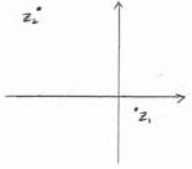






June 2009  
6667 Further Pure Mathematics FP1 (new)  
Mark Scheme

Question Number	Scheme	Marks
Q1 (a)	 <p>(b) <math> z_1  = \sqrt{2^2 + (-1)^2} = \sqrt{5}</math> (or awrt 2.24)</p> <p>(c) <math>\alpha = \arctan\left(\frac{1}{2}\right)</math> or <math>\arctan\left(-\frac{1}{2}\right)</math>  <math>\arg z_1 = -0.46</math> or <math>5.82</math> (awrt) (answer in degrees is A0 unless followed by correct conversion)</p> <p>(d) <math>\frac{-8+9i}{2-i} \times \frac{2+i}{2+i}</math>  <math>= \frac{-16-8i+18i-9}{5} = -5+2i</math> i.e. <math>a = -5</math> and <math>b = 2</math> or <math>-\frac{2}{3}a</math></p>	<p>B1 (1)</p> <p>M1 A1 (2)</p> <p>M1 A1 (2)</p> <p>M1 A1 A1ft (3)</p> <p>[8]</p>
Notes	<p>Alternative method to part (d)  <math>-8+9i = (2-i)(a+bi)</math>, and so <math>2a+b = -8</math> and <math>2b-a = 9</math> and attempt to solve as far as equation in one variable                  So <math>a = -5</math> and <math>b = 2</math></p> <p>(a) B1 needs both complex numbers as either points or vectors, in correct quadrants and with 'reasonably correct' relative scale</p> <p>(b) M1 Attempt at Pythagoras to find modulus of either complex number                  A1 condone correct answer even if negative sign not seen in (-1) term                  A0 for <math>\pm\sqrt{5}</math></p> <p>(c) <math>\arctan 2</math> is M0 unless followed by <math>\frac{3\pi}{2} + \arctan 2</math> or <math>\frac{\pi}{2} - \arctan 2</math> Need to be clear that <math>\arg z = -0.46</math> or <math>5.82</math> for A1</p> <p>(d) M1 Multiply numerator and denominator by conjugate of their denominator                  A1 for <math>-5</math> and A1 for <math>2i</math> (should be simplified)                  Alternative scheme for (d) Allow slips in working for first M1</p>	<p>M1</p> <p>A1 A1cao</p>



Question Number	Scheme	Marks
<p>Q2 (a)</p> <p>(b)</p>	$r(r+1)(r+3) = r^3 + 4r^2 + 3r, \text{ so use } \sum r^3 + 4\sum r^2 + 3\sum r$ $= \frac{1}{4}n^2(n+1)^2 + 4\left(\frac{1}{6}n(n+1)(2n+1)\right) + 3\left(\frac{1}{2}n(n+1)\right)$ $= \frac{1}{12}n(n+1)\{3n(n+1) + 8(2n+1) + 18\} \text{ or } = \frac{1}{12}n\{3n^3 + 22n^2 + 45n + 26\}$ $\text{or } = \frac{1}{12}(n+1)\{3n^3 + 19n^2 + 26n\}$ $= \frac{1}{12}n(n+1)\{3n^2 + 19n + 26\} = \frac{1}{12}n(n+1)(n+2)(3n+13) \quad (k=13)$ $\sum_{21}^{40} = \sum_1^{40} - \sum_1^{20}$ $= \frac{1}{12}(40 \times 41 \times 42 \times 133) - \frac{1}{12}(20 \times 21 \times 22 \times 73) = 763420 - 56210 = 707210$	<p>M1</p> <p>A1 A1</p> <p>M1 A1</p> <p>M1 A1cao (7)</p> <p>M1</p> <p>A1 cao (2)</p> <p>[9]</p>
<p>Notes</p>	<p>(a) M1 expand and must start to use at least one standard formula                      First 2 A marks: One wrong term A1 A0, two wrong terms A0 A0.                      M1: Take out factor <math>kn(n+1)</math> or <math>kn</math> or <math>k(n+1)</math> directly or from quartic                      A1: See scheme (cubics must be simplified)                      M1: Complete method including a quadratic factor and attempt to factorise it                      A1 Completely correct work.                      Just gives <math>k=13</math>, no working is <b>0</b> marks for the question.  <b>Alternative method</b>                      Expands <math>(n+1)(n+2)(3n+k)</math> and confirms that it equals  <math>\{3n^3 + 22n^2 + 45n + 26\}</math> together with statement <math>k=13</math> can earn last <b>M1A1</b>                      The previous <b>M1A1</b> can be implied if they are using a quartic.</p> <p>(b) M 1 is for substituting 40 and 20 into their <b>answer</b> to (a) and subtracting.                      (NB not 40 and 21)                      Adding terms is M0A0 as the question said "Hence"</p>	

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3.

$$f(x) = (x^2 + 4)(x^2 + 8x + 25)$$

(a) Find the four roots of  $f(x) = 0$ .

(5)

(b) Find the sum of these four roots.

(2)

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Question Number	Scheme	Marks
Q3 (a)	$x^2 + 4 = 0 \Rightarrow x = ki, \quad x = \pm 2i$ <p>Solving 3-term quadratic</p> $x = \frac{-8 \pm \sqrt{64 - 100}}{2} = -4 + 3i \text{ and } -4 - 3i$	M1, A1 M1 A1 A1ft (5)
Notes	<p>(b) <math>2i + (-2i) + (-4 + 3i) + (-4 - 3i) = -8</math></p> <p>Alternative method : Expands <math>f(x)</math> as quartic and chooses <math>\pm</math> coefficient of <math>x^3</math></p> <p>-8</p> <p>(a) Just <math>x = 2i</math> is M1 A0  <math>x = \pm 2</math> is M0A0  M1 for solving quadratic follows usual conventions, then A1 for a correct root (simplified as here) and A1ft for conjugate of first answer.  Accept correct answers with no working here. Do not give accuracy marks for factors unless followed by roots.</p> <p>(b) M1 for adding four roots of which at least two are complex conjugates and getting a real answer. A1 for <math>-8</math> following <b>correct</b> roots or the alternative method. If any incorrect working in part (a) this A mark will be A0</p>	M1 A1cso (2) [7] M1 A1 cso

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4. Given that  $\alpha$  is the only real root of the equation

$$x^3 - x^2 - 6 = 0$$

(a) show that  $2.2 < \alpha < 2.3$  **(2)**

(b) Taking 2.2 as a first approximation to  $\alpha$ , apply the Newton-Raphson procedure once to  $f(x) = x^3 - x^2 - 6$  to obtain a second approximation to  $\alpha$ , giving your answer to 3 decimal places. **(5)**

(c) Use linear interpolation once on the interval  $[2.2, 2.3]$  to find another approximation to  $\alpha$ , giving your answer to 3 decimal places. **(3)**

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Question Number	Scheme	Marks
<p>Q4 (a)</p> <p>(b)</p> <p>(c)</p>	$f(2.2) = 2.2^3 - 2.2^2 - 6 \quad (= -0.192)$ $f(2.3) = 2.3^3 - 2.3^2 - 6 \quad (= 0.877)$ <p>Change of sign <math>\Rightarrow</math> Root                      need numerical values correct (to 1 s.f.).</p> $f'(x) = 3x^2 - 2x$ $f'(2.2) = 10.12$ $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 2.2 - \frac{-0.192}{10.12}$ $= 2.219$ <p>(or equivalent such as <math>\frac{k}{\pm'0.192'} = \frac{0.1-k}{\pm'0.877'}</math> .)</p> $\alpha(0.877 + 0.192) = 2.3 \times 0.192 + 2.2 \times 0.877$ <p>or <math>k(0.877 + 0.192) = 0.1 \times 0.192</math>, where <math>\alpha = 2.2 + k</math>  so <math>\alpha \approx 2.218</math> (2.21796...)                      (Allow awrt)</p>	<p>M1</p> <p>A1      (2)</p> <p>B1</p> <p>B1</p> <p>M1 A1ft</p> <p>A1cao      (5)</p> <p>M1</p> <p>A1</p> <p>A1      (3)</p> <p>[10]</p>
<p>Alternative</p> <p>Notes</p>	<p>Uses equation of line joining (2.2, -0.192) to ( 2.3, 0.877) and substitutes <math>y = 0</math></p> $y + 0.192 = \frac{0.192 + 0.877}{0.1}(x - 2.2)$ and $y = 0$ , so $\alpha \approx 2.218$ or awrt as before (NB Gradient = 10.69) <p>(a) M1 for attempt at <math>f(2.2)</math> and <math>f(2.3)</math></p> <p>A1 need indication that there is a change of sign – (could be <math>-0.19 &lt; 0</math>, <math>0.88 &gt; 0</math>) and need conclusion. (These marks may be awarded in other parts of the question if not done in part (a))</p> <p>(b) B1 for seeing correct derivative (but may be implied by later correct work)</p> <p>B1 for seeing 10.12 or this may be implied by later work</p> <p>M1 Attempt Newton-Raphson with their values</p> <p>A1ft may be implied by the following answer (but does not require an evaluation)</p> <p>Final A1 must 2.219 exactly as shown.      So answer of 2.21897 would get 4/5</p> <p>If done twice ignore second attempt</p> <p>(c) M1 Attempt at ratio with their values of <math>\pm f(2.2)</math> and <math>\pm f(2.3)</math>.</p> <p>N.B. If you see <math>0.192 - \alpha</math> or <math>0.877 - \alpha</math> in the fraction then this is M0</p> <p>A1 correct linear expression and definition of variable if not <math>\alpha</math> (may be implied by final correct answer- does not need 3 dp accuracy)</p> <p>A1 for awrt 2.218</p> <p>If done twice ignore second attempt</p>	<p>M1</p> <p>A1, A1</p>



Question Number	Scheme	Marks
Q5 (a)             (b)	$\mathbf{R}^2 = \begin{pmatrix} a^2 + 2a & 2a + 2b \\ a^2 + ab & 2a + b^2 \end{pmatrix}$ <p>Puts their <math>a^2 + 2a = 15</math> or their <math>2a + b^2 = 15</math></p> <p>or their <math>(a^2 + 2a)(2a + b^2) - (a^2 + ab)(2a + 2b) = 225</math> ( or to 15) ,</p> <p>Puts their <math>a^2 + ab = 0</math> or their <math>2a + 2b = 0</math></p> <p>Solve to find either <math>a</math> or <math>b</math></p> <p><math>a = 3, b = -3</math></p>	<p>M1 A1 A1 (3)</p>  <p>M1,</p>  <p>M1</p>  <p>M1</p>  <p>A1, A1 (5) [8]</p>
Alternative for (b)    Notes	<p>Uses <math>\mathbf{R}^2 \times</math> column vector <math>= 15 \times</math> column vector, and equates rows to give two equations in <math>a</math> and <math>b</math> only</p> <p>Solves to find either <math>a</math> or <math>b</math> as above method</p> <p>(a) 1 term correct: M1 A0 A0 2 or 3 terms correct: M1 A1 A0</p> <p>(b) M1 M1 as described in scheme (In the alternative scheme column vector can be general or specific for first M1 but must be specific for 2<sup>nd</sup> M1)</p> <p>M1 requires solving equations to find <math>a</math> and/or <math>b</math> (though checking that correct answer satisfies the equations will earn this mark) This mark can be given independently of the first two method marks.</p> <p>So solving <math>\mathbf{M}^2 = 15\mathbf{M}</math> for example gives M0M0M1A0A0 in part (b)</p> <p>Also putting leading diagonal <math>= 0</math> and other diagonal <math>= 15</math> is M0M0M1A0A0 (No possible solutions as <math>a &gt; 0</math>)</p> <p>A1 A1 for correct answers only</p> <p><b>Any Extra answers given</b>, e.g. <math>a = -5</math> and <math>b = 5</math> or wrong answers – <b>deduct last A1 awarded</b></p> <p>So the two sets of answers would be A1 A0</p> <p>Just the answer <math>a = -5</math> and <math>b = 5</math> is A0 A0</p> <p>Stopping at two values for <math>a</math> or for <math>b</math> – no attempt at other is A0A0</p> <p>Answer with no working at all is 0 marks</p>	<p>M1, M1</p> <p>M1 A1 A1</p>



Question Number	Scheme	Marks
Q6 (a)	$y^2 = (8t)^2 = 64t^2$ and $16x = 16 \times 4t^2 = 64t^2$ Or identifies that $a = 4$ and uses general coordinates $(at^2, 2at)$	B1 (1)
(b)	(4, 0)	B1 (1)
(c)	$y = 4x^{\frac{1}{2}}$ $\frac{dy}{dx} = 2x^{-\frac{1}{2}}$  Replaces $x$ by $4t^2$ to give <b>gradient</b> $[2(4t^2)^{-\frac{1}{2}} = \frac{2}{2t} = \frac{1}{t}]$  Uses Gradient of normal is $-\frac{1}{\text{gradient of curve}}$ $[-t]$  $y - 8t = -t(x - 4t^2) \Rightarrow y + tx = 8t + 4t^3$ (*)	B1 M1, M1 M1 A1cso (5)
(d)	At $N$ , $y = 0$ , so $x = 8 + 4t^2$ or $\frac{8t + 4t^3}{t}$  Base $SN = (8 + 4t^2) - 4 (= 4 + 4t^2)$  Area of $\Delta PSN = \frac{1}{2}(4 + 4t^2)(8t) = 16t(1 + t^2)$ or $16t + 16t^3$ for $t > 0$  {Also Area of $\Delta PSN = \frac{1}{2}(4 + 4t^2)(-8t) = -16t(1 + t^2)$ for $t < 0$ } <i>this is not required</i>  <u>Alternatives:</u> (c) $\frac{dx}{dt} = 8t$ and $\frac{dy}{dt} = 8$ B1 $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt} = \frac{1}{t}$ M1 , then as in main scheme. (c) $2y \frac{dy}{dx} = 16$ B1 (or uses $x = \frac{y^2}{8}$ to give $\frac{dx}{dy} = \frac{2y}{8}$ ) $\frac{dy}{dx} = \frac{8}{y} = \frac{8}{8t} = \frac{1}{t}$ M1 , then as in main scheme.	B1 B1ft M1 A1 (4) [11]
Notes	(c) Second M1 – need not be function of $t$ Third M1 requires linear equation (not fraction) and should include the parameter $t$ but could be given for equation of tangent (So tangent equation loses 2 marks only and could gain B1M1M0M1A0) (d) Second B1 does not require simplification and may be a constant rather than an expression in $t$ . M1 needs correct area of triangle formula using $\frac{1}{2}$ ‘their $SN$ ’ $\times 8t$ Or may use two triangles in which case need $(4t^2 - 4)$ and $(4t^2 + 8 - 4t^2)$ for B1ft Then Area of $\Delta PSN = \frac{1}{2}(4t^2 - 4)(8t) + \frac{1}{2}(4t^2 + 8 - 4t^2)(8t) = 16t(1 + t^2)$ or $16t + 16t^3$	



Question Number	Scheme	Marks
Q7 (a) (b) (c)	<p>Use <math>4a - (-2 \times -1) = 0 \Rightarrow a = \frac{1}{2}</math></p> <p>Determinant: <math>(3 \times 4) - (-2 \times -1) = 10 \quad (\Delta)</math></p> $\mathbf{B}^{-1} = \frac{1}{10} \begin{pmatrix} 4 & 2 \\ 1 & 3 \end{pmatrix}$ <p><math>\frac{1}{10} \begin{pmatrix} 4 &amp; 2 \\ 1 &amp; 3 \end{pmatrix} \begin{pmatrix} k-6 \\ 3k+12 \end{pmatrix} = \frac{1}{10} \begin{pmatrix} 4(k-6) + 2(3k+12) \\ (k-6) + 3(3k+12) \end{pmatrix}</math></p> <p><math>\begin{pmatrix} k \\ k+3 \end{pmatrix}</math> Lies on <math>y = x + 3</math></p>	<p>M1, A1 (2)</p> <p>M1</p> <p>M1 A1cso (3)</p> <p>M1, A1ft</p> <p>A1 (3)</p> <p>[8]</p>
Notes	<p><u>Alternatives:</u></p> <p>(c) <math>\begin{pmatrix} 3 &amp; -2 \\ -1 &amp; 4 \end{pmatrix} \begin{pmatrix} x \\ x+3 \end{pmatrix} = \begin{pmatrix} 3x-2(x+3) \\ -x+4(x+3) \end{pmatrix}</math></p> <p><math>= \begin{pmatrix} x-6 \\ 3x+12 \end{pmatrix}</math>, which was of the form <math>(k-6, 3k+12)</math></p> <p>Or <math>\begin{pmatrix} 3 &amp; -2 \\ -1 &amp; 4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3x-2y \\ -x+4y \end{pmatrix} = \begin{pmatrix} k-6 \\ 3k+12 \end{pmatrix}</math>, and solves simultaneous equations</p> <p>Both equations correct and eliminate one letter to get <math>x = k</math> or <math>y = k + 3</math> or <math>10x - 10y = -30</math> or equivalent.</p> <p>Completely correct work ( to <math>x = k</math> and <math>y = k + 3</math>), and conclusion lies on <math>y = x + 3</math></p> <p>(a) Allow sign slips for first M1</p> <p>(b) Allow sign slip for determinant for first M1 (This mark may be awarded for 1/10 appearing in inverse matrix.)</p> <p>Second M1 is for correctly treating the 2 by 2 matrix, ie for <math>\begin{pmatrix} 4 &amp; 2 \\ 1 &amp; 3 \end{pmatrix}</math></p> <p>Watch out for determinant <math>(3 + 4) - (-1 + -2) = 10 - M0</math> then final answer is A0</p> <p>(c) M1 for multiplying matrix by appropriate column vector</p> <p>A1 correct work (ft wrong determinant)</p> <p>A1 for conclusion</p>	<p>M1, A1, A1</p> <p>M1</p> <p>A1</p> <p>A1</p>





Question Number	Scheme	Marks
<p>Q8 (a)</p> <p>(b)</p>	<p><math>f(1) = 5 + 8 + 3 = 16</math>, (which is divisible by 4). (<math>\therefore</math> True for <math>n = 1</math>).</p> <p>Using the formula to write down <math>f(k + 1)</math>, <math>f(k + 1) = 5^{k+1} + 8(k + 1) + 3</math></p> $f(k + 1) - f(k) = 5^{k+1} + 8(k + 1) + 3 - 5^k - 8k - 3$ $= 5(5^k) + 8k + 8 + 3 - 5^k - 8k - 3 = 4(5^k) + 8$ <p><math>f(k + 1) = 4(5^k + 2) + f(k)</math>, <b>which is divisible by 4</b></p> <p><math>\therefore</math> True for <math>n = k + 1</math> <b>if</b> true for <math>n = k</math>. True for <math>n = 1</math>, <math>\therefore</math> true for all <math>n</math>.</p> <p>For <math>n = 1</math>, <math>\begin{pmatrix} 2n+1 &amp; -2n \\ 2n &amp; 1-2n \end{pmatrix} = \begin{pmatrix} 3 &amp; -2 \\ 2 &amp; -1 \end{pmatrix} = \begin{pmatrix} 3 &amp; -2 \\ 2 &amp; -1 \end{pmatrix}^1</math> (<math>\therefore</math> True for <math>n = 1</math>.)</p> $\begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix}^{k+1} = \begin{pmatrix} 2k+1 & -2k \\ 2k & 1-2k \end{pmatrix} \begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix} = \begin{pmatrix} 2k+3 & -2k-2 \\ 2k+2 & -2k-1 \end{pmatrix}$ $= \begin{pmatrix} 2(k+1)+1 & -2(k+1) \\ 2(k+1) & 1-2(k+1) \end{pmatrix}$ <p><math>\therefore</math> True for <math>n = k + 1</math> <b>if</b> true for <math>n = k</math>. <b>True for <math>n = 1</math>, <math>\therefore</math> true for all <math>n</math></b></p>	<p>B1</p> <p>M1 A1</p> <p>M1 A1</p> <p>A1ft</p> <p>A1cso (7)</p> <p>B1</p> <p>M1 A1 A1</p> <p>M1 A1</p> <p>A1 cso (7) [14]</p>
<p>(a) Alternative for 2<sup>nd</sup> M:</p>	<p><math>f(k + 1) = 5(5^k) + 8k + 8 + 3</math> M1</p> <p><math>= 4(5^k) + 8 + (5^k + 8k + 3)</math> A1 or <math>= 5(5^k + 8k + 3) - 32k - 4</math></p> <p><math>= 4(5^k + 2) + f(k)</math>, or <math>= 5f(k) - 4(8k + 1)</math></p> <p>which is divisible by 4 A1 (or similar methods)</p>	
<p>Notes</p> <p>Part (b) Alternative</p>	<p>(a) B1 Correct values of 16 or 4 for <math>n = 1</math> or for <math>n = 0</math> (Accept “is a multiple of”)</p> <p>M1 Using the formula to write down <math>f(k + 1)</math> A1 Correct expression of <math>f(k+1)</math> (or for <math>f(n + 1)</math>)</p> <p>M1 Start method to connect <math>f(k+1)</math> with <math>f(k)</math> as shown</p> <p>A1 correct working toward multiples of 4, A1 ft result including <math>f(k + 1)</math> as subject, A1cso conclusion</p> <p>(b) B1 correct statement for <math>n = 1</math> or <math>n = 0</math></p> <p>First M1: Set up product of two appropriate matrices – product can be either way round</p> <p>A1 A0 for one or two slips in simplified result</p> <p>A1 A1 all correct simplified</p> <p>A0 A0 more than two slips</p> <p>M1: States in terms of <math>(k + 1)</math></p> <p>A1 Correct statement A1 for induction conclusion</p> <p>May write <math>\begin{pmatrix} 3 &amp; -2 \\ 2 &amp; -1 \end{pmatrix}^{k+1} = \begin{pmatrix} 2k+3 &amp; -2k-2 \\ 2k+2 &amp; -2k-1 \end{pmatrix}</math>. Then may or may not complete the proof.</p> <p>This can be awarded the second M (substituting <math>k + 1</math>) and following A (simplification) in part (b). The first three marks are awarded as before. Concluding that they have reached the same matrix and therefore a result will then be part of final A1 cso but also need other statements as in the first method.</p>	