



1.  $f(x) = 9x^3 - 33x^2 - 55x - 25$

Given that  $x=5$  is a solution of the equation  $f(x)=0$ , use an algebraic method to solve  $f(x)=0$  completely.

(5)

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---



**June 2015**  
**Further Pure Mathematics FP1 6667**  
**Mark Scheme**

Question Number	Scheme	Marks
1.	<p><math>(x - 5)</math> is a factor of <math>f(x)</math> so <math>f(x) = (x - 5)(9x^2 \dots)</math></p> <p><math>f(x) = (x - 5)(9x^2 + 12x + 5)</math></p> <p>Solve <math>(9x^2 + 12x + 5) = 0</math> to give <math>x =</math></p> <p><math>(x =) -\frac{2}{3} - \frac{1}{3}i, -\frac{2}{3} + \frac{1}{3}i</math> or <math>-\frac{2}{3} \pm \frac{1}{3}i</math> or <math>\frac{-2 \pm i}{3}</math> oe ( and 5)</p>	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1cao A1ft (5) <b>(5 marks)</b></p>
	<p style="text-align: center;"><b>Notes</b></p> <p>M1: Uses <math>(x-5)</math> as factor and begins division or process to obtain quadratic with <math>9x^2</math>. Award if no working but quadratic factor completely correct.</p> <p>A1: <math>9x^2 + 12x + 5</math></p> <p>M1: Solves their quadratic by usual rules leading to <math>x =</math> Award if one complex root correct with no working. Award for <math>(9x^2 + \dots)</math> incorrectly factorised to <math>(3x + p)(3x + q)</math>, where <math> pq  = 5</math></p> <p>A1: One correct complex root. Accept any exact equivalent form. Accept single fraction and <math>\pm</math></p> <p>A1ft: Conjugate of their first complex root.</p>	

Leave  
blank

2. In the interval  $13 < x < 14$ , the equation

$$3 + x \sin\left(\frac{x}{4}\right) = 0, \text{ where } x \text{ is measured in radians,}$$

has exactly one root,  $\alpha$ .

(a) Starting with the interval  $[13, 14]$ , use interval bisection twice to find an interval of width 0.25 which contains  $\alpha$ .

(3)

(b) Use linear interpolation once on the interval  $[13, 14]$  to find an approximate value for  $\alpha$ . Give your answer to 3 decimal places.

(4)

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---



Question Number	Scheme	Marks
<p>2. (a)</p>    <p>(b)</p>	<p>Let <math>f(x) = 3 + x \sin\left(\frac{x}{4}\right)</math> then <math>f(13) = 1.593</math> [and <math>f(14) = -1.911</math> need not be seen in (a)]</p> <p><math>f(13.5) = -0.122</math>, so root in <math>[13, 13.5]</math></p> <p><math>f(13.25) = 0.746</math> so root in <math>[13.25, 13.5]</math></p> <p><math>\frac{\alpha - 13}{14 - \alpha} = \frac{1.593}{1.911}</math> or <math>\frac{\alpha - 13}{1} = \frac{1.593}{1.593 + 1.911}</math></p> <p>So <math>\alpha(1.911 + 1.593) = 1.593 \times 14 + 13 \times 1.911</math> and <math>\alpha = \frac{47.145}{3.504} = 13.455</math></p>	<p>M1 A1</p> <p>A1</p> <p>M1 A1</p> <p>dM1 A1</p> <p>(3)</p> <p>(4)</p> <p><b>(7 marks)</b></p>
<p><b>Notes</b></p>		
<p>(a)</p> <p>M1: Evaluate <math>f(13)</math> and <math>f(13.5)</math> giving at least positive, negative <b>OR</b> evaluate <math>f(13.5)</math> and <math>f(13.25)</math> to give at least negative, positive. Do not award if using degrees.  A1: <math>f(13.5) = \text{awrt } -0.1</math>, <math>f(13.25) = \text{awrt } 0.7(5)</math>.  A1: Correct interval <math>[13.25, 13.5]</math> or equivalent form with or without boundaries.</p> <p>(b)</p> <p>M1: Attempt at linear interpolation on either side of equation with correct signs.  A1: Correct equivalent statement  dM1: Makes alpha subject of formula  A1: cao. Award A0 for 13.456 and 13.454</p> <p><b>ALT (b)</b></p> <p>Using equation of line</p> <p>M1: Attempt to find gradient <math>\frac{y_1 - y_0}{x_1 - x_0} = \frac{-1.911 - 1.593}{14 - 13} (= -3.504...)</math>, attempt to use <math>y - y_0 = m(x - x_0)</math></p> <p>with either 13 or 14 (gives <math>y = -3.504x + 47.145</math>) and substitute <math>y = 0</math></p> <p>A1: Correct statement after substituting <math>y = 0</math> in their equation i.e. <math>0 = -3.504x + 47.145</math></p> <p>dM1: Makes <math>x</math> the subject of the formula  A1: cao. Award A0 for 13.456 and 13.454</p>		

Leave blank

3. (a) Using the formulae for  $\sum_{r=1}^n r$  and  $\sum_{r=1}^n r^2$ , show that

$$\sum_{r=1}^n (r + 1)(r + 4) = \frac{n}{3} (n + 4)(n + 5)$$

for all positive integers  $n$ .

(5)

(b) Hence show that

$$\sum_{r=n+1}^{2n} (r + 1)(r + 4) = \frac{n}{3} (n + 1)(an + b)$$

where  $a$  and  $b$  are integers to be found.

(3)

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---



Question Number	Scheme	Marks
3. (a)	$\sum_{r=1}^n (r+1)(r+4)$ $= \sum_{r=1}^n r^2 + 5r + 4$ $= \frac{n}{6}(n+1)(2n+1) + 5\frac{n}{2}(n+1) + 4n$ $= \frac{n}{6}\{(n+1)(2n+1) + 15(n+1) + 24\}$ $= \frac{n}{6}\{2n^2 + 3n + 1 + 15n + 15 + 24\}$ $= \frac{n}{6}(2n^2 + 18n + 40) \text{ or } = \frac{n}{3}(n^2 + 9n + 20)$ $= \frac{n}{3}(n+4)(n+5) \text{ ** given answer**}$	<p>B1 M1 A1 dM1</p> <p>A1* (5)</p>
(b)	$\sum_{r=n+1}^{2n} (r+1)(r+4) = \frac{2n}{3}(2n+4)(2n+5) - \frac{n}{3}(n+4)(n+5)$ $= \frac{n}{3}\{8n^2 + 36n + 40 - n^2 - 9n - 20\}$ $= \frac{n}{3}\{7n^2 + 27n + 20\} = \frac{n}{3}(n+1)(7n+20) \text{ or } a = 7, b = 20$	<p>M1 dM1</p> <p>A1 (3)</p> <p><b>(8 marks)</b></p>

Notes

(a)

B1: Expands bracket correctly to  $r^2 + 5r + 4$

M1: Uses  $\frac{n}{6}(n+1)(2n+1)$  or  $\frac{n}{2}(n+1)$  correctly.

A1: Completely correct expression.

dM1: Attempts to remove factor  $\frac{n}{6}$  or  $\frac{n}{3}$  to obtain a quadratic factor. Need not be 3 term.

A1: Completely correct work including a step with a collected **3 term** quadratic prior in the bracket with correct printed answer.

Accept approach which starts with LHS and then RHS which meet at  $\frac{n^3}{3} + 3n^2 + \frac{20n}{3}$ . Award marks as above.

NB If induction attempted then typically this may only score the first B1.

However, consider the solution carefully and award as above if seen in the body of the induction attempt.

(b) M1: Uses  $f(2n) - f(n)$  or  $f(2n) - f(n+1)$  correctly. Require all 3 terms in  $2n$  (and  $n+1$  if used).

dM1: Attempts to remove factor  $\frac{n}{6}$  or  $\frac{n}{3}$  to obtain a quadratic factor. Need not be 3 term.

A1: Either in expression or as above.

Leave  
blank

4.

$$z_1 = 3i \text{ and } z_2 = \frac{6}{1 + i\sqrt{3}}$$

- (a) Express  $z_2$  in the form  $a + ib$ , where  $a$  and  $b$  are real numbers. (2)
  
- (b) Find the modulus and the argument of  $z_2$ , giving the argument in radians in terms of  $\pi$ . (4)
  
- (c) Show the three points representing  $z_1$ ,  $z_2$  and  $(z_1 + z_2)$  respectively, on a single Argand diagram. (2)

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

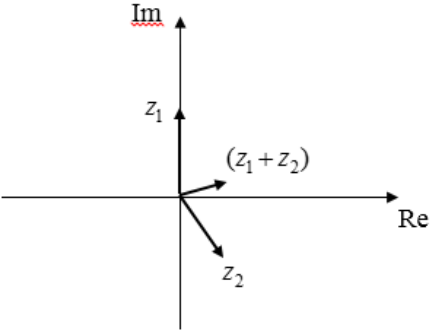
---

---

---





Question Number	Scheme	Marks
4. (a)	$z_2 = \frac{6(1-i\sqrt{3})}{(1+i\sqrt{3})(1-i\sqrt{3})} = \frac{6(1-i\sqrt{3})}{4}$ $z_2 = \frac{6(1-i\sqrt{3})}{4} \left( = \frac{3}{2} - i\frac{3}{2}\sqrt{3} \right)$	M1 A1 (2)
(b)	$ z_2  = \sqrt{\left(\frac{3}{2}\right)^2 + \left(\frac{3\sqrt{3}}{2}\right)^2} = \sqrt{\frac{9}{4} + \frac{27}{4}}$ <p>The modulus of <math>z_2</math> is 3</p> <p><math>\tan \theta = (\pm)\sqrt{3}</math> and attempts to find <math>\theta</math></p> <p>and the argument is <math>-\frac{\pi}{3}</math></p>	M1 A1 (4)
(c)		M1 A1 (2)
<b>Notes</b>		
<p>(a) M1: Multiplies numerator and denominator by <math>1-i\sqrt{3}</math>                      A1: any correct equivalent with real denominator.</p> <p>(b) M1: Uses correct method for modulus for their <math>z_2</math> in part (a)                      A1: for 3 only                      M1: Uses tan or inverse tan                      A1: <math>-\frac{\pi}{3}</math> accept <math>\frac{5\pi}{3}</math></p> <p>NB Answers only then award 4/4 but arg must be in terms of <math>\pi</math></p> <p>(c) M1:  <b>Either</b> <math>z_1</math> on imaginary axis and labelled with <math>z_1</math> or <math>3i</math> or <math>(0,3)</math> or axis labelled 3;  <b>or</b> their <math>z_2</math> in the correct quadrant labelled <math>z_2</math> or <math>\frac{3}{2} - i\frac{3}{2}\sqrt{3}</math> or <math>\left(\frac{3}{2}, -\frac{3}{2}\sqrt{3}\right)</math> or axes labelled                      or their <math>a+bi</math> or their <math>(a,b)</math> or axes labelled.                      Axes need not be labelled Re and Im.                      A1: All 3 correct ie <math>z_1</math> on positive imaginary axis, <math>z_2</math> in 4<sup>th</sup> quadrant and <math>z_1 + z_2</math> in the first quadrant.                      Accept points or lines. Arrows not required.</p>		
<b>(8 marks)</b>		

Leave  
blank

5. The rectangular hyperbola  $H$  has equation  $xy = 9$

The point  $A$  on  $H$  has coordinates  $\left(6, \frac{3}{2}\right)$ .

(a) Show that the normal to  $H$  at the point  $A$  has equation

$$2y - 8x + 45 = 0 \quad (5)$$

The normal at  $A$  meets  $H$  again at the point  $B$ .

(b) Find the coordinates of  $B$ . (4)

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---



Question Number	Scheme	Marks
<p>5. (a)</p> <p>(b)</p> <p><b>ALT</b></p>	<p><math>\frac{dy}{dx} = -\frac{9}{x^2}</math> or <math>\frac{dy}{dx} = -\frac{y}{x}</math> or <math>\frac{dy}{dx} = -\frac{1}{t^2}</math></p> <p>so gradient at <math>x = 6</math> or <math>t = 2</math> is <math>-\frac{9}{36}</math> or <math>-\frac{\frac{3}{2}}{6}</math> or <math>-\frac{1}{4}</math> o.e.</p> <p>Gradient of normal is <math>-\frac{1}{m}</math> (= 4)</p> <p>Equation of normal is <math>y - \frac{3}{2} = 4(x - 6)</math></p> <p>So <math>2y - 8x + 45 = 0</math> <b>**given answer**</b></p> <p><math>\frac{18}{x} - 8x + 45 = 0</math> or <math>2y - \frac{72}{y} + 45 = 0</math> or <math>x(4x - 22.5) = 9</math> or <math>y\left(\frac{y}{4} + \frac{45}{8}\right) = 9</math> o.e.</p> <p><math>8x^2 - 45x - 18 = 0</math> or <math>2y^2 + 45y - 72 = 0</math></p> <p>So <math>x = -\frac{3}{8}</math> or <math>y = -24</math></p> <p>Finds other ordinate: <math>\left(-\frac{3}{8}, -24\right)</math></p> <p>Sub <math>\left(3t, \frac{3}{t}\right)</math> in <math>2y - 8x + 45 = 0 \Rightarrow t = -\frac{1}{8}</math></p> <p>Sub <math>t = -\frac{1}{8}</math> in <math>\left(3t, \frac{3}{t}\right) \Rightarrow \left(-\frac{3}{8}, -24\right)</math></p>	<p>M1</p> <p>A1</p> <p>M1</p> <p>dM1</p> <p>A1 *</p> <p>(5)</p> <p>M1</p> <p>A1</p> <p>M1 A1</p> <p>M1A1</p> <p>M1A1</p> <p>(4)</p> <p><b>(9 marks)</b></p>
<p><b>Notes</b></p> <p>(a) M1: Differentiates to obtain <math>\frac{k}{x^2}</math> and substitutes <math>x = 6</math></p> <p>or uses implicit differentiation <math>\frac{dy}{dx} = -\frac{y}{x}</math> and substitutes <math>x</math> and <math>y</math></p> <p>or uses parametric differentiation <math>\frac{dy}{dx} = -\frac{1}{t^2}</math> and substitutes <math>t = 2</math></p> <p>A1: For grad of tangent – accept any equivalent i.e. - 0.25 etc</p> <p>M1: Uses negative reciprocal of their gradient.</p> <p>dM1: <math>y - y_1 = m(x - x_1)</math> with <math>\left(6, \frac{3}{2}\right)</math> or <math>y = mx + c</math> and sub <math>\left(6, \frac{3}{2}\right)</math> to find <math>c</math>.</p> <p>A1: cso: Correct answer with no errors seen in the solution.</p> <p>(b) M1: Obtains equation in one variable, <math>x</math> or <math>y</math></p> <p>A1: Correct value of <math>x</math> or correct value of <math>y</math></p> <p>M1: Finds second coordinate using <math>xy = 9</math> or solving second quadratic or equation of the normal</p> <p>A1: Correct coordinates that can be written as <math>x = \dots, y = \dots</math></p>		



Question Number	Scheme	Marks
<p>6. (i)</p> <p>(ii)</p>	<p>If <math>n = 1</math>, <math>\begin{pmatrix} 1 &amp; 0 \\ -1 &amp; 5 \end{pmatrix}^1 = \begin{pmatrix} 1 &amp; 0 \\ -\frac{1}{4}(5^1 - 1) &amp; 5^1 \end{pmatrix}</math> so <b>true</b> for <math>n = 1</math></p> <p>Assume result true for <math>n = k</math></p> $\begin{pmatrix} 1 & 0 \\ -1 & 5 \end{pmatrix}^{k+1} = \begin{pmatrix} 1 & 0 \\ -\frac{1}{4}(5^k - 1) & 5^k \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -1 & 5 \end{pmatrix} \text{ or } \begin{pmatrix} 1 & 0 \\ -1 & 5 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\frac{1}{4}(5^k - 1) & 5^k \end{pmatrix}$ $\begin{pmatrix} 1 & 0 \\ -1 & 5 \end{pmatrix}^{k+1} = \begin{pmatrix} 1 & 0 \\ -\frac{1}{4}(5^k - 1) - 5^k & 5 \times 5^k \end{pmatrix} \text{ or } \begin{pmatrix} 1 & 0 \\ -1 - 5 \cdot \frac{1}{4}(5^k - 1) & 5 \times 5^k \end{pmatrix}$ $= \begin{pmatrix} 1 & 0 \\ -\frac{1}{4}5^k + \frac{1}{4} - 5^k & 5^{k+1} \end{pmatrix} \text{ or } \begin{pmatrix} 1 & 0 \\ -1 - \frac{1}{4}5^{k+1} + \frac{5}{4} & 5^{k+1} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -\frac{1}{4}(5^{k+1} - 1) & 5^{k+1} \end{pmatrix}$ <p><b>True</b> for <math>n = k + 1</math> if <b>true</b> for <math>n = k</math>, (and <b>true</b> for <math>n = 1</math>) so <b>true</b> by induction for all <math>n \in \mathbf{Z}^+</math>.</p> <p>If <math>n = 1</math>, <math>\sum_{r=1}^n (2r-1)^2 = 1</math> and <math>\frac{1}{3}n(4n^2 - 1) = 1</math>, so <b>true</b> for <math>n = 1</math>.</p> <p>Assume result true for <math>n = k</math> so <math>\sum_{r=1}^{k+1} (2r-1)^2 = \frac{1}{3}k(4k^2 - 1) + (2(k+1) - 1)^2</math></p> $= \sum_{r=1}^{k+1} (2r-1)^2 = \frac{1}{3}(2k+1)\{(2k^2 - k) + (3(2k+1))\}$ $= \frac{1}{3}(2k+1)\{(2k^2 + 5k + 3)\} \text{ or } \frac{1}{3}(k+1)(4k^2 + 8k + 3) \text{ or } \frac{1}{3}((2k+3)(2k^2 + 3k + 1))$ $= \frac{1}{3}(k+1)(2k+1)(2k+3) = \frac{1}{3}(k+1)(4(k+1)^2 - 1)$ <p><b>True</b> for <math>n = k + 1</math> if <b>true</b> for <math>n = k</math>, ( and <b>true</b> for <math>n = 1</math>) so <b>true</b> by induction for all <math>n \in \mathbf{Z}^+</math></p>	<p>B1</p> <p>M1</p> <p>M1 A1</p> <p>A1</p> <p>A1cso</p> <p>B1</p> <p>M1</p> <p>M1 A1</p> <p>dA1</p> <p>A1cso</p> <p>(6)</p> <p>(6)</p> <p><b>12 marks</b></p>
<p><b>Notes</b></p> <p>(i) B1: Checks <math>n = 1</math> on both sides and states true for <math>n = 1</math> seen anywhere.  M1: Assumes true for <math>n = k</math> and indicates intention to multiply power <math>k</math> by power 1 either way around.  M1: Multiplies matrices. Condone one slip. A1: Correct unsimplified matrix  A1: Intermediate step required cao  A1: cso Makes correct induction statement including at least statements in bold.  Statement <b>true</b> for <math>n = 1</math> here could contribute to B1 mark earlier.</p> <p>(ii) B1: Checks <math>n = 1</math> on both sides and states true for <math>n = 1</math> seen anywhere.  M1: Assumes true for <math>n = k</math> and adds <math>(k+1)^{\text{th}}</math> term to sum of <math>k</math> terms. Accept <math>4(k+1)^2 - 4(k+1) + 1</math> or <math>(2k+1)^2</math> for <math>(k+1)^{\text{th}}</math> term. M1: Factorises out a linear factor of the three possible - usually <math>2k+1</math>  A1: Correct expression with one linear and one quadratic factor.  dA1: Need to see <math>\frac{1}{3}(k+1)(4(k+1)^2 - 1)</math> somewhere dependent upon previous A1.</p> <p>Accept assumption plus <math>(k+1)^{\text{th}}</math> term and <math>\frac{1}{3}(k+1)(4(k+1)^2 - 1)</math> both leading to <math>\frac{1}{3}(4k^3 + 12k^2 + 11k + 3)</math> then award for expressions seen as above.  A1: cso Makes correct complete induction statement including at least statements in bold. Statement true for <math>n = 1</math> here could contribute to B1 mark earlier.</p>		

7. (i)

$$\mathbf{A} = \begin{pmatrix} 5k & 3k-1 \\ -3 & k+1 \end{pmatrix}, \text{ where } k \text{ is a real constant.}$$

Given that  $\mathbf{A}$  is a singular matrix, find the possible values of  $k$ .

**(4)**

(ii)

$$\mathbf{B} = \begin{pmatrix} 10 & 5 \\ -3 & 3 \end{pmatrix}$$

A triangle  $T$  is transformed onto a triangle  $T'$  by the transformation represented by the matrix  $\mathbf{B}$ .

The vertices of triangle  $T'$  have coordinates  $(0, 0)$ ,  $(-20, 6)$  and  $(10c, 6c)$ , where  $c$  is a positive constant.

The area of triangle  $T'$  is 135 square units.

(a) Find the matrix  $\mathbf{B}^{-1}$

**(2)**

(b) Find the coordinates of the vertices of the triangle  $T$ , in terms of  $c$  where necessary.

**(3)**

(c) Find the value of  $c$ .

**(3)**

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---



Question Number	Scheme	Marks
7. (i)	$5k(k+1) - -3(3k-1)=0$ $5k^2 + 5k + 9k - 3 = 0$ $(5k - 1)(k + 3) = 0$ so $k =$ $k = \frac{1}{5}$ or $-3$	M1 A1 M1 A1 (4)
(ii)(a)	$B^{-1} = \frac{1}{45} \begin{pmatrix} 3 & -5 \\ 3 & 10 \end{pmatrix}$	M1 A1 (2)
(b)	$\frac{1}{45} \begin{pmatrix} 3 & -5 \\ 3 & 10 \end{pmatrix} \begin{pmatrix} 0 & -20 & 10c \\ 0 & 6 & 6c \end{pmatrix} =$ $\frac{1}{45} \begin{pmatrix} 0 & -90 & 0 \\ 0 & 0 & 90c \end{pmatrix}$ Vertices at $(0, 0)$ $(-2, 0)$ $(0, 2c)$	M1 A1,A1 (3)
<b>ALT</b>	$\begin{pmatrix} 10 & 5 \\ -3 & 3 \end{pmatrix} \begin{pmatrix} a & d & f \\ b & e & g \end{pmatrix} = \begin{pmatrix} 0 & -20 & 10c \\ 0 & 6 & 6c \end{pmatrix}$ and attempt to form simultaneous equations $10a + 5b = 0, -3a + 3b = 0$ $10d + 5e = -20, -3d + 3e = 6$ all correct oe $10f + 5g = 10c, -3f + 3g = 6c$ Vertices at $(0, 0)$ $(-2, 0)$ $(0, 2c)$	M1 A1 A1 B1 M1 A1 (3)
(c)	Area of $T$ is $\frac{1}{2} \times 2 \times 2c = 2c$ Area of $T \times \text{determinant} = 135$ So $c = \frac{3}{2}$	OR Area of $T' = \frac{1}{2} \begin{vmatrix} 0 & -20 & 10c & 0 \\ 0 & 6 & 6c & 0 \end{vmatrix} = 90c$ Their area = 135 So $c = \frac{3}{2}$ (12 marks)
<b>Notes</b>		
(i) M1: Puts determinant equal to zero A1: cao as three or four term quadratic M1: Solve their quadratic to find $k$ A1: cao – need both correct answers (ii) (a) M1 Uses correct method for inverse with fraction $\frac{1}{45}$ or $\frac{1}{\text{their det}}$ A1: All correct oe (b) M1: Post multiplies their inverse by <b>2 by 3 matrix or 2 by 2 matrix excluding the origin</b> or does not use inverse and attempts to form simultaneous equations. Can exclude origin. A1: $(-2,0)$ and $(0,2c)$ . Can be written as column vectors. Accept seen in final two columns of single matrix A1: $(0,0)$ . Can be written as column vectors. Award if seen as first column of single matrix. (c) B1: Area of $T$ given as $2c$ or area of $T' = 90c$ Accept $\pm$ M1: Either method using their area of $T$ and their det or their area of $T'$ A1: $c = \frac{3}{2}$ cao		

Leave blank

8. The point  $P(3p^2, 6p)$  lies on the parabola with equation  $y^2 = 12x$  and the point  $S$  is the focus of this parabola.

(a) Prove that  $SP = 3(1 + p^2)$  (3)

The point  $Q(3q^2, 6q)$ ,  $p \neq q$ , also lies on this parabola.

The tangent to the parabola at the point  $P$  and the tangent to the parabola at the point  $Q$  meet at the point  $R$ .

(b) Find the equations of these two tangents and hence find the coordinates of the point  $R$ , giving the coordinates in their simplest form. (8)

(c) Prove that  $SR^2 = SP \cdot SQ$  (3)

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---





Question Number	Scheme	Marks
8(a)	$SP = \sqrt{(3p^2 - a)^2 + 36p^2} \text{ , with } a = 3$ $SP = \sqrt{9p^4 + 18p^2 + 9} = 3(1 + p^2) \text{ **given answer**}$	M1, B1 A1 * (3)
<b>ALT</b>	For parabola, perpendicular distance from $P$ to directrix = $SP$ Directrix $x = -3$ So $SP = 3 + 3p^2 = 3(1 + p^2)$	M1 B1 A1
(b)	$y^2 = 12x \Rightarrow 2y \frac{dy}{dx} = 12 \text{ or } y = \sqrt{12x} \Rightarrow \frac{dy}{dx} = \sqrt{3x}^{-\frac{1}{2}} \text{ or } \frac{dy}{dx} = \frac{dp}{dx} \text{ or } \frac{dy}{dx} = \frac{dq}{dx}$ <p>The tangent at <math>P</math> has gradient <math>= \frac{1}{p}</math> or the tangent at <math>Q</math> has gradient <math>\frac{1}{q}</math></p> <p>and equation is <math>y - 6p = \frac{1}{p}(x - 3p^2)</math> or <math>py = x + 3p^2</math> o.e.</p> <p>Tangent at <math>Q</math> is <math>y - 6q = \frac{1}{q}(x - 3q^2)</math> or <math>qy = x + 3q^2</math> o.e.</p> <p>Eliminate <math>x</math> or <math>y</math>: So <math>x = 3pq</math> or <math>y = 3(p + q) = 3p + 3q</math></p> <p>Substitute for second variable so <math>x = 3pq</math> and <math>y = 3(p + q) = 3p + 3q</math></p>	M1 A1 A1 B1 M1 A1 M1 A1 (8)
(c)	$SR^2 = (3 - 3pq)^2 + (3p + 3q)^2 (= 9 + 9p^2q^2 + 9p^2 + 9q^2)$ $SP.SQ = 3(1 + p^2) 3(1 + q^2) (= 9 + 9p^2q^2 + 9p^2 + 9q^2)$ So $SR^2 = SP.SQ$ as required	M1 M1 A1 (3)
<p><b>Notes</b></p> <p>(a) M1: Uses distance between two points or states perpendicular distance from <math>P</math> to directrix required.                      B1: States or uses focus at <math>(3,0)</math> or focus at <math>a = 3</math> or directrix as <math>x = -3</math>                      A1: cso</p> <p>(b) M1: Calculus method for finding gradient and substitutes <math>x</math> value at either point                      A1: Either correct. Accept unsimplified.                      A1: One equation of tangent correct. B1: Both correct                      M1: Eliminate <math>x</math> or <math>y</math>. A1: Obtain first variable                      M1: Substitute or eliminate again. A1: Both variables correct in simplest form as above.</p> <p>(c) M1: Find their <math>SR^2 = (3 - 3pq)^2 + (3p + 3q)^2 (= 9 + 9p^2q^2 + 9p^2 + 9q^2)</math>                      M1: Find their <math>SP.SQ = 3(1 + p^2) 3(1 + q^2) (= 9 + 9p^2q^2 + 9p^2 + 9q^2)</math>                      A1: Deduce equal after no errors seen. Concluding statement required cso.</p> <p style="text-align: right;">(14 marks)</p>		