Centre No.						Paper Reference	Surname	Initial(s)
Candidate No.						6 6 6 7 / 0 1	Signature	

Paper Reference(s)

6667/01

Edexcel GCE

Further Pure Mathematics FP1

Advanced/Advanced Subsidiary

Tuesday 10 June 2014 – Morning

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
1	
2	
3	
4	
5	
6	
7	
8	
9	
Total	

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer to each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 9 questions in this question paper. The total mark for this paper is 75.
There are 28 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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1. The complex numbers z_1 and z_2 are given by

$$z_1 = p + 2\text{i} \text{ and } z_2 = 1 - 2\text{i}$$

where p is an integer.

(a) Find $\frac{z_1}{z_2}$ in the form $a + b\text{i}$ where a and b are real. Give your answer in its simplest form in terms of p.

(4)

Given that $\left|\frac{z_1}{z_2}\right| = 13$,

(b) find the possible values of p.

(4)

Question Number	Scheme	Notes	Marks
1.(a)	$\frac{z_1}{z_2}=\frac{p+2i}{1-2i}.\frac{1+2i}{1+2i}$	Multiplying top and bottom by conjugate	M1
	$=\frac{p+2pi+2i-4}{5}$	At least 3 correct terms in the numerator, evidence that $i^2=-1$ and denominator real.	M1
	$=\frac{p-4}{5}, \quad +\frac{2p+2}{5}\mathrm{i}$	Real + imaginary with i factored out. Accept single denominator with numerator in correct form. Accept 'a= ' and 'b='.	A1, A1
			(4)
(b)	$\left\|\frac{z_1}{z_2}\right\|^2=\left(\frac{p-4}{5}\right)^2+\left(\frac{2p+2}{5}\right)^2$.	Accept their answers to part (a). Any erroneous i or i^2 award M0	M1
	$\left(\frac{p-4}{5}\right)^2+\left(\frac{2p+2}{5}\right)^2=13^2$ or $\sqrt{\left(\frac{p-4}{5}\right)^2+\left(\frac{2p+2}{5}\right)^2}=13$	$\left\|\frac{z_1}{z_2}\right\|^2=13^2$ or $\left\|\frac{z_1}{z_2}\right\|=13$	dM1
	$\frac{p^2-8p+16}{25}+\frac{4p^2+8p+4}{25}=169 \text{ or } 13^2$		
	$5p^2+20=4225$		
	$p^2=841\Rightarrow p=\pm 29$	dM1:Attempt to solve their quadratic in *p*, dependent on both previous Ms. A1:**both** 29 **and** -29	dM1A1
	OR		
	$\frac{\|z_1\|}{\|z_2\|}=\frac{\sqrt{p^2+4}}{\sqrt{5}}$	Finding moduli Any erroneous i or i^2 award M0	M1
	$\frac{\sqrt{p^2+4}}{\sqrt{5}}=13$ oe	Equating to 13	dM1
	$\frac{p^2+4}{5}=169 \text{ or } 13^2$ oe		
	$p^2=841\Rightarrow p=\pm 29$	dM1:Attempt to solve their quadratic in *p*, dependent on both previous Ms	dM1A1
		A1:**both** 29 **and** -29	
			(4)
			Total 8

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2.

$$f(x) = x^3 - \frac{5}{2x^{\frac{3}{2}}} + 2x - 3, \quad x > 0$$

(a) Show that the equation f(x) = 0 has a root α in the interval [1.1, 1.5].

(2)

(b) Find f′(x).

(2)

(c) Using $x_0 = 1.1$ as a first approximation to α, apply the Newton-Raphson procedure once to f(x) to find a second approximation to α, giving your answer to 3 decimal places.

(3)

Question Number	Scheme	Notes	Marks
2.	$\mathrm{f}(x) = x^3 - \dfrac{5}{2x^{\frac{3}{2}}} + 2x - 3$		
(a)	$\mathrm{f}(1.1) = -1.6359604,$ $\mathrm{f}(1.5) = 2.0141723$	Attempts to evaluate both $\mathrm{f}(1.1)$ and $\mathrm{f}(1.5)$ and evaluates at least one of them correctly to awrt (or trunc.) 2 sf.	M1
	Sign change (and $\mathrm{f}(x)$ is continuous) therefore **a root / α** is between $x = 1.1$ and $x = 1.5$	Both values correct to awrt (or trunc.) 2 sf, sign change (or a statement which implies this e.g. -1.63.. < 0 < 2.014..) and conclusion.	A1
			(2)
(b)	$\mathrm{f}(x) = x^3 - \frac{5}{2}x^{-\frac{3}{2}} + 2x - 3$ $\Rightarrow \mathrm{f}'(x) = 3x^2 + \frac{15}{4}x^{-\frac{5}{2}} + 2$	M1: $x^n \to x^{n-1}$ for at least one term A1:Correct derivative oe	M1A1
			(2)
(c)	$\mathrm{f}'(1.1) = 3(1.1)^2 + \frac{15}{4}(1.1)^{-\frac{5}{2}} + 2 (= 8.585)$	Attempt to find $\mathrm{f}'(1.1)$. Accept $\mathrm{f}'(1.1)$ seen and their value.	M1
	$\alpha_2 = 1.1 - \left(\dfrac{\text{"}-1.6359604\text{"}}{\text{"}8.585\text{"}}\right)$	Correct application of N-R	M1
	$\alpha_2 = 1.291$	cao	A1
			(3)
			Total 7

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3. Given that 2 and $1 - 5\mathrm{i}$ are roots of the equation

$$x^3 + px^2 + 30x + q = 0, \qquad p, q \in \mathbb{R}$$

(a) write down the third root of the equation.

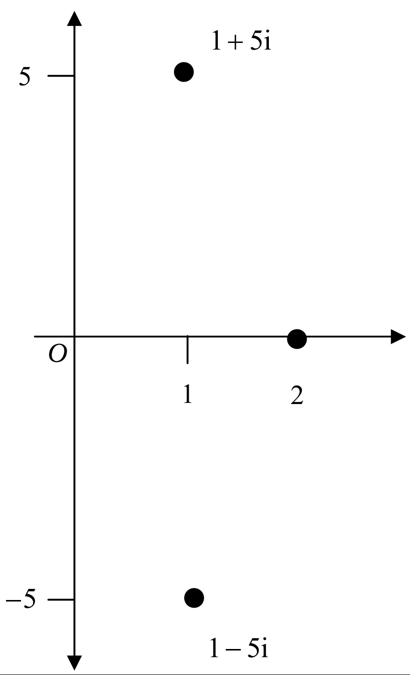
(1)

(b) Find the value of p and the value of q.

(5)

(c) Show the three roots of this equation on a single Argand diagram.

(2)

Question Number	Scheme	Notes	Marks
3.	$x^3 + px^2 + 30x + q = 0$		
(a)	$1+5i$		B1
			(1)
(b)	$((x-(1+5i))(x-(1-5i))) = x^2 - 2x + 26$ $((x-2)(x-(1\pm 5i))) = x^2 - (3\pm 5i)x + 2(1\pm 5i)$	M1: 1. Attempt to expand or 2. Use sum and product of the complex roots. A1: Correct expression	M1A1
	$(x^2 - 2x + 26)(x-2) = x^3 + px^2 + 30x + q$	Uses their third factor with their quadratic with at least 4 terms in the expansion	M1
	$p = -4, \quad q = -52$	May be seen in cubic	A1, A1
OR	f(1+5i)=0 or f(1-5i)=0	Substitute one complex root to achieve 2 equations in *p* and / or q	M1
	$q - 24p - 44 = 0$ and $10p + 40 = 0$	Both equations correct oe	A1
		Solving for *p* and *q*	M1
	$p = -4, \; q = -52$	May be seen in cubic	A1, A1
			(5)
(c)		B1: Conjugate pair correctly positioned and labelled with 1+5i, 1-5i or (1,5),(1,-5) or axes labelled 1 and 5. B1: The 2 correctly positioned relative to conjugate pair and labelled.	B1 B1
			(2) **Total 8**

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4. (i) Given that

$$\mathbf{A} = \begin{pmatrix} 1 & 2 \\ 3 & -1 \\ 4 & 5 \end{pmatrix} \text{ and } \mathbf{B} = \begin{pmatrix} 2 & -1 & 4 \\ 1 & 3 & 1 \end{pmatrix},$$

(a) find **AB**.

(b) Explain why $\mathbf{AB} \neq \mathbf{BA}$.

(4)

(ii) Given that

$$\mathbf{C} = \begin{pmatrix} 2k & -2 \\ 3 & k \end{pmatrix}, \text{ where } k \text{ is a real number}$$

find $\mathbf{C}^{-1}$, giving your answer in terms of k.

(3)

P 4 3 1 5 3 A 0 8 2 8

Question Number	Scheme	Notes	Marks
4.	$\mathbf{A}=\begin{pmatrix}1&2\\3&-1\\4&5\end{pmatrix}$, $\mathbf{B}=\begin{pmatrix}2&-1&4\\1&3&1\end{pmatrix}$		
(i)(a)	$\begin{pmatrix}1&2\\3&-1\\4&5\end{pmatrix}\begin{pmatrix}2&-1&4\\1&3&1\end{pmatrix}=\begin{pmatrix}4&5&6\\5&-6&11\\13&11&21\end{pmatrix}$	M1: 3x3 matrix with a number or numerical expression for each element A2:cao (-1 each error) Only 1 error award A1A0	M1A2
(b)	$\mathbf{BA}=\begin{pmatrix}2&-1&4\\1&3&1\end{pmatrix}\begin{pmatrix}1&2\\3&-1\\4&5\end{pmatrix}=\begin{pmatrix}15&25\\14&4\end{pmatrix}$	Allow any convincing argument. E.g.s BA is a 2x2 matrix (so AB ≠ BA) or dimensionally different. Attempt to evaluate product not required. NB 'Not commutative' only is B0	B1
			(4)
(ii)	$(\det\mathbf{C}=)2k\times k-3\times(-2)$	Correct attempt at determinant	M1
	$\mathbf{C}^{-1}=\frac{1}{2k^2+6}\begin{pmatrix}k&2\\-3&2k\end{pmatrix}$	M1: $\frac{1}{\text{their}\det\mathbf{C}}\begin{pmatrix}k&2\\-3&2k\end{pmatrix}$ A1:cao oe	M1A1
			(3)
			Total 7

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5. (a) Use the standard results for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$ to show that

$$\sum_{r=1}^{n} (2r-1)^2 = \frac{1}{3}n(4n^2-1)$$

(6)

(b) Hence show that

$$\sum_{r=2n+1}^{4n} (2r-1)^2 = an(bn^2-1)$$

where a and b are constants to be found.

(3)

Question Number	Scheme	Notes	Marks
5.(a)	$((2r-1)^2 =)4r^2-4r+1$		B1
	Proof by induction will usually score no more marks without use of standard results		
	$\sum_{r=1}^{n}(2r-1)^2=\sum_{r=1}^{n}\left(4r^2-4r+1\right)$		
	$=4\sum r^2-4\sum r+\sum 1$		
	$=\underline{4.\frac{1}{6}n(n+1)(2n+1)-4.\frac{1}{2}n(n+1)},+n$	M1: An attempt to use at least one of the standard results correctly in summing at least 2 terms of their expansion of $(2r-1)^2$ A1: Correct underlined expression oe B1: $\sum 1=n$	M1A1B1
	$=\frac{1}{3}n\left[4n^2+6n+2-6n-6+3\right]$	Attempt to factor out $\frac{1}{3}n$ before given answer	M1
	$=\frac{1}{3}n\left[4n^2-1\right]$	cso	A1
			(6)
(b)	$\sum_{r=2n+1}^{4n}(2r-1)^2=\mathrm{f}(4n)-\mathrm{f}(2n)\text{ or }\mathrm{f}(2n+1)$	Require some use of the result in part (a) for method.	M1
	$=\frac{1}{3}4n\left(4.(4n)^2-1\right)-\frac{1}{3}.2n\left(4.(2n)^2-1\right)$	Correct expression	A1
	$=\frac{2}{3}n\left[128n^2-2-16n^2+1\right]$		
	$=\frac{2}{3}n\left[112n^2-1\right]$	Accept $a=\frac{2}{3},b=112$	A1
			(3)
			Total 9

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6. The rectangular hyperbola H has cartesian equation $xy = c^2$.

The point $P\left(ct, \dfrac{c}{t}\right)$, $t > 0$, is a general point on H.

(a) Show that an equation of the tangent to H at the point P is

$$t^2 y + x = 2ct$$

(4)

An equation of the normal to H at the point P is $t^3 x - ty = ct^4 - c$

Given that the normal to H at P meets the x-axis at the point A and the tangent to H at P meets the x-axis at the point B,

(b) find, in terms of c and t, the coordinates of A and the coordinates of B.

(2)

Given that $c = 4$,

(c) find, in terms of t, the area of the triangle APB. Give your answer in its simplest form.

(3)

Question Number	Scheme	Notes	Marks
6.	$xy = c^2$ at $\left(ct, \frac{c}{t}\right)$.		
(a)	$y = \frac{c^2}{x} = c^2x^{-1} \Rightarrow \frac{dy}{dx} = -c^2x^{-2} = -\frac{c^2}{x^2}$	$\frac{dy}{dx} = kx^{-2}$	
	$xy = c^2 \Rightarrow x\frac{dy}{dx} + y = 0$	Correct use of product rule. The sum of two terms, one of which is correct **and** rhs = 0	M1
	$\frac{dy}{dx} = \frac{dy}{dt}.\frac{dt}{dx} = -\frac{c}{t^2}.\frac{1}{c}$	their $\frac{dy}{dt} \times \left(\frac{1}{\text{their } \frac{dx}{dt}}\right)$	
	$\frac{dy}{dx} = -c^2x^{-2}$ *or* $x\frac{dy}{dx} + y = 0$ *or* $\frac{dy}{dx} = \frac{-c}{t^2}.\frac{1}{c}$ or equivalent expressions	Correct differentiation	A1
	$y - \frac{c}{t} = -\frac{1}{t^2}(x - ct)$ $(\times t^2)$	$y - \frac{c}{t} =$ their $m_T(x - ct)$ or $y = mx + c$ with their m_T and $(ct, \frac{c}{t})$ in an attempt to find 'c'. **Their m_T must have come from calculus and should be a function of t or c or both c and t.**	dM1
	$t^2y + x = 2ct$ (Allow $x + t^2y = 2ct$)	Correct solution only.	A1*
			(4)
	(a) Candidates who derive $x + t^2y = 2ct$, by stating that $m_T = -\frac{1}{t^2}$, with no justification score <u>no</u> marks in (a).		
(b)	$y = 0 \Rightarrow x = \frac{ct^4 - c}{t^3} \Rightarrow A\left(\frac{ct^4 - c}{t^3}, 0\right)$	$\frac{ct^4 - c}{t^3}$ or equivalent form	B1
	$y = 0 \Rightarrow x = 2ct \Rightarrow B(2ct, 0)$.	$2ct$	B1
			(2)
(c)	$AB = "2ct" - "\frac{ct^4 - c}{t^3}"$ **or** $PA = ct^{-3}\sqrt{t^4 + 1}$ and $PB = ct^{-1}\sqrt{t^4 + 1}$	Attempt to subtract their x-coordinates either way around.	M1
	Area APB $= \frac{1}{2} \times$ *their AB* $\times \frac{c}{t}$	Valid complete method for the area of the triangle in terms of t or c and t.	M1
	$= \frac{1}{2}\left(2ct - \frac{ct^4 - c}{t^3}\right)\frac{c}{t} = \frac{c^2(t^4 + 1)}{2t^4}$		
	$= 8\left(1 + \frac{1}{t^4}\right)$ or $\frac{8(t^4 + 1)}{t^4}$ or $\frac{8t^4 + 8}{t^4}$ or $8 + \frac{8}{t^4}$	Use of $c = 4$ and completes to one of the given forms oe simplest form. Final answer should be positive for A mark.	A1
			(3)
			Total 9

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7. (i) In each of the following cases, find a 2×2 matrix that represents

(a) a reflection in the line $y = -x$,

(b) a rotation of 135° anticlockwise about (0, 0),

(c) a reflection in the line $y = -x$ followed by a rotation of 135° anticlockwise about (0, 0).

(4)

(ii) The triangle T has vertices at the points $(1, k)$, $(3, 0)$ and $(11, 0)$, where k is a constant.

Triangle T is transformed onto the triangle T' by the matrix

$$\begin{pmatrix} 6 & -2 \\ 1 & 2 \end{pmatrix}$$

Given that the area of triangle T' is 364 square units, find the value of k.

(6)

Question Number	Scheme	Notes	Marks
7.(i)(a)	$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$		B1
(b)	$\begin{pmatrix} -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$		B1
(c)	$\begin{pmatrix} -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$	M1: Multiplies their (b) x their (a) in the correct order A1: Correct matrix Correct matrix seen M1A1	M1A1
			(4)
(ii)	Area triangle T $= \frac{1}{2}\times(11-3)\times k = 4k$	M1: Correct method for area for *T* A1: 4k	M1A1
	$\det\begin{pmatrix} 6 & -2 \\ 1 & 2 \end{pmatrix} = 6\times 2 - 1\times(-2)(=14)$	M1: Correct method for determinant A1: 14	M1A1
	Area triangle $T = \frac{364}{\text{"}14\text{"}}(=26) \Rightarrow 4k = 26$	Uses 364 and their determinant correctly to form an equation in *k*.	M1
	$k = \frac{26}{4}\left(=\frac{13}{2}\right)$	Accept $k = \pm\frac{13}{2}$ or $k = -\frac{13}{2}$	A1
			(6)
			Total 10

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8. The points $P(4k^2, 8k)$ and $Q(k^2, 4k)$, where k is a constant, lie on the parabola C with equation $y^2 = 16x$.

The straight line l_1 passes through the points P and Q.

(a) Show that an equation of the line l_1 is given by

$$3ky - 4x = 8k^2$$

(4)

The line l_2 is perpendicular to the line l_1 and passes through the focus of the parabola C. The line l_2 meets the directrix of C at the point R.

(b) Find, in terms of k, the y coordinate of the point R.

(7)

Question Number	Scheme	Notes	Marks
8.(a)	$m=\frac{4k-8k}{k^2-4k^2}\left(=\frac{4}{3k}\right)$	Valid attempt to find gradient in terms of k	M1
	$y-8k=\frac{4}{3k}\left(x-4k^2\right)$ or $y-4k=\frac{4}{3k}\left(x-k^2\right)$ or $y=\frac{4}{3k}x+\frac{8k}{3}$	M1: Correct straight line method with their gradient in terms of k. If using $y=mx+c$ then award M provided they attempt to find c A1: Correct equation. If using $y=mx+c$, awardwhen they obtain $c=\frac{8k}{3}$ oe	M1A1
	$3ky-24k^2=4x-16k^2 \Rightarrow 3ky-4x=8k^2$ * or $3ky-12k^2=4x-4k^2 \Rightarrow 3ky-4x=8k^2$ *	Correct completion to printed answer with at least one intermediate step.	A1*
			(4)
(b)	(Focus) (4, 0)	Seen or implied as a number	B1
	(Directrix) x = -4	Seen or implied as a number	B1
	Gradient of l_2 is $-\frac{3k}{4}$	Attempt negative reciprocal of grad l_1 as a function of k	M1
	$y-0=\frac{-3k}{4}(x-4)$	Use of their changed gradient and numerical Focus in either formula, as printed oe	M1, A1
	$x=-4 \Rightarrow y=\frac{-3k}{4}(-4-4)$	Substitute numerical directrix into their line	M1
	$(y=)6k$	oe	A1
			(7)
			Total 11

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9. Prove by induction that, for $n \in \mathbb{Z}^+$,

$$f(n) = 8^n - 2^n$$

is divisible by 6

(6)

Question Number	Scheme	Notes	Marks
9.	$\text{f}(n) = 8^n - 2^n$ is divisible by 6.		
	$\text{f}(1) = 8^1 - 2^1 = 6$,	Shows that $\text{f}(1) = 6$	B1
	Assume that for $n = k$, $\text{f}(k) = 8^k - 2^k$ is divisible by 6.		
	$\text{f}(k+1) - \text{f}(k) = 8^{k+1} - 2^{k+1} - \left(8^k - 2^k\right)$	Attempt $\text{f}(k+1) - \text{f}(k)$	M1
	$= 8^k(8-1) + 2^k(1-2) = 7\times 8^k - 2^k$		
	$= 6\times 8^k + 8^k - 2^k \left(= 6\times 8^k + \text{f}(k)\right)$	M1: Attempt $\text{f}(k+1) - \text{f}(k)$ as a multiple of 6 A1: rhs a correct multiple of 6	M1A1
	$\text{f}(k+1) = 6\times 8^k + 2\text{f}(k)$	Completes to $\text{f}(k+1)$ = a multiple of 6	A1
	If the result is **true for** $n = k$, then it is now **true for** $n = k+1$. As the result has been shown to be **true for** $n = 1$, then the result is **true for all** n $(\in \mathbb{Z}^+.)$		A1cso
		Do not award final A if n defined incorrctly e.g. 'n is an integer' award A0	
			(6)
			Total 6
Way 2	$\text{f}(1) = 8^1 - 2^1 = 6$,	Shows that $\text{f}(1) = 6$	B1
	Assume that for $n = k$, $\text{f}(k) = 8^k - 2^k$ is divisible by 6.		
	$\text{f}(k+1) = 8^{k+1} - 2^{k+1} = 8\left(8^k - 2^k + 2^k\right) - 2.2^k$	Attempts $\text{f}(k+1)$ in terms of 2^k and 8^k	M1
	$\text{f}(k+1) = 8^{k+1} - 2^{k+1} = 8\left(\text{f}(k) + 2^k\right) - 2.2^k$	M1:Attempts $\text{f}(k+1)$ in terms of $\text{f}(k)$ A1: rhs correct and a multiple of 6	M1A1
	$\text{f}(k+1) = 8f(k) + 6.2^k$	Completes to $\text{f}(k+1)$ = a multiple of 6	A1
	If the result is **true for** $n = k$, then it is now **true for** $n = k+1$. As the result has been shown to be **true for** $n = 1$, then the result is **true for all** n $(\in \mathbb{Z}^+.)$		A1cso
Way 3	$\text{f}(1) = 8^1 - 2^1 = 6$,	Shows that $\text{f}(1) = 6$	B1
	Assume that for $n = k$, $\text{f}(k) = 8^k - 2^k$ is divisible by 6.		
	$\text{f}(k+1) - 8\text{f}(k) = 8^{k+1} - 2^{k+1} - 8.8^k + 8.2^k$	Attempt $\text{f}(k+1) - 8\text{f}(k)$	M1
		Any multiple m replacing 8 award M1	
	$\text{f}(k+1) - 8\text{f}(k) = 8^{k+1} - 8^{k+1} + 8.2^k - 2.2^k = 6.2^k$	M1: Attempt $\text{f}(k+1) - \text{f}(k)$ as a multiple of 6 A1: rhs a correct multiple of 6	M1A1
	$\text{f}(k+1) = 8f(k) + 6.2^k$	Completes to $\text{f}(k+1)$ = a multiple of 6	A1
		General Form for multiple m $\text{f}(k+1) = 6.8^k + (2-m)(8^k - 2^k)$	
	If the result is **true for** $n = k$, then it is now **true for** $n = k+1$. As the result has been shown to be **true for** $n = 1$, then the result is **true for all** n $(\in \mathbb{Z}^+.)$		A1cso