

Centre No.						Paper Reference							Surname	Initial(s)
Candidate No.						<b>6</b>	<b>6</b>	<b>6</b>	<b>7</b>	<b>/</b>	<b>0</b>	<b>1</b>	Signature	

Paper Reference(s)

**6667/01**

# Edexcel GCE

## Further Pure Mathematics FP1

## Advanced/Advanced Subsidiary

## Tuesday 10 June 2014 – Morning

Time: 1 hour 30 minutes

Examiner's use only

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Team Leader's use only

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### Materials required for examination

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### Mathematical Formulae (Pink)

### Items included with question papers

Nil

**Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.**

### Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature. Check that you have the correct question paper.

Answer ALL the questions.

You must write your answer to each question in the space following the question.

When a calculator is used, the answer should be given to an appropriate degree of accuracy.

## Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.

Full marks may be obtained for answers to ALL questions.

The marks for individual questions and the parts of questions are shown in round brackets: e.g. (2).

There are 9 questions in this question paper. The total mark for this paper is 75.

There are 28 pages in this question paper. Any blank pages are indicated.

## Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.

You should show sufficient working to make your methods clear to the Examiner.

Answers without working may not gain full credit.

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**1.** The complex numbers  $z_1$  and  $z_2$  are given by

$$z_1 = p + 2i \text{ and } z_2 = 1 - 2i$$

where  $p$  is an integer.

- (a) Find  $\frac{z_1}{z_2}$  in the form  $a + bi$  where  $a$  and  $b$  are real. Give your answer in its simplest form in terms of  $p$ .

Given that  $\left| \frac{z_1}{z_2} \right| = 13$ ,

- (b) find the possible values of  $p$ .



Question Number	Scheme	Notes	Marks
1.(a)	$\frac{z_1}{z_2} = \frac{p+2i}{1-2i} \cdot \frac{1+2i}{1+2i}$	Multiplying top and bottom by conjugate	M1
	$= \frac{p+2pi+2i-4}{5}$	At least 3 correct terms in the numerator, evidence that $i^2 = -1$ and denominator real.	M1
	$= \frac{p-4}{5}, \quad + \frac{2p+2}{5}i$	Real + imaginary with i factored out. Accept single denominator with numerator in correct form. Accept 'a=' and 'b='.	A1, A1
			(4)
(b)	$\left  \frac{z_1}{z_2} \right ^2 = \left( \frac{p-4}{5} \right)^2 + \left( \frac{2p+2}{5} \right)^2$	Accept their answers to part (a). Any erroneous i or $i^2$ award M0	M1
	$\left( \frac{p-4}{5} \right)^2 + \left( \frac{2p+2}{5} \right)^2 = 13^2$ or $\sqrt{\left( \frac{p-4}{5} \right)^2 + \left( \frac{2p+2}{5} \right)^2} = 13$	$\left  \frac{z_1}{z_2} \right ^2 = 13^2$ or $\left  \frac{z_1}{z_2} \right  = 13$	dM1
	$\frac{p^2-8p+16}{25} + \frac{4p^2+8p+4}{25} = 169$ or $13^2$		
	$5p^2 + 20 = 4225$		
	$p^2 = 841 \Rightarrow p = \pm 29$	dM1: Attempt to solve their quadratic in p, dependent on both previous Ms. A1: <b>both 29 and -29</b>	dM1A1
	OR		
	$\frac{ z_1 }{ z_2 } = \frac{\sqrt{p^2+4}}{\sqrt{5}}$	Finding moduli Any erroneous i or $i^2$ award M0	M1
	$\frac{\sqrt{p^2+4}}{\sqrt{5}} = 13$ oe	Equating to 13	dM1
	$\frac{p^2+4}{5} = 169$ or $13^2$ oe		
	$p^2 = 841 \Rightarrow p = \pm 29$	dM1: Attempt to solve their quadratic in p, dependent on both previous Ms A1: <b>both 29 and -29</b>	dM1A1
			(4)
			<b>Total 8</b>

2.

$$f(x) = x^3 - \frac{5}{2x^{\frac{3}{2}}} + 2x - 3, \quad x > 0$$

- (a) Show that the equation  $f(x) = 0$  has a root  $\alpha$  in the interval  $[1.1, 1.5]$ . (2)
- (b) Find  $f'(x)$ . (2)
- (c) Using  $x_0 = 1.1$  as a first approximation to  $\alpha$ , apply the Newton-Raphson procedure once to  $f(x)$  to find a second approximation to  $\alpha$ , giving your answer to 3 decimal places. (3)

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Question Number	Scheme	Notes	Marks
2.	$f(x) = x^3 - \frac{5}{2}x^2 + 2x - 3$		
(a)	$f(1.1) = -1.6359604,$ $f(1.5) = 2.0141723$	Attempts to evaluate both $f(1.1)$ and $f(1.5)$ and evaluates at least one of them correctly to awrt (or trunc.) 2 sf.	M1
	<b>Sign change</b> (and $f(x)$ is continuous) therefore a <b>root / <math>\alpha</math></b> is between $x = 1.1$ and $x = 1.5$	Both values correct to awrt (or trunc.) 2 sf, sign change (or a statement which implies this e.g. $-1.63.. < 0 < 2.014..$ ) and conclusion.	A1
			(2)
(b)	$f(x) = x^3 - \frac{5}{2}x^2 + 2x - 3$ $\Rightarrow f'(x) = 3x^2 + \frac{15}{4}x - 2$	M1: $x^n \rightarrow x^{n-1}$ for at least one term A1: Correct derivative oe	M1A1
			(2)
(c)	$f'(1.1) = 3(1.1)^2 + \frac{15}{4}(1.1) - 2 (= 8.585)$	Attempt to find $f'(1.1)$ . Accept $f'(1.1)$ seen and their value.	M1
	$\alpha_2 = 1.1 - \left( \frac{"-1.6359604"}{"8.585"} \right)$	Correct application of N-R	M1
	$\alpha_2 = 1.291$	cao	A1
			(3)
			<b>Total 7</b>

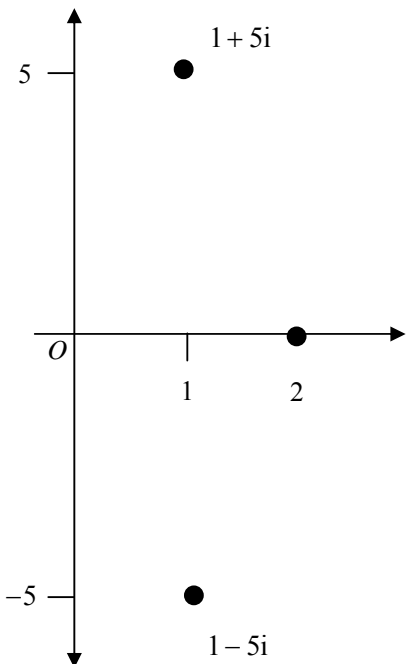
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- $$x^3 + px^2 + 30x + q = 0, \quad p, q \in \mathbb{R}$$

- (b) Find the value of  $p$  and the value of  $q$ . (5)

- (c) Show the three roots of this equation on a single Argand diagram. (2)

[illegible]

Question Number	Scheme	Notes	Marks
<b>3.</b>	$x^3 + px^2 + 30x + q = 0$		
<b>(a)</b>	$1 + 5i$		B1
			(1)
<b>(b)</b>	$((x - (1 + 5i))(x - (1 - 5i))) = x^2 - 2x + 26$ $((x - 2)(x - (1 \pm 5i))) = x^2 - (3 \pm 5i)x + 2(1 \pm 5i)$	M1: 1. Attempt to expand or 2. Use sum and product of the complex roots. A1: Correct expression	M1A1
	$(x^2 - 2x + 26)(x - 2) = x^3 + px^2 + 30x + q$	Uses their third factor with their quadratic with at least 4 terms in the expansion	M1
	$p = -4, \quad q = -52$	May be seen in cubic	A1, A1
OR	$f(1+5i)=0$ or $f(1-5i)=0$	Substitute one complex root to achieve 2 equations in $p$ and / or $q$	M1
	$q - 24p - 44 = 0$ and $10p + 40 = 0$	Both equations correct oe	A1
		Solving for $p$ and $q$	M1
	$p = -4, \quad q = -52$	May be seen in cubic	A1, A1
			(5)
<b>(c)</b>		B1: Conjugate pair correctly positioned and labelled with $1+5i$ , $1-5i$ or $(1,5)$ , $(1,-5)$ or axes labelled 1 and 5.  B1: The 2 correctly positioned relative to conjugate pair and labelled.	B1  B1
			(2)
			<b>Total 8</b>

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$$\mathbf{A} = \begin{pmatrix} 1 & 2 \\ 3 & -1 \\ 4 & 5 \end{pmatrix} \text{ and } \mathbf{B} = \begin{pmatrix} 2 & -1 & 4 \\ 1 & 3 & 1 \end{pmatrix},$$

(b) Explain why  $\mathbf{AB} \neq \mathbf{BA}$ .

(4)

$\mathbf{C} = \begin{pmatrix} 2k & -2 \\ 3 & k \end{pmatrix}$ , where  $k$  is a real number

find  $\mathbf{C}^{-1}$ , giving your answer in terms of  $k$ .

(3)

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Question Number	Scheme	Notes	Marks
4.	$\mathbf{A} = \begin{pmatrix} 1 & 2 \\ 3 & -1 \\ 4 & 5 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 2 & -1 & 4 \\ 1 & 3 & 1 \end{pmatrix}$		
(i)(a)	$\begin{pmatrix} 1 & 2 \\ 3 & -1 \\ 4 & 5 \end{pmatrix} \begin{pmatrix} 2 & -1 & 4 \\ 1 & 3 & 1 \end{pmatrix} = \begin{pmatrix} 4 & 5 & 6 \\ 5 & -6 & 11 \\ 13 & 11 & 21 \end{pmatrix}$	M1: 3x3 matrix with a number or numerical expression for each element	M1A2
		A2:cao (-1 each error) Only 1 error award A1A0	
(b)	$\mathbf{BA} = \begin{pmatrix} 2 & -1 & 4 \\ 1 & 3 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 3 & -1 \\ 4 & 5 \end{pmatrix} = \begin{pmatrix} 15 & 25 \\ 14 & 4 \end{pmatrix}$	Allow any convincing argument. E.g.s BA is a 2x2 matrix (so AB ≠ BA) or dimensionally different. Attempt to evaluate product not required. NB 'Not commutative' only is B0	B1
			(4)
(ii)	$(\det \mathbf{C} =) 2k \times k - 3 \times (-2)$	Correct attempt at determinant	M1
	$\mathbf{C}^{-1} = \frac{1}{2k^2 + 6} \begin{pmatrix} k & 2 \\ -3 & 2k \end{pmatrix}$	M1: $\frac{1}{\text{their } \det \mathbf{C}} \begin{pmatrix} k & 2 \\ -3 & 2k \end{pmatrix}$	M1A1
		A1:cao oe	
			(3)
			<b>Total 7</b>



Question Number	Scheme	Notes	Marks
5.(a)	$((2r-1)^2 =) 4r^2 - 4r + 1$		B1
	Proof by induction will usually score no more marks without use of standard results		
	$\sum_{r=1}^n (2r-1)^2 = \sum_{r=1}^n (4r^2 - 4r + 1)$		
	$= 4 \sum r^2 - 4 \sum r + \sum 1$		
	$= 4 \cdot \frac{1}{6} n(n+1)(2n+1) - 4 \cdot \frac{1}{2} n(n+1) + n$	M1: An attempt to use at least one of the standard results correctly in summing at least 2 terms of their expansion of $(2r-1)^2$ A1: Correct underlined expression oe B1: $\sum 1 = n$	M1A1B1
	$= \frac{1}{3} n [4n^2 + 6n + 2 - 6n - 6 + 3]$	Attempt to factor out $\frac{1}{3} n$ before given answer	M1
	$= \frac{1}{3} n [4n^2 - 1]$	cso	A1
			(6)
(b)	$\sum_{r=2n+1}^{4n} (2r-1)^2 = f(4n) - f(2n) \text{ or } f(2n+1)$	Require some use of the result in part (a) for method.	M1
	$= \frac{1}{3} 4n(4 \cdot (4n)^2 - 1) - \frac{1}{3} 2n(4 \cdot (2n)^2 - 1)$	Correct expression	A1
	$= \frac{2}{3} n [128n^2 - 2 - 16n^2 + 1]$		
	$= \frac{2}{3} n [112n^2 - 1]$	Accept $a = \frac{2}{3}, b = 112$	A1
			(3)
			<b>Total 9</b>



Question Number	Scheme	Notes	Marks
6.	$xy = c^2$ at $(ct, \frac{c}{t})$ .		
(a)	$y = \frac{c^2}{x} = c^2 x^{-1} \Rightarrow \frac{dy}{dx} = -c^2 x^{-2} = -\frac{c^2}{x^2}$	$\frac{dy}{dx} = k x^{-2}$	M1
	$xy = c^2 \Rightarrow x \frac{dy}{dx} + y = 0$	Correct use of product rule. The sum of two terms, one of which is correct <b>and</b> rhs = 0	
	$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx} = -\frac{c}{t^2} \cdot \frac{1}{c}$	their $\frac{dy}{dt} \times \left( \frac{1}{\text{their } \frac{dx}{dt}} \right)$	
	$\frac{dy}{dx} = -c^2 x^{-2}$ or $x \frac{dy}{dx} + y = 0$ or $\frac{dy}{dx} = \frac{-c}{t^2} \cdot \frac{1}{c}$ or equivalent expressions	Correct differentiation	A1
	$y - \frac{c}{t} = -\frac{1}{t^2}(x - ct) \quad (\times t^2)$	$y - \frac{c}{t} = \text{their } m_T (x - ct)$ or $y = mx + c$ with their $m_T$ and $(ct, \frac{c}{t})$ in an attempt to find 'c'. <b>Their <math>m_T</math> must have come from calculus and should be a function of <math>t</math> or <math>c</math> or both <math>c</math> and <math>t</math>.</b>	dM1
	$t^2 y + x = 2ct$ (Allow $x + t^2 y = 2ct$ )	Correct solution only.	A1*
			(4)
	<b>(a) Candidates who derive <math>x + t^2 y = 2ct</math>, by stating that <math>m_T = -\frac{1}{t^2}</math>, with no justification score no marks in (a).</b>		
(b)	$y = 0 \Rightarrow x = \frac{ct^4 - c}{t^3} \Rightarrow A\left(\frac{ct^4 - c}{t^3}, 0\right)$	$\frac{ct^4 - c}{t^3}$ or equivalent form	B1
	$y = 0 \Rightarrow x = 2ct \Rightarrow B(2ct, 0)$ .	$2ct$	B1
			(2)
(c)	$AB = "2ct" - \frac{ct^4 - c}{t^3}$ or $PA = ct^{-3}\sqrt{t^4 + 1}$ and $PB = ct^{-1}\sqrt{t^4 + 1}$	Attempt to subtract their $x$ -coordinates either way around.	M1
	$\text{Area APB} = \frac{1}{2} \times \text{their } AB \times \frac{c}{t}$	Valid complete method for the area of the triangle in terms of $t$ or $c$ and $t$ .	M1
	$= \frac{1}{2} \left( 2ct - \frac{ct^4 - c}{t^3} \right) \frac{c}{t} = \frac{c^2(t^4 + 1)}{2t^4}$		
	$= 8 \left( 1 + \frac{1}{t^4} \right)$ or $\frac{8(t^4 + 1)}{t^4}$ or $\frac{8t^4 + 8}{t^4}$ or $8 + \frac{8}{t^4}$	Use of $c = 4$ and completes to one of the given forms or simplest form. Final answer should be positive for A mark.	A1
			(3)
			<b>Total 9</b>



Question Number	Scheme	Notes	Marks
7.(i)(a)	$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$		B1
(b)	$\begin{pmatrix} -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$		B1
(c)	$\begin{pmatrix} -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$	M1: Multiplies their (b) x their (a) in the correct order A1: Correct matrix Correct matrix seen M1A1	M1A1
			(4)
(ii)	Area triangle $T = \frac{1}{2} \times (11 - 3) \times k = 4k$	M1: Correct method for area for $T$ A1: $4k$	M1A1
	$\det \begin{pmatrix} 6 & -2 \\ 1 & 2 \end{pmatrix} = 6 \times 2 - 1 \times (-2) (= 14)$	M1: Correct method for determinant A1: 14	M1A1
	Area triangle $T = \frac{364}{14} (= 26) \Rightarrow 4k = 26$	Uses 364 and their determinant correctly to form an equation in $k$ .	M1
	$k = \frac{26}{4} \left( = \frac{13}{2} \right)$	Accept $k = \pm \frac{13}{2}$ or $k = -\frac{13}{2}$	A1
			(6)
			<b>Total 10</b>

8. The points  $P(4k^2, 8k)$  and  $Q(k^2, 4k)$ , where  $k$  is a constant, lie on the parabola  $C$  with equation  $y^2 = 16x$ .

(a) Show that an equation of the line  $l_1$  is given by

$$3ky - 4x = 8k^2 \quad (4)$$

The line  $l_2$  is perpendicular to the line  $l_1$  and passes through the focus of the parabola  $C$ . The line  $l_2$  meets the directrix of  $C$  at the point  $R$ .

(b) Find, in terms of  $k$ , the  $y$  coordinate of the point  $R$ . (7)

[illegible]



Question Number	Scheme	Notes	Marks
8.(a)	$m = \frac{4k - 8k}{k^2 - 4k^2} \left( = \frac{4}{3k} \right)$	Valid attempt to find gradient in terms of $k$	M1
	$y - 8k = \frac{4}{3k}(x - 4k^2)$ or $y - 4k = \frac{4}{3k}(x - k^2)$ or $y = \frac{4}{3k}x + \frac{8k}{3}$	M1: Correct straight line method with their gradient in terms of $k$ . If using $y = mx + c$ then award M provided they attempt to find $c$ A1: Correct equation. If using $y = mx + c$ , award when they obtain $c = \frac{8k}{3}$ oe	M1A1
	$3ky - 24k^2 = 4x - 16k^2 \Rightarrow 3ky - 4x = 8k^2$ * or $3ky - 12k^2 = 4x - 4k^2 \Rightarrow 3ky - 4x = 8k^2$ *	Correct completion to printed answer with at least one intermediate step.	A1*
			(4)
(b)	(Focus) $(4, 0)$	Seen or implied as a number	B1
	(Directrix) $x = -4$	Seen or implied as a number	B1
	Gradient of $l_2$ is $-\frac{3k}{4}$	Attempt negative reciprocal of grad $l_1$ as a function of $k$	M1
	$y - 0 = \frac{-3k}{4}(x - 4)$	Use of their changed gradient and numerical Focus in either formula, as printed oe	M1, A1
	$x = -4 \Rightarrow y = \frac{-3k}{4}(-4 - 4)$	Substitute numerical directrix into their line	M1
	$(y =) 6k$	oe	A1
			(7)
			<b>Total 11</b>

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$$f(n) = 8^n - 2^n$$

is divisible by 6

(6)



Question Number	Scheme	Notes	Marks
<b>9.</b>	$f(n) = 8^n - 2^n$ is divisible by 6.		
	$f(1) = 8^1 - 2^1 = 6,$	Shows that $f(1) = 6$	B1
	Assume that for $n = k,$ $f(k) = 8^k - 2^k$ is divisible by 6.		
	$f(k+1) - f(k) = 8^{k+1} - 2^{k+1} - (8^k - 2^k)$	Attempt $f(k+1) - f(k)$	M1
	$= 8^k(8-1) + 2^k(1-2) = 7 \times 8^k - 2^k$		
	$= 6 \times 8^k + 8^k - 2^k (= 6 \times 8^k + f(k))$	M1: Attempt $f(k+1) - f(k)$ as a multiple of 6	M1A1
		A1: rhs a correct multiple of 6	
	$f(k+1) = 6 \times 8^k + 2f(k)$	Completes to $f(k+1) =$ a multiple of 6	A1
	If the result is <b>true for</b> $n = k,$ then it is now <b>true for</b> $n = k+1.$ As the result has been shown to be <b>true for</b> $n = 1,$ then the result is <b>true for all</b> $n (\in \mathbb{N}^+).$		A1cso
		Do not award final A if $n$ defined incorrectly e.g. ' $n$ is an integer' award A0	
			(6)
			<b>Total 6</b>
<b>Way 2</b>	$f(1) = 8^1 - 2^1 = 6,$	Shows that $f(1) = 6$	B1
	Assume that for $n = k,$ $f(k) = 8^k - 2^k$ is divisible by 6.		
	$f(k+1) = 8^{k+1} - 2^{k+1} = 8(8^k - 2^k + 2^k) - 2.2^k$	Attempts $f(k+1)$ in terms of $2^k$ and $8^k$	M1
	$f(k+1) = 8^{k+1} - 2^{k+1} = 8(f(k) + 2^k) - 2.2^k$	M1: Attempts $f(k+1)$ in terms of $f(k)$	M1A1
		A1: rhs correct and a multiple of 6	
	$f(k+1) = 8f(k) + 6.2^k$	Completes to $f(k+1) =$ a multiple of 6	A1
	If the result is <b>true for</b> $n = k,$ then it is now <b>true for</b> $n = k+1.$ As the result has been shown to be <b>true for</b> $n = 1,$ then the result is <b>true for all</b> $n (\in \mathbb{N}^+).$		A1cso
<b>Way 3</b>	$f(1) = 8^1 - 2^1 = 6,$	Shows that $f(1) = 6$	B1
	Assume that for $n = k,$ $f(k) = 8^k - 2^k$ is divisible by 6.		
	$f(k+1) - 8f(k) = 8^{k+1} - 2^{k+1} - 8.8^k + 8.2^k$	Attempt $f(k+1) - 8f(k)$	M1
		Any multiple $m$ replacing 8 award M1	
	$f(k+1) - 8f(k) = 8^{k+1} - 8.8^k + 8.2^k - 2.2^k = 6.2^k$	M1: Attempt $f(k+1) - f(k)$ as a multiple of 6	M1A1
		A1: rhs a correct multiple of 6	
	$f(k+1) = 8f(k) + 6.2^k$	Completes to $f(k+1) =$ a multiple of 6	A1
		General Form for multiple $m$ $f(k+1) = 6.8^k + (2-m)(8^k - 2^k)$	
	If the result is <b>true for</b> $n = k,$ then it is now <b>true for</b> $n = k+1.$ As the result has been shown to be <b>true for</b> $n = 1,$ then the result is <b>true for all</b> $n (\in \mathbb{N}^+).$		A1cso