Centre No.						Paper Reference	Surname	Initial(s)
Candidate No.						6 6 6 7 / 0 1	Signature	

Paper Reference(s)

6667/01

Edexcel GCE

Further Pure Mathematics FP1

Advanced/Advanced Subsidiary

Wednesday 22 June 2011 – Morning

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
1	
2	
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9	
Total	

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer to each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 9 questions in this question paper. The total mark for this paper is 75.
There are 32 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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1. $$f(x) = 3^x + 3x - 7$$

(a) Show that the equation $f(x) = 0$ has a root α between $x = 1$ and $x = 2$.

(2)

(b) Starting with the interval $[1, 2]$, use interval bisection twice to find an interval of width 0.25 which contains α.

(3)

June 2011
6667 Further Pure Mathematics FP1
Mark Scheme

Question Number	Scheme	Notes	Marks
1.	$f(x) = 3^x + 3x - 7$		
(a)	$f(1) = -1$ $f(2) = 8$	Either any one of f(1) = -1 or f(2) = 8.	M1
	Sign change (positive, negative) (and $f(x)$ is continuous) therefore (a **root**) α is between $x = 1$ and $x = 2$.	Both values correct, sign change and conclusion	A1
			(2)
(b)	$f(1.5) = 2.696152423...$ $\{\Rightarrow 1 \leq \alpha \leq 1.5\}$	$f(1.5) = $ awrt 2.7 (or truncated to 2.6)	B1
	$f(1.25) = 0.698222038...$	Attempt to find $f(1.25)$.	M1
	$\Rightarrow 1 \leq \alpha \leq 1.25$	$f(1.25) = $ awrt 0.7 with $1 \leq \alpha \leq 1.25$ or $1 < \alpha < 1.25$ or $[1, 1.25]$ or $(1, 1.25)$. or equivalent in words.	A1
	In (b) there is no credit for linear interpolation and a correct answer with no working scores no marks.		(3)
			5

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2. $$z_1 = -2 + \mathrm{i}$$

(a) Find the modulus of z_1.

(1)

(b) Find, in radians, the argument of z_1, giving your answer to 2 decimal places.

(2)

The solutions to the quadratic equation

$$z^2 - 10z + 28 = 0$$

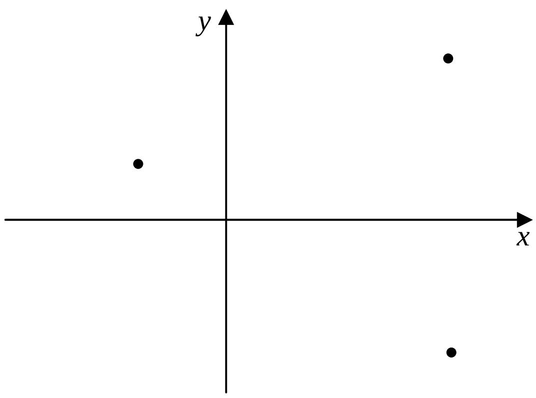
are z_2 and z_3.

(c) Find z_2 and z_3, giving your answers in the form $p \pm \mathrm{i}\sqrt{q}$, where p and q are integers.

(3)

(d) Show, on an Argand diagram, the points representing your complex numbers z_1, z_2 and z_3.

(2)

Question Number	Scheme	Notes	Marks
2. (a)	$\lvert z_1 \rvert = \sqrt{(-2)^2 + 1^2} = \sqrt{5} = 2.236...$	$\sqrt{5}$ or awrt 2.24	B1
			(1)
(b)	$\arg z = \pi - \tan^{-1}\left(\frac{1}{2}\right)$	$\tan^{-1}\left(\frac{1}{2}\right)$ or $\tan^{-1}\left(\frac{2}{1}\right)$ or $\cos^{-1}\left(\frac{2}{\sqrt{5}}\right)$ or $\sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$ or $\sin^{-1}\left(\frac{1}{\sqrt{5}}\right)$ or $\cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$	M1
	$= 2.677945045... = 2.68$ (2 dp)	awrt 2.68	A1 oe
	Can work in degrees for the method mark ($\arg z = 153.4349488^{\circ}$)		(2)
	$\arg z = \tan^{-1}\left(\frac{1}{-2}\right) = -0.46$ on its own is M0 but $\pi + \tan^{-1}\left(\frac{1}{-2}\right) = 2.68$ scores M1A1 $\pi - \tan^{-1}\left(\frac{1}{-2}\right) =$ is M0 as is $\pi - \tan\left(\frac{1}{2}\right)$ (2.60)		
(c)	$z^2 - 10z + 28 = 0$		
	$z = \dfrac{10 \pm \sqrt{100 - 4(1)(28)}}{2(1)}$	An attempt to use the quadratic formula (usual rules)	M1
	$= \dfrac{10 \pm \sqrt{100 - 112}}{2}$		
	$= \dfrac{10 \pm \sqrt{-12}}{2}$		
	$= \dfrac{10 \pm 2\sqrt{3}\,\mathrm{i}}{2}$	Attempt to simplify their $\sqrt{-12}$ in terms of i. E.g. $\mathrm{i}\sqrt{12}$ or $\mathrm{i}\sqrt{3 \times 4}$	M1
	If their b^2-4ac >0 then only the first M1 is available.		
	So, $z = 5 \pm \sqrt{3}\,\mathrm{i}$. $\{p = 5,\ q = 3\}$	$5 \pm \sqrt{3}\,\mathrm{i}$	A1 oe
	Correct answers with no working scores full marks. **See appendix for alternative solution by completing the square**		(3)
(d)		Note that the points are $(-2, 1)$, $(5, \sqrt{3})$ and $(5, -\sqrt{3})$.	
		The point $(-2, 1)$ plotted correctly on the Argand diagram with/without label.	B1
		The distinct points z_2 and z_3 plotted correctly and symmetrically about the x-axis on the Argand diagram with/without label.	B1 $\sqrt{}$
	The points must be correctly placed relative to each other. If you are in doubt about awarding the marks then consult your team leader or use review.		(2)
	NB the second B mark in (d) depends on having obtained complex numbers in (c)		
			8

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3. (a) Given that

$$\mathbf{A} = \begin{pmatrix} 1 & \sqrt{2} \\ \sqrt{2} & -1 \end{pmatrix}$$

(i) find $\mathbf{A}^2$,

(ii) describe fully the geometrical transformation represented by $\mathbf{A}^2$.

(4)

(b) Given that

$$\mathbf{B} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$$

describe fully the geometrical transformation represented by **B**.

(2)

(c) Given that

$$\mathbf{C} = \begin{pmatrix} k+1 & 12 \\ k & 9 \end{pmatrix}$$

where k is a constant, find the value of k for which the matrix **C** is singular.

(3)

P 3 8 1 6 8 A 0 8 3 2

Question Number	Scheme	Notes	Marks
3. (a)	$\mathbf{A} = \begin{pmatrix} 1 & \sqrt{2} \\ \sqrt{2} & -1 \end{pmatrix}$		
(i)	$\mathbf{A}^2 = \begin{pmatrix} 1 & \sqrt{2} \\ \sqrt{2} & -1 \end{pmatrix}\begin{pmatrix} 1 & \sqrt{2} \\ \sqrt{2} & -1 \end{pmatrix}$		
	$= \begin{pmatrix} 1+2 & \sqrt{2}-\sqrt{2} \\ \sqrt{2}-\sqrt{2} & 2+1 \end{pmatrix}$	A correct method to multiply out two matrices. Can be implied by two out of four correct elements.	M1
	$= \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$	Correct answer	A1
			(2)
(ii)	**Enlargement**; scale factor 3, centre $(0, 0)$.	**Enlargement**;	B1;
		scale factor **3**, centre **(0, 0)**	B1
	Allow 'from' or 'about' for centre and 'O' or 'origin' for (0, 0)		
			(2)
(b)	$\mathbf{B} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$		
	Reflection; in the line $y = -x$.	**Reflection**;	B1;
		$y = -x$	B1
	Allow 'in the axis' 'about the line' $y = -x$ etc.		
	The question does not specify a <u>single</u> transformation so we would need to accept any combinations that are correct e.g. Anticlockwise rotation of 90° about the origin followed by a reflection in the *x*-axis is acceptable. In cases like these, the combination has to be <u>completely</u> correct and scored as B2 (no part marks). If in doubt consult your Team Leader.		(2)
(c)	$\mathbf{C} = \begin{pmatrix} k+1 & 12 \\ k & 9 \end{pmatrix}$, k is a constant.		
	$\mathbf{C}$ is singular $\Rightarrow \det\mathbf{C} = 0$. (Can be implied)	$\det\mathbf{C} = 0$	B1
	Special Case $\frac{1}{9(k+1)-12k} = 0$ **B1(implied)M0A0**		
	$9(k+1) - 12k \; (= 0)$	Applies $9(k+1) - 12k$	M1
	$9k + 9 = 12k$		
	$9 = 3k$		
	$k = 3$	$k = 3$	A1
	***k* = 3 with no working can score full marks**		(3)
			9

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4. $$f(x) = x^2 + \frac{5}{2x} - 3x - 1, \quad x \neq 0$$

(a) Use differentiation to find $f'(x)$.

(2)

The root α of the equation $f(x) = 0$ lies in the interval $[0.7, 0.9]$.

(b) Taking 0.8 as a first approximation to α, apply the Newton-Raphson process once to $f(x)$ to obtain a second approximation to α. Give your answer to 3 decimal places.

(4)

Question Number	Scheme	Notes	Marks
4.	$f(x) = x^2 + \frac{5}{2x} - 3x - 1, \quad x \neq 0$		
(a)	$f(x) = x^2 + \frac{5}{2}x^{-1} - 3x - 1$		
	$f'(x) = 2x - \frac{5}{2}x^{-2} - 3\ \{+0\}$	At least two of the four terms differentiated correctly.	M1
		Correct differentiation. (Allow any correct unsimplified form)	A1
	$\left\{f'(x) = 2x - \frac{5}{2x^2} - 3\right\}$		(2)
(b)	$f(0.8) = 0.8^2 + \frac{5}{2(0.8)} - 3(0.8) - 1 (= 0.365)\left(= \frac{73}{200}\right)$	A correct numerical expression for f(0.8)	B1
	$f'(0.8) = -5.30625\left(= \frac{-849}{160}\right)$	Attempt to insert $x = 0.8$ into their f'(x). Does not require an evaluation. (If $f'(0.8)$ is incorrect for their derivative and there is no working score M0)	M1
	$\alpha_2 = 0.8 - \left(\frac{"0.365"}{"-5.30625"}\right)$	Correct application of Newton-Raphson using their values. Does not require an evaluation.	M1
	$= 0868786808...$		
	$= 0.869\ (3\text{dp})$	0.869	A1 **cao**
	A correct answer only with no working scores no marks. N-R must be seen. Ignore any further applications of N-R		(4)
	A derivative of $2x - 5(2x)^{-2} - 3$ is quite common and leads to f'(0.8) = -3.353125 and a final answer of 0.909. This would normally score M1A0B1M1M1A0 (4/6) Similarly for a derivative of $2x - 10x^{-2} - 3$ where the corresponding values are f'(0.8) = -17.025 and answer 0.821		
			6

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5. $$\mathbf{A} = \begin{pmatrix} -4 & a \\ b & -2 \end{pmatrix}, \text{ where } a \text{ and } b \text{ are constants.}$$

Given that the matrix **A** maps the point with coordinates $(4, 6)$ onto the point with coordinates $(2, -8)$,

(a) find the value of a and the value of b.

(4)

A quadrilateral R has area 30 square units.
It is transformed into another quadrilateral S by the matrix **A**.
Using your values of a and b,

(b) find the area of quadrilateral S.

(4)

Question Number	Scheme	Notes	Marks
5.	$\mathbf{A} = \begin{pmatrix} -4 & a \\ b & -2 \end{pmatrix}$, where a and b are constants**.**		
(a)	$\mathbf{A}\begin{pmatrix} 4 \\ 6 \end{pmatrix} = \begin{pmatrix} 2 \\ -8 \end{pmatrix}$		
	Therefore, $\begin{pmatrix} -4 & a \\ b & -2 \end{pmatrix}\begin{pmatrix} 4 \\ 6 \end{pmatrix} = \begin{pmatrix} 2 \\ -8 \end{pmatrix}$	Using the information in the question to form the matrix equation. Can be implied by both correct equations below.	M1
	Do not allow this mark for other incorrect statements unless interpreted correctly later e.g. $\begin{pmatrix} 4 \\ 6 \end{pmatrix}\begin{pmatrix} -4 & a \\ b & -2 \end{pmatrix} = \begin{pmatrix} 2 \\ -8 \end{pmatrix}$ would be M0 unless followed by correct equations or $\begin{pmatrix} -16+6a \\ 4b-12 \end{pmatrix} = \begin{pmatrix} 2 \\ -8 \end{pmatrix}$		
	So, $-16 + 6a = 2$ and $4b - 12 = -8$ Allow $\begin{pmatrix} -16+6a \\ 4b-12 \end{pmatrix} = \begin{pmatrix} 2 \\ -8 \end{pmatrix}$	Any one correct equation. Any correct horizontal line	M1
	giving $a = 3$ and $b = 1$.	Any one of $a = 3$ or $b = 1$.	A1
		Both $a = 3$ and $b = 1$.	A1
			(4)
(b)	$\det \mathbf{A} = 8 - (3)(1) = 5$	Finds determinant by applying $8 -$ their ab.	M1
		$\det \mathbf{A} = 5$	A1
	Special case: The equations -16 + 6b = 2 and 4a – 12 = -8 give a = 1 and b = 3. This comes from incorrect matrix multiplication. This will score nothing in (a) but allow all the marks in (b).		
	Note that $\det \mathbf{A} = \dfrac{1}{8-ab}$ scores M0 here but the following 2 marks are available. However, beware $\det \mathbf{A} = \dfrac{1}{8-ab} = \dfrac{1}{5} \Rightarrow area\,S = \dfrac{30}{\frac{1}{5}} = 150$ This scores M0A0 M1A0		
	Area $S = (\det \mathbf{A})(\text{Area } R)$		
	Area $S = 5 \times 30 = 150 \text{ (units)}^2$	$\dfrac{30}{\text{their } \det \mathbf{A}}$ or $30 \times (\text{their } \det \mathbf{A})$	M1
		150 or ft answer	A1 $\sqrt{}$
	If their det A < 0 then allow ft provided final answer > 0		(4)
	In (b) Candidates may take a more laborious route for the area scale factor and find the area of the unit square, for example, after the transformation represented by A. This needs to be a complete method to score any marks. Correctly establishing the area scale factor M1. Correct answer 5 A1. Then mark as original scheme.		
			8

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6. Given that $z = x + \mathrm{i}y$, find the value of x and the value of y such that

$$z + 3\mathrm{i}z^{*} = -1 + 13\mathrm{i}$$

where z^{*} is the complex conjugate of z.

(7)

Question Number	Scheme	Notes	Marks
6.	$z + 3\mathrm{i}z^* = -1 + 13\mathrm{i}$		
	$(x + \mathrm{i}y) + 3\mathrm{i}(x - \mathrm{i}y)$	$z^* = x - \mathrm{i}y$	B1
		Substituting $z = x + \mathrm{i}y$ and their z^* into $z + 3\mathrm{i}z^*$	M1
	$x + \mathrm{i}y + 3\mathrm{i}x + 3y = -1 + 13\mathrm{i}$	Correct equation in x and y with $\mathrm{i}^2 = -1$. Can be implied.	A1
	$(x + 3y) + \mathrm{i}(y + 3x) = -1 + 13\mathrm{i}$		
	Re part: $x + 3y = -1$ Im part: $y + 3x = 13$	An attempt to equate real **and** imaginary parts.	M1
		Correct equations.	A1
	$3x + 9y = -3$ $3x + y = 13$		
	$8y = -16 \Rightarrow y = -2$	Attempt to solve simultaneous equations to find one of x or y. **At least one of the equations must contain both x and y terms.**	M1
	$x + 3y = -1 \Rightarrow x - 6 = -1 \Rightarrow x = 5$	Both $x = 5$ and $y = -2$.	A1 (7)
	$\{z = 5 - 2\mathrm{i}\}$		**7**

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7. (a) Use the results for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$ to show that

$$\sum_{r=1}^{n}(2r-1)^2 = \frac{1}{3}n(2n+1)(2n-1)$$

for all positive integers n.

(6)

(b) Hence show that

$$\sum_{r=n+1}^{3n}(2r-1)^2 = \frac{2}{3}n\left(an^2+b\right)$$

where a and b are integers to be found.

(4)

Question Number	Scheme	Notes	Marks
7.	$\{S_n =\} \sum_{r=1}^{n} (2r-1)^2$		
(a)	$= \sum_{r=1}^{n} 4r^2 - 4r + 1$	Multiplying out brackets and an attempt to use at least one of the two standard formulae correctly.	M1
	$= 4.\frac{1}{6}n(n+1)(2n+1) - 4.\frac{1}{2}n(n+1) + n$	First two terms correct.	A1
		$+n$	B1
	$= \frac{2}{3}n(n+1)(2n+1) - 2n(n+1) + n$		
	$= \frac{1}{3}n\{2(n+1)(2n+1) - 6(n+1) + 3\}$	Attempt to factorise out $\frac{1}{3}n$	M1
		Correct expression with $\frac{1}{3}n$ factorised out with no errors seen.	A1
	$= \frac{1}{3}n\{2(2n^2+3n+1) - 6(n+1) + 3\}$ $= \frac{1}{3}n\{4n^2+6n+2 - 6n - 6 + 3\}$ $= \frac{1}{3}n(4n^2-1)$		
	$= \frac{1}{3}n(2n+1)(2n-1)$	Correct proof. No errors seen.	A1 *
			(6)
	Note that there are no marks for proof by induction.		
(b)	$\sum_{r=n+1}^{3n} (2r-1)^2 = S_{3n} - S_n$		
	$= \frac{1}{3}.3n(6n+1)(6n-1) - \frac{1}{3}n(2n+1)(2n-1)$	Use of $S_{3n} - S_n$ or $S_{3n} - S_{n+1}$ with the result from (a) used at least once.	M1
		Correct unsimplified expression. E.g. Allow $2(3n)$ for $6n$.	A1
	Note that (b) says **hence** so they have to be using the result from (a)		
	$= n(36n^2-1) - \frac{1}{3}n(4n^2-1)$		
	$= \frac{1}{3}n(108n^2 - 3 - 4n^2 + 1)$	Factorising out $\frac{1}{3}n$ (or $\frac{2}{3}n$)	dM1
	$= \frac{1}{3}n(104n^2 - 2)$		
	$= \frac{2}{3}n(52n^2-1)$	$\frac{2}{3}n(52n^2-1)$	A1
	$\{a = 52,\ b = -1\}$		(4)
			10

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8. The parabola C has equation $y^2 = 48x$.

The point $P(12t^2, 24t)$ is a general point on C.

(a) Find the equation of the directrix of C.
(2)

(b) Show that the equation of the tangent to C at $P(12t^2, 24t)$ is

$$x - ty + 12t^2 = 0$$
(4)

The tangent to C at the point $(3, 12)$ meets the directrix of C at the point X.

(c) Find the coordinates of X.
(4)

Question Number	Scheme	Notes	Marks
8.	$C: y^2 = 48x$ with general point $P\left(12t^2, 24t\right)$.		
(a)	$y^2 = 4ax \;\Rightarrow\; a = \frac{48}{4} = 12$	Using $y^2 = 4ax$ to find a.	M1
	So, directrix has the equation $x + 12 = 0$	$x + 12 = 0$	A1 oe
	Correct answer with no working allow full marks		(2)
(b)	$y = \sqrt{48}\, x^{\frac{1}{2}} \;\Rightarrow\; \frac{dy}{dx} = \frac{1}{2}\sqrt{48}\, x^{-\frac{1}{2}} \left(= 2\sqrt{3}\, x^{-\frac{1}{2}}\right)$ or (implicitly) $y^2 = 48x \Rightarrow 2y\frac{dy}{dx} = 48$ or (chain rule) $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = 24 \times \frac{1}{24t}$	$\frac{dy}{dx} = \pm k\, x^{-\frac{1}{2}}$ $ky\frac{dy}{dx} = c$ their $\frac{dy}{dt} \times \left(\frac{1}{\text{their } \frac{dx}{dt}}\right)$	M1
	When $x = 12t^2$, $\frac{dy}{dx} = \frac{\sqrt{48}}{2\sqrt{12t^2}} = \frac{\sqrt{4}}{2t} = \frac{1}{t}$ or $\frac{dy}{dx} = \frac{48}{2y} = \frac{48}{48t} = \frac{1}{t}$	$\frac{dy}{dx} = \frac{1}{t}$	A1
	T: $y - 24t = \frac{1}{t}\left(x - 12t^2\right)$	Applies $y - 24t = \text{their } m_T\left(x - 12t^2\right)$ or $y = \left(\text{their } m_T\right)x + c$ using $x = 12t^2$ *and* $y = 24t$ in an attempt to find c. **Their m_T must be a function of t.**	M1
	T: $ty - 24t^2 = x - 12t^2$		
	T: $x - ty + 12t^2 = 0$	Correct solution.	A1 **cso** *
	Special case: If the gradient is <u>quoted</u> as 1/t, this can score M0A0M1A1		(4)
(c)	Compare $P\left(12t^2, 24t\right)$ with $(3, 12)$ gives $t = \frac{1}{2}$.	$t = \frac{1}{2}$	B1
	NB $x - ty + 12t^2 = 0$ with $x = 3$ and $y = 12$ gives $4t^2 - 4t + 1 = 0 = (2t-1)^2 \Rightarrow t = \frac{1}{2}$		
	$t = \frac{1}{2}$ into **T** gives $x - \frac{1}{2}y + 3 = 0$	Substitutes their t into **T**.	M1
	See Appendix for an alternative approach to find the tangent		
	At X, $x = -12 \Rightarrow -12 - \frac{1}{2}y + 3 = 0$	Substitutes their x from (a) into **T**.	M1
	So, $-9 = \frac{1}{2}y \;\Rightarrow\; y = -18$		
	So the coordinates of X are $(-12, -18)$.	$(-12, -18)$	A1
	The coordinates must be together at the end for the final A1 e.g. as above or $x = -12$, $y = -18$		(4)
			10

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9. Prove by induction, that for $n \in \mathbb{Z}^+$,

(a) $\begin{pmatrix} 3 & 0 \\ 6 & 1 \end{pmatrix}^n = \begin{pmatrix} 3^n & 0 \\ 3(3^n - 1) & 1 \end{pmatrix}$,

(6)

(b) $f(n) = 7^{2n-1} + 5$ is divisible by 12.

(6)

Question Number	Scheme	Notes	Marks
9. (a)	$n=1;\quad \text{LHS} = \begin{pmatrix} 3 & 0 \\ 6 & 1 \end{pmatrix}^1 = \begin{pmatrix} 3 & 0 \\ 6 & 1 \end{pmatrix}$ $\text{RHS} = \begin{pmatrix} 3^1 & 0 \\ 3(3^1-1) & 1 \end{pmatrix} = \begin{pmatrix} 3 & 0 \\ 6 & 1 \end{pmatrix}$ As LHS = RHS, the matrix result is true for $n = 1$.	Check to see that the result is true for $n = 1$.	B1
	Assume that the matrix equation is true for $n = k$, ie. $\begin{pmatrix} 3 & 0 \\ 6 & 1 \end{pmatrix}^k = \begin{pmatrix} 3^k & 0 \\ 3(3^k-1) & 1 \end{pmatrix}$		
	With $n = k+1$ the matrix equation becomes $\begin{pmatrix} 3 & 0 \\ 6 & 1 \end{pmatrix}^{k+1} = \begin{pmatrix} 3 & 0 \\ 6 & 1 \end{pmatrix}^k \begin{pmatrix} 3 & 0 \\ 6 & 1 \end{pmatrix}$		
	$= \begin{pmatrix} 3^k & 0 \\ 3(3^k-1) & 1 \end{pmatrix}\begin{pmatrix} 3 & 0 \\ 6 & 1 \end{pmatrix}$ or $\begin{pmatrix} 3 & 0 \\ 6 & 1 \end{pmatrix}\begin{pmatrix} 3^k & 0 \\ 3(3^k-1) & 1 \end{pmatrix}$	$\begin{pmatrix} 3^k & 0 \\ 3(3^k-1) & 1 \end{pmatrix}$ by $\begin{pmatrix} 3 & 0 \\ 6 & 1 \end{pmatrix}$	M1
	$= \begin{pmatrix} 3^{k+1}+0 & 0+0 \\ 9(3^k-1)+6 & 0+1 \end{pmatrix}$ or $\begin{pmatrix} 3^{k+1}+0 & 0+0 \\ 6.3^k+3(3^k-1) & 0+1 \end{pmatrix}$	Correct unsimplified matrix with no errors seen.	A1
	$= \begin{pmatrix} 3^{k+1} & 0 \\ 9(3^k)-3 & 1 \end{pmatrix}$		
	$= \begin{pmatrix} 3^{k+1} & 0 \\ 3\left(3(3^k)-1\right) & 1 \end{pmatrix}$		
	$= \begin{pmatrix} 3^{k+1} & 0 \\ 3(3^{k+1}-1) & 1 \end{pmatrix}$	Manipulates so that $k \to k+1$ on at least one term.	dM1
		Correct result with no errors seen with some working between this and the previous A1	A1
	If the result is true for $n = k$, (1) then it is now true for $n = k+1$. (2) As the result has shown to be true for $n = 1$, (3) then the result is true for all n. (4) **All 4 aspects need to be mentioned at some point for the last A1.**	Correct conclusion with all previous marks earned	A1 cso
			(6)

Question Number	Scheme	Notes	Marks
9. (b)	$f(1) = 7^{2-1} + 5 = 7 + 5 = 12$,	Shows that $f(1) = 12$.	B1
	{which is divisible by 12}. {$\therefore$ f (n) is divisible by 12 when $n = 1$.}		
	Assume that for $n = k$, $f(k) = 7^{2k-1} + 5$ is divisible by 12 for $k \in ¢^{+}$.		
	So, $f(k+1) = 7^{2(k+1)-1} + 5$	Correct unsimplified expression for $f(k+1)$.	B1
	giving, $f(k+1) = 7^{2k+1} + 5$		
	$\therefore f(k+1) - f(k) = \left(7^{2k+1} + 5\right) - \left(7^{2k-1} + 5\right)$	Applies $f(k+1) - f(k)$. No simplification is necessary and condone missing brackets.	M1
	$= 7^{2k+1} - 7^{2k-1}$		
	$= 7^{2k-1}\left(7^2 - 1\right)$	Attempting to isolate 7^{2k-1}	M1
	$= 48\left(7^{2k-1}\right)$	$48\left(7^{2k-1}\right)$	A1cso
	$\therefore f(k+1) = f(k) + 48\left(7^{2k-1}\right)$, which is divisible by 12 as both $f(k)$ and $48\left(7^{2k-1}\right)$ are both divisible by 12.(1) If the result is true for $n = k$, (2) then it is now true for $n = k+1$. (3) As the result has shown to be true for $n = 1$,(4) then the result is true for all n. (5). **All 5 aspects need to be mentioned at some point for the last A1.**	Correct conclusion with no incorrect work. Don't condone missing brackets.	A1 cso
	There are other ways of proving this by induction. See appendix for 3 alternatives. If you are in any doubt consult your team leader and/or use the review system.		(6)
			12