Centre No.						Paper Reference							Surname	Initial(s)
Candidate No.						6	6	6	7	/	0	1	Signature	

Paper Reference(s)

6667/01

Edexcel GCE

Further Pure Mathematics FP1

Advanced/Advanced Subsidiary

Monday 10 June 2013 – Morning

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
1	
2	
3	
4	
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7	
8	
9	
Total	

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer to each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 9 questions in this question paper. The total mark for this paper is 75.
There are 32 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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1.

$$\mathbf{M} = \begin{pmatrix} x & x-2 \\ 3x-6 & 4x-11 \end{pmatrix}$$

Given that the matrix **M** is singular, find the possible values of x.

(4)

Question Number	Scheme	Notes	Marks
1.	$\mathbf{M}=\begin{pmatrix} x & x-2 \\ 3x-6 & 4x-11 \end{pmatrix}$		
	$\det\mathbf{M} = x(4x-11)-(3x-6)(x-2)$	Correct attempt at determinant	M1
	$x^2+x-12\ (=0)$	Correct 3 term quadratic	A1
	$(x+4)(x-3)\ (=0) \rightarrow x = \ldots$	Their 3TQ = 0 and attempts to solve relevant quadratic using factorisation or completing the square or correct quadratic formula leading to x =	M1
	$x=-4,\ x=3$	Both values correct	A1
			(4)
			Total 4
Notes			
	$x(4x-11)=(3x-6)(x-2)$ award first M1		
	$\pm(x^2+x-12)$ seen award first M1A1		
	Method mark for solving 3 term quadratic: 1. <u>Factorisation</u> $(x^2+bx+c)=(x+p)(x+q)$, where $\lvert pq\rvert=\lvert c\rvert$, leading to x = $(ax^2+bx+c)=(mx+p)(nx+q)$, where $\lvert pq\rvert=\lvert c\rvert$ and $\lvert mn\rvert=\lvert a\rvert$, leading to x = 2. <u>Formula</u> Attempt to use <u>correct</u> formula (with values for *a*, *b* and *c*). 3. <u>Completing the square</u> Solving $x^2+bx+c=0$: $\left(x\pm\frac{b}{2}\right)^2 \pm q\pm c,\quad q\neq 0,$ leading to x =...		
	Both correct with no working 4/4, only one correct 0/4		

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2. $$f(x) = \cos(x^2) - x + 3, \qquad 0 < x < \pi$$

(a) Show that the equation $f(x) = 0$ has a root α in the interval [2.5, 3].

(2)

(b) Use linear interpolation once on the interval [2.5, 3] to find an approximation for α, giving your answer to 2 decimal places.

(3)

Question Number	Scheme	Notes	Marks
2	$\text{f}(x)=\cos\left(x^2\right)-x+3$		
(a)	f(2.5) = 1.499..... f(3) = -0.9111.....	Either any one of f(2.5) = awrt 1.5 or f(3) = awrt -0.91	M1
	Sign change (positive, negative) (and $\text{f}(x)$ is continuous) therefore root or equivalent.	Both f(2.5) = awrt 1.5 and f(3) = awrt -0.91, sign change and conclusion.	A1
	Use of degrees gives f(2.5) = 1.494 and f(3) = 0.988 which is awarded M1A0		**(2)**
(b)	$\dfrac{3-\alpha}{"0.91113026188"}=\dfrac{\alpha-2.5}{"1.4994494182"}$	Correct linear interpolation method – accept equivalent equation - ensure signs are correct.	M1 A1ft
	$\alpha=\dfrac{3\times1.499...+2.5\times0.9111....}{1.499...+0.9111....}$		
	$\alpha=2.81\,(\text{2d.p.})$	cao	A1
			(3)
			Total 5
Notes	Alternative (b)		
	Gradient of line is $-\dfrac{'1.499...'+'0.9111...'}{0.5}(=-4.82)\ (3\text{sf})$. Attempt to find equation of straight line and equate y to 0 award M1 and A1ft for their gradient awrt 3sf.		

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3. Given that $x = \frac{1}{2}$ is a root of the equation

$$2x^3 - 9x^2 + kx - 13 = 0, \qquad k \in \mathbb{R}$$

find

(a) the value of k,

(3)

(b) the other 2 roots of the equation.

(4)

Question Number	Scheme	Notes	Marks
3(a)	**Ignore part labels and mark part (a) and part (b) together.**		
	$f\left(\frac{1}{2}\right)=2\left(\frac{1}{2}\right)^3-9\left(\frac{1}{2}\right)^2+k\left(\frac{1}{2}\right)-13$	Attempts f(0.5)	M1
	$\left(\frac{1}{4}\right)-\left(\frac{9}{4}\right)+\left(\frac{k}{2}\right)-13=0\Rightarrow k=......$	Sets f(0.5) = 0 **and** leading to k=	dM1
	k = 30	cao	A1
	Alternative using long division:		
	$2x^3-9x^2+kx-13\div(2x-1)$ $=x^2-4x+\frac{1}{2}k-2\,(\text{Quotient})$ $\text{Remainder}\ \frac{1}{2}k-15$	Full method to obtain a remainder as a function of k	M1
	$\frac{1}{2}k-15=0$	Their remainder = 0	dM1
	$k=30$		A1
	Alternative by inspection:		
	$(2x-1)(x^2-4x+13)=2x^3-9x^2+30x-13$	First M for $(2x-1)(x^2+bx+c)$ or $(x-\frac{1}{2})(2x^2+bx+c)$ Second M1 for ax^2+bx+c where ($b=-4$ or $c=13$) or ($b=-8$ or $c=26$)	M1dM1
	k = 30		A1
			(3)
(b)	$f(x)=(2x-1)(x^2-4x+13)$ *or* $\left(x-\frac{1}{2}\right)(2x^2-8x+26)$	M1: $(x^2+bx\pm13)$ or $(2x^2+bx\pm26)$ Uses inspection or long division or compares coefficients **and** $(2x-1)$ or $\left(x-\frac{1}{2}\right)$ to obtain a quadratic factor of this form.	M1
	$x^2-4x+13$ or $2x^2-8x+26$	A1 $(x^2-4x+13)$ or $(2x^2-8x+26)$ seen	A1
	$x=\frac{4\pm\sqrt{4^2-4\times13}}{2}$ or equivalent	Use of correct quadratic formula for their <u>3TQ</u> or completes the square.	M1
	$x=\frac{4\pm6i}{2}=2\pm3i$	oe	A1
			(4)
			Total 7

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4. The rectangular hyperbola H has Cartesian equation $xy = 4$

The point $P\left(2t, \frac{2}{t}\right)$ lies on H, where $t \neq 0$

(a) Show that an equation of the normal to H at the point P is

$$ty - t^3x = 2 - 2t^4$$

(5)

The normal to H at the point where $t = -\frac{1}{2}$ meets H again at the point Q.

(b) Find the coordinates of the point Q.

(4)

Question Number	Scheme	Notes	Marks
4(a)	$y = \frac{4}{x} = 4x^{-1} \Rightarrow \frac{dy}{dx} = -4x^{-2} = -\frac{4}{x^2}$	$\frac{dy}{dx} = k\,x^{-2}$	
	$xy = 4 \Rightarrow x\frac{dy}{dx} + y = 0$	Use of the product rule. The sum of two terms including dy/dx, one of which is correct.	M1
	$\frac{dy}{dx} = \frac{dy}{dt}.\frac{dt}{dx} = -\frac{2}{t^2}.\frac{1}{2}$	their $\frac{dy}{dt} \times \left(\frac{1}{\text{their } \frac{dx}{dt}}\right)$	
	$\frac{dy}{dx} = -4x^{-2}$ *or* $x\frac{dy}{dx} + y = 0$ or $\frac{dy}{dx} = -\frac{2}{t^2}.\frac{1}{2}$ or equivalent expressions	Correct derivative $-4x^{-2}$, $-\frac{y}{x}$ or $\frac{-1}{t^2}$	A1
	So, $m_N = t^2$	Perpendicular gradient rule $m_N m_T = -1$	M1
	$y - \frac{2}{t} = t^2(x - 2t)$	$y - \frac{2}{t} =$ their $m_N(x - 2t)$ or $y = mx + c$ with their m_N and $(2t, \frac{2}{t})$ in an attempt to find 'c'. **Their gradient of the normal must be different from their gradient of the tangent and have come from calculus and should be a function of *t*.**	M1
	$ty - t^3x = 2 - 2t^4$ *		A1* cso
			(5)
(b)	$t = -\frac{1}{2} \Rightarrow -\frac{1}{2}y - \left(-\frac{1}{2}\right)^3 x = 2 - 2\left(-\frac{1}{2}\right)^4$	Substitutes the given value of t into the normal	M1
	$4y - x + 15 = 0$		
	$y = \frac{4}{x} \Rightarrow x^2 - 15x - 16 = 0$ or $\left(2t, \frac{2}{t}\right) \rightarrow \frac{8}{t} - 2t + 15 = 0 \Rightarrow 2t^2 - 15t - 8 = 0$ or $x = \frac{4}{y} \Rightarrow 4y^2 + 15y - 4 = 0$.	Substitutes to give a quadratic	M1
	$(x+1)(x-16) = 0 \Rightarrow x =$ or $(2t+1)(t-8) = 0 \Rightarrow t =$ or $(4y-1)(y+4) = 0 \Rightarrow y =$	Solves their 3TQ	M1
	$(P: x = -1, y = -4)(Q:) x = 16, y = \frac{1}{4}$	Correct values for x and y	A1
			(4)
			Total 9

Leave blank

5. (a) Use the standard results for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$ to show that

$$\sum_{r=1}^{n} (r + 2)(r + 3) = \frac{1}{3}n(n^2 + 9n + 26)$$

for all positive integers n.

(6)

(b) Hence show that

$$\sum_{r=n+1}^{3n} (r + 2)(r + 3) = \frac{2}{3}n(an^2 + bn + c)$$

where a, b and c are integers to be found.

(4)

Question Number	Scheme	Notes	Marks
5(a)	$(r+2)(r+3)=r^2+5r+6$		B1
	$\sum(r^2+5r+6)=\frac{1}{6}n(n+1)(2n+1)+5\times\frac{1}{2}n(n+1),+6n$	M1: Use of correct expressions for $\sum r^2$ **and** $\sum r$ B1ft: $\sum k=nk$	M1,B1ft
	$=\frac{1}{3}n\left[\frac{1}{2}(n+1)(2n+1)+\frac{15}{2}(n+1)+18\right]$	M1:Factors out n ignoring treatment of constant. A1: Correct expression with $\frac{1}{3}n$ or $\frac{1}{6}n$ factored out, allow recovery.	M1 A1
	$\left(=\frac{1}{3}n\left[n^2+\frac{3}{2}n+\frac{1}{2}+\frac{15}{2}n+\frac{15}{2}+18\right]\right)$ $=\frac{1}{3}n\left[n^2+9n+26\right]$ *	Correct completion to printed answer	A1*cso
			(6)
5(b)	$\sum_{r=n+1}^{3n}=\frac{1}{3}3n\left((3n)^2+9(3n)+26\right)-\frac{1}{3}n\left(n^2+9n+26\right)$	M1: f(**3*n***) – f(n or n+1) and attempt to use part (a). A1: Equivalent correct expression	M1A1
	3f(***n***) – f(n or n+1) is M0		
	$(=n(9n^2+27n+26)-\frac{1}{3}n(n^2+9n+26))$		
	$=\frac{2}{3}n\left(\frac{27}{2}n^2+\frac{81}{2}n+39-\frac{1}{2}n^2-\frac{9}{2}n-13\right)$	Factors out $=\frac{2}{3}n$ dependent on previous M1	dM1
	$=\frac{2}{3}n\left(13n^2+36n+26\right)$	Accept correct expression.	A1
	$(a=13,b=36,c=26)$		
			(4)
			Total 10

Leave blank

6. A parabola C has equation $y^2 = 4ax$, $a > 0$

The points $P(ap^2, 2ap)$ and $Q(aq^2, 2aq)$ lie on C, where $p \neq 0$, $q \neq 0$, $p \neq q$.

(a) Show that an equation of the tangent to the parabola at P is

$$py - x = ap^2$$

(4)

(b) Write down the equation of the tangent at Q.

(1)

The tangent at P meets the tangent at Q at the point R.

(c) Find, in terms of p and q, the coordinates of R, giving your answers in their simplest form.

(4)

Given that R lies on the directrix of C,

(d) find the value of pq.

(2)

Question Number	Scheme	Notes	Marks
6(a)	$y = 2a^{\frac{1}{2}}x^{\frac{1}{2}} \Rightarrow \frac{dy}{dx} = a^{\frac{1}{2}}x^{-\frac{1}{2}}$	$x^{\frac{1}{2}} \to x^{-\frac{1}{2}}$	
	$y^2 = 4ax \Rightarrow 2y\frac{dy}{dx} = 4a$	$ky\frac{dy}{dx} = c$	M1
	or $\frac{dy}{dx} = \frac{dy}{dt}.\frac{dt}{dx} = 2a.\frac{1}{2ap}$	$\frac{dy}{dt} \times \frac{1}{\frac{dx}{dt}}$. Can be a function of *p or t*.	
	$\frac{dy}{dx} = a^{\frac{1}{2}}x^{-\frac{1}{2}}$ or $2y\frac{dy}{dx} = 4a$ or $\frac{dy}{dx} = 2a.\frac{1}{2ap}$	Differentiation is accurate.	A1
	$y - 2ap = \frac{1}{p}(x - ap^2)$	Applies $y - 2ap =$ their $m(x - ap^2)$ or $y = (\text{their } m)x + c$ using $x = ap^2$ *and* $y = 2ap$ in an attempt to find c. **Their *m* must be a function of *p* from calculus.**	M1
	$py - x = ap^2$ *	Correct completion to printed answer*	A1 cso
			(4)
(b)	$qy - x = aq^2$		B1
			(1)
(c)	$qy - aq^2 = py - ap^2$	Attempt to obtain an **equation** in one variable *x* or *y*	M1
	$y(q-p) = aq^2 - ap^2$ $y = \frac{aq^2 - ap^2}{q - p}$	Attempt to isolate *x* or *y*	M1
	$y = a(p+q)$ *or* $ap + aq$ $x = apq$	A1: Either one correct simplified coordinate A1: Both correct simplified coordinates	A1,A1
	$(R(apq, ap + aq))$		
			(4)
(d)	$'apq' = -a$	Their *x* coordinate of $R = -a$	M1
	$pq = -1$	**Answer only**: Scores 2/2 if *x* coordinate of *R* is *apq* otherwise 0/2.	A1
			(2)
			Total 11

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7. $z_1 = 2 + 3\text{i}, \quad z_2 = 3 + 2\text{i}, \quad z_3 = a + b\text{i}, \quad a, b \in \mathbb{R}$

(a) Find the exact value of $|z_1 + z_2|$.
(2)

Given that $w = \dfrac{z_1 z_3}{z_2}$,

(b) find w in terms of a and b, giving your answer in the form $x + \text{i}y, \quad x, y \in \mathbb{R}$
(4)

Given also that $w = \dfrac{17}{13} - \dfrac{7}{13}\text{i}$,

(c) find the value of a and the value of b,
(3)

(d) find arg w, giving your answer in radians to 3 decimal places.
(2)

Question Number	**Scheme**	**Notes**	**Marks**
7	$z_1 = 2+3\text{i}, \quad z_2 = 3+2\text{i}$		
(a)	$z_1 + z_2 = 5+5\text{i} \Rightarrow \lvert z_1 + z_2 \rvert = \sqrt{5^2+5^2}$	Adds z_1 and z_2 and correct use of Pythagoras. i under square root award M0.	M1
	$\sqrt{50}\ (= 5\sqrt{2})$		A1 cao
			(2)
(b)	$\frac{z_1 z_3}{z_2} = \frac{(2+3\text{i})(a+b\text{i})}{3+2\text{i}}$ $= \frac{(2+3\text{i})(a+b\text{i})(3-2i)}{(3+2\text{i})(3-2\text{i})}$	Substitutes for z_1, z_2 and z_3 and multiplies by $\frac{3-2\text{i}}{3-2\text{i}}$	M1
	$(3+2\text{i})(3-2\text{i}) = 13$	13 seen.	B1
	$\frac{z_1 z_3}{z_2} = \frac{(12a-5b)+(5a+12b)\text{i}}{13}$	M1: Obtains a numerator with 2 real and 2 imaginary parts. A1: As stated or $\frac{(12a-5b)}{13} + \frac{(5a+12b)}{13}\text{i}$ ONLY.	dM1A1
			(4)
(c)	$12a-5b=17$ $5a+12b=-7$	Compares real and imaginary parts to obtain 2 equations which both involve *a* and *b*. Condone sign errors only.	M1
	$60a-25b=85$ $60a+144b=-84$ $\Rightarrow$ $b=-1$	Solves as far as *a* = or *b* =	dM1
	a = 1, *b* = -1	Both correct	A1
		Correct answers with no working award 3/3.	
			(3)
(d)	$\arg(w) = -\tan^{-1}\left(\frac{7}{17}\right)$	Accept use of $\pm\tan^{-1}$ or $\pm\tan$. awrt ± 0.391 or ± 5.89 implies M1.	M1
	=awrt -0.391 or awrt 5.89		A1
			(2)
			Total 11

Leave blank

8.

$$\mathbf{A} = \begin{pmatrix} 6 & -2 \\ -4 & 1 \end{pmatrix}$$

and **I** is the 2 × 2 identity matrix.

(a) Prove that

$$\mathbf{A}^2 = 7\mathbf{A} + 2\mathbf{I}$$

(2)

(b) Hence show that

$$\mathbf{A}^{-1} = \frac{1}{2}(\mathbf{A} - 7\mathbf{I})$$

(2)

The transformation represented by **A** maps the point P onto the point Q.

Given that Q has coordinates $(2k + 8, -2k - 5)$, where k is a constant,

(c) find, in terms of k, the coordinates of P.

(4)

Question Number	Scheme	Notes	Marks
8(a)	$\mathbf{A}^2 = \begin{pmatrix} 6 & -2 \\ -4 & 1 \end{pmatrix}\begin{pmatrix} 6 & -2 \\ -4 & 1 \end{pmatrix} = \begin{pmatrix} 44 & -14 \\ -28 & 9 \end{pmatrix}$	M1:Attempt both $\mathbf{A}^2$ and $7\mathbf{A} + 2\mathbf{I}$	M1A1
	$7\mathbf{A} + 2\mathbf{I} = \begin{pmatrix} 42 & -14 \\ -28 & 7 \end{pmatrix} + \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} 44 & -14 \\ -28 & 9 \end{pmatrix}$	A1: Both matrices correct	
	OR $\mathbf{A}^2 - 7\mathbf{A} = \mathbf{A}(\mathbf{A} - 7\mathbf{I})$	M1 for expression and attempt to substitute and multiply (2x2)(2x2)=2x2	
	$\mathbf{A}(\mathbf{A} - 7\mathbf{I}) = \begin{pmatrix} 6 & -2 \\ -4 & 1 \end{pmatrix}\begin{pmatrix} -1 & -2 \\ -4 & -6 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = 2\mathbf{I}$	A1 cso	
			(2)
(b)	$\mathbf{A}^2 = 7\mathbf{A} + 2\mathbf{I} \Rightarrow \mathbf{A} = 7\mathbf{I} + 2\mathbf{A}^{-1}$	Require one correct line using accurate expressions involving $\mathbf{A}^{-1}$ and identity matrix to be clearly stated as $\mathbf{I}$.	M1
	$\mathbf{A}^{-1} = \frac{1}{2}(\mathbf{A} - 7\mathbf{I})$*		A1* cso
	Numerical approach award 0/2.		
			(2)
(c)	$\mathbf{A}^{-1} = \frac{1}{2}\begin{pmatrix} -1 & -2 \\ -4 & -6 \end{pmatrix}$	Correct inverse matrix or equivalent	B1
	$\frac{1}{2}\begin{pmatrix} -1 & -2 \\ -4 & -6 \end{pmatrix}\begin{pmatrix} 2k+8 \\ -2k-5 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} -2k-8+4k+10 \\ -8k-32+12k+30 \end{pmatrix}$	Matrix multiplication involving their inverse and k: (2x2)(2x1)=2x1. N.B. $\begin{pmatrix} 6 & -2 \\ -4 & 1 \end{pmatrix}\begin{pmatrix} 2k+8 \\ -2k-5 \end{pmatrix}$ is M0	M1
	$\begin{pmatrix} k+1 \\ 2k-1 \end{pmatrix}$ or $(k+1,\ 2k-1)$	$(k+1)$ first A1, $(2k-1)$ second A1	A1,A1
	Or:		
	$\begin{pmatrix} 6 & -2 \\ -4 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2k+8 \\ -2k-5 \end{pmatrix}$	Correct matrix equation.	B1
	$6x - 2y = 2k + 8$ $-4x + y = -2k - 5 \Rightarrow x = \ldots or\ y = \ldots$	Multiply out and attempt to solve simultaneous equations for x or y in terms of k.	M1
	$\begin{pmatrix} k+1 \\ 2k-1 \end{pmatrix}$ or $(k+1,\ 2k-1)$	$(k+1)$ first A1, $(2k-1)$ second A1	A1,A1
			(4)
			Total 8

Leave blank

9. (a) A sequence of numbers is defined by

$$u_1 = 8$$
$$u_{n+1} = 4u_n - 9n, \quad n \geqslant 1$$

Prove by induction that, for $n \in \mathbb{Z}^+$,

$$u_n = 4^n + 3n + 1$$

(5)

(b) Prove by induction that, for $m \in \mathbb{Z}^+$,

$$\begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix}^m = \begin{pmatrix} 2m+1 & -4m \\ m & 1-2m \end{pmatrix}$$

(5)

Question Number	Scheme	Notes	Marks
9(a)	$u_1 = 8$ given $n=1 \Rightarrow u_1 = 4^1 + 3(1) + 1 = 8 \quad (\therefore \text{ true for } n = 1)$	$4^1 + 3(1) + 1 = 8$ seen	B1
	Assume true for $n = k$ so that $u_k = 4^k + 3k + 1$		
	$u_{k+1} = 4\left(4^k + 3k + 1\right) - 9k$	Substitute u_k into u_{k+1} as $u_{k+1} = 4u_k - 9k$	M1
	$= 4^{k+1} + 12k + 4 - 9k = 4^{k+1} + 3k + 4$	Expression of the form $4^{k+1} + ak + b$	A1
	$= 4^{k+1} + 3(k+1) + 1$	Correct completion to an expression in terms of $k + 1$	A1
	If <u>true for $n = k$</u> then <u>true for $n = k + 1$</u> and as <u>true for $n = 1$</u> <u>true for all n</u>	Conclusion with all 4 underlined elements that can be seen anywhere in the solution; n defined incorrectly award A0.	A1 cso
			(5)
(b)	**Condone use of *n* here.**		
	$lhs = \begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix}^1 = \begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix}$ $rhs = \begin{pmatrix} 2(1)+1 & -4(1) \\ 1 & 1-2(1) \end{pmatrix} = \begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix}$	Shows true for $m = 1$	B1
	Assume $\begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix}^k = \begin{pmatrix} 2k+1 & -4k \\ k & 1-2k \end{pmatrix}$		
	$\begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix}^{k+1} = \begin{pmatrix} 2k+1 & -4k \\ k & 1-2k \end{pmatrix}\begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix}$	$\begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 2k+1 & -4k \\ k & 1-2k \end{pmatrix}$ award M1	M1
	$= \begin{pmatrix} 6k+3-4k & -8k-4+4k \\ 3k+1-2k & -4k-1+2k \end{pmatrix}$	Or equivalent 2x2 matrix. $\begin{pmatrix} 6k+3-4k & -12k-4+8k \\ 2k+1-k & -4k-1+2k \end{pmatrix}$ award A1from above.	A1
	$= \left(\begin{pmatrix} 2k+3 & -4k-4 \\ k+1 & -2k-1 \end{pmatrix}\right)$		
	$= \begin{pmatrix} 2(k+1)+1 & -4(k+1) \\ k+1 & 1-2(k+1) \end{pmatrix}$	Correct completion to a matrix in terms of $k + 1$	A1
	If <u>true for $m = k$</u> then <u>true for $m = k + 1$</u> and as <u>true for $m = 1$</u> true <u>for all m</u>	Conclusion with all 4 underlined elements that can be seen anywhere in the solution; m defined incorrectly award A0.	A1 cso
			(5)
			Total 10