]**
(a) Alternative for 2[nd] M:	$f(k+1) = 5(5^k) + 8k + 8 + 3$ M1 $= 4(5^k) + 8 + (5^k + 8k + 3)$ A1 or $= 5(5^k + 8k + 3) - 32k - 4$ $= 4(5^k + 2) + f(k)$, or $= 5f(k) - 4(8k+1)$ which is divisible by 4 A1 (or similar methods)	
Notes	(a) B1 Correct values of 16 or 4 for n = 1 or for n = 0 (Accept "is a multiple of") M1 Using the formula to write down f(k + 1) A1 Correct expression of f(k+1) (or for f(n +1) M1 Start method to connect f(k+1) with f(k) as shown A1 correct working toward multiples of 4, A1 ft result including f(k +1) as subject, A1cso conclusion (b) B1 correct statement for n = 1 or n = 0 First M1: Set up product of two appropriate matrices – product can be either way round A1 A0 for one or two slips in simplified result A1 A1 all correct simplified A0 A0 more than two slips M1: States in terms of (k + 1) A1 Correct statement A1 for induction conclusion	
Part (b) Alternative	May write $\begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix}^{k+1} = \begin{pmatrix} 2k+3 & -2k-2 \\ 2k+2 & -2k-1 \end{pmatrix}$**.** Then may or may not complete the proof. This can be awarded the second M (substituting k + 1)and following A (simplification) in part (b). The first three marks are awarded as before . Concluding that they have reached the same matrix and therefore a result will then be part of final A1 cso but also need other statements as in the first method.	