Write your name here

Surname	Other names

Pearson Edexcel GCE

Centre Number	Candidate Number

Further Pure Mathematics FP1

Advanced/Advanced Subsidiary

Friday 19 May 2017 – Morning
Time: 1 hour 30 minutes

Paper Reference
6667/01

You must have:
Mathematical Formulae and Statistical Tables (Pink)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

Pearson

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1. $$f(x) = \frac{1}{3}x^2 + \frac{4}{x^2} - 2x - 1, \quad x > 0$$

(a) Show that the equation $f(x) = 0$ has a root α in the interval $[6, 7]$

(2)

(b) Taking 6 as a first approximation to α, apply the Newton-Raphson process once to $f(x)$ to obtain a second approximation to α. Give your answer to 2 decimal places.

(5)

Question Number	Scheme		Marks
1.	$f(x) = \frac{1}{3}x^2 + \frac{4}{x^2} - 2x - 1,\ x > 0$		
(a)	$f(6) = -0.88888888...$	Either any one of $f(6) =$ awrt -0.9 or	M1
	$f(7) = 1.414965986...$	$f(7) =$ awrt 1.4	
	Sign change or $f(6) = -$ve and $f(7) = +$ve or $f(6) \times f(7) = -$ve o.e. (and $f(x)$ is continuous) therefore a root / α (exists between $x = 6$ and $x = 7$) o.e.	Both $f(6) =$ awrt -0.9 and $f(7) =$ awrt 1.4, sign change and conclusion. Allow $f(6) = -\frac{8}{9}$ and $f(7) = \frac{208}{147}$.	A1
			[2]
(b)	$f'(x) = \frac{2}{3}x - \frac{8}{x^3} - 2$	$\frac{1}{3}x^2 \to \pm Ax$ or $\frac{4}{x^2} \to \pm Bx^{-3}$ or $-2x - 1 \to -2$	M1
		At least two of these terms differentiated correctly.	A1
		Correct derivative.	A1
	$\{f'(6) = 1.962962963...\}$	$f'(6) = \frac{53}{27}$	
	$\alpha \simeq 6 - \left(\frac{"-0.88888888..."}{"1.962962963..."}\right)$	Correct application of Newton-Raphson using their values.	M1
	$= 6.452830189...$	Exact form of α is $\frac{342}{53}$	
	$= 6.45\ (2\text{dp})$	6.45	A1 **cso**
			[5]
			7

	Question 1 Notes	
1. (a)	**Note**	Accept at least 'sign change therefore root' o.e. for A1. Any incorrect statements made in the conclusion award A0.
(b)	**Note**	Denominator in NR calculation may contain evidence for first 3 marks. Correct answer of 6.45 with minimal working will imply earlier marks for elements not explicitly stated. However, incorrect values leading to a correct final answer should be marked accordingly.

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2.

$$\mathbf{A} = \begin{pmatrix} 2 & -1 \\ 4 & 3 \end{pmatrix}, \quad \mathbf{P} = \begin{pmatrix} 3 & 6 \\ 11 & -8 \end{pmatrix}$$

(a) Find $\mathbf{A}^{-1}$

(2)

The transformation represented by the matrix **B** followed by the transformation represented by the matrix **A** is equivalent to the transformation represented by the matrix **P**.

(b) Find **B**, giving your answer in its simplest form.

(3)

Question Number	Scheme		Marks
2.	$\mathbf{A}=\begin{pmatrix}2 & -1\\4 & 3\end{pmatrix}$, $\mathbf{P}=\begin{pmatrix}3 & 6\\11 & -8\end{pmatrix}$		
(a)	$\mathbf{A}^{-1}=\frac{1}{10}\begin{pmatrix}3 & 1\\-4 & 2\end{pmatrix}$	Either $\frac{1}{10}$ or $\begin{pmatrix}3 & 1\\-4 & 2\end{pmatrix}$	M1
		Correct matrix seen.	A1
			[2]
(b) **Way 1**	$\mathbf{P}=\mathbf{AB}$ $\Rightarrow\ \mathbf{A}^{-1}\mathbf{P}=\mathbf{A}^{-1}\mathbf{AB}\ \Rightarrow\ \mathbf{B}=\mathbf{A}^{-1}\mathbf{P}$		
	$\mathbf{B}=\frac{1}{10}\begin{pmatrix}3 & 1\\-4 & 2\end{pmatrix}\begin{pmatrix}3 & 6\\11 & -8\end{pmatrix}$	Multiplies **their** $\mathbf{A}^{-1}$ by $\mathbf{P}$ in correct order. This substituted statement is sufficient.	M1
	$=\begin{pmatrix}2 & 1\\1 & -4\end{pmatrix}$	At least 2 elements correct or $k\begin{pmatrix}20 & 10\\10 & -40\end{pmatrix}$ oe. May be unsimplified	A1
		Correct simplified matrix.	A1
			[3]
(b) **Way 2**	$\{\mathbf{P}=\mathbf{AB}\Rightarrow\}$ $\begin{pmatrix}3 & 6\\11 & -8\end{pmatrix}=\begin{pmatrix}2 & -1\\4 & 3\end{pmatrix}\begin{pmatrix}a & b\\c & d\end{pmatrix}$		
	$\begin{pmatrix}3 & 6\\11 & -8\end{pmatrix}=\begin{pmatrix}2a-c & 2b-d\\4a+3c & 4b+3d\end{pmatrix}$	Attempt to multiply **A** by **B** in the correct order and puts equal to **P**	M1
	$\Rightarrow a=2,\ c=1,\ b=1,\ d=-4$		
	So, $\mathbf{B}=\begin{pmatrix}2 & 1\\1 & -4\end{pmatrix}$	At least 2 elements are correct.	A1
		Correct matrix.	A1
			[3] **5**

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3. The rectangular hyperbola H has parametric equations

$$x = 4t, \quad y = \frac{4}{t} \qquad t \neq 0$$

The points P and Q on this hyperbola have parameters $t = \frac{1}{4}$ and $t = 2$ respectively.

The line l passes through the origin O and is perpendicular to the line PQ.

(a) Find an equation for l.

(3)

(b) Find a cartesian equation for H.

(1)

(c) Find the exact coordinates of the two points where l intersects H.
Give your answers in their simplest form.

(3)

Question Number	Scheme		Marks
3. (a)	$x = 4t,\ y = \frac{4}{t},\ t \neq 0$		
	$t = \frac{1}{4} \Rightarrow P(1, 16),\quad t = 2 \Rightarrow Q(8, 2)$	Coordinates for either P or Q are correctly stated. (Can be implied).	B1
	$m(PQ) = \frac{2-16}{8-1}\ \{= -2\}$	Finds the gradient of the chord PQ with $\frac{y_2 - y_1}{x_2 - x_1}$ then uses in $y = -\frac{1}{m}x$. Condone incorrect sign of gradient.	M1
	$m(l) = \frac{1}{2}$		
	So, $l: y = \frac{1}{2}x$ or $2y = x$	$y = \frac{1}{2}x$ or $2y = x$	A1 oe
			[3]
(b)	$xy = 16$ or $y = \frac{16}{x}$ or $x = \frac{16}{y}$	Correct Cartesian equation. Accept $\frac{4}{y} = \frac{x}{4}$ or $xy = 4^2$	B1 oe
			[1]
(c)	**Way 1**: $\frac{1}{2}x = \frac{16}{x}$ $\{x^2 = 32\}$ **Way 2**: $\frac{4}{t} = \frac{1}{2}(4t)$ $\{t^2 = 2\}$ **Way 3**: $2y = \frac{16}{y}$ $\{y^2 = 8\}$	Attempts to substitute their l into either their Cartesian equation or parametric equations of H	M1
	$(4\sqrt{2},\ 2\sqrt{2}),\ (-4\sqrt{2},\ -2\sqrt{2})$	At least one set of coordinates (simplified or un-simplified) or $x = \pm 4\sqrt{2},\ y = \pm 2\sqrt{2}$	A1
		Both sets of simplified coordinates. Accept written in pairs as $x = 4\sqrt{2},\ y = 2\sqrt{2}$ $x = -4\sqrt{2},\ y = -2\sqrt{2}$	A1
			[3] **7**

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4. (i) The complex number w is given by

$$w = \frac{p - 4\text{i}}{2 - 3\text{i}}$$

where p is a real constant.

(a) Express w in the form $a + b\text{i}$, where a and b are real constants. Give your answer in its simplest form in terms of p.

(3)

Given that $\arg w = \frac{\pi}{4}$

(b) find the value of p.

(2)

(ii) The complex number z is given by

$$z = (1 - \lambda\text{i})(4 + 3\text{i})$$

where λ is a real constant.

Given that

$$|z| = 45$$

find the possible values of λ.
Give your answers as exact values in their simplest form.

(3)

Question Number	Scheme		Marks
4. (i)		Mark (i)(a) and (i)(b) together.	
	$w = \dfrac{p-4\text{i}}{2-3\text{i}} \qquad \arg w = \dfrac{\pi}{4}$		
(a) **Way 1**	$w = \dfrac{(p-4\text{i})}{(2-3\text{i})} \times \dfrac{(2+3\text{i})}{(2+3\text{i})}$	Multiplies by $\dfrac{(2+3\text{i})}{(2+3\text{i})}$	M1
.	$= \left(\dfrac{2p+12}{13}\right) + \left(\dfrac{3p-8}{13}\right)\text{i}$	At least one of either the real or imaginary part of w is correct. Must be expanded but could be unsimplified e.g. expressed as single fraction. Condone $a+\text{i}b$.	A1
		Correct w in its simplest form.	A1
			[3]
(a) **Way 2**	$(a+\text{i}b)(2-3\text{i}) = (p-4\text{i})$		
	$2a+3b=p$ $3a-2b=4$	Multiplies out to obtain 2 equations in two unknowns.	M1
	$= \left(\dfrac{2p+12}{13}\right) + \left(\dfrac{3p-8}{13}\right)\text{i}$	At least one of either the real or imaginary part of w is correct. Must be expanded but could be unsimplified e.g. expressed as single fraction. Condone $a+\text{i}b$.	A1
		Correct w in its simplest form.	A1
			[3]
(b)	$\left\{\arg w = \dfrac{\pi}{4} \Rightarrow\right\} \quad 2p+12 = 3p-8$ o.e. seen anywhere.	Sets the numerators of the real part of their w equal to the imaginary part of their w or if arctan used, require evidence of $\tan\dfrac{\pi}{4} = 1$	M1
	$\Rightarrow p = 20$	$p = 20$	A1
			[2]
(ii) **Way 1**	$z = (1-\lambda\text{i})(4+3\text{i})$ and $\lvert z \rvert = 45$		
	$\sqrt{1+\lambda^2}\,\sqrt{4^2+3^2}$	Attempts to apply $\lvert(1-\lambda\text{i})(4+3\text{i})\rvert = \sqrt{1+\lambda^2}\,\sqrt{4^2+3^2}$	M1
	$\sqrt{1+\lambda^2}\,\sqrt{4^2+3^2} = 45$	Correct equation.	A1
	$\{\lambda^2 = 9^2 - 1 \Rightarrow\}\ \lambda = \pm 4\sqrt{5}$	$\lambda = \pm 4\sqrt{5}$	A1
			[3]
Way 2	$z = (4+3\lambda) + (3-4\lambda)\text{i}$ $\sqrt{(4+3\lambda)^2 + (3-4\lambda)^2}$	Attempt to multiply out, group real and imaginary parts and apply the modulus.	M1
	$(4+3\lambda)^2 + (3-4\lambda)^2 = 45^2$ or $\sqrt{(4+3\lambda)^2 + (3-4\lambda)^2} = 45$	Correct equation.	A1
	$\{16+24\lambda+9\lambda^2+9-24\lambda+16\lambda^2 = 2025\}$	Condone if middle terms in expansions not explicitly stated.	
	$\{25\lambda^2 = 2000 \Rightarrow\}\ \lambda = \pm 4\sqrt{5}$	$\lambda = \pm 4\sqrt{5}$	A1
			[3] 8

	Question 4 Notes	
(ii)	**M1**	Also allow $(1+\lambda^2)(4^2+3^2)$ for M1.
	M1	Also allow $(4+3\lambda)^2 + (3-4\lambda)^2$ for M1.

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5. (i)

$$\mathbf{A} = \begin{pmatrix} p & 2 \\ 3 & p \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} -5 & 4 \\ 6 & -5 \end{pmatrix}$$

where p is a constant.

(a) Find, in terms of p, the matrix **AB**

(2)

Given that

$$\mathbf{AB} + 2\mathbf{A} = k\mathbf{I}$$

where k is a constant and **I** is the 2×2 identity matrix,

(b) find the value of p and the value of k.

(4)

(ii)

$$\mathbf{M} = \begin{pmatrix} a & -9 \\ 1 & 2 \end{pmatrix}, \text{ where } a \text{ is a real constant}$$

Triangle T has an area of 15 square units.

Triangle T is transformed to the triangle T' by the transformation represented by the matrix **M**.

Given that the area of triangle T' is 270 square units, find the possible values of a.

(5)

Question Number	Scheme		Marks
5. (i)	$\mathbf{A}=\begin{pmatrix}p & 2\\ 3 & p\end{pmatrix}$, $\mathbf{B}=\begin{pmatrix}-5 & 4\\ 6 & -5\end{pmatrix}$, $\mathbf{M}=\begin{pmatrix}a & -9\\ 1 & 2\end{pmatrix}$	p, a are constants.	
(a)	$\{\mathbf{AB}\}=\begin{pmatrix}-5p+12 & 4p-10\\ -15+6p & 12-5p\end{pmatrix}$	At least 2 elements are correct.	M1
		Correct matrix.	A1
			[2]
(b)	$\{\mathbf{AB}+2\mathbf{A}=k\mathbf{I}\}$		
	$\begin{pmatrix}-5p+12 & 4p-10\\ -15+6p & 12-5p\end{pmatrix}+2\begin{pmatrix}p & 2\\ 3 & p\end{pmatrix}=k\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$	If 'simultaneous equations' used, allocate marks as below.	
	$\begin{pmatrix}-3p+12 & 4p-6\\ -9+6p & 12-3p\end{pmatrix}=\begin{pmatrix}k & 0\\ 0 & k\end{pmatrix}$		
	$"4p-10"+4=0$ or $"-15+6p"+6=0$ or $"-9+6p"="4p-6"$	Forms an equation in p	M1
	$\Rightarrow p=\frac{3}{2}$	$p=\frac{3}{2}$ o.e.	A1
	$k=-5\left(\frac{3}{2}\right)+12+2\left(\frac{3}{2}\right)\Rightarrow k=...$	Substitutes their $p=\frac{3}{2}$ into "their $(-5p+12)$"$+2p$ to find a value for k or eliminates p to find k.	M1
	$k=\frac{15}{2}$	$k=\frac{15}{2}$ oe	A1
			[4]
(ii) **Way 1**	$\pm\frac{270}{15}$ $\{=\pm 18\}$	Can be implied from calculations.	B1
	$\det\mathbf{M}=(a)(2)-(-9)(1)$	Applies $ad-bc$ to $\mathbf{M}$. Require clear evidence of correct formula being used for M1 if errors seen.	M1
	$\Rightarrow 2a+9=18$ or $2a+9=-18$	Equates their det $\mathbf{A}$ to either 18 or -18	M1
	$\Rightarrow a=4.5$ or $a=-13.5$	At least one of either $a=4.5$ or $a=-13.5$	A1
		Both $a=4.5$ and $a=-13.5$	A1
			[5]
(ii) **Way 2**	Consider vertices of triangle with area 15 units e.g. (0,0),(15,0) and (0,2) and attempting 2 values of a.		B1
	e.g. $\begin{pmatrix}a & -9\\ 1 & 2\end{pmatrix}\begin{pmatrix}0 & 15 & 0\\ 0 & 0 & 2\end{pmatrix}=\begin{pmatrix}0 & 15a & -18\\ 0 & 15 & 4\end{pmatrix}$	Pre-multiplies their matrix by $\mathbf{M}$ and obtains single matrix	M1
	e.g. $\frac{1}{2}\begin{vmatrix}0 & 15a & -18 & 0\\ 0 & 15 & 4 & 0\end{vmatrix}=270$	Equates their determinant to 270 and attempts to solve.	M1
	$\Rightarrow a=4.5$ or $a=-13.5$	At least one of either $a=4.5$ or $a=-13.5$	A1
		Both $a=4.5$ and $a=-13.5$	A1
			[5]
			11

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6. Given that 4 and 2i – 3 are roots of the equation

$$x^3 + ax^2 + bx - 52 = 0$$

where a and b are real constants,

(a) write down the third root of the equation,

(1)

(b) find the value of a and the value of b.

(5)

Question Number	Scheme		Marks
6.	$x^3+ax^2+bx-52=0$, $a, b\in\mathbb{R}$, 4 and $2\text{i}-3$ are roots		
(a)	$-2\text{i}-3$	$-2\text{i}-3$ seen anywhere in solution for Q6.	B1
			[1]
(b) **Way 1**	$(x-(2\text{i}-3))(x-"(-2\text{i}-3)"); = x^2+6x+13$ or $x=-3\pm 2\text{i} \Rightarrow (x+3)^2=-4$; $=x^2+6x+13(=0)$ $(x-4)(x-(2\text{i}-3)); = x^2-(1+2\text{i})x+4(2\text{i}-3)$ $(x-4)(x-"(-2\text{i}-3)"); = x^2-(1-2\text{i})x+4(-2\text{i}-3)$	Must follow from their part (a). Any incorrect signs for their part (a) in initial statement award M0; accept any equivalent expanded expression for A1.	M1; A1
	$(x-4)(x^2+6x+13)$ $\{=x^3+ax^2+bx-52\}$	$(x-3^{\text{rd}}\text{ root})(\text{their quadratic})$. Could be found by comparing coefficients from long division.	M1
	$a=2$, $b=-11$ or $x^3+2x^2-11x-52$	At least one of $a=2$ or $b=-11$	A1
		Both $a=2$ and $b=-11$	A1
			[5]
(b) **Way 2**	Sum $=(2\text{i}-3)+"(-2\text{i}-3)"=-6$ Product $=(2\text{i}-3)\times"(-2\text{i}-3)"=13$ So quadratic is $x^2+6x+13$	Attempts to apply either $x^2-(\text{sum roots})x+(\text{product roots})=0$ or $x^2-2\text{Re}(\alpha)x+\lvert\alpha^2\rvert=0$	M1
		$x^2+6x+13$	A1
	$(x-4)(x^2+6x+13)$ $\{=x^3+ax^2+bx-52\}$	$(x-3^{\text{rd}}\text{ root})(\text{their quadratic})$	M1
	$a=2$, $b=-11$ or $x^3+2x^2-11x-52$	At least one of $a=2$ or $b=-11$	A1
		Both $a=2$ and $b=-11$	A1
			[5]
(b) **Way 3**	$(2\text{i}-3)^3+a(2\text{i}-3)^2+b(2\text{i}-3)-52=0$ $5a-3b=43$ (real parts) and $6a-b=23$ (imaginary parts) or uses f(4) = 0 and f(a complex root) = 0 to form equations in *a* and *b*.	Substitutes $2\text{i}-3$ into the displayed equation and equates both real and imaginary parts.	M1
		$5a-3b=43$ and $6a-b=23$ **or** $16a+4b=-12$ and $(2i-3)^3+a(2i-3)^2+b(2i-3)-52=0$ / $(-2i-3)^3+a(-2i-3)^2+b(-2i-3)-52=0$	A1
	So $a=2, b=-11$ or $x^3+2x^2-11x-52$	Solves these equations simultaneously to find at least one of either $a=...$ or $b=...$	M1
		At least one of $a=2$ or $b=-11$	A1
		Both $a=2$ and $b=-11$	A1
			[5]
(b) **Way 4**	$b=$ sum of product pairs	Attempts sum of product pairs.	M1
	$=4(2\text{i}-3)+4"(-2\text{i}-3)"+(2\text{i}-3)"(-2\text{i}-3)"$	All pairs correct o.e.	A1
	$a=-(\text{sum of 3 roots})=-(4+2\text{i}\ -3"-2\text{i}-3")$	Adds up all 3 roots	M1
	$a=2$, $b=-11$ or $x^3+2x^2-11x-52$	At least one of $a=2$ or $b=-11$	A1
		Both $a=2$ and $b=-11$	A1
			[5]

(b) **Way 5**	Uses f(4) = 0	M1
	$16a+4b=-12$	A1
	$a=-(\text{sum of 3 roots})=-(4+2\text{i} \ -3"-2\text{i}-3")$ — Adds up all 3 roots	M1
	$a=2,\ b=-11$ or $x^3+2x^2-11x-52$ — At least one of $a=2$ or $b=-11$	A1
	Both $a=2$ and $b=-11$	A1
		[5]
		6

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7. The parabola C has equation $y^2 = 4ax$, where a is a constant and $a > 0$
The point $Q(aq^2, 2aq)$, $q > 0$, lies on the parabola C.

(a) Show that an equation of the tangent to C at Q is

$$qy = x + aq^2$$

(4)

The tangent to C at the point Q meets the x-axis at the point $X\left(-\frac{1}{4}a, 0\right)$ and meets the directrix of C at the point D.

(b) Find, in terms of a, the coordinates of D.

(4)

Given that the point F is the focus of the parabola C,

(c) find the area, in terms of a, of the triangle FXD, giving your answer in its simplest form.

(2)

Questin Number	Scheme		Marks
7.	$y^2 = 4ax$, at $Q(aq^2, 2aq)$		
(a)	$y = 2\sqrt{a}\,x^{\frac{1}{2}} \Rightarrow \dfrac{dy}{dx} = \sqrt{a}\,x^{-\frac{1}{2}}$ or $2y\dfrac{dy}{dx} = 4a$ or $\dfrac{dy}{dx} = 2a \times \dfrac{1}{2aq}$	$\dfrac{dy}{dx} = \pm k\,x^{-\frac{1}{2}}$ or $ky\dfrac{dy}{dx} = c$ or $\dfrac{\text{their } \frac{dy}{dq}}{\text{their } \frac{dx}{dq}}$	M1
	When $x = aq^2$, $m_T = \dfrac{dy}{dx} = \dfrac{\sqrt{a}}{\sqrt{aq^2}} = \dfrac{\sqrt{a}}{\sqrt{a}\,q} = \dfrac{1}{q}$	$\dfrac{dy}{dx} = \dfrac{1}{q}$	A1
	or when $y = 2aq$, $m_T = \dfrac{dy}{dx} = \dfrac{4a}{2(2aq)} = \dfrac{1}{q}$		
	T: $y - 2aq = \dfrac{1}{q}(x - aq^2)$	Applies $y - 2aq = (\text{their } m_T)(x - aq^2)$ or $y = (\text{their } m_T)x + c$ and an attempt to find c with gradient from calculus.	dM1
	T: $qy - 2aq^2 = x - aq^2$		
	T: $qy = x + aq^2$ *	**cso**	A1 *
			[4]
(b)	$X\left(-\frac{1}{4}a, 0\right) \Rightarrow 0 = -\frac{1}{4}a + aq^2$	Substitutes $x = -\frac{1}{4}a$ and $y = 0$ into **T**	M1
	$\Rightarrow \left\{q^2 = \dfrac{1}{4} \Rightarrow q = -\dfrac{1}{2} \text{ (reject)}\right\}$ $q = \dfrac{1}{2}$	$q = \dfrac{1}{2}$ oe	A1
	So, $\dfrac{1}{2}y = -a + a\left(\dfrac{1}{2}\right)^2$	Substitutes their "$q = \frac{1}{2}$" and $x = -a$ in **T** or finds $y_D = \dfrac{1}{q}(-a + aq^2)$	M1
	giving, $y = -\dfrac{3a}{2}$. So $D\left(-a, -\frac{3}{2}a\right)$ o.e.	$D\left(-a, -\frac{3}{2}a\right)$ o.e.	A1
			[4]
(c)	$\{\text{focus } F(a, 0)\}$		
Way 1	$\text{Area}(FXD) = \dfrac{1}{2}\left(\dfrac{5a}{4}\right)\left(\dfrac{3a}{2}\right) = \dfrac{15a^2}{16}$	Applies $\dfrac{1}{2}(\text{their } \lvert FX \rvert)(\text{their } \lvert y_D \rvert)$. If their $\left\lvert y_D = \dfrac{1}{q}(-a + aq^2) \right\rvert$ then require an attempt to sub for q to award M.	M1
		$\dfrac{15a^2}{16}$ or $0.9375a^2$	A1 cso
			[2]

(c) **Way 2**	$\text{Area}(FXD) = \frac{1}{2}\begin{vmatrix} a & -\frac{1}{4}a & -a & a \\ 0 & 0 & -\frac{3}{2}a & 0 \end{vmatrix}$		
	$= \frac{1}{2}\left\lvert\left(0+\frac{3}{8}a^2+0\right)-\left(0+0-\frac{3}{2}a^2\right)\right\rvert = \frac{15}{16}a^2$	A correct attempt to apply the shoelace method.	M1
		$\frac{15a^2}{16}$ or $0.9375a^2$	A1cao
			[2]
(c) **Way 3**	Rectangle – triangle 1 – triangle 2		
	$= 2a.\frac{3a}{2} - \frac{1}{2}.\frac{3a}{4}.\frac{3a}{2} - \frac{1}{2}.2a.\frac{3a}{2} = 3a^2 - \frac{9a^2}{16} - \frac{3a^2}{2}$		M1
		$\frac{15a^2}{16}$ or $0.9375a^2$	A1cao
(c) **Way 4**	Attempts sine rule using appropriate choice from $FX = \frac{5a}{4},\ FD = \frac{5a}{2}, DX = \frac{3\sqrt{5}a}{4},\ \sin F = \frac{3}{5}, \sin X = \frac{2}{\sqrt{5}}$	Uses Area $= \frac{1}{2}ab\sin C$	M1
		$\frac{15a^2}{16}$ or $0.9375a^2$	A1cao
			10

	Question 7 Notes
(c) **Way 1**	Do not award M1 if area of wrong triangle found e.g. $\frac{1}{2}.2a.\frac{3a}{2} = \frac{3a^2}{2}$

Leave blank

8. (a) Use the standard results for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$ to show that

$$\sum_{r=1}^{n}(3r^2 + 8r + 3) = \frac{1}{2}n(2n + 5)(n + 3)$$

for all positive integers n.

(5)

Given that

$$\sum_{r=1}^{12}\left(3r^2 + 8r + 3 + k(2^{r-1})\right) = 3520$$

(b) find the exact value of the constant k.

(4)

Question Number	Scheme		Marks
8. (a)	$\sum_{r=1}^{n}\left(3r^2+8r+3\right)$		
	$=\frac{3}{6}n(n+1)(2n+1)+\frac{8}{2}n(n+1)+3n$	An attempt to use at least one of the correct standard formulae for first two terms.	M1
		Correct first two terms.	A1
		$3\to 3n$	B1
	$=\frac{1}{2}n(n+1)(2n+1)+4n(n+1)+3n$		
	$=\frac{1}{2}n\left((2n+1)(n+1)+8(n+1)+6\right)$	Factorise out at least n from all terms at any point. There must be a factor of n in every term.	M1
	$=\frac{1}{2}n\left(2n^2+3n+1+8n+8+6\right)$		
	$=\frac{1}{2}n\left(2n^2+11n+15\right)$		
	$=\frac{1}{2}n(2n+5)(n+3)$ (*)	Achieves the correct answer, no errors seen.	A1*cso
			[5]
	$\sum_{r=1}^{12}\left(3r^2+8r+3+k(2^{r-1})\right)=3520$		
(b)	$\sum_{r=1}^{12}\left(3r^2+8r+3\right)=\frac{1}{2}(12)(29)(15)\ \{=2610\}$	Attempt to evaluate $\sum_{r=1}^{12}\left(3r^2+8r+3\right)$	M1
	$\sum_{r=1}^{12}(2^{r-1})=\frac{1(1-2^{12})}{1-2}\ \{=4095\}$	Attempt to apply the sum to 12 terms of a GP or adds up all 12 terms.	M1
		$\frac{1(1-2^{12})}{1-2}$ o.e. or 4095.	A1
	So, $2610+4095k=3520\Rightarrow 4095k=910$		
	giving, $k=\frac{2}{9}$	$k=\frac{2}{9}$ or $0.\dot{2}$	A1
			[4] **9**

	Question 8 Notes
8. (b)	**Note** 2nd M1 1st A1: These two marks can be implied by seeing 4095 or 4095k

Leave blank

9. (i) A sequence of numbers is defined by

$$u_1 = 6, \qquad u_2 = 27$$

$$u_{n+2} = 6u_{n+1} - 9u_n \qquad n \geqslant 1$$

Prove by induction that, for $n \in \mathbb{Z}^+$

$$u_n = 3^n(n + 1)$$

(6)

(ii) Prove by induction that, for $n \in \mathbb{Z}^+$

$$f(n) = 3^{3n-2} + 2^{3n+1} \text{ is divisible by 19}$$

(6)

Question Number	Scheme		Marks
9.	$u_{n+2} = 6u_{n+1} - 9u_n,\ n \geqslant 1,\ u_1 = 6,\ u_2 = 27;\quad u_n = 3^n(n+1)$		
(i)	$n=1;\quad u_1 = 3(2) = 6$	Check that $u_1 = 6$ and $u_2 = 27$	B1
	$n=2;\quad u_2 = 3^2(2+1) = 27$		
	So u_n is true when $n=1$ and $n=2$.		
	Assume that $u_k = 3^k(k+1)$ and $u_{k+1} = 3^{k+1}(k+2)$ are true.	Could assume for $n=k, n=k-1$ and show for $n=k+1$	
	Then $u_{k+2} = 6u_{k+1} - 9u_k$		
	$= 6(3^{k+1})(k+2) - 9(3^k)(k+1)$	Substituting u_k and u_{k+1} into $u_{k+2} = 6u_{k+1} - 9u_k$	M1
		Correct expression	A1
	$= 2(3^{k+2})(k+2) - (3^{k+2})(k+1)$	Achieves an expression in 3^{k+2}	M1
	$= (3^{k+2})(2k+4-k-1)$		
	$= (3^{k+2})(k+3)$		
	$= (3^{k+2})(k+2+1)$	$(3^{k+2})(k+2+1)$ or $(3^{k+2})(k+3)$	A1
	If the result is **true for *n* = *k* and *n* = *k*+1** then it is now **true for *n* = *k*+2**. As it is **true for *n* = 1 and *n* = 2** then it is **true for all *n*** $(\in \mathbb{Z}^+)$.	Correct conclusion seen at the end. Condone true for n = 1 and n = 2 seen anywhere. This should be compatible with assumptions.	A1 cso
			[6]
(ii)	$f(n) = 3^{3n-2} + 2^{3n+1}$ is divisible by 19		
	In all ways, first M is for applying $f(k+1)$ with at least 1 power correct. The second M is dependent on at least one accuracy being awarded and making $f(k+1)$ the subject and the final A is correct solution only.		
(ii) **Way 1**	$f(1) = 3^1 + 2^4 = 19$ {which is divisible by 19}.	Shows $f(1) = 19$	B1
	{$\therefore f(n)$ is divisible by 19 when $n=1$ }		
	{Assume that for $n=k$,		
	$f(k) = 3^{3k-2} + 2^{3k+1}$ is divisible by 19 for $k \in \mathbb{Z}^+$.}		
	$f(k+1) - f(k) = 3^{3(k+1)-2} + 2^{3(k+1)+1} - (3^{3k-2} + 2^{3k+1})$	Applies $f(k+1)$ with at least 1 power correct	M1
	$f(k+1) - f(k) = 27(3^{3k-2}) + 8(2^{3k+1}) - (3^{3k-2} + 2^{3k+1})$		
	$f(k+1) - f(k) = 26(3^{3k-2}) + 7(2^{3k+1})$		
	$= 7(3^{3k-2} + 2^{3k+1}) + 19(3^{3k-2})$	Either $7(3^{3k-2} + 2^{3k+1})$ or $7f(k); 19(3^{3k-2})$	A1;
	or $= 26(3^{3k-2} + 2^{3k+1}) - 19(2^{3k+1})$	or $26(3^{3k-2} + 2^{3k+1})$ or $26f(k); -19(2^{3k+1})$	A1
	$= 7f(k) + 19(3^{3k-2})$		
	or $= 26f(k) - 19(2^{3k+1})$		
	$\therefore f(k+1) = 8f(k) + 19(3^{3k-2})$	Dependent on at least one of the previous accuracy marks being awarded.	dM1
	or $f(k+1) = 27f(k) - 19(2^{3k+1})$	Makes Applies $f(k+1)$ with at least 1 power correct the subject	
	{$\therefore f(k+1) = 8f(k) + 19(3^{3k-2})$ is divisible by 19 as both		
	$8f(k)$ and $19(3^{3k-2})$ are both divisible by 19}		

	If the result is **true for $n = k$**, then it is now **true for $n = k + 1$**. As the result has shown to be **true for $n = 1$**, then the result is **true for all n** $(\in \mathbb{Z}^+)$.	Correct conclusion seen at the end. Condone true for $n = 1$ stated earlier.	A1 cso
			[6]
(ii) **Way 2**	$\text{f}(1) = 3^1 + 2^4 = 19$ {which is divisible by 19}.	Shows $\text{f}(1) = 19$	B1
	$\{\therefore \text{f}(n)$ is divisible by 19 when $n = 1\}$		
	Assume that for $n = k$,		
	$\text{f}(k) = 3^{3k-2} + 2^{3k+1}$ is divisible by 19 for $k \in \mathbb{Z}^+$.		
	$\text{f}(k+1) = 3^{3(k+1)-2} + 2^{3(k+1)+1}$	Applies $\text{f}(k+1)$ with at least 1 power correct	M1
	$\text{f}(k+1) = 27(3^{3k-2}) + 8(2^{3k+1})$		
	$= 8(3^{3k-2} + 2^{3k+1}) + 19(3^{3k-2})$	Either $8(3^{3k-2} + 2^{3k+1})$ or $8\text{f}(k)$; $19(3^{3k-2})$	A1; A1
	or $= 27(3^{3k-2} + 2^{3k+1}) - 19(2^{3k+1})$	or $27(3^{3k-2} + 2^{3k+1})$ or $27\text{f}(k)$; $-19(2^{3k+1})$	
	$\therefore \text{f}(k+1) = 8\text{f}(k) + 19(3^{3k-2})$ **or** $\text{f}(k+1) = 27\text{f}(k) - 19(2^{3k+1})$	**Dependent on at least one of the previous accuracy marks being awarded.**	dM1
	$\{\therefore \text{f}(k+1) = 8\text{f}(k) + 19(3^{3k-2})$ is divisible by 19 as both $8\text{f}(k)$ and $19(3^{3k-2})$ are both divisible by 19}		
	If the result is **true for $n = k$**, then it is now **true for $n = k + 1$**. As the result has shown to be **true for $n = 1$**, then the result is **true for all n** $(\in \mathbb{Z}^+)$.	Correct conclusion seen at the end. Condone true for $n = 1$ stated earlier.	A1 cso
			[6]
	$\text{f}(n) = 3^{3n-2} + 2^{3n+1}$ is divisible by 19		
(ii) **Way 3**	$\text{f}(1) = 3^1 + 2^4 = 19$ {which is divisible by 19}.	Shows $\text{f}(1) = 19$	B1
	$\{\therefore \text{f}(n)$ is divisible by 19 when $n = 1\}$		
	Assume that for $n = k$,		
	$\text{f}(k) = 3^{3k-2} + 2^{3k+1}$ is divisible by 19 for $k \in \mathbb{Z}^+$.		
	$\text{f}(k+1) - \alpha\text{f}(k) = 3^{3(k+1)-2} + 2^{3(k+1)+1} - \alpha(3^{3k-2} + 2^{3k+1})$	Applies $\text{f}(k+1)$ with at least 1 power correct	M1
	$\text{f}(k+1) - \alpha\text{f}(k) = (27-\alpha)(3^{3k-2}) + (8-\alpha)2^{3k+1}$		
	$= (8-\alpha)(3^{3k-2} + 2^{3k+1}) + 19(3^{3k-2})$	$(8-\alpha)(3^{3k-2} + 2^{3k+1})$ or $(8-\alpha)\text{f}(k)$; $19(3^{3k-2})$ NB choosing $\alpha = 8$ makes first term disappear.	A1; A1
	or $= (27-\alpha)(3^{3k-2} + 2^{3k+1}) - 19(2^{3k+1})$	$(27-\alpha)(3^{3k-2} + 2^{3k+1})$ or $(27-\alpha)\text{f}(k)$; $-19(2^{3k+1})$ NB choosing $\alpha = 27$ makes first term disappear.	
	$\therefore \text{f}(k+1) = 8\text{f}(k) + 19(3^{3k-2})$ **or** $\text{f}(k+1) = 27\text{f}(k) - 19(2^{3k+1})$	**Dependent on at least one of the previous accuracy marks being awarded.** Makes $\text{f}(k+1)$ the subject.	dM1
	$\{\therefore \text{f}(k+1) = 27\text{f}(k) - 19(2^{3k+1})$ is divisible by 19 as both $27\text{f}(k)$ and $19(2^{3k+1})$ are both divisible by 19}		
	If the result is **true for $n = k$**, then it is now **true for $n = k + 1$**. As the result has shown to be **true for $n = 1$**, then the result is true **for all n** $(\in \mathbb{Z}^+)$.	Correct conclusion seen at the end. Condone true for $n = 1$ stated earlier.	A1 cso
			[6]
			12

	Question 9 Notes
(ii)	Accept use of $\text{f}(k) = 3^{3k-2} + 2^{3k+1} = 19m$ o.e. and award method and accuracy as above.