

Write your name here

Surname	Other names
---------	-------------

**Pearson  
Edexcel GCE**

Centre Number

--	--	--	--	--

Candidate Number

--	--	--	--

# Further Pure Mathematics FP1

## Advanced/Advanced Subsidiary

Friday 20 May 2016 – Morning  
**Time: 1 hour 30 minutes**

Paper Reference

**6667/01**

**You must have:**

Mathematical Formulae and Statistical Tables (Pink)

Total Marks

--

**Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

### Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

### Information

- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

### Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ►

P46681A

©2016 Pearson Education Ltd.

5/1/1/1/1/1/1/1/



**PEARSON**

Leave blank

1. Given that  $k$  is a real number and that

$$A = \begin{pmatrix} 1+k & k \\ k & 1-k \end{pmatrix}$$

find the exact values of  $k$  for which  $A$  is a singular matrix. Give your answers in their simplest form.

(3)

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



Question Number	Scheme	Marks
1.	Determinant of $\mathbf{A} = (1 - k)(1 + k) - k^2 = 0$	M1
	$1 - k + k - k^2 - k^2 (= 0)$ $1 - 2k^2 (= 0)$ $\text{So } k = \frac{\pm\sqrt{2}}{2}$	A1  A1 (3)
<b>3 marks</b>		
<p style="text-align: center;">Notes</p> <p>M1: for attempting <math>ad - bc = 0</math> with '=' seen or <b>used</b> later in the solution.</p> <p>A1: Correct (unsimplified) expression on LHS or correct equation after brackets expanded.</p> <p>A1: Accept <math>\pm\frac{\sqrt{2}}{2}</math>, <math>\pm\frac{1}{\sqrt{2}}</math>, <math>\pm\sqrt{\frac{1}{2}}</math>, <math>\pm\sqrt{0.5}</math>. Must have <math>\pm</math> for mark.</p>		

Leave blank

2.

$$f(x) = 3x^{\frac{3}{2}} - 25x^{-\frac{1}{2}} - 125, \quad x > 0$$

(a) Find  $f'(x)$ . (2)

The equation  $f(x) = 0$  has a root  $\alpha$  in the interval  $[12, 13]$ .

(b) Using  $x_0 = 12.5$  as a first approximation to  $\alpha$ , apply the Newton-Raphson procedure once to  $f(x)$  to find a second approximation to  $\alpha$ , giving your answer to 3 decimal places. (4)

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



Question Number	Scheme	Marks
2.	<p>(a) <math>f'(x) = \frac{9}{2}x^{\frac{1}{2}} + \frac{25}{2}x^{-\frac{3}{2}}</math></p> <p>(b) <math>f(12.5) = 0.5115\dots</math> (at least 0.51...) and <math>f'(12.5) = 16.1927\dots</math> (at least 16... seen)</p> $x_1 = 12.5 - \frac{f(12.5)}{f'(12.5)} = 12.5 - \frac{0.5115}{16.1927\dots} = 12.468$	<p>M1 A1</p> <p>(2)</p> <p>B1, B1</p> <p>M1 A1</p> <p>(4)</p>
<b>6 marks</b>		
<p style="text-align: center;">Notes</p> <p>(a) M1: for attempting differentiation i.e. decrease a power by 1                      A1 Accept equivalent expression i.e. condone equivalent fractions.</p> <p>(b) B1: One correct, must be explicitly seen if final answer incorrect, may be implied by correct final answer.                      B1: Both correct; must be explicitly seen if final answer incorrect, may be implied by correct final answer.                      M1: for attempting Newton- Raphson with their values for <math>f(12.5)</math> and <math>f'(12.5)</math>                      A1: cao correct to 3dp                      Newton Raphson used more than once – isw.</p>		



Question Number	Scheme	Marks
3.	<p>(a) <math>\sum_{r=1}^{3n} r^2 = \frac{1}{6}3n(3n+1)(6n+1)</math> or <math>\sum_{r=1}^{3n} r^2 = \frac{1}{2}n(3n+1)(6n+1)</math> or equivalent</p> <p>(b) See <math>\sum_{r=1}^{2n} r^2 = \frac{1}{3}n(2n+1)(4n+1)</math> or equivalent</p> <p>Attempt to use <math>\sum_{r=1}^{3n} r^2 - \sum_{r=1}^{2n} r^2 = \frac{n}{6}\{3(3n+1)(6n+1) - 2(2n+1)(4n+1)\}</math></p> $= \frac{n}{6}\{(54n^2 + 27n + 3) - (16n^2 + 12n + 2)\}$ $= \frac{n}{6}\{(38n^2 + 15n + 1)\}$ <p>(a = 38, b = 15, c = 1)</p>	<p>B1 (1)</p> <p>B1</p> <p>M1</p> <p>dM1</p> <p>A1</p> <p>(4)</p> <p><b>5 marks</b></p>

**Notes**

(a) B1: Either right hand side or exact equivalent - isw if expanded

(b) B1: States or uses  $\sum_{r=1}^{2n} r^2 = \frac{1}{3}n(2n+1)(4n+1)$

M1: Subtracts their sum to  $2n$  or  $2n - 1$  **and** attempts to factorise by  $\frac{n}{6}$  seen anywhere.

dM1: Expands two quadratics dependent on first M1

A1: cao





Question Number	Scheme	Marks
4.	<p>(a) <math>z = \frac{4(1-i)}{(1+i)(1-i)}</math></p> <p><math>z = 2(1-i)</math> or <math>2 - 2i</math> or exact equivalent.</p> <p>(b) <math>z^2 = (2 - 2i)(2 - 2i) = 4 - 8i + 4i^2</math></p> <p><math>= -8i</math></p> <p>(c) If <math>z</math> is a root so is <math>z^*</math> So <math>(x - 2 + 2i)(x - 2 - 2i)</math> (or <math>x^2 - 2\text{Re}(z).x +  z ^2</math>)</p> <p>So <math>(x - 2 + 2i)(x - 2 - 2i) = 0</math> (or <math>x^2 - 2\text{Re}(z).x +  z ^2 = 0</math>) and so <math>p = q =</math></p> <p>Equation is <math>x^2 - 4x + 8(= 0)</math> or <math>p = -4</math> and <math>q = 8</math></p>	<p>M1</p> <p>A1</p> <p>(2)</p> <p>M1</p> <p>A1 cao</p> <p>(2)</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>(3)</p> <p>(7 marks)</p>
ALT 1	<p>(c)</p> <p>Substitutes <math>z = 2 - 2i</math> and <math>z^2 = -8i</math> into quadratic and equates real and imaginary parts to obtain <math>2p + q = 0</math> and <math>-2p - 8 = 0</math></p> <p>Attempts to solve simultaneous equations to obtain <math>p = -4</math> and <math>q = 8</math></p>	<p>M1</p> <p>M1A1</p>
ALT 2	<p>(c)</p> <p>Attempts to obtain <math>p = -</math> sum of roots</p> <p>Attempts product of roots to obtain <math>q =</math></p> <p>Equation is <math>x^2 - 4x + 8(= 0)</math> or <math>p = -4</math> and <math>q = 8</math></p>	<p>M1</p> <p>M1</p> <p>A1</p>
ALT 3	<p>(c) <math>x - 2 = \pm 2i</math> either sign acceptable</p> <p><math>(x - 2)^2 = -4 \Rightarrow x^2 - 4x + 4 = -4</math> i.e square and attempt to expand to give 3-term quadratic</p> <p>Equation is <math>x^2 - 4x + 8(= 0)</math> or <math>p = -4</math> and <math>q = 8</math></p>	<p>M1</p> <p>M1</p> <p>A1</p>

**Notes**

(a) M1: Multiplies numerator and denominator by  $1 - i$  or by  $-1 + i$

A1: cao

(b) M1: Squares their  $z$ , or the given  $z = \frac{4}{1+i}$ , to produce at least 3 terms which can be implied by the correct answer.

A1:  $-8i$  or  $0 - 8i$  only

(c) M1: Uses their  $z$  and  $z^*$  in  $(x - z)(x - z^*)$

M1: Multiplies two factors and obtains  $p =$  or  $q =$

A1: Both correct required – can be implied by  $x^2 - 4x + 8$

ALT 1

(c) M1: Substitutes their  $z$  and their  $z^2$  into the quadratic and equates real and imaginary parts to obtain two equations in  $p$  and  $q$

M1: Attempts to solve for one unknown to obtain  $p =$  or  $q =$

A1: Both correct required – can be implied by  $x^2 - 4x + 8(= 0)$



Question Number	Scheme	Marks
5.	<p>(a) Gradient = <math>\frac{2ap - 2aq}{ap^2 - aq^2}</math> seen</p> <p><math>\left( \frac{2ap - 2aq}{ap^2 - aq^2} = \frac{2a(p - q)}{a(p - q)(p + q)} \right) = \frac{2}{p + q}</math> or seen in an equation</p> <p>uses <math>y - y_1 = m(x - x_1)</math> to give <math>(y - 2aq) = "m"(x - aq^2)</math> or equivalent with <math>p</math></p> <p>or uses <math>y = mx + c</math> to give <math>y = "m"x + c</math> and substitute a point to find <math>c</math></p> <p>or uses <math>\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}</math> to give <math>\frac{(y - 2aq)}{2ap - 2aq} = \frac{(x - aq^2)}{ap^2 - aq^2}</math> or equivalent with <math>p</math></p> <p>so <math>(y - 2aq) = \frac{2}{p + q}(x - aq^2)</math> or <math>(y - 2ap) = \frac{2}{p + q}(x - ap^2)</math> or <math>y = \frac{2}{p + q}x + \frac{2apq}{p + q}</math> or</p> <p><math>\frac{(y - 2aq)}{2a} = \frac{(x - aq^2)}{a(p + q)}</math></p> <p>See <math>2aq^2</math> or <math>2ap^2</math> term appear and disappear to give <math>y(p + q) = 2x + 2apq</math> *</p> <p>(b) Substitute <math>(a, 0)</math> into line equation, to give <math>0 = 2a + 2apq</math> so <math>pq = -1</math></p> <p>(c) <math>y = 2a^{\frac{1}{2}}x^{\frac{1}{2}} \Rightarrow \frac{dy}{dx} = a^{\frac{1}{2}}x^{-\frac{1}{2}}</math> or <math>y^2 = 4ax \Rightarrow 2y \frac{dy}{dx} = 4a</math> or</p> <p><math>\frac{dy}{dx} = \frac{dy}{dp} \times \frac{dp}{dx} \Rightarrow \frac{dy}{dx} = 2a \times \frac{1}{2ap}</math></p> <p>So at <math>P</math> tangent gradient = <math>\frac{1}{p}</math></p> <p>(d) At <math>Q</math> tangent gradient = <math>\frac{1}{q}</math></p> <p><math>\frac{1}{p} \times \frac{1}{q} = \frac{1}{pq} = \frac{1}{-1} = -1</math> with at least one intermediate step, the tangents are <b>perpendicular</b> or at <b>right angles</b></p>	<p>B1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>A1 cso (5)</p> <p>B1 (1)</p> <p>M1</p> <p>A1 (2)</p> <p>B1</p> <p>B1cso (2)</p> <p>(10 marks)</p>

Notes

(a) B1: Correct statement for gradient (isw) B1:  $\frac{2}{p + q}$  - can be seen later in the solution.

M1: use of a correct formula for a line equation through  $P$  or  $Q$  with their gradient. Must be finding a chord, not a tangent or a normal.

A1: for a correct line equation with simplified gradient in any equivalent form

A1: cso (as given answer)

(b) B1: For using  $(a, 0)$  to show that  $pq = -1$

(c) M1: Use calculus to find an expression for  $dy/dx$  and substitute coordinates of  $P$ . They may use chord gradient and let  $p$  tend to  $q$ .

(d) B1:  $1/q$  seen B1:  $\frac{1}{p} \times \frac{1}{q} = -1$  or  $\frac{1}{p} = -\frac{1}{1/q}$  or  $\frac{1}{q} = -\frac{1}{1/p}$  and at least words in bold with no errors seen.

Leave blank

6.

$$\mathbf{P} = \begin{pmatrix} -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

- (a) Describe fully the single geometrical transformation  $U$  represented by the matrix  $\mathbf{P}$ . (2)

The transformation  $U$  maps the point  $A$ , with coordinates  $(p, q)$ , onto the point  $B$ , with coordinates  $(6\sqrt{2}, 3\sqrt{2})$ .

- (b) Find the value of  $p$  and the value of  $q$ . (3)

The transformation  $V$ , represented by the  $2 \times 2$  matrix  $\mathbf{Q}$ , is a reflection in the line with equation  $y = x$ .

- (c) Write down the matrix  $\mathbf{Q}$ . (1)

The transformation  $U$  followed by the transformation  $V$  is the transformation  $T$ . The transformation  $T$  is represented by the matrix  $\mathbf{R}$ .

- (d) Find the matrix  $\mathbf{R}$ . (3)

- (e) Deduce that the transformation  $T$  is self-inverse. (1)

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



Question Number	Scheme	Marks
6.	<p>(a) <b>Rotation</b>, 135 degrees or <math>\frac{3\pi}{4}</math> radians (anticlockwise) about <math>O</math> or 225 degrees or <math>\frac{5\pi}{4}</math> clockwise about <math>O</math>.</p> <p>(b) <math display="block">\begin{pmatrix} -\frac{1}{\sqrt{2}} &amp; -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} &amp; -\frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} 6\sqrt{2} \\ 3\sqrt{2} \end{pmatrix}</math> <math display="block">-p - q = 12 \text{ and } p - q = 6 \text{ or equivalent}</math> <math display="block">p = -3 \text{ and } q = -9 \text{ or } \begin{pmatrix} -3 \\ -9 \end{pmatrix}</math> <p>ALT Uses Inverse matrix <math>\mathbf{P}^{-1}</math> with vector <math>= \begin{pmatrix} -\frac{1}{\sqrt{2}} &amp; \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} &amp; -\frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 6\sqrt{2} \\ 3\sqrt{2} \end{pmatrix}</math> <math display="block">p = -3 \text{ and } q = -9 \text{ or } \begin{pmatrix} -3 \\ -9 \end{pmatrix}</math></p> <p>(c) <math>\mathbf{Q} = \begin{pmatrix} 0 &amp; 1 \\ 1 &amp; 0 \end{pmatrix}</math></p> <p>Accept <math>\mathbf{T}</math> if used instead of <math>\mathbf{R}</math></p> <p>(d) <math>\mathbf{R} = \mathbf{Q}\mathbf{P} = \begin{pmatrix} 0 &amp; 1 \\ 1 &amp; 0 \end{pmatrix} \begin{pmatrix} -\frac{1}{\sqrt{2}} &amp; -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} &amp; -\frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} &amp; -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} &amp; -\frac{1}{\sqrt{2}} \end{pmatrix}</math></p> <p>(e) <math>\mathbf{R}^{-1} = \frac{1}{-1} \begin{pmatrix} -\frac{1}{\sqrt{2}} &amp; +\frac{1}{\sqrt{2}} \\ +\frac{1}{\sqrt{2}} &amp; +\frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} &amp; -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} &amp; -\frac{1}{\sqrt{2}} \end{pmatrix}</math></p> <p><math>= \mathbf{R}</math> (so matrix is self inverse and so transformation is self inverse)</p> <p>ALT 1 (e) <math>\mathbf{R}\mathbf{R} = \begin{pmatrix} \frac{1}{\sqrt{2}} &amp; -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} &amp; -\frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} &amp; -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} &amp; -\frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} 1 &amp; 0 \\ 0 &amp; 1 \end{pmatrix}</math> (so <math>\mathbf{R}</math> is self inverse and so transformation is self inverse)</p> <p>ALT 2 (e) Matrix represents a reflection (so is self inverse)</p> </p>	<p>M1, A1 (2)</p> <p>M1A1 B1 cso (3)</p> <p>M1A1 B1 cso (3)</p> <p>B1 (1)</p> <p>M1 A1 A1 (3)</p> <p>B1 (1) <b>10 marks</b></p> <p>B1</p> <p>B1</p>
<b>Notes</b>		
<p>(a) M1: Rotation only A1: 135 degrees about <math>O</math> SC: 135 degrees about <math>O</math> only award M1A0.</p> <p>(b) M1: Multiplies matrices in correct order to obtain two equations in <math>p</math> and <math>q</math>. A1: Two correct equations B1 cso: <math>p</math> and <math>q</math> both correct, may be in vector form. No errors seen in solution. ALT (b) M1: Attempt to find Inverse Matrix and pre-multiply A1: Correct Inverse Matrix used B1 cso: <math>p</math> and <math>q</math> both correct, may be in vector form. No errors seen in solution.</p> <p>(d) M1: Sets matrix product correct way round and obtains one correct term for their <math>\mathbf{Q}</math> A1: Two correct terms from a correct <math>\mathbf{Q}</math>. <math>\mathbf{Q}</math> incorrect award A0 here. A1: Completely correct matrix</p> <p>(e) B1: Calculates <math>\mathbf{R}^{-1}</math> and indicates that <math>\mathbf{R}^{-1} = \mathbf{R}</math> or calculates <math>\mathbf{R}^2</math> and indicates that <math>\mathbf{R}^2 = \mathbf{I}</math> or states that <math>\mathbf{R}</math> represents a reflection.</p>		



Question Number	Scheme	Marks
7.	<p>(a) <math>z^2 = (a + 2i)(a + 2i) = (a^2 - 4) + 4ia</math>                      So <math>z^2 + 2z = (a^2 - 4 + 2a) + i(4a + 4)</math> or <math>x = (a^2 + 2a - 4)</math> and <math>y = 4a + 4</math></p> <p>(b) and so <math>4a + 4 = 0 \rightarrow a = -1</math></p> <p>ALT (b)                      Substitute <math>a = -1</math> and show that <math>y = 0</math></p> <p>(c) <math> z  = \sqrt{5}</math> or awrt 2.24  <math>\arctan(-2) = 2.03</math></p> <p>(d)</p> <p>(e) <math>OP</math> and <math>QR</math> are parallel, and <math>QR</math> is twice the length of <math>OP</math>                      Or Enlargement with Scale Factor 2 (centre <math>O</math>), followed by translation <math>\begin{pmatrix} -3 \\ -4 \end{pmatrix}</math>                      Or Enlargement with Scale Factor 2, centre <math>(3, 4)</math> or centre <math>3 + 4i</math>  <math>\overline{QR} = 2\overline{OP}</math> with clear indication of vectors award B1B1, without vectors award B0B1</p>	<p>M1                      M1 A1 A1                      (4)</p> <p>B1                      (1)</p> <p>B1</p> <p>B1</p> <p>M1, A1 cao                      (3)</p> <p>M1 A1                      B1ft                      (3)</p> <p>B1, B1                      (2)</p> <p><b>13 marks</b></p>

**Notes:**

- (a) M1: Squares  $z$  to produce at least 3 terms which can be implied by the correct answer.  
 M1: Adds  $2z$  to their  $z^2$   
 A1: Correct  $x$  A1 Correct  $y$  accept  $4ai+4i$
- (b) B1: Completely accurate cao
- (c) B1:  $\sqrt{5}$  or 2.24 or awrt 2.24  
 M1 for using  $\tan$  or  $\arctan$   
 A1 cao 2.03
- (d) M1: **Either** their  $OP$  in the correct quadrant labelled  $P$  or  $z$  or their  $-1 + 2i$  or their  $(-1, 2)$  or axes labelled **or** their  $OQ$  in the correct quadrant labelled  $Q$  or  $z^2$  or their  $-3 - 4i$  or their  $(-3, -4)$  or axes labelled  
 A1: Both  $OP$  and  $OQ$  correct i.e. in the 2<sup>nd</sup> and 3<sup>rd</sup> quadrants respectively.  
 B1ft:  $OR : z^2 + 2z (= -5)$  **on real axis** to left of the origin.  
 Accept points or lines. Arrows not required. Axes need not be labelled Re and Im.  
 Treat correct quadrant (or on axis) as important aspect for accuracy, lengths of lines if present can be accepted as correct.

Leave blank

8. (i) Prove by induction that, for  $n \in \mathbb{Z}^+$

$$\sum_{r=1}^n \frac{2r+1}{r^2(r+1)^2} = 1 - \frac{1}{(n+1)^2} \quad (5)$$

(ii) A sequence of positive rational numbers is defined by

$$\begin{aligned} u_1 &= 3 \\ u_{n+1} &= \frac{1}{3}u_n + \frac{8}{9}, \quad n \in \mathbb{Z}^+ \end{aligned}$$

Prove by induction that, for  $n \in \mathbb{Z}^+$

$$u_n = 5 \times \left(\frac{1}{3}\right)^n + \frac{4}{3} \quad (5)$$

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA





Question Number	Scheme	Marks
8.	<p>(i) If <math>n=1</math>, <math>\sum_{r=1}^n \frac{2r+1}{r^2(r+1)^2} = \frac{3}{4}</math> <b>and</b> <math>1 - \frac{1}{(n+1)^2} = \frac{3}{4}</math>, so true for <math>n = 1</math>.</p> <p>Assume result true for <math>n = k</math> <b>and</b> consider <math>\sum_{r=1}^{k+1} \frac{2r+1}{r^2(r+1)^2} = 1 - \frac{1}{(k+1)^2} + \frac{2(k+1)+1}{(k+1)^2(k+2)^2}</math></p> $= 1 - \left( \frac{(k+2)^2}{(k+1)^2(k+2)^2} - \frac{2(k+1)+1}{(k+1)^2(k+2)^2} \right) = 1 - \left( \frac{(k^2+2k+1)}{(k+1)^2(k+2)^2} \right)$ $= 1 - \left( \frac{(k+1)^2}{(k+1)^2(k+2)^2} \right) = 1 - \left( \frac{1}{(k+1+1)^2} \right)$ <p><b>True</b> for <math>n = k + 1</math> if <b>true</b> for <math>n = k</math>, ( and <b>true</b> for <math>n = 1</math>) so <b>true</b> by induction for all <math>n \in \mathbb{Z}^+</math></p>	<p>B1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1cso</p> <p>(5)</p>
	<p>(ii) <math>n = 1</math>: <math>u_1 = 5 \times \left(\frac{1}{3}\right)^1 + \frac{4}{3} = 3</math> so expression for <math>u_n</math> true for <math>n = 1</math></p> <p>Assume result true for <math>n = k</math> <b>and</b> consider <math>u_{k+1} = \frac{1}{3} \left( 5 \times \left(\frac{1}{3}\right)^k + \frac{4}{3} \right) + \frac{8}{9}</math></p> <p>Obtain <math>u_{k+1} = 5 \times \left(\frac{1}{3}\right)^{k+1} + \frac{4}{9} + \frac{8}{9}</math></p> $5 \times \left(\frac{1}{3}\right)^{k+1} + \frac{4}{3}$ <p>and deduce that result is true for <math>n = k + 1</math></p> <p><b>True</b> for <math>n = k + 1</math> if <b>true</b> for <math>n = k</math>, ( and <b>true</b> for <math>n = 1</math>) so <b>true</b> by induction for all <math>n \in \mathbb{Z}^+</math></p>	<p>B1</p> <p>M1</p> <p>A1</p> <p>dM1</p> <p>A1 cso</p> <p>(5)</p> <p><b>10 marks</b></p>

**Notes:**

(i) B1: Checks  $n = 1$  on both sides **and** states true for  $n = 1$  seen anywhere

M1: (Assumes true for)  $n = k$  **and** adds  $(k+1)^{\text{th}}$  term to sum of  $k$  terms

A1:  $1 - \left( \frac{(k^2+2k+1)}{(k+1)^2(k+2)^2} \right)$  seen (linked to 2<sup>nd</sup> M)

M1:  $(k+1)^2(k+2)^2$  attempted as common denominator of two fractions.

A1cso: Makes correct complete induction statement including at least statements in bold. Accept  $n \geq 1$  or  $n = 1, 2, 3, \dots$  or all positive Integers or all  $n$ . Statement true for  $n = 1$  here could contribute to B1 mark earlier.

(ii) B1: Checks  $n = 1$  in  $u_n$  and states true for  $n = 1$  seen anywhere.

M1: (Assumes result for)  $n = k$  **and** substitutes  $u_k$  into correct expression for  $u_{k+1}$

A1:  $\frac{4}{9} + \frac{8}{9}$  or  $\frac{1}{3} \cdot \frac{4}{3} + \frac{8}{9}$  seen

dM1: Obtains  $5 \times \left(\frac{1}{3}\right)^{k+1} + \frac{4}{3}$  and statement true for  $n = k + 1$  or equivalent seen anywhere dependent on previous M.

A1cso: Makes correct complete induction statement including at least statements in bold. Accept  $n \geq 1$  or  $n = 1, 2, 3, \dots$  or all positive Integers or all  $n$ . Statement true for  $n = 1$  here could contribute to B1 mark earlier.

Leave blank

9. The rectangular hyperbola,  $H$ , has cartesian equation  $xy = 25$

(a) Show that an equation of the normal to  $H$  at the point  $P\left(5p, \frac{5}{p}\right)$ ,  $p \neq 0$ , is

$$y - p^2x = \frac{5}{p} - 5p^3 \tag{5}$$

This normal meets the line with equation  $y = -x$  at the point  $A$ .

(b) Show that the coordinates of  $A$  are

$$\left(-\frac{5}{p} + 5p, \frac{5}{p} - 5p\right) \tag{3}$$

The point  $M$  is the midpoint of the line segment  $AP$ .  
Given that  $M$  lies on the positive  $x$ -axis,

(c) find the exact value of the  $x$  coordinate of point  $M$ . (3)

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

DO NOT WRITE IN THIS AREA



Question Number	Scheme	Marks
9 (a)	$y = \frac{25}{x} \Rightarrow \frac{dy}{dx} = -25x^{-2},$ <p><b>or</b> <math>y + x \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{y}{x}</math> <b>or</b> <math>\dot{x} = 5, \dot{y} = -\frac{5}{t^2}</math> so <math>\frac{dy}{dx} = -\frac{1}{t^2}</math></p> <p>and at <math>P</math> <math>\frac{dy}{dx} = -\frac{1}{p^2}</math> so gradient of normal is <math>p^2</math></p> <p><b>Either</b> <math>y - \frac{5}{p} = p^2(x - 5p)</math> <b>or</b> <math>y = p^2x + k</math> and use <math>x = 5p, y = \frac{5}{p}</math></p> $\Rightarrow y - p^2x = \frac{5}{p} - 5p^3 \quad (*)$	<p>B1</p> <p>M1 A1</p> <p>M1</p> <p>A1cso (5)</p>
(b)	<p>At the point <math>A: y + p^2y = \frac{5}{p} - 5p^3</math> <b>or</b> <math>-x - p^2x = \frac{5}{p} - 5p^3</math></p> $y(1 + p^2) = \frac{5}{p}(1 - p^4) \quad \text{or} \quad -x(1 + p^2) = \frac{5}{p}(1 - p^4)$ $y = \frac{\frac{5}{p}(1 - p^2)(1 + p^2)}{(1 + p^2)} = \frac{5}{p}(1 - p^2) = \frac{5}{p} - 5p \quad \text{or} \quad x = \frac{-\frac{5}{p}(1 - p^2)(1 + p^2)}{(1 + p^2)} = \frac{-5}{p}(1 - p^2) = \frac{-5}{p} + 5p^*$ <p>so <math>x = -\frac{5}{p}(1 - p^2) = -\frac{5}{p} + 5p</math> and <math>y = \frac{5}{p} - 5p^*</math></p>	<p>M1</p> <p>M1</p> <p>A1cso (3)</p>
(c)	<p><math>M</math> has coordinates <math>\left(-\frac{5}{2p} + 5p, \frac{5}{p} - \frac{5p}{2}\right)</math> o.e.</p> <p>So when <math>y = 0, \frac{5}{p} - \frac{5p}{2} = 0</math> and <math>p = \sqrt{2}</math> so <math>M</math> has <math>x</math> coordinate <math>\frac{15}{4}\sqrt{2}</math> o.e.</p>	<p>B1</p> <p>M1 A1 (3)</p> <p><b>11 marks</b></p>

Notes

- (a) B1: Any correct expression for gradient of tangent
- M1: Substitutes values into **derived expression using calculus** to give gradient of normal at  $P$
- A1: cao. Can be implied by use in equation of a straight line
- M1: Use of formula for the equation of a straight line with their changed gradient
- A1: cso
- (b) M1: Replaces  $x$  by  $-y$  or  $y$  by  $-x$
- M1: Factorises  $(1 - p^4)$  to simplify answer in first variable
- A1 cso: Obtains both  $x$  and  $y$
- ALT (b) Accept Verification.
- M1: Substitutes the coordinates of  $A$  into the equation of the normal
- M1: Substitutes the coordinates of  $A$  into both the normal and  $y = -x$ .
- A1 cso: No errors seen
- (c) B1: Correct  $x$ - coordinate of midpoint (may be implied) and correct  $y$  coordinate, accept equivalent forms
- M1: Puts their  $y = 0$  and finds value for  $p$  to use in  $x =$
- A1:  $+\frac{15}{4}\sqrt{2}$  or equivalent only