Write your name here

Surname	Other names

Pearson Edexcel GCE

Centre Number | Candidate Number

Further Pure Mathematics FP1

Advanced/Advanced Subsidiary

Monday 14 May 2018 – Afternoon
Time: 1 hour 30 minutes

Paper Reference **6667/01**

You must have:
Mathematical Formulae and Statistical Tables (Pink)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

Pearson

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1. $$f(z) = 2z^3 - 4z^2 + 15z - 13$$

Given that $f(z) \equiv (z-1)(2z^2 + az + b)$, where a and b are real constants,

(a) find the value of a and the value of b.

(2)

(b) Hence use algebra to find the three roots of the equation $f(z) = 0$

(4)

Question Number	Scheme	Notes	Marks
1.	$\text{f}(z) = 2z^3 - 4z^2 + 15z - 13 \equiv (z-1)(2z^2 + az + b)$		
(a)	$a = -2,\ b = 13$	At least one of either $a = -2$ or $b = 13$ or seen as their coefficients.	B1
		Both $a = -2$ and $b = 13$ or seen as their coefficients.	B1
			[2]
(b)	$\{z =\}$ 1 is a root	1 is a root, seen anywhere.	B1
	$\left\{2z^2 - 2z + 13 = 0 \Rightarrow z^2 - z + \frac{13}{2} = 0\right\}$		
	Either • $z = \frac{2 \pm \sqrt{4 - 4(2)(13)}}{2(2)}$	Correct method for solving a 3-term quadratic equation. Do not allow M1 here for an attempt at factorising.	M1
	or • $\left(z - \frac{1}{2}\right)^2 - \frac{1}{4} + \frac{13}{2} = 0$ and $z = ...$		
	or • $(2z-1)^2 - 1 + +13 = 0$ and $z = ...$		
	So, $\{z =\}$ $\frac{1}{2} + \frac{5}{2}\text{i},\ \frac{1}{2} - \frac{5}{2}\text{i}$	At least one of either $\frac{1}{2} + \frac{5}{2}\text{i}$ or $\frac{1}{2} - \frac{5}{2}\text{i}$ or any equivalent form.	A1
		For conjugate of first complex root	A1ft
			[4]
			Total 6

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2. $$f(x) = \frac{3}{2}x^2 + \frac{4}{3x} + 2x - 5, \qquad x < 0$$

The equation $f(x) = 0$ has a single root α.

(a) Show that α lies in the interval $[-3, -2.5]$

(2)

(b) Taking -3 as a first approximation to α, apply the Newton-Raphson procedure once to $f(x)$ to obtain a second approximation to α. Give your answer to 3 decimal places.

(5)

(c) Use linear interpolation once on the interval $[-3, -2.5]$ to find another approximation to α, giving your answer to 3 decimal places.

(3)

Question Number	Scheme	Notes	Marks
2. (a)	$\text{f}(-3) = 2.05555555...$ $\text{f}(-2.5) = -1.15833333...$	Attempt **both** of $\text{f}(-3) = $ awrt 2.1 or trunc 2 or 2.0 or $\frac{37}{18}$ **and** $\text{f}(-2.5) = $ awrt -1.2 or trunc -1.1 or $-\frac{139}{120}$	M1
	Sign change oe (and $\text{f}(x)$ is continuous) therefore a root α {exists in the interval $[-3, -2.5]$.}	Both $\text{f}(-3) = $ awrt 2.1 and $\text{f}(-2.5) = $ awrt -1.2, sign change and 'root' or 'α'. Any errors award A0.	A1
			[2]
(b)	$\text{f}'(x) = 3x - \frac{4}{3x^2} + 2$	$\frac{3}{2}x^2 \to \pm Ax$ or $\frac{4}{3x} \to \pm Bx^{-2}$ or $2x - 5 \to 2$ Calculus must be seen for this to be awarded.	M1
		At least two terms differentiated correctly	A1
		Correct derivative.	A1
	$\alpha = -3 - \left(\frac{\text{" }2.055...\text{"}}{\text{"}-7.148...\text{"}}\right)$	Correct application of Newton-Raphson **using their values from calculus**.	M1
	$= -2.71243523...$ or $-\frac{1047}{386}$ or $-2\frac{275}{386}$	Exact value or awrt -2.712	A1
			[5]
(c)	$\frac{-2.5-\alpha}{\text{"}1.158...\text{"}} = \frac{\alpha - -3}{\text{"}2.055...\text{"}}$ or $\frac{\alpha - -3}{\text{"}2.055...\text{"}} = \frac{-2.5 - -3}{\text{"}2.055...\text{"} + \text{"}1.158...\text{"}}$	A correct linear interpolation statement with correct signs. $\frac{-2.5+\alpha}{\text{"}1.158...\text{"}} = \frac{-\alpha - -3}{\text{"}2.055...\text{"}}$ provided α sign changed at the end. Do not award until α is seen.	M1
	$\alpha = -3 + \left(\frac{\text{" }2.055...\text{"}}{\text{" }2.055...\text{"} + \text{"}1.158...\text{"}}\right)(0.5)$ or $\alpha = -3 + \left(\frac{\text{"}2.055...\text{"}}{\text{"}3.213...\text{"}}\right)(0.5)$ or $\alpha = \left(\frac{(-2.5)(\text{" }2.055...\text{"}) - 3(\text{"}1.158...\text{"})}{\text{" }2.055...\text{"} + \text{"}1.158...\text{"}}\right)$	Achieves a correct linear interpolation statement with correct signs for $\alpha = ...$ dependent on the previous method mark.	dM1
	$= -2.68020743...$ or $-\frac{3101}{1157}$ or $-2\frac{787}{1157}$		
	$= -2.680$ (3 dp)	-2.680: only penalise accuracy once in (b) and (c), but must be to at least 3sf.	A1 **cao**

ALT (c)	The gradient of the line between (-3, 2.055…) and (-2.5, -1.158…) is $\frac{2.055...--1.158...}{-3--2.5}=-6.427...$		
	Equation of the line joining the points $y-2.055...=-6.427...(x--3)$	Correct attempt to find the equation of a line between the two points.	M1
	At $y=0$, $0-2.055...=-6.427...(x--3)$	Subs $y=0$ in their line and achieves $x=...$	dM1
	$\Rightarrow x=-2.680$	-2.680: only penalise accuracy once in (b) and (c), but must be to at least 3sf.	A1 cao
			[3]
			Total 10

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3. (i) Given that

$$\mathbf{A} = \begin{pmatrix} -2 & 3 \\ 1 & 1 \end{pmatrix}, \quad \mathbf{AB} = \begin{pmatrix} -1 & 5 & 12 \\ 3 & -5 & -1 \end{pmatrix}$$

(a) find $\mathbf{A}^{-1}$

(2)

(b) Hence, or otherwise, find the matrix **B**, giving your answer in its simplest form.

(3)

(ii) Given that

$$\mathbf{C} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

(a) describe fully the single geometrical transformation represented by the matrix **C**.

(2)

(b) Hence find the matrix $\mathbf{C}^{39}$

(2)

Question Number	Scheme	Notes	Marks
3. (i) (a)	$\mathbf{A}^{-1} = \frac{1}{-2-3}\begin{pmatrix}1 & -3\\ -1 & -2\end{pmatrix}$	Either $\frac{1}{-2-3}$ or $-\frac{1}{5}$ or $\begin{pmatrix}1 & -3\\ -1 & -2\end{pmatrix}$	M1
		Correct expression for $\mathbf{A}^{-1}$	A1
			[2]
(b)	$\{\mathbf{B} = \mathbf{A}^{-1}(\mathbf{AB})\}$		
	$\mathbf{B} = -\frac{1}{5}\begin{pmatrix}1 & -3\\ -1 & -2\end{pmatrix}\begin{pmatrix}-1 & 5 & 12\\ 3 & -5 & -1\end{pmatrix}$	Writing down their $\mathbf{A}^{-1}$ multiplied by $\mathbf{AB}$	M1
	$= \left\{-\frac{1}{5}\right\}\begin{pmatrix}-10 & 20 & 15\\ -5 & 5 & -10\end{pmatrix}$	At least one correct row or at least two correct columns of $\begin{pmatrix}\dots\\ \dots\end{pmatrix}$. (Ignore $-\frac{1}{5}$).	A1
	$= \begin{pmatrix}2 & -4 & -3\\ 1 & -1 & 2\end{pmatrix}$	Correct simplified matrix for **B**	A1
			[3]
ALT (b)	Let $\mathbf{B} = \begin{pmatrix}a & b & c\\ d & e & f\end{pmatrix}$		
	$-2a+3d=-1$ $-2b+3e=5$ $a+d=3$ $b+e=-5$ $-2c+3f=12$ $c+f=-1$	Writes down at least 2 correct sets of simultaneous equations	M1
	$\{a=2,\ d=1,\ b=-4,\ e=-1,\ c=-3,\ f=2\}$		
	$\mathbf{B} = \begin{pmatrix}2 & -4 & -3\\ 1 & -1 & 2\end{pmatrix}$	At least one correct row or at least two correct columns for the matrix **B**	A1
		Correct matrix for **B**	A1
			[3]
(ii) (a)	Rotation	Rotation only.	M1
	90° clockwise about the origin	**90º** $\left(\text{or } \frac{\pi}{2}\right)$ **clockwise** about the **origin** or **270º** $\left(\text{or } \frac{3\pi}{2}\right)$ (anti-clockwise) about the **origin.** **-90º** $\left(\text{or } -\frac{\pi}{2}\right)$ **(**anticlockwise**)** about the **origin.** Origin can be written as $(0, 0)$ or O.	A1
			[2]
(b)	$\{\mathbf{C}^{39}\} = \mathbf{C}^{-1}$ or $\mathbf{C}^{3} = \begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}$	For stating $\mathbf{C}^{-1}$ or $\mathbf{C}^{3}$ or 'rotation of **270º** clockwise o.e. about the **origin.** Can be implied by correct matrix.	M1
		$\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}$ Correct answer with no working award M1A1	A1
			[2]
			Total 9

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4. (a) Use the standard results for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$ to show that, for all positive integers n,

$$\sum_{r=1}^{n}\left(r^2 - r - 8\right) = \frac{1}{3}n(n-a)(n+a)$$

where a is a positive integer to be determined.

(4)

(b) Hence, or otherwise, state the positive value of n that satisfies

$$\sum_{r=1}^{n}\left(r^2 - r - 8\right) = 0$$

(1)

Given that

$$\sum_{r=3}^{17}\left(kr^3 + r^2 - r - 8\right) = 6710 \qquad \text{where } k \text{ is a constant}$$

(c) find the exact value of k.

(4)

Question Number	Scheme	Notes	Marks
4. (a)	$\sum_{r=1}^{n}\left(r^2 - r - 8\right)$		
	$= \frac{1}{6}n(n+1)(2n+1) - \frac{1}{2}n(n+1) - 8n$	At least one of the first two terms is correct.	M1
		Correct expression	A1
	$= \frac{1}{6}n\left((2n+1)(n+1) - 3(n+1) - 48\right)$	An attempt to factorise out at least n.	M1
	$= \frac{1}{6}n\left(2n^2 + 3n + 1 - 3n - 3 - 48\right)$		
	$= \frac{1}{6}n\left(2n^2 - 50\right)$		
	$= \frac{2}{6}n\left(n^2 - 25\right)$		
	$= \frac{1}{3}n(n-5)(n+5)$	Achieves the correct answer.	A1
			[4]
(b)	$n = 5$	5. Give B0 for 2 or more possible values of n.	B1 cao
			[1]
(c)	$\left(\frac{k}{4}(17^2)(18^2) - \frac{k}{4}(3^2)(2^2)\right) + \left(\frac{1}{3}(17)(22)(12) - \frac{1}{3}(2)(-3)(7)\right)$	Applying ***at least one of*** $n = 17$ **or** $n = 2$ to **both** $\frac{1}{4}n^2(n+1)^2$ and their $\frac{1}{3}n(n-5)(n+5)$	M1
		Applying $n = 17$ **and** $n = 2$ **only** to **both** $\frac{1}{4}n^2(n+1)^2$ and their $\frac{1}{3}n(n-5)(n+5)$. Require differences only for both brackets.	M1
	$\{\sum = 6710 \Rightarrow\}\ 23409k - 9k + 1496 + 14 = 6710 \Rightarrow k = \frac{2}{9}$	Sets their sum to 6710 and solves to give $k = ...$	ddM1
		$k = \frac{2}{9}$ or $0.\dot{2}$	A1 **cso**
			[4]
			Total 9

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5. The rectangular hyperbola H has equation $xy = c^2$, where c is a positive constant.

Given that $P\left(ct, \frac{c}{t}\right)$, $t \neq 0$, is a general point on H,

(a) use calculus to show that the equation of the tangent to H at P can be written as

$$t^2y + x = 2ct$$

(4)

The points A and B lie on H.

The tangent to H at A and the tangent to H at B meet at the point $\left(-\frac{8c}{5}, \frac{3c}{5}\right)$.

Given that the x coordinate of A is positive,

(b) find, in terms of c, the coordinates of A and the coordinates of B.

(5)

Question Number	Scheme	Notes	Marks
5. (a)	$y = c^2x^{-1} \Rightarrow \frac{dy}{dx} = -c^2x^{-2}$ or (implicitly) $y + x\frac{dy}{dx} = 0$ or (chain rule) $\frac{dy}{dx} = -ct^{-2} \times \frac{1}{c}$	$\frac{dy}{dx} = \pm k x^{-2}$ or $y + x\frac{dy}{dx} = 0$ or $\frac{\text{their } \frac{dy}{dt}}{\text{their } \frac{dx}{dt}}$	M1
	When $x = ct$, $m_T = \frac{dy}{dx} = \frac{-c^2}{(ct)^2} = -\frac{1}{t^2}$ or at $P\left(ct, \frac{c}{t}\right)$, $m_T = \frac{dy}{dx} = -\frac{y}{x} = -\frac{ct^{-1}}{ct} = -\frac{1}{t^2}$	$\frac{dy}{dx} = -\frac{1}{t^2}$	A1
	T: $y - \frac{c}{t} = -\frac{1}{t^2}(x - ct)$	Applies $y - \frac{c}{t} = (\text{their } m_T)(x - ct)$ where their m_T has come from calculus	M1
	T: $t^2y - ct = -x + ct$	At least one line of working.	
	T: $t^2y + x = 2ct$ *	Correct solution.	A1 **cso** *
			[4]
(b)	$t^2\left(\frac{3c}{5}\right) + \left(-\frac{8c}{5}\right) = 2ct$	Substitutes $\left(-\frac{8c}{5}, \frac{3c}{5}\right)$ into tangent.	M1
	$3t^2 - 8 = 10t$	Correct 3TQ in terms of t Can include uncancelled c.	A1
	$\{3t^2 - 10t - 8 = 0 \Rightarrow\}$ $(t-4)(3t+2) = 0 \Rightarrow t = ...$	Attempt to solve their 3TQ for t	M1
	$t = 4,\ -\frac{2}{3} \Rightarrow A\left(4c, \frac{c}{4}\right), B\left(-\frac{2}{3}c, -\frac{3c}{2}\right)$	Uses one of their values of t to find A or B	M1
		Correct coordinates. Condone A and B swapped or missing.	A1
			[5]
			Total 9
ALT 1 (b)	$y - \frac{3c}{5} = -\frac{1}{t^2}\left(x - -\frac{8c}{5}\right)$ $\Rightarrow \frac{c}{t} - \frac{3c}{5} = -\frac{1}{t^2}\left(ct + \frac{8c}{5}\right)$	Substitutes $\left(ct, \frac{c}{t}\right)$ into their $y - \frac{3c}{5} = -\frac{1}{t^2}\left(x - -\frac{8c}{5}\right)$	M1
	$3t^2 - 10t = 8$	Correct 3TQ in terms of t . Can include uncancelled c.	A1
	then apply the original mark scheme.		
ALT 2 (b)	$A\left(ct_1, \frac{c}{t_1}\right), B\left(ct_2, \frac{c}{t_2}\right)$ $t_1^2y + x = 2ct_1$ $t_2^2y + x = 2ct_2$	Substitutes A and B into the equation of the tangent, solves for x and y	M1
	$t_1 + t_2 = \frac{10}{3}$, $t_1t_2 = -\frac{8}{3}$		
	$3t^2 - 8 = 10t$	Correct 3TQ in terms of t_1 or t_2 Can include uncancelled c.	A1
	then apply original scheme		

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6. $$\mathbf{M} = \begin{pmatrix} 8 & -1 \\ -4 & 2 \end{pmatrix}$$

(a) Find the value of det **M**

(1)

The triangle T has vertices at the points (4, 1), (6, k) and (12, 1), where k is a constant.

The triangle T is transformed onto the triangle T' by the transformation represented by the matrix **M**.

Given that the area of triangle T' is 216 square units,

(b) find the possible values of k.

(5)

Question Number	Scheme		Marks
6. (a)	$\{\det \mathbf{M} = (8)(2) - (-1)(-4)\} \Rightarrow \det \mathbf{M} = 12$	12	B1
			[1]
(b)	Area $T = \frac{216}{12}$ $\{=18\}$	Area $T = \frac{216}{\text{their "}\det \mathbf{M}\text{"}}$	M1
	$h = \pm(1-k)$	Uses $(k-1)$ or $(1-k)$ in their solution.	M1
	$\frac{1}{2}8(k-1) = 18$ or $\frac{1}{2}8(1-k) = 18$ or $(k-1) = \frac{18}{4}$ or $(1-k) = \frac{18}{4}$ or $\{\frac{1}{2}8h = 18\} \Rightarrow h = \frac{9}{2}, k = 1 \pm \frac{9}{2}$	**dependent on the two previous M marks** $\frac{1}{2}8(k-1)$ or $\frac{1}{2}8(1-k)$ $= \frac{216}{\text{their "}\det \mathbf{M}\text{"}}$ or $(k-1)$ or $(1-k)$ $= \frac{216}{4(\text{their "}\det \mathbf{M}\text{"})}$ or $h = \frac{216}{4(\text{their "}\det \mathbf{M}\text{"})}, k = 1 \pm \frac{216}{4(\text{their "}\det \mathbf{M}\text{"})}$	ddM1
	$\Rightarrow$ $k = 5.5$ or $k = -3.5$	At least one of either $k = 5.5$ or $k = -3.5$	A1
		Both $k = 5.5$ and $k = -3.5$	A1
			[5]
ALT (b)	$\mathbf{T}' = \begin{pmatrix} 8 & -1 \\ -4 & 2 \end{pmatrix}\begin{pmatrix} 4 & 6 & 12 \\ 1 & k & 1 \end{pmatrix}$		
	$\mathbf{T}' = \begin{pmatrix} 31 & 48-k & 95 \\ -14 & -24+2k & -46 \end{pmatrix}$ or 18 seen	At least 5 out of 6 elements are correct or 18 seen..	M1
	$\frac{1}{2}\begin{vmatrix} 31 & 48-k & 95 & 31 \\ -14 & -24+2k & -46 & -14 \end{vmatrix} = 216$ **or** $\frac{1}{2}\begin{vmatrix} 4 & 6 & 12 & 4 \\ 1 & k & 1 & 1 \end{vmatrix} = 18$	$\frac{1}{2}\lvert\text{their } \mathbf{T}'\rvert = 216$ or $\frac{1}{2}\begin{vmatrix} 4 & 6 & 12 & 4 \\ 1 & k & 1 & 1 \end{vmatrix} = 18$	M1
	$\frac{1}{2}\begin{vmatrix} -744 + 62k + 672 - 14k - 2208 + 46k \\ +2280 - 190k - 1330 + 1426 \end{vmatrix} = 216$ $\frac{1}{2}\lvert 4k - 6 + 6 - 12k + 12 - 4\rvert = 18$	**Dependent on the two previous M marks.** Full method of evaluating a determinant.	ddM1
	$\frac{1}{2}\lvert 96 - 96k\rvert = 216$ or $\frac{1}{2}\lvert 8 - 8k\rvert = 18$		
	So, $1-k = 4.5$ or $k-1 = 4.5$		
	$\Rightarrow$ $k = -3.5$ or $k = 5.5$	At least one of either $k = -3.5$ or $k = 5.5$	A1
		Both $k = -3.5$ and $k = 5.5$	A1
			[5]
			Total 6

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7. The parabola C has equation $y^2 = 4ax$, where a is a positive constant.
The point S is the focus of C.

The straight line l passes through the point S and meets the directrix of C at the point D.

Given that the y coordinate of D is $\dfrac{24a}{5}$,

(a) show that an equation of the line l is

$$12x + 5y = 12a$$

(2)

The point $P(ak^2, 2ak)$, where k is a positive constant, lies on the parabola C.

Given that the line segment SP is perpendicular to l,

(b) find, in terms of a, the coordinates of the point P.

(6)

Question Number	Scheme	Notes	Marks
7.	$y^2 = 4ax,\ S(a,0),\ D\left(-a, \frac{24a}{5}\right),\ P(ak^2, 2ak)$		
(a)	$m_l = \dfrac{\frac{24a}{5} - 0}{-a-a}\ \left\{= \dfrac{\frac{24a}{5} - 0}{-2a} = -\frac{12}{5}\right\}$ $\dfrac{y - \frac{24a}{5}}{0 - \frac{24a}{5}} = \dfrac{x - -a}{a - -a}$ or $\dfrac{y-0}{\frac{24a}{5} - 0} = \dfrac{x-a}{-a-a}$	Uses $S(a, 0)$ and $D\left(\text{their "}-a\text{"}, \frac{24a}{5}\right)$ to find an expression for the gradient of l or applies the formula $\dfrac{y-y_1}{y_2-y_1} = \dfrac{x-x_1}{x_2-x_1}$ Can be un-simplified or simplified.	M1
	$l:\ y - 0 = -\frac{12}{5}(x-a) \Rightarrow\ 5y = -12x + 12a$ $l:\ 12x + 5y = 12a$ (*)	Correct solution only leading to $12x + 5y = 12a$ No errors seen.	A1 *
			[2]
ALT (a)	$y = mx + c$ At S, $0 = ma + c$ At D, $\frac{24a}{5} = -ma + c$ $\Rightarrow c = \frac{12a}{5}, m = -\frac{12}{5}$	Uses $S(a, 0)$ and $D\left(\text{their "}-a\text{"}, \frac{24a}{5}\right)$ to find 2 simultaneous equations and solves to achieve $c = ..., m = ...$	M1
	$y = -\frac{12}{5}x + \frac{12a}{5} \Rightarrow 12x + 5y = 12a$ *	Correct solution only leading to $12x + 5y = 12a$	A1*
			[2]
(b)	$m_{SP} = \dfrac{2ak}{ak^2 - a}\ \left\{= \dfrac{2k}{k^2-1}\right\}$	Attempts to find the gradient of SP	M1
	$m_l = -\left(\dfrac{ak^2 - a}{2ak}\right)$ or $m_{SP} = -\dfrac{1}{(-\frac{12}{5})}\ \left\{= \frac{5}{12}\right\}$	Some evidence of applying $m_1 m_2 = -1$	M1
	So $\left\{\dfrac{2k}{k^2-1} = \dfrac{5}{12} \Rightarrow\right\}\ 24k = 5k^2 - 5$	Correct 3TQ in terms of k in any form.	A1
	$\{5k^2 - 24k - 5 = 0 \Rightarrow\}\ (k-5)(5k+1) = 0 \Rightarrow k = ...$	Attempt to solve their 3TQ for k	M1
	$\{\text{As } k > 0, \text{ so } k = 5\} \Rightarrow (25a, 10a)$	Uses their k to find P	M1
		$(25a, 10a)$	A1
			[6]

ALT 1 (b)	SP: $y-0=\frac{5}{12}(x-a)$	$y-0=m_{SP}(x-a)$	M1
		$m_{SP}=-\frac{1}{(-\frac{12}{5})}\left\{=\frac{5}{12}\right\}$	M1
	$\left\{y^2=4ax\Rightarrow\right\}\left(\frac{5}{12}(x-a)\right)^2=4ax$	Can sub for x and achieve $\frac{12}{5}y+a$	
	$25(x^2-2ax+a^2)=576ax$		
	$25x^2-626ax+25a^2=0$	Correct 3TQ in terms of a and x or $5y^2-48ay-20a^2=0$	A1
	$(25x-a)(x-25a)=0\Rightarrow x=...$	Attempt to solve their 3TQ for x	M1
	$x=\frac{a}{25}\Rightarrow y=\frac{5}{12}\left(\frac{a}{25}-a\right)\left\{=-\frac{2a}{5}\right\}$ $x=25a\Rightarrow y=\frac{5}{12}(25a-a)\ \{=10a\}$	Uses their x to find y	M1
	$\{\text{As } k>0,\}\Rightarrow(25a,10a)$	$(25a,10a)$	A1
			[6]
ALT 2 (b)	$0=m_{SP}a+c$	Subs S into $y=m_{SP}x+c$ to find c	M1
	$m_{SP}=-\frac{1}{(-\frac{12}{5})}\left\{=\frac{5}{12}\right\}$	Some evidence of applying $m_1m_2=-1$	M1
	$y=\frac{5}{12}x-\frac{5}{12}a$		
	At P, $2ak=\frac{5}{12}ak^2-\frac{5}{12}a$	Correct 3TQ in terms of k	A1
	then as part (b)		
			Total 8

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8. Prove by induction that

$$f(n) = 2^{n+2} + 3^{2n+1}$$

is divisible by 7 for all positive integers n.

(6)

Question Number	Scheme	Notes	Marks
8.	$\text{f}(n) = 2^{n+2} + 3^{2n+1}$ divisible by 7		
	$\text{f}(1) = 2^3 + 3^3 = 35$ {which is divisible by 7}.	Shows $\text{f}(1) = 35$	B1
	$\{\therefore \text{f}(n)$ is divisible by 7 when $n=1$ }		
	{Assume that for $n = k$,		
	$\text{f}(k) = 2^{k+2} + 3^{2k+1}$ is divisible by 7 for $k \in \mathbb{Z}^+$.}		
	$\text{f}(k+1) - \text{f}(k) = 2^{k+1+2} + 3^{2(k+1)+1} - (2^{k+2} + 3^{2k+1})$	Applies $\text{f}(k+1)$ with at least 1 power correct	M1
	$\text{f}(k+1) - \text{f}(k) = 2(2^{k+2}) + 9(3^{2k+1}) - (2^{k+2} + 3^{2k+1})$		
	$\text{f}(k+1) - \text{f}(k) = 2^{k+2} + 8(3^{2k+1})$		
	$= (2^{k+2} + 3^{2k+1}) + 7(3^{2k+1})$	$(2^{k+2} + 3^{2k+1})$ or $\text{f}(k)$; $7(3^{2k+1})$	A1; A1
	or $= 8(2^{k+2} + 3^{2k+1}) - 7(2^{k+2})$	or $8(2^{k+2} + 3^{2k+1})$ or $8\text{f}(k)$; $-7(2^{k+2})$	
	$= \text{f}(k) + 7(3^{2k+1})$ **or** $= 8\text{f}(k) - 7(2^{k+2})$		
	$\therefore \text{f}(k+1) = 2\text{f}(k) + 7(3^{2k+1})$ **or** $\text{f}(k+1) = 9\text{f}(k) - 7(2^{k+2})$	**Dependent on at least one of the previous accuracy marks being awarded.** Makes $\text{f}(k+1)$ the subject	dM1
	$\{\therefore \text{f}(k+1) = 2\text{f}(k) + 7(3^{2k+1})$ is divisible by 7 as both $2\text{f}(k)$ and $7(3^{2k+1})$ are both divisible by 7}		
	If the result is **true for** $n = k$, then it is now **true for** $n = k+1$. As the result has shown to be **true for** $n = 1$, then the result is true **for all *n*** $(\in \mathbb{Z}^+)$.	Correct conclusion seen at the end. Condone true for *n* = 1 stated earlier**.**	A1 cso
			[6]
ALT	$\text{f}(k+1) - \alpha\text{f}(k) = 2^{k+3} + 3^{2k+3} - \alpha(2^{k+2} + 3^{2k+1})$	Applies $\text{f}(k+1)$ with at least 1 power correct	M1
	$\text{f}(k+1) - \alpha\text{f}(k) = (2-\alpha)2^{k+2} + (9-\alpha)3^{2k+1}$		
	$\text{f}(k+1) - \alpha\text{f}(k) = (2-\alpha)(2^{k+2} + 3^{2k+1}) + 7.3^{2k+1}$ or	$(2-\alpha)(2^{k+2} + 3^{2k+1})$ or $(2-\alpha)\text{f}(k)$; 7.3^{2k+1} or	A1;A1
	$\text{f}(k+1) - \alpha\text{f}(k) = (9-\alpha)(2^{k+2} + 3^{2k+1}) - 7.2^{k+2}$	$(9-\alpha)(2^{k+2} + 3^{2k+1})$ or $(9-\alpha)\text{f}(k)$; -7.2^{k+2}	
		NB: Choosing $\alpha = 0, \alpha = 2, \alpha = 9$ will make relevant terms disappear, but marks should be awarded accordingly.	
			Total 6

Leave blank

9. (i) Given that

$$\frac{3w+7}{5} = \frac{p-4\text{i}}{3-\text{i}} \qquad \text{where } p \text{ is a real constant}$$

(a) express w in the form $a + b\text{i}$, where a and b are real constants. Give your answer in its simplest form in terms of p.

(5)

Given that $\arg w = -\frac{\pi}{2}$

(b) find the value of p.

(1)

(ii) Given that

$$(z + 1 - 2\text{i})^* = 4\text{i}z$$

find z, giving your answer in the form $z = x + \text{i}y$, where x and y are real constants.

(6)

Question Number	Scheme		Marks
9.(i) (a)	$\frac{3w+7}{5} = \frac{(p-4\text{i})}{(3-\text{i})} \times \frac{(3+\text{i})}{(3+\text{i})}$	Multiplies by $\frac{(3+\text{i})}{(3+\text{i})}$ or divide by $(9-3\text{i})$ then multiply by $\frac{(9+3\text{i})}{(9+3\text{i})}$	M1
	$= \left(\frac{3p+4}{10}\right) + \left(\frac{p-12}{10}\right)\text{i}$	Evidence of $(3-\text{i})(3+\text{i}) = 10$ or 3^2+1^2 or 9^2+3^2	B1
	So, $w = \left(\frac{3p-10}{6}\right) + \left(\frac{p-12}{6}\right)\text{i}$	Rearranges to $w = ...$	dM1
		At least one of either the real or imaginary part of w is correct in any equivalent form.	A1
		Correct w in the form $a+b\text{i}$. Accept $a+\text{i}b$.	A1
			[5]
ALT (i) (a)	$(3-\text{i})(3w+7) = 5(p-4\text{i})$		
	$9w+21-3\text{i}w-7\text{i} = 5p-20\text{i}$		
	$w(9-3\text{i}) = 5p-21-13\text{i}$		
	Let $w = a+b\text{i}$, so $(a+b\text{i})(9-3\text{i}) = 5p-21-13\text{i}$		
	$9a+3b-3a\text{i}+9b\text{i} = 5p-21-13\text{i}$		
	Real: $9a+3b = 5p-21$ Imaginary: $-3a+9b = -13$	Sets $w = a+b\text{i}$ and equates at least either the real or imaginary part.	M1
		$9a+3b = 5p-21$	B1
	$b = \frac{p-12}{6}$, $a = \frac{3p-10}{6}$	Solves to finds $a = ...$ and $b = ...$	dM1
		At least one of a or b is correct in any equivalent form.	A1
	$w = \left(\frac{3p-10}{6}\right) + \left(\frac{p-12}{6}\right)\text{i}$	Correct w in the form $a+b\text{i}$. Accept $a+\text{i}b$.	A1
			[5]
(b)	$\left\{\arg w = -\frac{\pi}{2} \Rightarrow \left(\frac{3p-10}{6}\right) = 0\right\} \Rightarrow p = \frac{10}{3}$	$p = \frac{10}{3}$ Follow through provided $p < 12$	B1ft
			[1]

(ii)	$(x+\mathrm{i}y+1-2\mathrm{i})^* = 4\mathrm{i}(x+\mathrm{i}y)$	Replaces z with $x+\mathrm{i}y$ on both sides of the equation	M1
	$x-\mathrm{i}y+1+2\mathrm{i} = 4\mathrm{i}(x+\mathrm{i}y)$ or $x+\mathrm{i}y+1-2\mathrm{i} = -4\mathrm{i}(x-\mathrm{i}y)$	Fully correct method for applying the conjugate	M1
	$x-\mathrm{i}y+1+2\mathrm{i} = 4\mathrm{i}x-4y$		
	Real: $x+1=-4y$ Imaginary: $-y+2=4x$	$x+1=-4y$ and $-y+2=4x$	A1
	$4x+16y=-4$ $4x+y=2$ $\Rightarrow 15y=-6 \Rightarrow y=...$	Solves two equations in x and y to obtain at least one of x or y	ddM1
	So, $x=\frac{3}{5},\ y=-\frac{2}{5}$ $\left\{z=\frac{3}{5}-\frac{2}{5}\mathrm{i}\right\}$	At least one of either x or y are correct	A1
		Both x and y are correct	A1
			[6]
			Total 12