Centre No.						Paper Reference	Surname	Initial(s)
Candidate No.						6 6 6 7 / 0 1	Signature	

Paper Reference(s)

6667/01

Edexcel GCE

Further Pure Mathematics FP1

Advanced/Advanced Subsidiary

Friday 1 June 2012 – Morning

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
1	
2	
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4	
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6	
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8	
9	
10	
Total	

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer to each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 10 questions in this question paper. The total mark for this paper is 75.
There are 32 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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1. $$f(x) = 2x^3 - 6x^2 - 7x - 4$$

(a) Show that $f(4) = 0$

(1)

(b) Use algebra to solve $f(x) = 0$ completely.

(4)

Summer 2012
6667 Further Pure FP1
Mark Scheme

Question Number	Scheme	Notes	Marks
	$f(x) = 2x^3 - 6x^2 - 7x - 4$		
1. (a)	$f(4) = \underline{128 - 96 - 28 - 4 = 0}$	$\underline{128 - 96 - 28 - 4 = 0}$	B1
	<u>Just</u> $2(4)^3 - 6(4)^2 - 7(4) - 4 = 0$ or $2(64) - 6(16) - 7(4) - 4 = 0$ is B0 But $2(64) - 6(16) - 7(4) - 4 = 128 - 128 = 0$ or $2(4)^3 - 6(4)^2 - 7(4) - 4 = 4 - 4 = 0$ is B1		
	There must be sufficient working to <u>show</u> that f(4) = 0		
			[1]
(b)	$f(4) = 0 \Rightarrow (x - 4)$ is a factor.		
	$f(x) = (x - 4)(2x^2 + 2x + 1)$	M1: $(2x^2 + kx + 1)$ Uses inspection or long division or compares coefficients **and** $(x - 4)$ (**not** $(\boldsymbol{x} + \mathbf{4})$) to obtain a quadratic factor of this form. A1: $(2x^2 + 2x + 1)$ cao	M1A1
	So, $x = \dfrac{-2 \pm \sqrt{4 - 4(2)(1)}}{2(2)}$ $(2)\left(x^2 + x + \frac{1}{2}\right) = 0 \Rightarrow (2)\left(\left(x \pm \frac{1}{2}\right)^2 \pm k \pm \frac{1}{2}\right)\ k \neq 0 \Rightarrow x =$	Use of correct quadratic formula for their <u>3TQ</u> or completes the square.	M1
	Allow an attempt at factorisation provided the usual conditions are satisfied and proceeds as far as *x* = ..		
	$\Rightarrow x = \dfrac{-2 \pm \sqrt{-4}}{2(2)}$		
	$\Rightarrow x = 4,\ \dfrac{-2 \pm 2i}{4}$	**All <u>three</u> roots stated somewhere in (b).** Complex roots must be at least as given but apply isw if necessary.	A1
			[4]
			5 marks

Leave blank

2. (a) Given that

$$\mathbf{A} = \begin{pmatrix} 3 & 1 & 3 \\ 4 & 5 & 5 \end{pmatrix} \quad \text{and} \quad \mathbf{B} = \begin{pmatrix} 1 & 1 \\ 1 & 2 \\ 0 & -1 \end{pmatrix}$$

find **AB**.

(2)

(b) Given that

$$\mathbf{C} = \begin{pmatrix} 3 & 2 \\ 8 & 6 \end{pmatrix}, \quad \mathbf{D} = \begin{pmatrix} 5 & 2k \\ 4 & k \end{pmatrix}, \text{ where } k \text{ is a constant}$$

and

$$\mathbf{E} = \mathbf{C} + \mathbf{D}$$

find the value of k for which **E** has no inverse.

(4)

Question Number	Scheme	Notes	Marks
2. (a)	$\mathbf{A} = \begin{pmatrix} 3 & 1 & 3 \\ 4 & 5 & 5 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 1 & 1 \\ 1 & 2 \\ 0 & -1 \end{pmatrix}$		
	$\mathbf{AB} = \begin{pmatrix} 3 & 1 & 3 \\ 4 & 5 & 5 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & 2 \\ 0 & -1 \end{pmatrix}$		
	$= \begin{pmatrix} 3+1+0 & 3+2-3 \\ 4+5+0 & 4+10-5 \end{pmatrix}$	A correct method to multiply out two matrices. Can be implied by two out of four correct (unsimplified) elements in a dimensionally correct matrix. A 2x2 matrix with a number or a calculation at each corner.	M1
	$= \begin{pmatrix} 4 & 2 \\ 9 & 9 \end{pmatrix}$	Correct answer	A1
	A correct answer with no working can score both marks		
			[2]
(b)	$\mathbf{C} = \begin{pmatrix} 3 & 2 \\ 8 & 6 \end{pmatrix}$, $\mathbf{D} = \begin{pmatrix} 5 & 2k \\ 4 & k \end{pmatrix}$, where k is a constant,		
	$\mathbf{C} + \mathbf{D} = \begin{pmatrix} 3 & 2 \\ 8 & 6 \end{pmatrix} + \begin{pmatrix} 5 & 2k \\ 4 & k \end{pmatrix} = \begin{pmatrix} 8 & 2k+2 \\ 12 & 6+k \end{pmatrix}$	An attempt to add C to D. Can be implied by two out of four correct (unsimplified) elements in a dimensionally correct matrix.	M1
	$\mathbf{E}$ does not have an inverse $\Rightarrow \det \mathbf{E} = 0$.		
	$8(6+k) - 12(2k+2)$	Applies "$ad - bc$" to $\mathbf{E}$ where $\mathbf{E}$ is a 2x2 matrix.	M1
	$8(6+k) - 12(2k+2) = 0$	States or applies $\det(\mathbf{E}) = 0$ where $\det(\mathbf{E}) = ad - bc$ or $ad + bc$ only and $\mathbf{E}$ is a 2x2 matrix.	M1
	Note $8(6+k) - 12(2k+2) = 0$ or $8(6+k) = 12(2k+2)$ could score both M's		
	$48 + 8k = 24k + 24$ $24 = 16k$		
	$k = \frac{3}{2}$		A1 oe
			[4]
			6 marks

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3. $$f(x) = x^2 + \frac{3}{4\sqrt{x}} - 3x - 7, \quad x > 0$$

A root α of the equation $f(x) = 0$ lies in the interval $[3, 5]$.

Taking 4 as a first approximation to α, apply the Newton-Raphson process once to $f(x)$ to obtain a second approximation to α. Give your answer to 2 decimal places.

(6)

Question Number	Scheme	Notes	Marks
3.	$\mathrm{f}(x) = x^2 + \dfrac{3}{4\sqrt{x}} - 3x - 7, \quad x > 0$		
	$\mathrm{f}(x) = x^2 + \dfrac{3}{4}x^{-\frac{1}{2}} - 3x - 7$		
	$\mathrm{f}'(x) = 2x - \dfrac{3}{8}x^{-\frac{3}{2}} - 3\ \{+0\}$	M1: $x^n \to x^{n-1}$ on at least one term A1: Correct differentiation.	M1A1
	$\mathrm{f}(4) = -2.625 = -\dfrac{21}{8} = -2\dfrac{5}{8}$ *or* $4^2 + \dfrac{3}{4\sqrt{4}} - 3\times4 - 7$	$\mathrm{f}(4) = -2.625$ A correct evaluation of f(4) or a correct numerical expression for f(4). This can be implied by a correct answer below but in all other cases, f(4) must be seen explicitly evaluated or as an expression.	B1
	$\mathrm{f}'(4) = 4.953125 = \dfrac{317}{64} = 4\dfrac{61}{64}$	Attempt to insert $x = 4$ into their $\mathrm{f}'(x)$. Not dependent on the first M but must be what they think is $\mathrm{f}'(x)$.	M1
	$\alpha_2 = 4 - \left(\dfrac{\text{"}-2.625\text{"}}{\text{"}4.953125\text{"}}\right)$	Correct application of Newton-Raphson using their values.	M1
	$= 4.529968454... \quad \left(= \dfrac{1436}{317} = 4\tfrac{168}{317}\right)$		
	$= 4.53 \ (2\,\mathrm{dp})$	4.53 **cso**	A1 **cao**
	Note that the kind of errors that are being made in differentiating are sometimes giving 4.53 but the final mark is cso and the final A1 should not be awarded in these cases.		
	Ignore any further iterations		
	A correct derivative followed by $\alpha_2 = 4 - \dfrac{f(4)}{f'(4)} = 4.53$ can score full marks.		
			[6]
			6 marks

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4. (a) Use the standard results for $\sum_{r=1}^{n} r^3$ and $\sum_{r=1}^{n} r$ to show that

$$\sum_{r=1}^{n}\left(r^3 + 6r - 3\right) = \frac{1}{4}n^2(n^2 + 2n + 13)$$

for all positive integers n.

(5)

(b) Hence find the exact value of

$$\sum_{r=16}^{30}\left(r^3 + 6r - 3\right)$$

(2)

Question Number	Scheme	Notes	Marks
4. (a)	$\sum_{r=1}^{n}(r^3 + 6r - 3)$		
	$= \underline{\frac{1}{4}n^2(n+1)^2 + 6.\frac{1}{2}n(n+1)} - 3n$	M1; An attempt to use at least one of the standard formulae correctly in summing at least 2 terms of $r^3 + 6r - 3$ A1: Correct underlined expression. B1: $-3 \to -3n$	M1A1B1
	$= \frac{1}{4}n^2(n+1)^2 + 3n^2 + 3n - 3n$		
	If any marks have been lost, no further marks are available in part (a)		
	$= \frac{1}{4}n^2(n+1)^2 + 3n^2$ $= \frac{1}{4}n^2\left((n+1)^2 + 12\right)$	Cancels out the $3n$ and attempts to factorise out at least $\frac{1}{4}n$.	dM1
	$= \frac{1}{4}n^2\left(n^2 + 2n + 13\right)$ **(AG)**	Correct answer **with no errors seen**.	A1 *
	Provided the first 3 marks are scored, allow the next two marks for correctly showing the algebraic equivalence. E.g. showing that both $\frac{1}{4}n^2(n+1)^2 + 6.\frac{1}{2}n(n+1) - 3n$ and $\frac{1}{4}n^2\left(n^2 + 2n + 13\right) = \frac{1}{4}n^4 + \frac{1}{2}n^3 + \frac{13}{4}n^2$		
	There are no marks for proof by induction but apply the scheme if necessary.		
			[5]
(b)	$S_n = \sum_{r=16}^{30}(r^3 + 6r - 3) = S_{30} - S_{15}$		
	$= \frac{1}{4}(30)^2\left(30^2 + 2(30) + 13\right) - \frac{1}{4}(15)^2\left(15^2 + 2(15) + 13\right)$	<u>Use</u> of $S_{30} - S_{15}$ or $S_{30} - S_{16}$	M1
	NB They must be using $S_n = \frac{1}{4}n^2\left(n^2 + 2n + 13\right)$ **not** $S_n = n^3 + 6n - 3$		
	$= 218925 - 15075$		
	$= 203850$	203850	A1 **cao**
	NB $S_{30} - S_{16}$ =218925 – 19264 = 199661 (Scores M1 A0)		
			[2]
			7 marks

Leave blank

5.

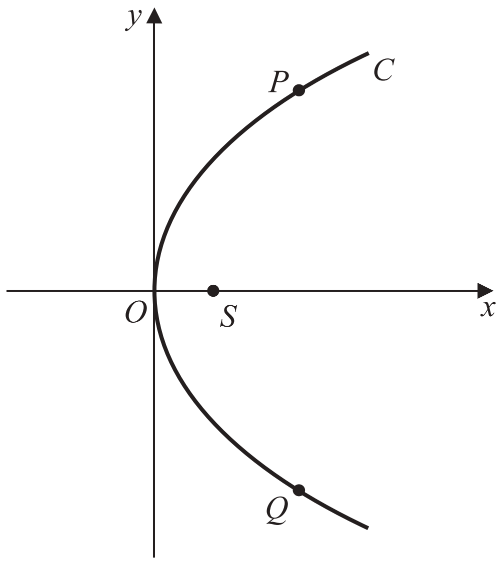

Figure 1

Figure 1 shows a sketch of the parabola C with equation $y^2 = 8x$.
The point P lies on C, where $y > 0$, and the point Q lies on C, where $y < 0$
The line segment PQ is parallel to the y-axis.

Given that the distance PQ is 12,

(a) write down the y-coordinate of P,
(1)

(b) find the x-coordinate of P.
(2)

Figure 1 shows the point S which is the focus of C.
The line l passes through the point P and the point S.

(c) Find an equation for l in the form $ax + by + c = 0$, where a, b and c are integers.
(4)

Question Number	Scheme	Notes	Marks
5.	$C: y^2 = 8x \Rightarrow a = \frac{8}{4} = 2$		
(a)	$PQ = 12 \Rightarrow$ By symmetry $y_P = \frac{12}{2} = \underline{6}$	$y = \underline{6}$	B1
			[1]
(b)	$y^2 = 8x \Rightarrow 6^2 = 8x$	Substitutes their y-coordinate into $y^2 = 8x$.	M1
	$\Rightarrow x = \underline{\frac{36}{8}} = \underline{\frac{9}{2}}$ (So P has coordinates $\left(\frac{9}{2}, 6\right)$)	$\Rightarrow x = \underline{\frac{36}{8}}$ or $\underline{\frac{9}{2}}$	A1 oe
			[2]
(c)	Focus $S(2, 0)$	Focus has coordinates $(2, 0)$. Seen or implied. Can score anywhere.	B1
	Gradient $PS = \dfrac{6-0}{\frac{9}{2}-2}$ $\left\{= \dfrac{6}{\left(\frac{5}{2}\right)} = \dfrac{12}{5}\right\}$	Correct method for finding the gradient of the line segment PS. If no gradient formula is quoted and the gradient is incorrect, score M0 but allow this mark if there is a clear use of $\dfrac{y_2 - y_1}{x_2 - x_1}$ even if their coordinates are 'confused'.	M1
	Either $y - 0 = \frac{12}{5}(x-2)$ or $y - 6 = \frac{12}{5}(x - \frac{9}{2})$; - - - - - - - - - - - - - - - **or** $y = \frac{12}{5}x + c$ and $0 = \frac{12}{5}(2) + c \Rightarrow c = -\frac{24}{5}$;	$y - y_1 = m(x - x_1)$ with 'their PS gradient' and their (x_1, y_1) **Their PS gradient must have come from using P and S (not calculus) and they must use their P or S as (x_1, y_1).** - - - - - - - - - - - - - - - or uses $y = mx + c$ with 'their gradient' in an attempt to find c. **Their PS gradient must have come from using P and S (not calculus) and they must use their P or S as (x_1, y_1).**	M1
	l: $\underline{12x - 5y - 24 = 0}$	$\underline{12x - 5y - 24 = 0}$	A1
	Allow any equivalent form e.g. $k(12x - 5y - 24) = 0$ where k is an integer		
			[4]
			7 marks

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6. $$\mathrm{f}(x) = \tan\left(\frac{x}{2}\right) + 3x - 6, \quad -\pi < x < \pi$$

(a) Show that the equation $\mathrm{f}(x) = 0$ has a root α in the interval $[1, 2]$.

(2)

(b) Use linear interpolation once on the interval $[1, 2]$ to find an approximation to α. Give your answer to 2 decimal places.

(3)

Question Number	Scheme	Notes	Marks
6.	$f(x) = \tan\left(\frac{x}{2}\right) + 3x - 6, \quad -\pi < x < \pi$		
(a)	$f(1) = -2.45369751...$ $f(2) = 1.557407725...$	Attempts to evaluate both $f(1)$ and $f(2)$ and evaluates at least one of them correctly to awrt (or trunc.) 2 sf. Nm	M1
	Sign change (and $f(x)$ is continuous) therefore a root α is between $x = 1$ and $x = 2$.	Both values correct to awrt (or trunc.) 2 sf, sign change (or a statement which implies this e.g. -2.453.. < 0 < 1.5574..) and conclusion.	A1
			[2]
(b)	$\frac{\alpha - 1}{"2.45369751..."} = \frac{2 - \alpha}{"1.557407725..."}$ **or** $\frac{"2.45369751..." + "1.557407725"}{1} = \frac{"2.45369751..."}{\alpha - 1}$	Correct linear interpolation method. It must be a <u>correct statement</u> using their f(2) and f(1). Can be implied by working below.	M1
	If any "negative lengths" are used, score M0		
	$\alpha = 1 + \left(\frac{"2.45369751..."}{"1.557407725..." + "2.45369751..."}\right)1$ $= \frac{6.464802745}{4.011105235}$	Correct follow through expression to find α .Method can be implied here. (Can be implied by awrt 1.61.)	A1 $\sqrt{}$
	$= 1.611726037...$	awrt 1.61	A1
			[3]
			5 marks
	Special Case – Use of Degrees		
	$f(1) = -2.991273132...$ $f(2) = 0.017455064...$	Attempts to evaluate both $f(1)$ and $f(2)$ and evaluates at least one of them correctly to awrt (or trunc.) 2 sf.	M1A0
	$\frac{\alpha - 1}{"2.991273132..."} = \frac{2 - \alpha}{"0.017455064..."}$	Correct linear interpolation method. It must be a <u>correct statement</u> using their f(2) and f(1). Can be implied by working below.	M1
	If any "negative lengths" are used, score M0		
	$\alpha = 1 + \left(\frac{"2.99127123..."}{"0.017455064..." + "2.99127123..."}\right)1$	Correct follow through expression to find α .Method can be implied here. (Can be implied by awrt 1.99.)	A1 $\sqrt{}$
	$= 1.994198523...$		A0

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7.

$$z = 2 - \mathrm{i}\sqrt{3}$$

(a) Calculate $\arg z$, giving your answer in radians to 2 decimal places. **(2)**

Use algebra to express

(b) $z + z^2$ in the form $a + b\mathrm{i}\sqrt{3}$, where a and b are integers, **(3)**

(c) $\dfrac{z+7}{z-1}$ in the form $c + d\mathrm{i}\sqrt{3}$, where c and d are integers. **(4)**

Given that

$$w = \lambda - 3\mathrm{i}$$

where λ is a real constant, and $\arg(4 - 5\mathrm{i} + 3w) = -\dfrac{\pi}{2}$,

(d) find the value of λ. **(2)**

Question Number	Scheme	Notes	Marks
7. (a)	$\arg z = -\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$	$\tan^{-1}\left(\pm\frac{\sqrt{3}}{2}\right)$ or $\tan^{-1}\left(\pm\frac{2}{\sqrt{3}}\right)$ seen or evaluated	M1
	Awrt ±0.71 or awrt ±0.86 can be taken as evidence for the method mark. Or ±40.89 or ±49.10 if working in degrees		
	$= -0.7137243789.. = -0.71$ (2 dp)	awrt -0.71 or awrt 5.57	A1
	NB $\tan\left(\frac{\sqrt{3}}{2}\right) = 1.18$ and $\tan\left(\frac{2}{\sqrt{3}}\right) = 2.26$ **and both score M0**		
			[2]
(b)	$z^2 = \left(2 - \mathrm{i}\sqrt{3}\right)\left(2 - \mathrm{i}\sqrt{3}\right)$ $= 4 - 2\mathrm{i}\sqrt{3} - 2\mathrm{i}\sqrt{3} + 3\mathrm{i}^2$	An attempt to multiply out the brackets to give four terms (or four terms implied).	M1
	$= 2 - \mathrm{i}\sqrt{3} + \left(4 - 4\mathrm{i}\sqrt{3} - 3\right)$ $= 2 - \mathrm{i}\sqrt{3} + \left(1 - 4\mathrm{i}\sqrt{3}\right)$ $= 3 - 5\mathrm{i}\sqrt{3}$ (Note: $a = 3, b = -5$.)	M1: An understanding that $\mathrm{i}^2 = -1$ and an attempt to add z and put in the form $a + b\mathrm{i}\sqrt{3}$ A1: $3 - 5\mathrm{i}\sqrt{3}$	M1A1
	$z + z^2 = 2 - \mathrm{i}\sqrt{3} + \left(4 - 4\mathrm{i}\sqrt{3} + 3\right) = 9 - 5\mathrm{i}\sqrt{3}$ scores M1M0A0 (No evidence of $\mathrm{i}^2 = -1$)		
			[3]
(c)	$\frac{z+7}{z-1} = \frac{2 - \mathrm{i}\sqrt{3} + 7}{2 - \mathrm{i}\sqrt{3} - 1}$	Substitutes $z = 2 - \mathrm{i}\sqrt{3}$ into both numerator and denominator.	M1
	$= \frac{\left(9 - \mathrm{i}\sqrt{3}\right)}{\left(1 - \mathrm{i}\sqrt{3}\right)} \times \frac{\left(1 + \mathrm{i}\sqrt{3}\right)}{\left(1 + \mathrm{i}\sqrt{3}\right)}$	Simplifies $\frac{z+7}{z-1}$ **and** multiplies by $\frac{\textbf{their } \left(1 + \mathrm{i}\sqrt{3}\right)}{\textbf{their } \left(1 + \mathrm{i}\sqrt{3}\right)}$	**d**M1
	$= \frac{9 + 9\mathrm{i}\sqrt{3} - \mathrm{i}\sqrt{3} + 3}{1 + 3}$ $= \frac{12 + 8\mathrm{i}\sqrt{3}}{4}$	Simplifies realising that a real number is needed in the denominator and applies $\mathrm{i}^2 = -1$ in their numerator expression and denominator expression.	M1
	$= 3 + 2\mathrm{i}\sqrt{3}$ (Note: $c = 3, d = 2$.)	$3 + 2\mathrm{i}\sqrt{3}$	A1
			[4]
(d)	$w = \lambda - 3\mathrm{i}$, and $\arg(4 - 5\mathrm{i} + 3w) = -\frac{\pi}{2}$		
	$\left(4 - 5i + 3w = 4 + 3\lambda - 14\mathrm{i}\right)$		
	So real part of $(4 - 5\mathrm{i} + 3w) = 0$ or $4 + 3\lambda = 0$	States real part of $(4 - 5\mathrm{i} + 3w) = 0$ or $4 + 3\lambda = 0$	M1
	So, $\lambda = -\frac{4}{3}$	$-\frac{4}{3}$	A1
			[2]
	Allow $\pm\left(\frac{14}{3\lambda + 4}\right) = \pm\infty \Rightarrow 3\lambda + 4 = 0$ M1 $\Rightarrow \lambda = -\frac{4}{3}$ A1		
			11 marks

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8. The rectangular hyperbola H has equation $xy = c^2$, where c is a positive constant.

The point $P\left(ct, \frac{c}{t}\right)$, $t \neq 0$, is a general point on H.

(a) Show that an equation for the tangent to H at P is

$$x + t^2y = 2ct$$

(4)

The tangent to H at the point P meets the x-axis at the point A and the y-axis at the point B.

Given that the area of the triangle OAB, where O is the origin, is 36,

(b) find the exact value of c, expressing your answer in the form $k\sqrt{2}$, where k is an integer.

(4)

Question Number	Scheme	Notes	Marks
8.	$xy = c^2$ at $\left(ct, \frac{c}{t}\right)$.		
(a)	$y = \frac{c^2}{x} = c^2x^{-1} \Rightarrow \frac{dy}{dx} = -c^2x^{-2} = -\frac{c^2}{x^2}$	$\frac{dy}{dx} = k\,x^{-2}$	
	$xy = c^2 \Rightarrow x\frac{dy}{dx} + y = 0$	Correct use of product rule. The sum of two terms, one of which is correct **and** rhs = 0	M1
	$\frac{dy}{dx} = \frac{dy}{dt}.\frac{dt}{dx} = -\frac{c}{t^2}.\frac{1}{c}$	their $\frac{dy}{dt} \times \left(\frac{1}{\text{their } \frac{dx}{dt}}\right)$	
	$\frac{dy}{dx} = -c^2x^{-2}$ *or* $x\frac{dy}{dx} + y = 0$ *or* $\frac{dy}{dx} = \frac{-c}{t^2}.\frac{1}{c}$ or equivalent expressions	Correct differentiation	A1
	So, $m_T = \frac{dy}{dx} = -\frac{1}{t^2}$	$-\frac{1}{t^2}$	
	$y - \frac{c}{t} = -\frac{1}{t^2}(x - ct)$ $(\times t^2)$	$y - \frac{c}{t} =$ their $m_T(x - ct)$ or $y = mx + c$ with their m_T and $(ct, \frac{c}{t})$ in an attempt to find 'c'. **Their m_T must have come from calculus and should be a function of t or c or both c and t.**	M1
	$x + t^2y = 2ct$ (Allow $t^2y + x = 2ct$)	Correct solution.	A1 *
	(a) Candidates who derive $x + t^2y = 2ct$, by stating that $m_T = -\frac{1}{t^2}$, with no justification score <u>no</u> marks in (a).		
			[4]
(b)	$y = 0 \Rightarrow x = 2ct \Rightarrow A(2ct, 0)$.	$x = 2ct$, seen or implied.	B1
	$x = 0 \Rightarrow y = \frac{2ct}{t^2} \Rightarrow B\left(0, \frac{2c}{t}\right)$.	$y = \frac{2ct}{t^2}$ or $\frac{2c}{t}$, seen or implied.	B1
	Area $OAB = 36 \Rightarrow \frac{1}{2}(2ct)\left(\frac{2c}{t}\right) = 36$	Applies $\frac{1}{2}$(their x)(their y) = 36 where x and y are functions of c or t or both (not x or y) and some attempt was made to substitute both $x = 0$ **and** $y = 0$ in the tangent to find A and B.	M1
	Do not allow the x and y coordinates of P to be used for the dimensions of the triangle.		
	$\Rightarrow 2c^2 = 36 \Rightarrow c^2 = 18 \Rightarrow c = 3\sqrt{2}$	$c = 3\sqrt{2}$	A1
		Do <u>not</u> allow $c = \pm 3\sqrt{2}$	**[4]**
			8 marks

Leave blank

9.

$$\mathbf{M} = \begin{pmatrix} 3 & 4 \\ 2 & -5 \end{pmatrix}$$

(a) Find det **M**.

(1)

The transformation represented by **M** maps the point $S(2a - 7, a - 1)$, where a is a constant, onto the point $S'(25, -14)$.

(b) Find the value of a.

(3)

The point R has coordinates (6, 0).

Given that O is the origin,

(c) find the area of triangle ORS.

(2)

Triangle ORS is mapped onto triangle $OR'S'$ by the transformation represented by **M**.

(d) Find the area of triangle $OR'S'$.

(2)

Given that

$$\mathbf{A} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

(e) describe fully the single geometrical transformation represented by **A**.

(2)

The transformation represented by **A** followed by the transformation represented by **B** is equivalent to the transformation represented by **M**.

(f) Find **B**.

(4)

Question Number	Scheme	Notes	Marks
9. (a)	$\det \mathbf{M} = 3(-5) - (4)(2) = -15 - 8 = \underline{-23}$	$\underline{-23}$	B1
			[1]
(b)	Therefore, $\begin{pmatrix} 3 & 4 \\ 2 & -5 \end{pmatrix}\begin{pmatrix} 2a-7 \\ a-1 \end{pmatrix} = \begin{pmatrix} 25 \\ -14 \end{pmatrix}$	Using the information in the question to form the matrix equation. Can be implied by any of the correct equations below.	M1
	Either, $3(2a-7) + 4(a-1) = 25$ or $2(2a-7) - 5(a-1) = -14$ or $\begin{pmatrix} 3(2a-7)+4(a-1) \\ 2(2a-7)-5(a-1) \end{pmatrix} = \begin{pmatrix} 25 \\ -14 \end{pmatrix}$	Any one correct equation (unsimplified) inside or outside matrices	A1
	giving $a = 5$	$a = 5$	A1
			[3]
(c)	$\text{Area}(ORS) = \frac{1}{2}(6)(4); = \underline{12}$ (units)^2	M1: $\frac{1}{2}(6)(\text{Their } a - 1)$ A1: 12 **cao and cso**	M1A1
	Note A(6, 0) is sometimes misinterpreted as (0, 6) – this is the wrong triangle and scores M0 e.g.1/2x6x3 = 9		
			[2]
(d)	$\text{Area}(OR'S') = \pm 23 \times (12)$	$\pm \det \mathbf{M} \times$ (their part (c) answer)	M1
		$\underline{276}$ (follow through provided area > 0)	A1 $\sqrt{}$
	A method not involving the determinant requires the coordinates of R$'$ to be calculated ((18, 12)) and then a <u>correct</u> method for the area e.g. (26x25 – 7x13 – 9x12 – 7x25) M1 = 276 A1		
			[2]
(e)	Rotation; 90° anti-clockwise (or 270° clockwise) about $(0, 0)$.	B1: Rotation, Rotates, Rotate, Rotating (not **turn**) B1: 90° anti-clockwise (or 270° clockwise) about (around/from etc.) $(0, 0)$	B1;B1
			[2]
(f)	$\mathbf{M} = \mathbf{BA}$	$\mathbf{M} = \mathbf{BA}$, seen or implied.	M1
	$\mathbf{A}^{-1} = \frac{1}{(0)(0)-(1)(-1)}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}; = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$	$\mathbf{A}^{-1} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$	A1
	$\mathbf{B} = \begin{pmatrix} 3 & 4 \\ 2 & -5 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$	Applies $\mathbf{M}(\text{their } \mathbf{A}^{-1})$	M1
	$\mathbf{B} = \begin{pmatrix} -4 & 3 \\ 5 & 2 \end{pmatrix}$		A1
	NB some candidates state **M** = **AB** and then calculate **MA**$^{-1}$ or state **M** = **BA** and then calculate **A**$^{-1}$**M**. These could score M0A0 M1A1ft and M1A1M0A0 respectively.		**[4]**
			14 marks
	Special case		
(f)	$\mathbf{M} = \mathbf{AB}$	$\mathbf{M} = \mathbf{AB}$, seen or implied.	M0
		$\mathbf{A}^{-1} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$	A0
	$\mathbf{B} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 3 & 4 \\ 2 & -5 \end{pmatrix} = \begin{pmatrix} 2 & -5 \\ -3 & -4 \end{pmatrix}$	Applies (their $\mathbf{A}^{-1}$)$\mathbf{M}$	M1A1ft

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10. Prove by induction that, for $n \in \mathbb{Z}^+$,

$$f(n) = 2^{2n-1} + 3^{2n-1} \text{ is divisible by 5.}$$

(6)

Question Number	Scheme	Notes	Marks
10.	$f(n) = 2^{2n-1} + 3^{2n-1}$ is divisible by 5.		
	$f(1) = 2^1 + 3^1 = 5,$	Shows that $f(1) = 5$.	B1
	Assume that for $n = k$, $f(k) = 2^{2k-1} + 3^{2k-1}$ is divisible by 5 for $k \in ¢^{+}$.		
	$f(k+1) - f(k) = 2^{2(k+1)-1} + 3^{2(k+1)-1} - \left(2^{2k-1} + 3^{2k-1}\right)$	M1: Attempts $f(k+1) - f(k)$. A1: Correct expression for <u>f(k + 1)</u> (Can be unsimplified)	M1A1
	$= 2^{2k+1} + 3^{2k+1} - 2^{2k-1} - 3^{2k-1}$		
	$= 2^{2k-1+2} + 3^{2k-1+2} - 2^{2k-1} - 3^{2k-1}$		
	$= 4\left(2^{2k-1}\right) + 9\left(3^{2k-1}\right) - 2^{2k-1} - 3^{2k-1}$	Achieves an expression in 2^{2k-1} and 3^{2k-1}	M1
	$= 3\left(2^{2k-1}\right) + 8\left(3^{2k-1}\right)$		
	$= 3\left(2^{2k-1}\right) + 3\left(3^{2k-1}\right) + 5\left(3^{2k-1}\right)$		
	$= 3f(k) + 5\left(3^{2k-1}\right)$		
	$\therefore f(k+1) = 4f(k) + 5\left(3^{2k-1}\right)$ or $4(2^{2k-1} + 3^{2k-1}) + 5\left(3^{2k-1}\right)$	Where f(k + 1) is correct and is clearly a multiple of 5.	A1
	If the result is true for $n = k$, then it is now true for $n = k+1$. As the result has shown to be true for $n = 1$, then the result is true for all *n*.	Correct conclusion **at the end,** at least as given, and all previous marks scored.	A1 cso
			[6]
			6 marks
	All methods should complete to f(k + 1) = ... where f(k +1) is clearly shown to be divisible by 5 to enable the final 2 marks to be available.		
Note that there are many different ways of proving this result by induction.			